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International Mathematics

for the Middle Years

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PEARSON
Education

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Drag and Drops

Chapter 1: Powers of numbers
Chapter 2: Percentages, Money
Chapter 3: Order of operations, Number puzzles
Chapter 4: Writing fractions as decimals
Chapter 5: Mathematical terms 5, Addition and subtraction of directed numbers, Multiplication and division of directed numbers, Plotting coordinates
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Chapter 10: Types of angles, Finding the size of angles a, b and c
Chapter 11: Recognising plane shapes, Properties of 2D shapes, Recognising 3D shapes
Chapter 12: Units of length, Perimeter of shapes
Chapter 13: Units of capacity and volume, Area puzzle
Chapter 15: Venn Diagram

Chapter Review Questions

These can be used as a diagnostic tool or for revision. They include multiple-choice, pattern-matching and fill-in-the-gaps style questions.

Destinations

Links to useful websites which relate directly to the chapter content.

Features of International Mathematics

International Mathematics for the Middle Years is organised with the international student in mind. Examples and exercises are not restricted to a particular syllabus and so provide students with a global perspective.

Each edition has a review section for students who may have gaps in the Mathematics they have studied previously. Sections on the language of Mathematics and terminology will help students for whom English is a second language.

Areas of Interaction are given for each chapter and Assessment Grids for Investigations provide teachers with aids to assessing Analysis and Reasoning, Communication, and Reflection and Evaluation as part of the International Baccalaureate Middle Years Program (IBMYP). The Assessment Grids will also assist students in addressing these criteria and will enhance their understanding of the subject content.

How is International Mathematics for the Middle Years organised?

As well as the student coursebook, additional support for both students and teachers is provided:

- Interactive Student CD — **free** with each coursebook
- Companion Website
- Teacher's Resource — printout and CD

Coursebook

Chapter-opening pages summarise the key content and present the learning outcomes addressed in each chapter.

Areas of Interaction references are included in the chapter-opening pages. For example, **Homo Faber**.



Prep Quizzes review skills needed to complete a topic. These anticipate problems and save time in the long run. These quizzes offer an excellent way to start a lesson.

Well-graded exercises — within each exercise, **levels of difficulty** are indicated by the colour of the question number.

1 green . . . foundation **4** blue . . . core **9** red . . . extension

2 Write each as a power.
a 7×7 **b** $8 \times 8 \times 8$

7 A farmer has chickens and cows. If there are 18 heads and 52 feet on these altogether, how many cows are there?

16 Place the numbers below into two groups of four, so that the sum of one group is as close as possible to the sum of the other group. {9, 10, 18, 21, 25, 42, 49, 51}

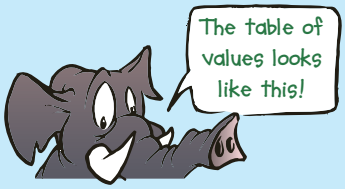
Worked examples are used extensively and are easy for students to identify.

worked examples

1 392 = CCCXCII **2** 1987 = MCMLXXXVII **3** 56 049 = $\overline{\text{LV}}$MXLIX

Important rules and concepts are clearly highlighted at regular intervals throughout the text.

Cartoons are used to give students friendly advice or tips.



Foundation Worksheets provide alternative exercises for students who need to consolidate earlier work or who need additional work at an easier level. Students can access these on the CD by clicking on the Foundation Worksheet icons. These can also be copied from the Teacher's Resource or from the Teacher's Resource Centre on the Companion Website.

Foundation Worksheet 3:01A

Grouping symbols NS4.1

1 a $(3 + 2) \times 10$

2 a $(8 - 2) \times 3$

3 a $10 - (4 + 3)$

Challenge activities and worksheets provide more difficult investigations and exercises. They can be used to extend more able students.

Fun Spots provide amusement and interest, while often reinforcing course work. They encourage creativity and divergent thinking, and show that Mathematics is enjoyable.

Investigations and Practical Activities encourage students to seek knowledge and develop research skills. They are an essential part of any Mathematics course. Each Investigation is accompanied by a sample Assessment Grid which can be modified or used as it stands to assist teachers in assessing criteria B, C and D as prescribed by the Middle Years Program (MYP).

Students should be made aware of the criteria *before* beginning the Investigation so that they know what is required of them.

Diagnostic Tests at the end of each chapter test students' achievement of learning outcomes. More importantly, they indicate the weaknesses that need to be addressed by going back to the section in the text or on the CD listed beside the test question.

Assignments are provided at the end of each chapter. Where there are two assignments, the first revises the content of previous chapters, while the second concentrates on developing the student's ability to work mathematically.

The **See** cross-references direct students to other sections of the coursebook relevant to a particular section.

Arithmetic/Algebra Cards (see pp xx–xxi) are used to practise basic arithmetic and algebra skills. Corresponding terms in columns can be added, subtracted, multiplied or divided by each other or by other numbers. This is a great way to start a lesson.

The Language of Mathematics

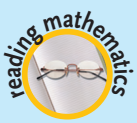
Within the coursebook, Mathematics literacy is addressed in three specific ways:



ID Cards (see pp. xiv–xix) review the language of Mathematics by asking students to identify common terms, shapes and symbols. They should be used as often as possible, either at the beginning of a lesson or as part of a test or examination.



Mathematical Terms met during the chapter are defined at the end of each chapter. These terms are also tested in a **Drag and Drop** interactive activity that follows this section in each chapter.



Reading Mathematics help students to develop mathematics literacy skills and provide opportunities for students to communicate mathematical ideas. They present Mathematics in the context of everyday experiences.

An **Answers** section provides answers to all the exercises in the coursebook, including the ID Cards.



Interactive Student CD

This is provided at the back of the coursebook and is an important part of the total learning package.

Bookmarks and links allow easy navigation within and between the different electronic components of the CD that contains:

- A copy of the student coursebook.
- Appendixes A–I for enrichment and review work, linked from the coursebook.
- Printable copies of the Foundation Worksheets and Challenge Worksheets, linked from the coursebook.
- An archived, offline version of the Companion Website, including:
  - Chapter Review Questions and Quick Quizzes
  - All the Technology Applications: activities and investigations, drag-and-drops and animations
  - Destination (links to useful websites).

All these items are clearly linked from the coursebook via the Companion Website.

- A link to the live Companion Website.

Companion Website

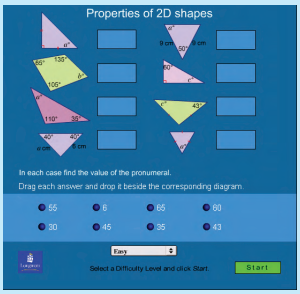


The Companion Website contains a wealth of support material for students and teachers:

- **Chapter Review Questions** which can be used as a diagnostic tool or for revision. These are self-correcting and include multiple-choice, pattern-matching and fill-in the-gaps style questions. Results can be emailed directly to the teacher or parents.
- **Quick Quizzes** for each chapter.
- **Destinations** — links to useful websites which relate directly to the chapter content.

- **Technology Applications** — activities that apply concepts covered in each chapter and are designed for students to work independently:

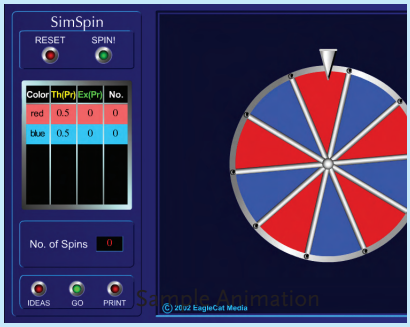
Activity — Activities and investigations using technology such as Excel spreadsheets and The Geometer's Sketchpad.



Sample Drag and Drop

Drag and Drop Interactive — Drag and Drop interactives to improve speed in basic skills.

Interactive Animation — Animations to develop key skills by manipulating visually stimulating and interactive demonstrations of key mathematical concepts.



- **Teacher's Resource Centre** — provides a wealth of teacher support material and is password protected:
  - — Coursebook corrections
  - — Topic Review Tests and answers
  - — Foundation and Challenge worksheets and answers
  - — Answers to the exercises in the Homework Book.

Teacher's Resource

Teacher's Resource Chapter Test — This material is provided as both a printout and as an electronic copy on CD:

- Electronic copy of the complete Student Coursebook in PDF format
- Teaching Program, including treatment of syllabus outcomes, in both PDF and editable Microsoft Word formats
- Practice Tests and Answers
- Foundation and Challenge Worksheets and Answers
- Answers to some of the Technology Application Activities and investigations.

Most of this material is also available in the Teacher's Resource Centre of the Companion Website.

Using this Book for Teaching MYP for the IB

- **Holistic Learning**
- **Intercultural Awareness**
- **Communication**

These elements of the MYP Mathematics course are integrated throughout the text. Links are made possible between subjects, and different methods of communicating solutions to problems through investigations allow students to explore their own ideas.

The Areas of Interaction

- **Approaches to Learning**
- **Community Awareness**
- **Health and Social Education**
- **Environment**
- **Homo Faber**

Areas of Interaction covered are outlined at the start of each chapter, allowing teachers to develop links between subjects and formulate their own Interdisciplinary Units.

Addressing the Objectives

Assessment grids are provided for Investigations throughout the text to not only help teachers assess criteria B, C and D of the MYP, but also to assist students in addressing the criteria. The assessment grids should be modified to suit the student where necessary.

A **Knowledge and Understanding**
This criterion is addressed in the Diagnostic Tests and Revision Assignments that accompany each chapter. Teachers can also use the worksheets from the CD to add to material for this criterion.

B **Application and Reasoning**
It is possible to address this criterion using the Working Mathematically sections accompanying each chapter, and also using the Investigations throughout the text.

C **Communication**
This can be assessed using the Investigations throughout the book.

D **Reflection and Evaluation**
This can be assessed using the Investigations throughout the book.

Fulfilling the Framework for Mathematics

The content of the text covers the five broad areas required to fulfil the Framework:

- **Number**
- **Algebra**
- **Geometry**
- **Statistics**
- **Discrete Mathematics**

Although the material in the text is not exhaustive, it covers the required areas in sufficient depth. Teachers can use the text as a resource to build on as they develop their own scheme of work within their school.

Metric Equivalents

Length
1 m = 1000 mm
= 100 cm
= 10 dm
1 cm = 10 mm
1 km = 1000 m
Area
1 m^2 = 10 000 cm^2
1 ha = 10 000 m^2
1 km^2 = 100 ha
Mass
1 kg = 1000 g
1 t = 1000 kg
1 g = 1000 mg
Volume
1 m^3 = 1 000 000 cm^3
= 1000 dm^3
1 L = 1000 mL
1 kL = 1000 L
1 m^3 = 1 kL
1 cm^3 = 1 mL
1000 cm^3 = 1 L
Time
1 min = 60 s
1 h = 60 min
1 day = 24 h
1 year = 365 days
1 leap year = 366 days

Months of the year

30 days each has September,
April, June and November.
All the rest have 31, except February alone,
Which has 28 days clear and 29 each leap year.

Seasons

Northern Hemisphere

Summer:	June, July, August
Fall/Autumn:	September, October, November
Winter:	December, January, February
Spring:	March, April, May

Southern Hemisphere

Summer:	December, January, February
Autumn/Fall:	March, April, May
Winter:	June, July, August
Spring:	September, October, November



The Language of Mathematics

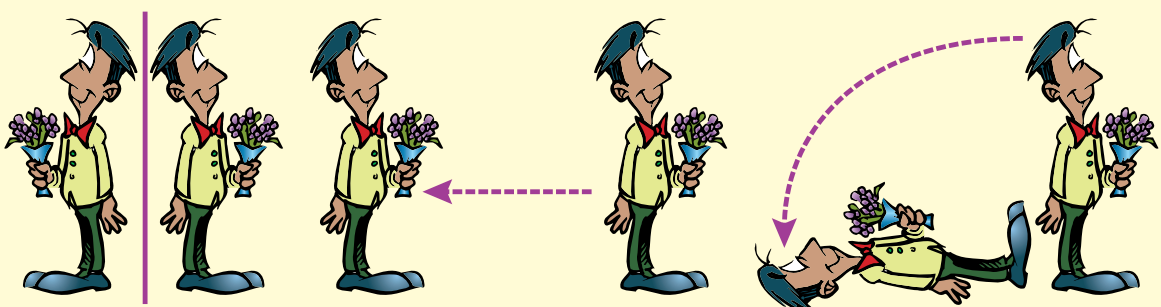
You should regularly test your knowledge by identifying the items on each card.

ID Card 1 (Metric Units)			
1 m	2 dm	3 cm	4 mm
5 km	6 m^2	7 cm^2	8 km^2
9 ha	10 m^3	11 cm^3	12 s
13 min	14 h	15 m/s	16 km/h
17 g	18 mg	19 kg	20 t
21 L	22 mL	23 kL	24 °C

See page 529 for answers.

ID Card 2 (Symbols)			
1 +	2 −	3 ×	4 ÷
5 =	6 $\doteqdot$ or ≈	7 ≠	8 <
9 ⩽	10 ⊀	11 >	12 ⩾
13 ⊁	14 ⩾̸	15 4^2	16 4^3
17 $\sqrt{25}$	18 $\sqrt[3]{27}$	19 %	20 ∴
21 eg	22 ie	23 $n^{\frac{3}{4}}$	24 $d^{\frac{3}{4}}$

See page 529 for answers.



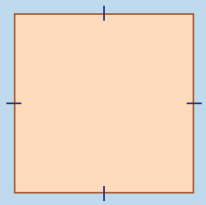
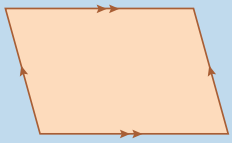
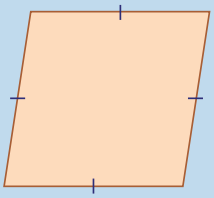
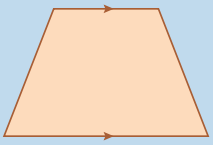
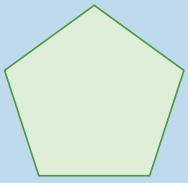
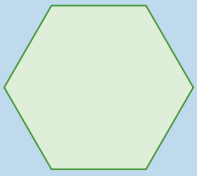
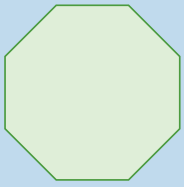
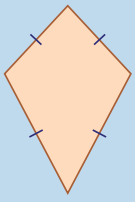
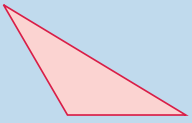
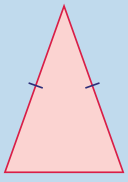
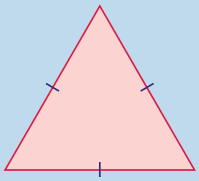
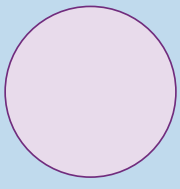
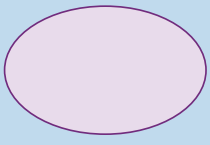
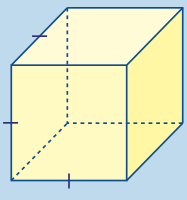
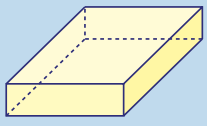
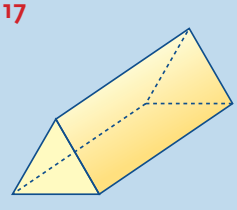
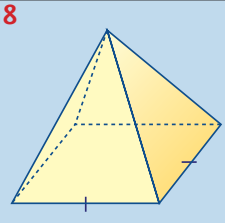
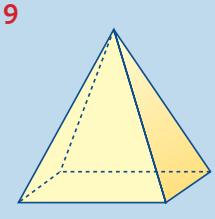
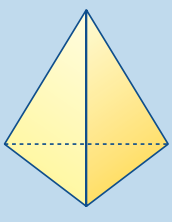
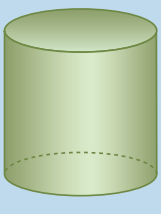
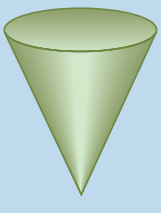
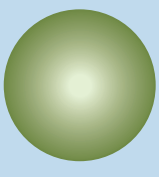
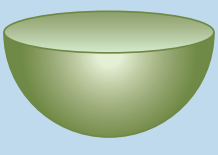
ID Card 3 (Language)			
1 6 minus 2	**2** the sum of 6 and 2	**3** divide 6 by 2	**4** subtract 2 from 6
5 the quotient of 6 and 2	**6** $2\overline{)6}$ the divisor is	**7** $2\overline{)6}$ the dividend is	**8** 6 lots of 2
9 decrease 6 by 2	**10** the product of 6 and 2	**11** 6 more than 2	**12** 2 less than 6
13 6 squared	**14** the square root of 36	**15** 6 take away 2	**16** multiply 6 by 2
17 average of 6 and 2	**18** add 6 and 2	**19** 6 to the power of 2	**20** 6 less 2
21 the difference between 6 and 2	**22** increase 6 by 2	**23** share 6 between 2	**24** the total of 6 and 2

See page 529 for answers.



id

ID Card 4 (Language)

1 	2 	3 	4 
5 	6 	7 	8 
9 	10  All sides different	11 	12 
13 	14 	15 	16 
17 	18 	19 	20 
21 	22 	23 	24 

See page 529 for answers.

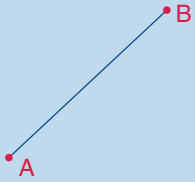
ID Card 5 (Language)

1



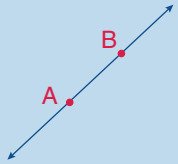
.....

2



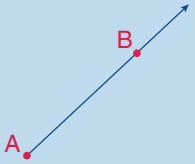
.....

3



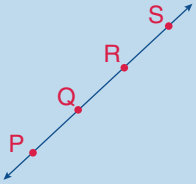
.....

4



.....

5



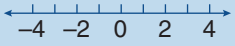
..... points

6



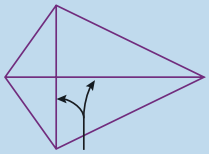
C is the .....

7



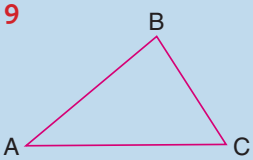
.....
.....

8



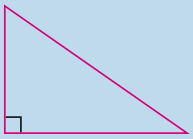
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9



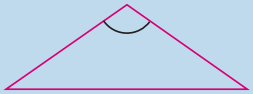
all angles less than 90°

10



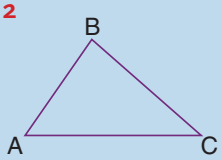
one angle 90°

11



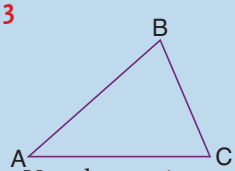
one angle greater than 90°

12



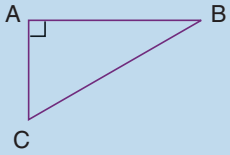
A, *B* and *C* are ..... of the triangle.

13



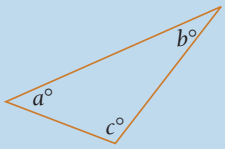
Use the vertices to name the Δ

14



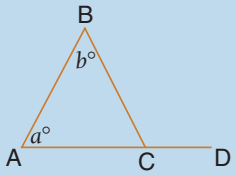
BC is the ..... of the right-angled Δ.

15



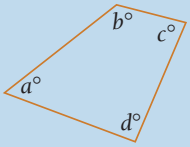
$a° + b° + c°$ = ...

16



$\angle BCD$ = ...

17



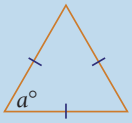
$a° + b° + c° + d°$ = ...

18



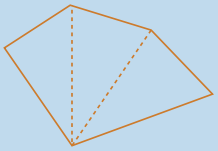
Which is true?
(a) $a° < b°$
(b) $a° = b°$
(c) $a° > b°$

19



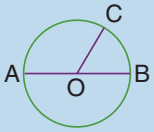
$a°$ = ...

20



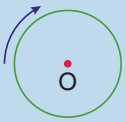
Angle sum = ...

21



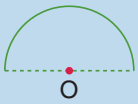
AB is a .....
OC is a .....

22



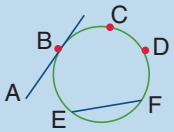
Name of distance around the circle.
.....

23



.....

24

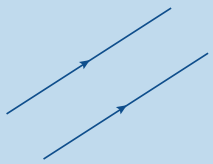


AB is a .....
CD is an .....
EF is a.....

See page 529 for answers.

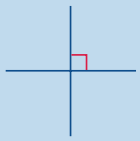
ID Card 6 (Language)

1



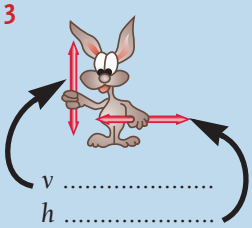
..... lines

2



..... lines

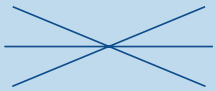
3



v .....

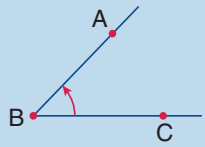
h .....

4



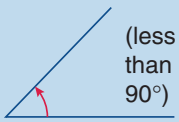
..... lines

5



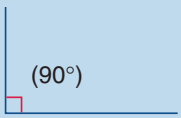
angle .....

6



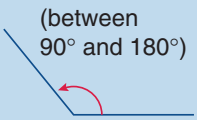
..... angle

7



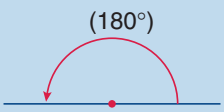
..... angle

8



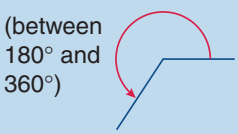
..... angle

9



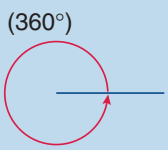
..... angle

10



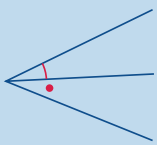
..... angle

11



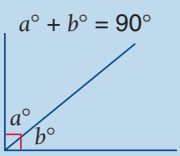
.....

12



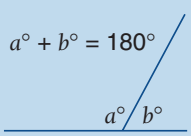
..... angles

13



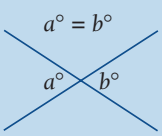
..... angles

14



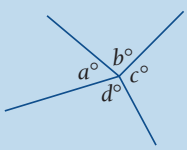
..... angles

15



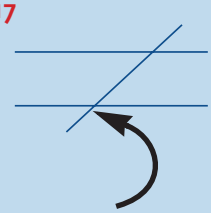
..... angles

16



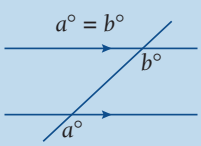
$a° + b° + c° + d°$ = ...

17



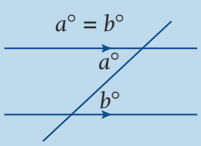
.....

18



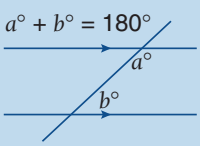
..... angles

19



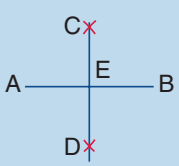
..... angles

20



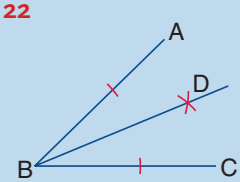
..... angles

21



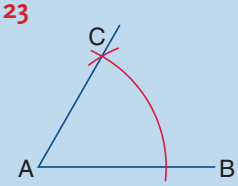
b..... an interval

22



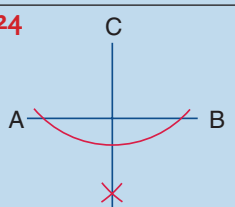
b..... an angle

23



$\angle CAB$ = ...

24



CD is *p*..... to *AB*.

See page 529 for answers.

ID Card 7 (Language)

1

AD

a..... *D*.....

2

BC

b..... *C*.....

3

am

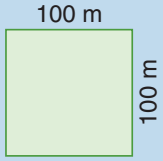
a..... *M*.....

4

pm

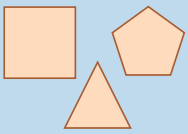
p..... *m*.....

5



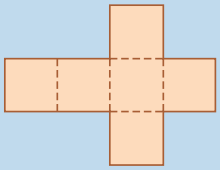
area is 1 .....

6



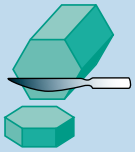
r..... shapes

7



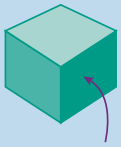
..... of a cube

8



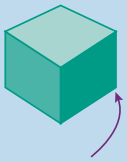
c.....-*s*.....

9



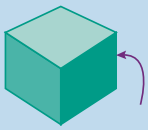
f.....

10



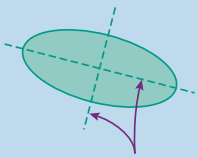
v.....

11



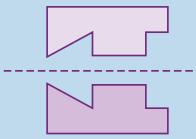
e.....

12



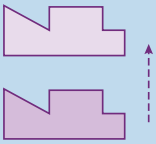
axes of .....

13



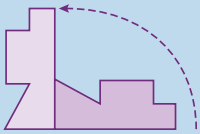
r.....

14



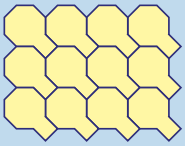
t.....

15



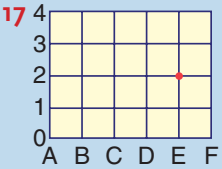
r.....

16



t.....

17



The *c*..... of the dot are E2.

18

Cars sold

Mon	𝍸 I
Tues	𝍸 𝍸 𝍸 I
Wed	𝍸 𝍸 II
Thurs	𝍸 𝍸 𝍸
Fri	𝍸 𝍸 II

t.....

19

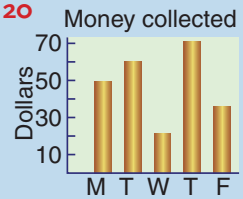
Money collected

Mon	●●●●
Tues	●●●◔
Wed	●◖
Thurs	●●●●
Fri	●●◢

● Stands for $10

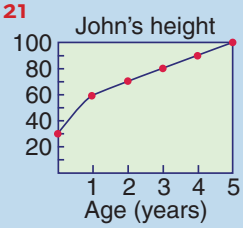
p..... graph

20



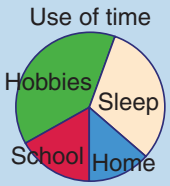
c..... graph

21



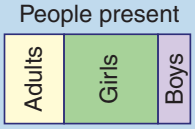
l..... graph

22



s..... graph

23



b..... graph

24



s..... *d*.....

See page 529 for answers.

Arithmetic and Algebra Cards

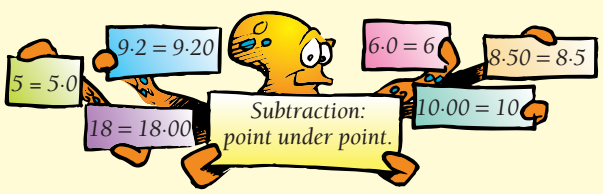
Arithmetic card

	A	B	C	D	E	F	G	H	I	J	K	L
1	2	5	$\frac{1}{2}$	100	0·2	5%	$\frac{2}{3}$	90	0·9	15%	13	56
2	3	8	$\frac{1}{3}$	90	0·1	40%	$\frac{1}{7}$	30	0·5	35%	18	36
3	9	7	$\frac{3}{4}$	80	0·7	20%	$\frac{1}{5}$	20	0·3	1%	15	21
4	5	12	$\frac{1}{5}$	20	0	60%	$\frac{1}{6}$	80	0·1	44%	10	96
5	4	15	$\frac{3}{5}$	80	0·5	25%	$\frac{1}{8}$	100	0·6	95%	19	88
6	1	18	$\frac{2}{3}$	30	0·4	10%	$\frac{1}{4}$	60	0·2	65%	12	24
7	0	4	$\frac{4}{5}$	60	0·8	50%	$\frac{1}{2}$	10	0·4	90%	20	43
8	11	3	$\frac{1}{4}$	40	0·3	80%	$\frac{4}{5}$	60	0·2	100%	14	62
9	10	10	$\frac{3}{10}$	70	0·6	30%	$\frac{2}{5}$	30	0·8	$33\frac{1}{3}$%	18	75
10	7	9	$\frac{2}{5}$	50	0·9	75%	$\frac{1}{3}$	70	0·7	85%	16	45

How to use these cards

As an example, if the instruction given for the Arithmetic Card is 'column B + column K', then you write down answers for the following problems:

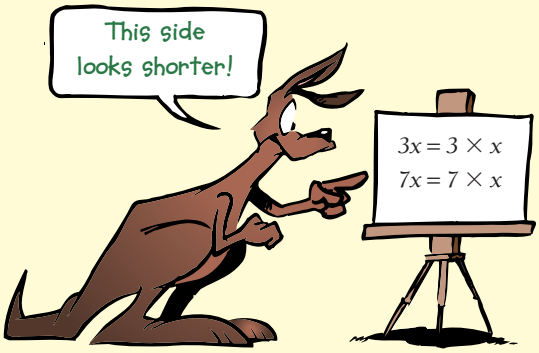
1 5 + 13 **2** 8 + 18 **3** 7 + 15 **4** 12 + 10 **5** 15 + 19
6 18 + 12 **7** 4 + 20 **8** 3 + 14 **9** 10 + 18 **10** 9 + 16



Algebra card

	A	B	C	D	E	F	G	H	I	J	K	L
1	4	-7	-11	x	$2y$	$-3x$	$-6y$	x^2	$2m$	x^7	$-5y$	-4
2	3	-6	7	$5x$	$7y$	$-x$	$-5y$	x^5	$3d$	x^2	$8y$	-3
3	5	-8	-9	$2x$	$10y$	$4x$	$4y$	x^3	$-2c$	x^8	$-12y$	-2
4	8	-2	-7	$3x$	$4y$	$-9x$	$-9y$	x^6	$-8h$	x^4	$0y$	-1
5	7	-10	-8	$10x$	$5y$	$-5x$	$-2y$	x	$9w$	x^3	$-7y$	0
6	2	-12	6	$7x$	$15y$	$-7x$	$-3y$	x^8	$-4d$	x^5	$0y$	1
7	0	-15	-5	$4x$	$6y$	$5x$	$-8y$	x^4	$-9n$	x	$-17y$	2
8	1	-5	9	$11x$	$3y$	$8x$	$-y$	x^7	$3x$	x^9	$-9y$	3
9	9	-3	8	$6x$	y	$-4x$	$-12y$	x^3	$-2w$	x^6	$5y$	4
10	10	-4	-3	$8x$	$9y$	$-8x$	$-7y$	x^4	$-7m$	x^{10}	$-8y$	5

For convenience, stick copies of these pages in your book.



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Klinge, Sven: pp. 13, 77, 160, 162, 164t, 164c, 190, 208, 237, 239, 249, 351r, 433, 449.

McSeveny, Alan: pp. 191bl, 191r, 342, 388.

Pearson Education Australia: Nada Jolic, p. 289; Lisa Piemonte, pp. 53, 154, 164b, 171, 172, 193, 205, 215, 266br, 266l, 337, 338, 412, 438.

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1

Beginnings in Number



Chapter Contents

1:01 The history of number
1:02 Place value
Investigation: Estimation
Fun Spot: A nest of squares
1:03 The four operations
1:04 Long division
1:05 Speed and accuracy
1:06 Powers of numbers
1:07 Expanded notation
1:08 Estimating answers
Fun Spot: What's the difference between a boxer and a telephone?
1:09 Diagnostic checkups
A Operations checkup
B Fractions checkup
C Decimals and money checkup
D Percentages checkup
Fun Spot: Magic squares
Mathematical Terms, Diagnostic Test, Revision Assignment, Working Mathematically

Learning Outcomes

Students will:

- Recognise the properties of special groups of whole numbers and apply a range of strategies to aid computation.
- Order, read and write numbers of any size.
- Select and apply appropriate strategies for addition and subtraction with counting numbers of any size.
- Select and apply appropriate strategies for multiplication and division.
- Compare, order and calculate with decimals, simple fractions and simple percentages.

Areas of Interaction

Approaches to Learning, Homo Faber

1:01 | The History of Number

Many cultures throughout the world have had little need to use numerals. Making notches on a stick or bone, using knots in string, or using pebbles to represent numbers may have completely satisfied the needs of many communities. Remember that the number system used does not reflect the ability to reason but, rather, the need for such symbols. Early Aboriginal tribes of Australia had little need of a number system, as trade was in the form of barter, where tribes would trade with neighbouring tribes on a community basis. Most tribes had simple needs. Many Aboriginal languages used concrete methods of indicating numbers of persons or objects, such as the use of fingers to show the number. Words for 'one' and 'two' were common and these were sometimes combined to make three and four. 'Two-one' would be three and 'two-two' would be four. Larger numbers of objects were described as 'big mob' or 'little mob'. The life and culture of the Aboriginal people was rich in elements needed to ensure survival and harmony. A complex number system was not one of those elements.

As the need for larger numbers grew, many civilisations invented symbols for large numbers. The Egyptian number system is a good example of this, as is the system of Roman numerals which is often used in the prefaces of books, on the faces of clocks and can be seen on certain buildings showing the date of construction.

Egyptian numerals

About 3000 years before the birth of Jesus Christ, the Egyptians had developed a tally system based on ten. Ten of one symbol can be replaced by one of another.

Number	*Symbol*	*Meaning*
1		a vertical staff
10		a heel bone
100		a coiled rope
1000		a lotus flower
10 000		a bent reed or pointing finger
100 000		a burbot fish or tadpole
1 000 000		an amazed man or God of infinity
10 000 000		a religious symbol

- The order of symbols does not affect the value of the numeral.
- The value of a numeral can be found by adding the values of the symbols used.

examples

1 364 = 

or



2 11 321 143 =



3  = 1 070 101

Roman numerals

With the rise to power of the Romans, their number system spread to many other countries. The Roman system developed into one that included a subtraction idea. When a smaller unit appears before a larger one, it is subtracted from the larger one. The position of the symbols is therefore important.

Number	*Symbol*	*Meaning*
1	I	one finger
5	V	one hand
10	X	two Vs:
50	L	half a C
100	C	*centum* = hundred
500	D	half an Ⅿ : Ⅿ
1000	M	Ⅿ

- LX means 50 and 10.
- XL means 50 less 10.
- Larger numerals are formed by placing a stroke above the symbol:

$\overline{V}$ = 5000
$\overline{X}$ = 10 000
$\overline{L}$ = 50 000
$\overline{C}$ = 100 000
$\overline{D}$ = 500 000
$\overline{M}$ = 1 000 000



worked examples

1 392 = CCCXCII

2 1987 = MCMLXXXVII

3 56 049 = $\overline{LV}$MXLIX

Hindu–Arabic numerals

These numerals, which we use today, were invented by Hindus in India around 300 BC and were carried to Europe by Arabs who had invaded Spain in the eighth century. Despite attempts to keep this remarkable number system a secret from the rest of Europe, it gradually spread and gained recognition. The Hindu–Arabic system was well known in Europe by AD 1100.

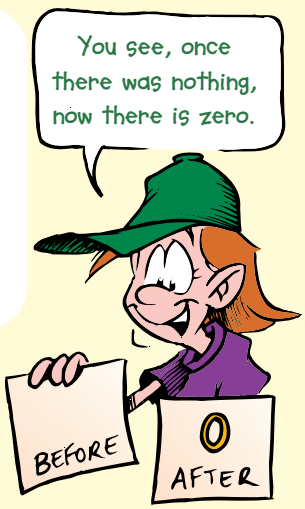
Now	*300 BC*	*AD 876*	*11th C.*	*15th C.*
1	—	)	1	1
2	=	?	2	2
3	≡	3	3	3
4	⫩	8	4	4
5	⫫	4	5	5
6	6	6	6	6
7	7	7	7	7
8	5	8	8	8
9	9	9	9	9
0		o		o

- The position of a symbol is very important. The system has *place value*, based on ten.
- The invention of a symbol for zero was a significant step, as an empty space for zero could be misunderstood.

examples

1 The eleventh century numeral for 453 is 5G7

2 The AD 876 numeral for 54 901 is 58907



Exercise 1:01

1 Write answers in your own words. Refer to previous pages if necessary.

a Explain why a native tribe from Papua New Guinea may not have symbols for large numbers.

b Would an early native Papuan tribe need a complex number system?

c Upon which number was the Egyptian number system based?

d Does the position of a symbol affect its value in an Egyptian numeral?

e In a Roman numeral, what is meant when a symbol for a smaller unit appears before a symbol for a larger unit?



f Where do you think the Roman symbol V came from?

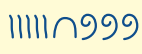
g Why did the Romans choose an X to represent ten?

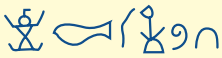
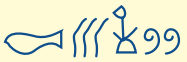
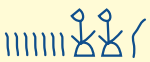
h In which country did our number system have its beginnings?

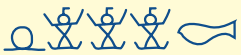
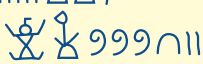
i Which people first carried the Hindu–Arabic numerals to Europe?

j Which of the number systems dealt with in this book had a symbol for zero? Why was the zero important to that system?

2 Change these Egyptian numerals into our modern Hindu–Arabic numerals.

a  **b**  **c**  **d** 

e  **f**  **g**  **h** 

i  **j**  **k**  **l** 

3 Change these to Egyptian numerals.

a 48	**b** 91	**c** 706	**d** 1989	**e** 3965
f 31 024	**g** 93 708	**h** 102 420	**i** 150 000	**j** 371 213
k 4 600 000	**l** 23 000 000	**m** 11 111 111	**n** 3 053 000	**o** 15 680 000

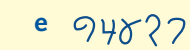
4 Change these Roman numerals into our own numerals.

a CCCXV	**b** XXXIV	**c** CXXVIII	**d** DCLXXXII	**e** DCCIX
f MDCCXXIV	**g** MMMCD	**h** DXLVIII	**i** CMXLVII	**j** DCCCVIII
k MDV	**l** $\overline{V}$DLV	**m** $\overline{L}$MMD	**n** CDXCIX	**o** CMXCIX

5 Change these Hindu–Arabic numerals to Roman numerals.

a 37	**b** 213	**c** 86	**d** 637	**e** 684
f 49	**g** 290	**h** 645	**i** 1452	**j** 778
k 988	**l** 1989	**m** 8489	**n** 5384	**o** 543 627

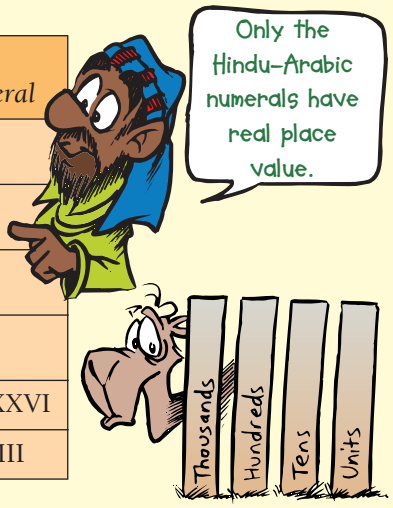
6 Write these AD 876 Hindu–Arabic numerals as modern numerals.

a  **b**  **c**  **d**  **e** 

f  **g**  **h**  **i**  **j**

7 Copy this table into your book and complete it.

Hindu–Arabic numeral (AD 876)	*Egyptian numeral*	*Roman numeral*
387		
1904		
	𓏺𓏺𓏺𓏺𓏺𓏺𓏺𓏺𓏺	
	𓆐𓏺𓏺𓏺𓏺𓏺𓏺𓏺𓏺	
		MMMDCCLXXVI
		MCMLXXXVIII



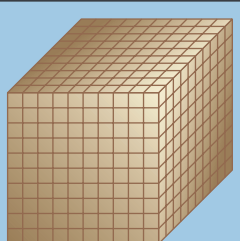
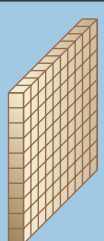
8 What advantages does the modern Hindu–Arabic system have over the Egyptian and Roman systems of numeration?

9 What is the longest Egyptian numeral between 500 and 1500?

10 What is the longest Roman numeral between 500 and 1500?

1:02 | Place Value

We have seen that our Hindu–Arabic numerals are more useful than either the Egyptian or the Roman numerals because we use place value.

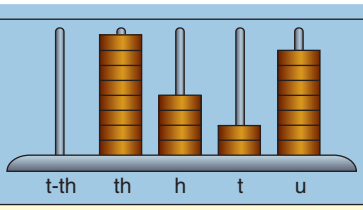
Place value headings	*Thousands*	*Hundreds*	*Tens*	*Units*
Multi-attribute blocks illustration (Dienes' blocks)				
	$10 \times (100)$ (1000)	$10 \times (10)$ (100)	$10 \times (1)$ (10)	1 (1)



example

	8	4	2	7
	8 thousands +	4 hundreds +	2 tens +	7 units
expanded notation:	(8×1000) +	(4×100) +	(2×10) +	(7×1)

'Spike' abacus



■ Gaps in numerals are used as 'thousands' markers.

worked examples

1 Write 314 903 213 in words.

2 Write the value of the 5, the 6 and the 7 in 546 703.

Solutions

1 314 million

903 thousand

213 units

314	903	213

We write 'three hundred and fourteen million, nine hundred and three thousand, two hundred and thirteen'.

2 546 thousand

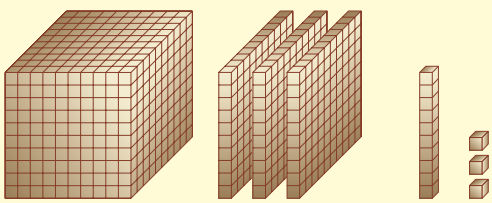
7 hundred and 3

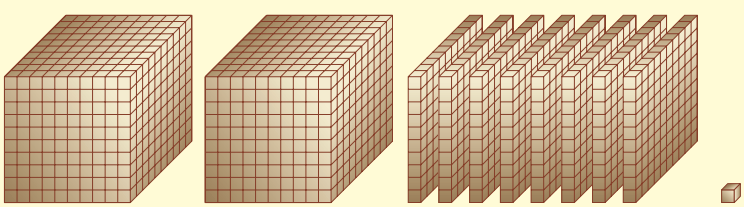
100 000	10 000	1000	100	10	1
5	4	6	7	0	3

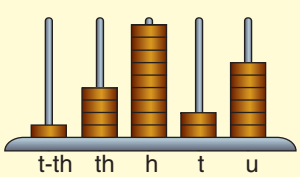
The value of the 5 is 500 000.
The value of the 6 is 6000.
The value of the 7 is 700.

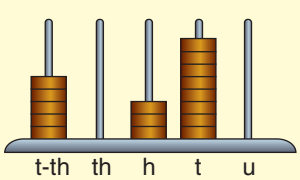
Exercise 1:02

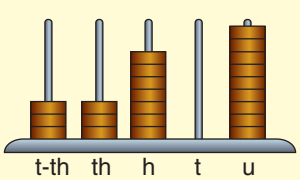
1 What numbers are represented by each of the following?

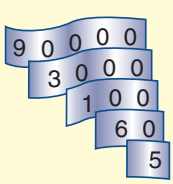
a 

b 

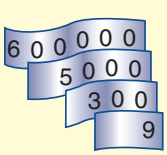
c 

d 

e 

f 

g 

h 

2 In each of the following, write as a numeral the number written in words.

a Five thousand, eight hundred and fifteen.
b Three hundred and twenty-four thousand, six hundred and seventy-nine.
c Eight hundred and six thousand and fifty.
d One million, nine hundred and twenty-seven thousand, four hundred and sixty-three.
e Fifty-six million, two thousand, nine hundred and fourteen.
f Eighty-three million, nine hundred and seven thousand, two hundred and one.
g Thirty-seven million, seventy thousand, eight hundred and forty-seven.
h Nine hundred and twenty-seven million, one hundred thousand and seventy.
i Two hundred and four million, forty-two thousand, four hundred and twenty.

3 Write each of the following numerals in words.

a 8405	**b** 43 627	**c** 90 046
d 130 215	**e** 927 004	**f** 6 360 064
g 17 004 988	**h** 9 302 850	**i** 443 200 000

■ A *number* is the idea of 'how many'. A *numeral* is what we write down to stand for the number.

4 Write the value of the 5, the 6 and the 7, in each of the following.

a 567	**b** 5607	**c** 53 067
d 570 600	**e** 63 075	**f** 635 700
g 6 354 073	**h** 1 567 214	**i** 58 673 121

Example: 234 000

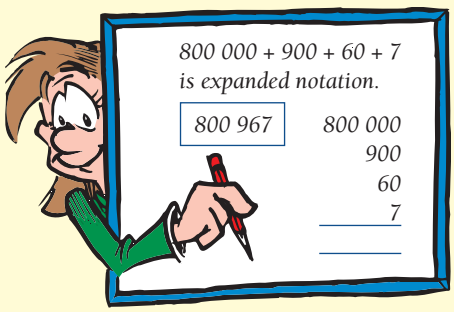
Digit	*Value*
2	200 000
3	30 000
4	4000
0	0

5 Write the value of each of the digits in the following.

a 6421	**b** 80 179	**c** 786
d 91 032	**e** 6094	**f** 1 340 627
g 7 346 912	**h** 27 000 000	**i** 675 000 000

6 Write each of these as a simple (basic) numeral.

a 50 000 + 7000 + 600 + 50 + 7
b 900 000 + 20 000 + 8000 + 600 + 70 + 8
c 3 000 000 + 800 000 + 60 000 + 7000 + 9
d 1 000 000 + 70 000 + 4000 + 600 + 10 + 2
e 800 000 + 900 + 60 + 7
f 5 000 000 + 800 000 + 1000 + 600 + 50 + 7
g 3 000 000 + 800 000 + 60 000 + 70 + 2
h 400 000 + 9000 + 800 + 70 + 1



7 Write these numbers in expanded notation.

a 59 675 **b** 806 307 **c** 9 137 826

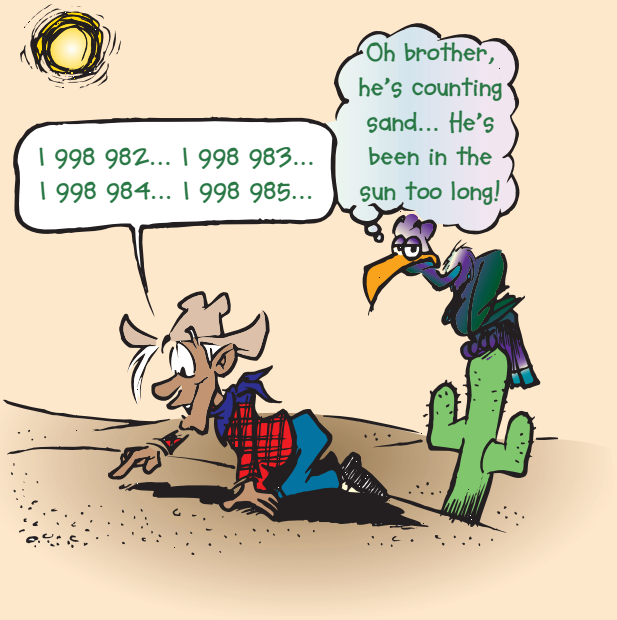
8 Rewrite the numerals in each part, putting them in order from smallest to largest.

a 47 341, 9841, 63 425, 120 070, 1688
b 1903, 24 106, 100 520, 91 000, 65 125
c 635 188, 86 314, 219 414, 9999, 10 112
d 132 145, 58 096, 8014, 72 143, 88 000
e 1 090 040, 938 497, 138 096, 365 214
f 77 717, 8987, 637 114, 123 000, 97 312
g 47 314, 100 000, 9060, 8914, 621 114
h 3 156 214, 1 500 000, 5 937 193, 980 000

investigation 1:02

Investigation 1:02 | Estimation

Please use the Assessment Grid on the following page to help you understand what is required for this Investigation.



1 **Estimating large numbers**
In groups of two or four, discuss and then complete these estimations. (You may use a calculator.)
   - **a** Given a thimble, a cup and a bucket of sand, it is possible to find an estimate of the number of grains of sand in the bucket without trying to count each grain of sand. Write down in full sentences the steps you would use to find your estimate.
   - **b** Estimate the number of telephone numbers that would be found in a telephone book of your choice. Explain a method that would allow you to find the approximate number in that book.

2 **Counting to 1 000 000**
Time how long it takes you to count from 1000 to 1050; from 121 150 to 121 200; and from 999 950 to 1 000 000. Estimate the time it would take you to count from 1 to 1 000 000.

fun spot 1:02

Fun Spot 1:02 | A nest of squares

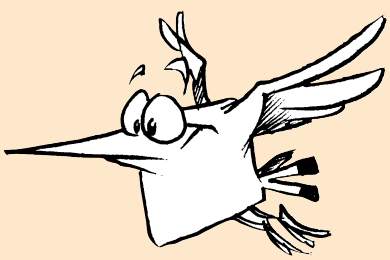
The diagram shows a nest of squares.

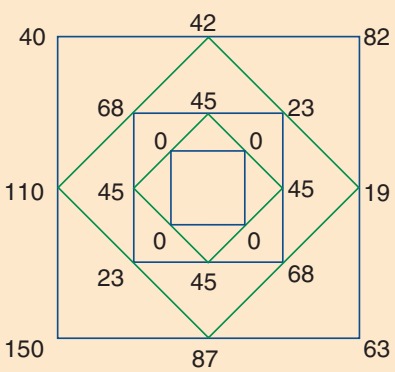
On the four corners (or *vertices*) of the largest square any four numerals can be written. The positive difference of neighbouring numerals is written between them.

If this process is repeated, eventually you will have a square with a zero on each corner (or *vertex*).

Choose any four numbers and show that this happens for your nest of squares.

Will it also happen for a nest of triangles? Pentagons? Octagons? Why don't you draw some of these nests and find out.



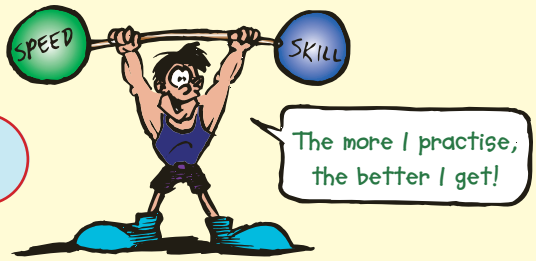


Assessment Grid for Investigation 1:02 | Estimation

The following is a sample assessment grid for this investigation. You should carefully read the criteria *before* beginning the investigation so that you know what is required.

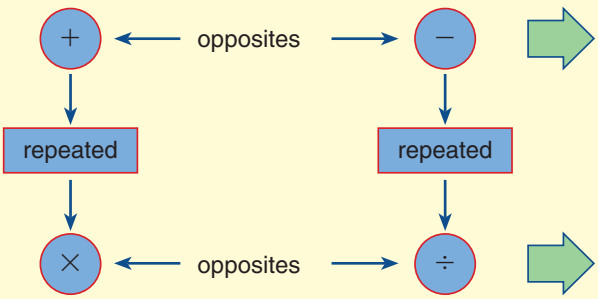
Assessment Criteria (B, C, D)				Achieved ✓
Criterion B Application & Reasoning	a	No systematic approach has been used.	1	
			2	
	b	An organised approach has been attempted but the problem has not been solved.	3	
			4	
	c	A systematic approach has been used and the answer is reasonable.	5	
			6	
	d	A systematic and organised approach has been used and the majority of the answers are reasonable.	7	
			8	
	e	All the answers obtained are within reason, well explained and demonstrate the use of a systematic approach.	9	
			10	
Criterion C Communication	a	Not many explanations are given and the steps undertaken are hard to follow.	1	
			2	
	b	Some working out is shown and an attempt has been made to show the results in an organised manner.	3	
			4	
	c	Working out is shown and is easy to follow. The results are arranged in an organised manner.	5	
			6	
Criterion D Reflection & Evaluation	a	Some attempt has been made to explain the steps taken.	1	
			2	
	b	The steps undertaken are explained with some success and some results have been checked for reasonableness.	3	
			4	
	c	All the steps are explained successfully and all results have been checked for their reasonableness.	5	
			6	
	d	All steps are explained and results have been checked. Alternatives may have been discussed.	7	
			8	

1:03 | The Four Operations



Constant review is the secret to removing errors and increasing your speed.

The four operations are related:



■ $(123 + 7) - 7 = 123$
Adding 7 and subtracting 7 are opposites.

■ $(175 \times 5) \div 5 = 175$
Multiplying by 5 and dividing by 5 are opposites.

Exercise 1:03

Foundation Worksheet 1:03A, B

Know your tables

1	7+	0	1	4	3	7	2	8	6	5	9
2	11−	2	8	5	1	9	6	3	10	7	4
3	7×	0	2	1	5	10	3	7	4	8	6	9	11

1 Complete:

a $6 + 6 + 6 + 6 + 6 = \ldots$.
5 lots of $6 = \ldots$.
$5 \times 6 = \ldots$.

b $9 + 9 + 9 + 9 + 9 + 9 + 9 + 9 = \ldots$.
8 lots of $9 = \ldots$.
$8 \times 9 = \ldots$.

Multiplication can be done by repeated addition.

c How many 10s can be subtracted from 80?
How many lots of 10 in 80?
What is $80 \div 10$?
Division can be done by repeated subtraction.

d How many 6s can be subtracted from 24?
How many lots of 6 in 24?
What is $24 \div 6$?



2

×	4	1	3	6	0	10	2	5	9	7	11	8
2												
8												
4												
7												

■ If $7 \times 9 = 63$ then $63 \div 7 = 9$ and $63 \div 9 = 7$.

3 Each box should be completed in 20 seconds.

a

$36 \div 6$	
$56 \div 8$	
$32 \div 4$	
$27 \div 9$	
$42 \div 7$	

b

$36 \div 4$	
$49 \div 7$	
$48 \div 8$	
$45 \div 5$	
$63 \div 9$	

c

$42 \div 6$	
$24 \div 8$	
$36 \div 9$	
$16 \div 4$	
$56 \div 7$	

d

$24 \div 4$	
$63 \div 7$	
$64 \div 8$	
$72 \div 9$	
$35 \div 5$	

e

$48 \div 6$	
$28 \div 4$	
$21 \div 7$	
$72 \div 8$	
$81 \div 9$	

4 Division can be seen as sharing.

a Share 48 lollies between 4 girls. How many lollies does each get?

b \$84 is to be shared by 6 students. How much will each student receive?

c 66 kg of fertiliser is to be used to treat 6 paddocks. If the same amount is used on each paddock, how much will be used on each paddock?

d How many slices of bread would each of 8 sailors get if they shared 120 slices?

5 Use the fact that division is the opposite of multiplication to find answers to:

a $(156 \times 9) \div 9$ **b** $(409 \times 7) \div 7$ **c** $(1196 \div 6) \times 6$ **d** $(6784 \times 11) \div 11$

e $(86 \times 13) \div 13$ **f** $(1160 \div 8) \times 8$ **g** $(318 \times 97) \div 97$ **h** $(3156 \div 4) \times 4$

6 To find $\frac{1}{8}$ of 160 we would divide 160 by 8. Use this method to find:

a $\frac{1}{4}$ of 48 **b** $\frac{1}{2}$ of 366 **c** $\frac{1}{3}$ of 300 **d** $\frac{1}{5}$ of 55

e $\frac{1}{8}$ of 160 **f** $\frac{1}{10}$ of 460 **g** $\frac{1}{9}$ of 36 **h** $\frac{1}{7}$ of 63

7 Division can be seen as repeated subtraction.

a How many times can 8 be subtracted from 33? What is the remainder?

b How many times can 14 be subtracted from 140? Is there a remainder?

c How many times can 9 be subtracted from 47? What is the remainder?

d How many 4 cm long pieces of wood can be cut from a length of 47 cm?

8 **a** $33 - 8 - 8 - 8 - 8$ **b** $138 - (10 \times 6) - (10 \times 6) - (2 \times 6) - (1 \times 6)$

c $2354 - (100 \times 14) - (50 \times 14) - (15 \times 14) - (3 \times 14)$

worked example

Four people are to share 19 slices of toast. How much should each receive?

Solution

$4\overline{)19}$ $4\text{ r }3$ or $4\overline{)19}$ $4\frac{3}{4}$ or $\frac{19}{4} = 4\frac{3}{4}$

Each should receive 4 slices.

Remainders can be expressed as fractions. $\frac{19}{4}$ means $19 \div 4$.

9 Write each answer as a mixed number.

a $8 \div 3$ **b** $11 \div 4$ **c** $37 \div 10$ **d** $33 \div 5$

e $7\overline{)11}$ **f** $6\overline{)5}$ **g** $\frac{13}{5}$ **h** $\frac{8}{3}$

i $5\overline{)608}$ **j** $4\overline{)375}$ **k** $10\overline{)108}$ **l** $9\overline{)757}$

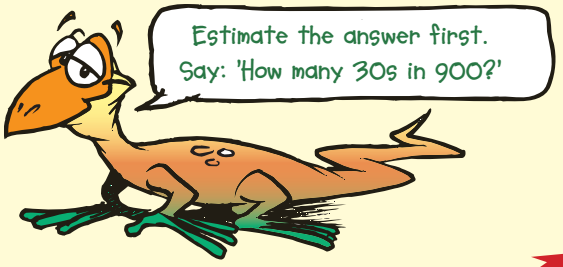
10 Write remainders in your answers as fractions.

a $10\overline{)1089}$	**b** $10\overline{)36\,000}$	**c** $10\overline{)51\,086}$	**d** $10\overline{)71\,052}$
e $7\overline{)1086}$	**f** $5\overline{)9135}$	**g** $3\overline{)27\,111}$	**h** $4\overline{)61\,041}$
i $6\overline{)36\,066}$	**j** $7\overline{)71\,449}$	**k** $5\overline{)15\,007}$	**l** $8\overline{)245\,761}$

11 To find the average of a set of scores, add them up and divide your answer by the number of scores. The average of 4, 6 and 14 is $(4 + 6 + 14) \div 3$. Find the average of:

a 7, 4 and 10	**b** 9 and 21	**c** 16, 4 and 9	**d** 18, 8, 6 and 8
e 18 and 36	**f** 2, 11, 9 and 2	**g** 1, 1, 1, 1 and 6	**h** 22, 15 and 80

1:04 | Long division



■ Once we have placed the '2' above the '7', we must also place a number above the '8'.

■ Method 1
Contracted form

$$\begin{array}{r} 28 \text{ r } 10 \\ 31\overline{)878} \\ -\,62 \\ \hline 258 \\ 248 \\ \hline 10 \end{array}$$

■ Method 2
Preferred multiples

$$\begin{array}{r|l} 28 \text{ r } 10 & \\ 31\overline{)878} & \\ -\,620 & 20 \\ \hline 258 & \\ 248 & 8 \\ \hline 10 & 28 \end{array}$$

Answer: 28 remainder 10 or $28\frac{10}{31}$

Exercise 1:04

1 Do not use a calculator for these. Use method 1 or 2, above.

a $98 \div 14$	**b** $91 \div 13$	**c** $63 \div 12$	**d** $79 \div 11$
e $214 \div 15$	**f** $196 \div 17$	**g** $238 \div 18$	**h** $256 \div 16$
i $647 \div 23$	**j** $555 \div 24$	**k** $858 \div 13$	**l** $973 \div 16$

2 If $838 \div 12 = 69$ r 10 then $838 = 69 \times 12 + 10$. Complete:

a $88 \div 44 = 2$, $\therefore 88 = 2 \times \ldots$	**b** $708 \div 23 = 30$ r 18, $\therefore 708 = 30 \times \ldots + \ldots$
c $182 \div 13 = 14$, $\therefore 182 = 14 \times \ldots$	**d** $654 \div 36 = 18$ r 6, $\therefore 654 = 18 \times \ldots + \ldots$

3 We could change a division question into multiplication. $356 \div 52 = \ldots$ becomes $52 \times \ldots = 356$. We then guess and check.

a $96 \div 31$	**b** $128 \div 39$	**c** $99 \div 22$	**d** $107 \div 18$
e $345 \div 99$	**f** $281 \div 47$	**g** $152 \div 29$	**h** $288 \div 62$

Check your answers to these using *method 1* or *method 2*.

1:05 | Speed and Accuracy

Exercise 1:05

Use your watch to time how quickly you can do each of the following sets of questions.
Add 10 seconds for each mistake. Record your times in a table like the one below.

1

a	$6 + 8$	b	9×9
c	$35 \div 5$	d	$15 - 7$
e	14×100	f	$\$2 - \$1.35$

g Find the cost of 3 cups at $2.15 per cup.
h John earned five times as much as $4.20. How much did he earn?
i How many 50-cent stamps can I buy for $6.20?
j $16.80 was shared by four girls. How much did each get?

2

a	$7 + 5$	b	8×6
c	$42 \div 6$	d	$14 - 8$
e	18×100	f	$\$2 - \$1.85$

g Find the cost of 4 fans at $3.15 per fan.
h Kylie earned six times as much as $5.10. How much did she earn?
i How many 50-cent stamps can I buy for $5.30?
j $18.60 was shared by three boys. How much did each get?

3

a	$9 + 4$	b	6×9
c	$21 \div 7$	d	$16 - 9$
e	17×100	f	$\$2 - \$1.55$

g Find the cost of 5 pens at $1.15 per pen.
h Spiro earned eight times as much as $2.20. How much did he earn?
i How many 50-cent stamps can I buy for $8.10?
j $15.50 was shared by five boys. How much did each get?

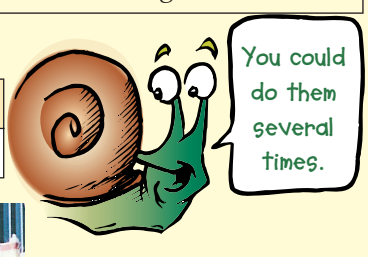
4

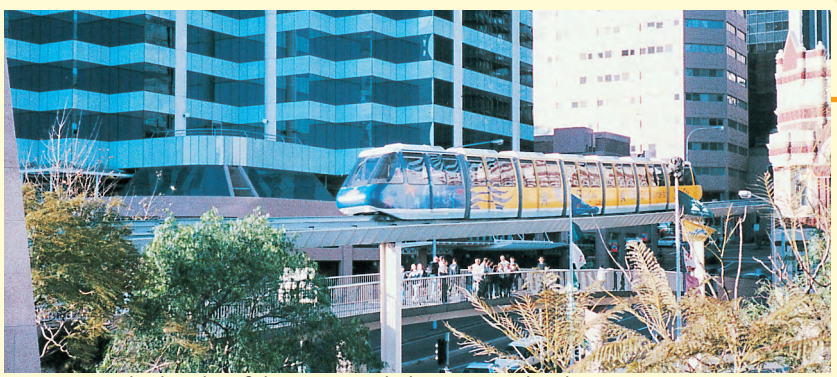
a	$8 + 7$	b	7×7
c	$28 \div 4$	d	$13 - 7$
e	13×100	f	$\$2 - \$1.65$

g Find the cost of 3 books at $2.25 per book.
h Sandra earned four times as much as $3.40. How much did she earn?
i How many 50-cent stamps can I buy for $7.40?
j $20.40 was shared by four girls. How much did each get?

Record of results

Number	1	2	3	4	1	2	3	4	1	2	3	4
Time												





- Estimate the height of this monorail above street level.

1:06 | Powers of Numbers

prep quiz 1:06

1	$3 \times 3 \times 3$	2	5×5	3	$2 \times 2 \times 2 \times 2 \times 2$	4	$5 \times 5 \times 5$
5	10×10	6	$10 \times 10 \times 10$	7	$10 \times 10 \times 10 \times 10$		
8	$10 \times 10 \times 10 \times 10 \times 10$			9	$7 \times 10 \times 10$	10	$4 \times 10 \times 10 \times 10$

We use a *power* when a number is used more than once in a product.

$5^2 = 5 \times 5$



This is five squared, or five to the power of two.

$10^3 = 10 \times 10 \times 10$



This is ten cubed, or ten to the power of three.

worked examples

1 Write 5^3 in expanded form and write its basic numeral.
2 Write $10 \times 10 \times 10 \times 10$ as a power of ten.
3 What is the basic numeral of 7×10^3?

A basic numeral is the simplest answer.

Solutions

1 5^3
$= 5 \times 5 \times 5$
$= 25 \times 5$
$= 125$

2 $10 \times 10 \times 10 \times 10$
$= 10^4$

3 7×10^3
$= 7 \times (10 \times 10 \times 10)$
$= 7 \times 1000$
$= 7000$

Exercise 1:06

Foundation Worksheet 1:06
Powers of numbers
1 $4 \times 4 = \ldots$
2 $7^2 = \ldots$
3 a $10 \times 10 \times 10 = \ldots$

1 Write each in expanded form and as a basic numeral.

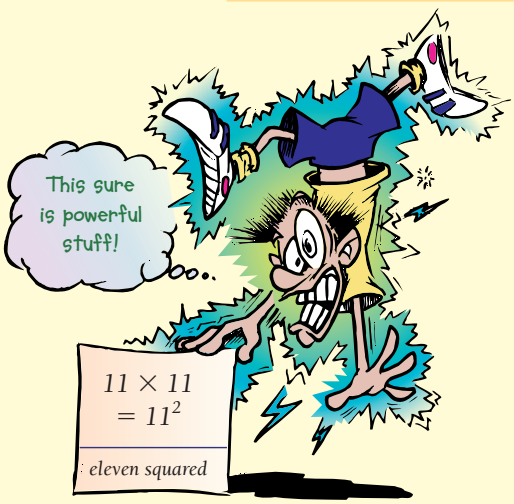
a	4^2	b	10^2	c	2^2
d	1^2	e	7^2	f	9^2
g	2^3	h	5^4	i	3^3
j	10^3	k	10^4	l	10^5

2 Write each as a power.

a	7×7	b	$8 \times 8 \times 8$
c	$10 \times 10 \times 10 \times 10$	d	$10 \times 10 \times 10$
e	$4 \times 4 \times 4 \times 4 \times 4 \times 4$	f	$2 \times 2 \times 2 \times 2$
g	$10 \times 10 \times 10 \times 10 \times 10$		

3 Write the basic numeral for each.

a	6×10^1	b	3×10^2	c	5×10^3
d	2×10^4	e	7×10^3	f	1×10^2
g	4×10^3	h	9×10^2	i	8×10^4
j	$3^2 \times 10^3$	k	$2^3 \times 10^4$	l	$7^2 \times 2^3$



1:07 | Expanded Notation

prep quiz 1:07

If $10^2 = 10 \times 10$, find the simplest numeral for:

1 10^2 **2** 5×10^2 **3** 8×10^2

If $10^3 = 10 \times 10 \times 10$, find the simplest numeral for:

4 10^3 **5** 2×10^3 **6** 9×10^3

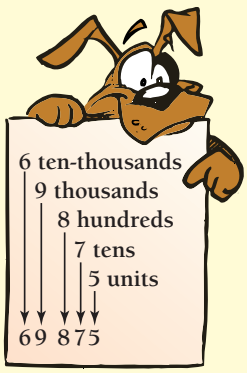
Write in simplest form:

7 $(6 \times 10) + 3$ **8** $(2 \times 10^2) + (5 \times 10) + 1$

9 $(7 \times 10^3) + (9 \times 10^2) + (3 \times 10) + 5$

10 $(8 \times 10^3) + (1 \times 10^2) + (3 \times 10) + 2$

Ten-thousands *10 000*	*Thousands* *1000*	*Hundreds* *100*	*Tens* *10*	*Units* *1*
$10 \times 10 \times 10 \times 10$ 10^4	$10 \times 10 \times 10$ 10^3	10×10 10^2	10 10^1	1 1
6	9	8	7	5



$69\,875 = (6 \times 10\,000) + (9 \times 1\,000) + (8 \times 100) + (7 \times 10) + (5 \times 1)$

$= (6 \times 10^4) + (9 \times 10^3) + (8 \times 10^2) + (7 \times 10) + (5 \times 1)$

worked examples

1 Write 6 millions + 9 thousands + 2 hundreds + 5 tens as a simple numeral.

2 Write $(5 \times 10^4) + (7 \times 10^2) + (2 \times 10^1) + (1 \times 1)$ as a numeral in its simplest form.

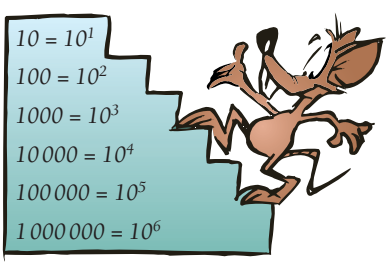
3 Write 932 014 in expanded notation.

Solutions

1 6 millions + 9 thousands + 2 hundreds + 5 tens.

Column values						
1 000 000	100 000	10 000	1 000	100	10	1
6	0	0	9	2	5	0

Zeros act as place holders to allow the other digits to be in their right columns.



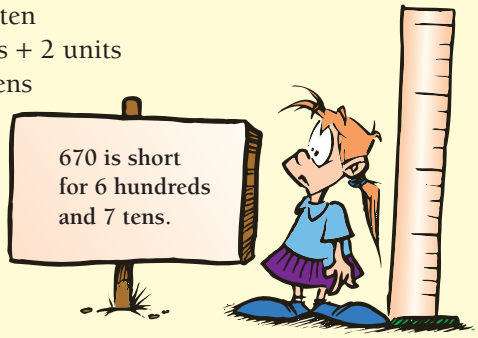
2 $(5 \times 10\,000) + (0 \times 1000) + (7 \times 100) + (2 \times 10) + (1 \times 1)$
$= 50\,721$

3 $932\,014$
$= (9 \times 100\,000) + (3 \times 10\,000) + (2 \times 1000) + (0 \times 100) + (1 \times 10) + (4 \times 1)$
$= (9 \times 10^5) + (3 \times 10^4) + (2 \times 10^3) + (0 \times 10^2) + (1 \times 10^1) + (4 \times 1)$

Exercise 1:07

1 Write each as a simple numeral (a basic numeral).

a 6 thousands + 4 hundreds + 5 tens + 9 units
b 2 ten-thousands + 8 thousands + 6 hundreds + 3 tens
c 9 hundred-thousands + 8 hundreds + 7 tens + 4 units
d 4 millions + 3 hundred-thousands + 8 ten-thousands
e 7 ten-thousands + 4 thousands + 5 hundreds + 8 tens + 6 units
f 1 million + 1 hundred-thousand + 1 thousand + 1 ten
g 5 millions + 6 ten-thousands + 8 hundreds + 4 tens + 2 units
h 4 ten-thousands + 8 thousands + 3 hundreds + 9 tens



2 Write these numerals in simplest form.

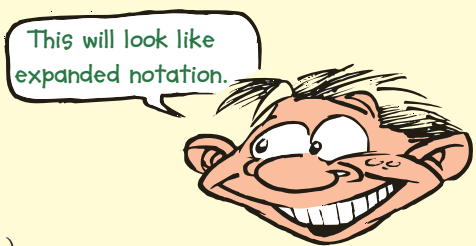
a $(8 \times 1000) + (5 \times 100) + (3 \times 10) + (9 \times 1)$
b $(7 \times 100) + (3 \times 10) + (8 \times 1)$
c $(7 \times 1000) + (3 \times 100) + (0 \times 10) + (4 \times 1)$
d $(9 \times 1000) + (0 \times 100) + (6 \times 10) + (7 \times 1)$
e $(9 \times 1000) + (0 \times 100) + (0 \times 10) + (3 \times 1)$
f $(8 \times 1000) + (2 \times 100) + (1 \times 10) + (0 \times 1)$
g $(6 \times 1\,000\,000) + (8 \times 100\,000) + (1 \times 10\,000) + (9 \times 1000) + (3 \times 100) + (4 \times 10) + (6 \times 1)$
h $(1 \times 1\,000\,000) + (4 \times 100\,000) + (0 \times 10\,000) + (2 \times 1000) + (0 \times 100) + (7 \times 10) + (5 \times 1)$
i $(3 \times 1\,000\,000) + (0 \times 100\,000) + (9 \times 10\,000) + (4 \times 1000) + (3 \times 100) + (6 \times 10) + (3 \times 1)$
j $(6 \times 1\,000\,000) + (9 \times 100\,000) + (0 \times 10\,000) + (0 \times 1000) + (0 \times 100) + (0 \times 10) + (0 \times 1)$

3 Write each of the following in the form of question 1.

a 1360	**b** 53 200	**c** 1 600 000	**d** 9386
e 4 307 200	**f** 800 305	**g** 1989	**h** 7 345 831

4 Write each of the following in expanded form as in question 2.

a 6709	**b** 13 304	**c** 9084
d 103 243	**e** 97 670	**f** 207 116
g 1 344 562	**h** 9 765 813	



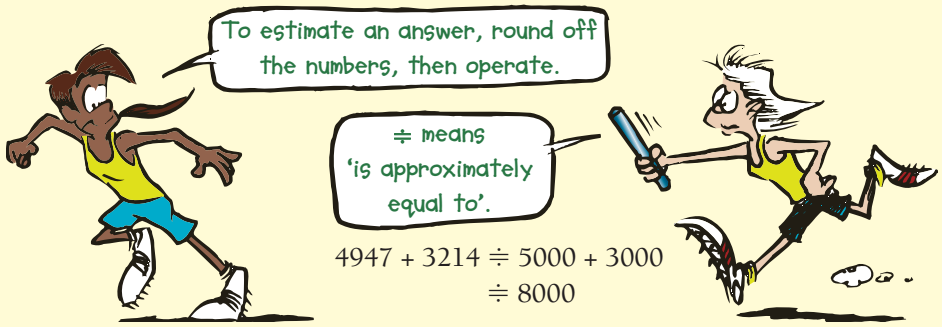
5 Write each of the numerals of question 2 in expanded notation using powers of ten.
eg **a** becomes $(8 \times 10^3) + (5 \times 10^2) + (3 \times 10^1) + (9 \times 1)$

6 Write each of the numerals of question 3 in expanded form using powers of ten.



- Estimate the number of shapes shown here.

1:08 | Estimating Answers



prep quiz 1:08

Round off each to the nearest ten.

1 84 **2** 47 **3** 16 **4** 38·15

Round off each to the nearest hundred.

5 435 **6** 851 **7** 118 **8** 274·85

Round off each to the nearest thousand.

9 6469 **10** 9710

Foundation Worksheet 1:08

Rounding numbers

1 Round off to the nearest ten:
a 368

2 Round off to the nearest hundred:
a 8731

Leading figure estimation

To use leading figure estimation, we round off each number involved to its leading figure (as we have in Prep Quiz 1:07) and then perform the operation.

worked examples

Use leading figure estimation to estimate an answer for each of these.

1 618 + 337 + 159 + 409

2 38 346 − 16 097

3 3250 × 11·4

4 1987 ÷ 4



■ *Note:* It is a printing convention that no space be left for 'thousands' in a four-digit number.

■ *Remember:*
If the digit that follows the leading figure is 5 or more, ROUND UP.

Solutions

1
600
300
200
400
1500

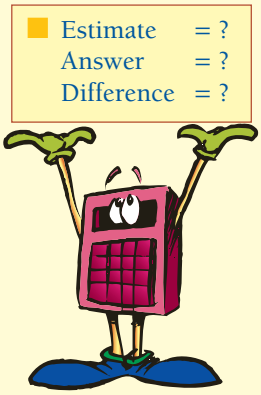
2
40 000
− 20 000
20 000

3
4000
× 10
40 000

4
500
4)2000

Exercise 1:08

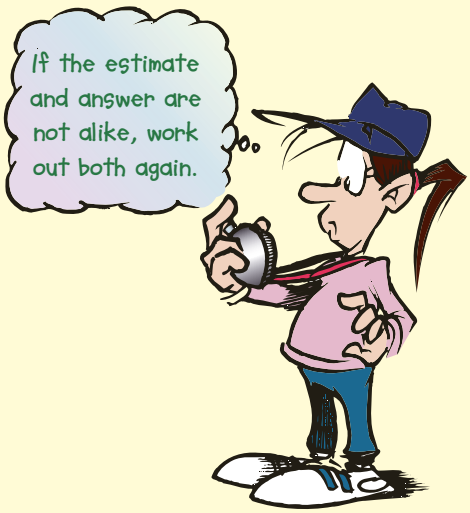
Use leading figure estimation to estimate an answer.
Then use a calculator to find the answer and the difference between the estimate and the answer.



1
- **a** 659 + 527 + 305
- **b** 1963 + 3211 + 2416
- **c** 65 342 + 31 450
- **d** 3 140 942 + 6 095 811
- **e** 947 – 382
- **f** 8342 – 2625
- **g** 8611 – 8093
- **h** 340 321 – 127 999
- **i** 3813 × 3
- **j** 1716 × 6
- **k** 84 694 × 1·85
- **l** 736 308 × 2·65
- **m** 8192 ÷ 4
- **n** 64 035 ÷ 3
- **o** 3870 ÷ 1·25
- **p** 479 352 ÷ 4·8

2 For each part, estimate the answer before you calculate, then work out the difference between your estimate and answer.

- **a** Crowd attendances at the National Tennis Centre over three days were 8146, 7964 and 9193. What was the total attendance?
- **b** Minh took 88 410 mousetraps to his home town but sold only 2365. How many mousetraps were not sold?
- **c** There are 86 400 seconds in one day. How many seconds are there in 7 days?
- **d** Three prospectors found a gold nugget which they sold for $27 360. They shared the money equally. How much did each receive?
- **e** How many groups of 175 soldiers can be formed by 24 290 soldiers?
- **f** Alan received two cheques in April, one for $13 714.93 and the other for $9854.19. He received two more in October, for $19 142.55 and $12 740.02. How much did he receive altogether?
- **g** Of the money Alan received in **f**, he had to pay the government $23 657.40 in taxes. How much did Alan get to keep?
- **h** Kylie sold seven cars to Firststate Ltd for $56 315 each. What was the total amount she received for the sale?



3 Estimation speed test.

- Have a friend measure the time it takes you (in seconds) to complete the ten *estimates* on the next page.
- Try to finish all ten *estimates* in four minutes.
- When you have finished:
  - **a** work out the answers with a pencil and paper or with a calculator.
  - **b** work out the difference between the estimate and the correct answer for each one.

	Question	Working	Estimate	Answer	Difference
a	11 615 + 58 107				
b	92 094 ÷ 6				
c	48 314 − 27 199				
d	37 118 × 7				
e	2 416 809 + 4 369 170				
f	567 180 ÷ 5				
g	69 847 × 9				
h	486·76 ÷ 1·72				
i	735·15 − 275·35				
j	2315 × 7·8				

4

- Where two amounts have different numbers of digits, we usually estimate to a similar place for both numbers when doing addition or subtraction.
  eg 14 384 + 4256 ≒ 14 000 + 4000
  ≒ 18 000
- If I used leading digit estimation it would have been:
  14 384 + 4256 ≒10 000 + 4000
  ≒ 14 000
  This last estimate is even smaller than the first of the original numbers!
- Which example in question 2 would be better suited to this method?

■ ≒ means 'is approximately equal to'.



- Estimate the number of people who live in this city.
- Estimate the number of dollars that workers in this city might earn in one day.

fun spot 1:08

Fun Spot 1:08 | What's the difference between a boxer and a telephone?

Work out the answer to each part and put the letter for that part in the box that is above the correct answer.

N 6×5	**O** 0×8	**S** 17×1	**N** 4×7
R 8×3	**N** 2×13	**X** 8×8	**O** 9×9
B $9 + 4$	**I** $7 + 8$	**I** $6 + 5$	**E** $8 + 6$
E $17 - 8$	**E** $13 - 6$	**I** $20 - 17$	**I** $100 - 53$
R $125 \div 5$	**N** $24 \div 3$	**X** $42 \div 7$	**O** $100 \div 20$
T $\frac{1}{5}$ of 20	**T** $\frac{1}{10}$ of 100	**H** $\frac{1}{2}$ of 58	**H** $\frac{1}{3}$ of 66
R 3^3	**G** 5×10^2	**S** 2^4	**N** 5^3
A $\$1.20 \times 5$	**O** $\$2.00 \div 4$	**B** $\$12 \div 3$	**E** $\$8 \times 7$

Change these Roman numerals into our own numerals.

A CCCLXIV **G** MDXCI



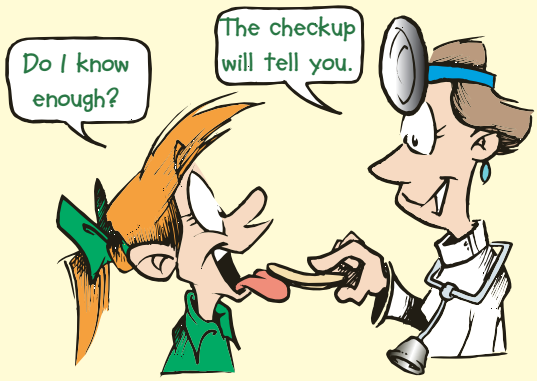
5	30	$56	13	0	6	14	17	11	125	$6	27	15	28	500

4	22	9	81	10	29	7	24	25	3	26	1591	16	47	8	364	$4	$0.50	64



- How many different patterns could you make using 2 yellow and 2 green apples, placed in a line?

1:09 | Diagnostic Checkups



Checkup and treatment

- Complete each Diagnostic Checkup.
- Correct your work, using the answers at the back of the book.
- If you have more than one mistake in any row, get help by going to the CD reference shown at the end of that row. There you will find an explanation and practice exercise.
- *Note:* Many students will not have covered all the work included in these checkups. They are designed to make teachers and students aware of what is known and what still needs attention.



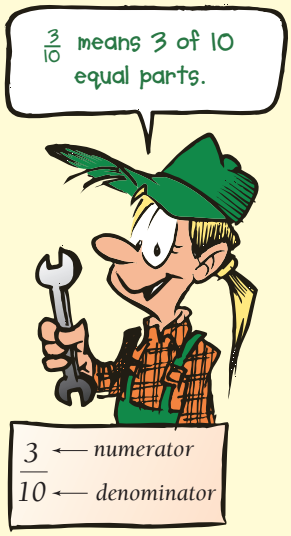
1:09A Operations Checkup

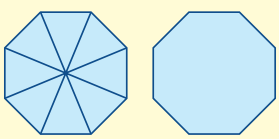
					CD Appendix
1	a 64 26 37 9	b 515 307 96 983	c 19 208 7 537 38 690	d 76 215 4 806 193 521 60 433	B:01
2	a 647 − 193	b 3862 − 1477	c 56 312 − 17 294	d 491 625 − 38 163	B:02
3	a 659 × 6	b 2506 × 8	c 789 × 10	d 1305 × 70	B:03
4	a 78 × 32	b 1907 × 93	c 8653 × 45	d 473 × 580	B:03
5	a $6\overline{)660}$	b $5\overline{)654}$	c $7\overline{)2374}$	d $4\overline{)1624}$	B:04
6	a $20\overline{)8950}$	b $90\overline{)9630}$	c $400\overline{)6800}$	d $40\overline{)73\,400}$	B:04

1:09B Fractions Checkup

		CD Appendix
1	Complete the following to make equivalent fractions. **a** $\frac{3}{4} = \frac{\square}{8}$ **b** $\frac{1}{5} = \frac{\square}{10}$ **c** $\frac{3}{10} = \frac{\square}{100}$	B:05
2	Write each fraction in its simplest form. **a** $\frac{8}{10}$ **b** $\frac{6}{8}$ **c** $\frac{20}{50}$	B:05
3	Which of the two fractions is smaller? **a** $\frac{1}{5}, \frac{1}{10}$ **b** $\frac{3}{4}, \frac{6}{10}$ **c** $\frac{2}{5}, \frac{1}{2}$	B:06
4	Arrange in order, from smallest to largest: **a** $\{\frac{1}{5}, \frac{1}{2}, \frac{1}{10}\}$ **b** $\{\frac{3}{10}, \frac{7}{10}, \frac{1}{10}\}$	B:06
5	Write these mixed numbers as improper fractions. **a** $2\frac{1}{2}$ **b** $5\frac{3}{10}$ **c** $1\frac{1}{4}$	B:07
6	Write these improper fractions as mixed numbers. **a** $\frac{7}{4}$ **b** $\frac{87}{10}$ **c** $\frac{9}{2}$	B:07
	Give the simplest answer for:	
7	**a** $\frac{3}{10} + \frac{4}{10}$ **b** $\frac{1}{5} + \frac{2}{5}$ **c** $\frac{8}{10} + \frac{3}{10}$	B:08
8	**a** $\frac{9}{10} - \frac{8}{10}$ **b** $\frac{3}{4} - \frac{1}{4}$ **c** $\frac{5}{8} - \frac{1}{8}$	B:08
9	**a** $\frac{1}{2} + \frac{1}{4}$ **b** $\frac{2}{3} + \frac{2}{9}$ **c** $\frac{3}{10} + \frac{2}{5}$	B:08
10	**a** $\frac{9}{10} - \frac{1}{5}$ **b** $\frac{1}{3} - \frac{1}{6}$ **c** $\frac{7}{16} - \frac{1}{4}$	B:08
11	**a** $4 \times \frac{3}{4}$ **b** $5 \times \frac{3}{10}$ **c** $10 \times \frac{2}{5}$	B:09
12	**a** $\frac{1}{24}$ of 24 **b** $\frac{1}{10}$ of 90 **c** $\frac{1}{8}$ of 160	B:10

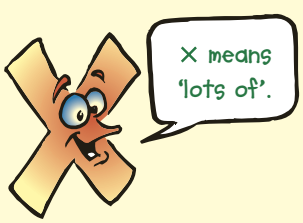






$\frac{8}{8} = 1$

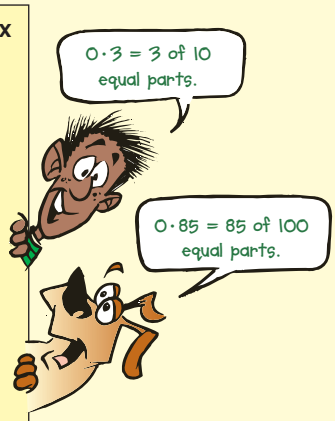
8 eighths make 1 whole.



Do each Checkup without getting help. If mistakes occur in a line, go to the CD reference shown on that line for an explanation and a practice exercise.

1:09C Decimals and Money Checkup

		CD Appendix
1	Write these fractions as decimals. **a** $\frac{7}{10}$ **b** $2\frac{13}{100}$ **c** $\frac{9}{1000}$	B:11
2	Write these decimals as fractions in simplest form. **a** 0·317 **b** 0·59 **c** 0·5	B:11
3	Which of the two decimals is smaller? **a** 0·3, 0·1 **b** 0·11, 0·2 **c** 0·5, 0·49	B:12
4	Put in order, smallest to largest: **a** 1, 0·3, 0·8 **b** 0·11, 0·91, 0·51 **b** 0·12, 0·8, 0·509	B:12
5	**a** 1·9 + 3·6 **b** 2·74 + 0·4 **c** 5·18 + 3	B:13
6	**a** 0·65 + 4·2 + 63·4 **b** 5·85 + 0·6 + 7 **c** 7·2 + 16 + 4·1	B:13
7	**a** 9·2 − 1·25 **b** 10·63 − 6·3 **c** 18·415 − 9·31	B:14
8	**a** 1·6 × 5 **b** 0·75 × 4 **c** 0·181 × 3	B:15
9	**a** $3\overline{)48.6}$ **b** $6\overline{)1.8}$ **c** $3\overline{)0.123}$	B:16
10	**a** 3·1 ÷ 10 **b** 0·05 × 100 **c** 0·8 × 1000	B:17
11	Write each amount as dollars, using a decimal. **a** 5 dollars 65 cents **b** 8 dollars 5 cents **c** 7 dollars 40 cents	B:18
12	**a** \$5.80 + \$9 **b** \$63.20 + \$2.50 **c** \$201 + \$1.20	B:18
13	**a** \$9.60 − \$3.40 **b** \$50 − \$3.50 **c** \$5 − \$1.85	B:18
14	**a** \$4.50 × 4 **b** \$1.85 × 6 **c** \$9.05 × 5	B:18
15	**a** \$18 ÷ 4 **b** \$12.15 ÷ 3 **c** \$70 ÷ 8	B:18

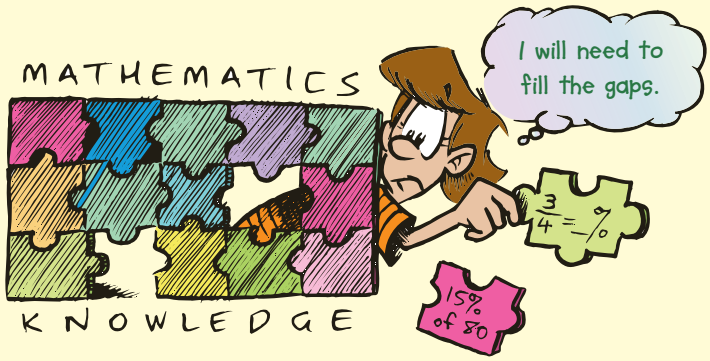


2 tens
5 units
3 tenths
8 hundredths
7 thousandths

25·387

10	1		$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
2	5	·	3	8	7

20 ·
5 ·
· 3
· 08
· 007
25 · 387



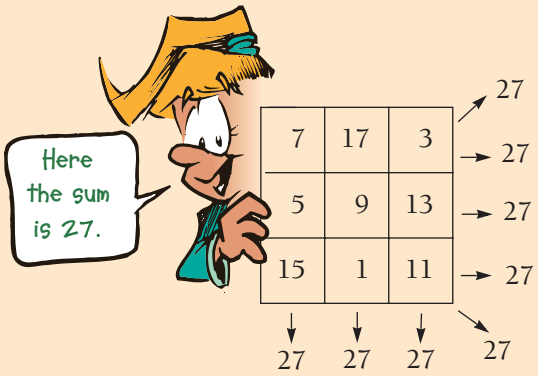
1:09D Percentages Checkup

				CD Appendix
1	Write these as hundredths.			B:19
	a 7%	**b** 33%	**c** 15%	
2	Write each as a fraction in its simplest form.			B:19
	a 20%	**b** 30%	**c** 15%	
3	Write each as a whole number.			B:20
	a 100%	**b** 300%	**c** 200%	
4	Write each as a mixed number in simplest form.			B:20
	a 250%	**b** 105%	**c** 125%	
	Write each as a percentage.			
5	**a** $\frac{8}{100}$	**b** $\frac{37}{100}$	**c** $\frac{50}{100}$	B:21
6	**a** $\frac{3}{20}$	**b** $\frac{7}{50}$	**c** $\frac{7}{10}$	B:21
7	**a** 4	**b** 1	**c** 3	B:21
8	**a** $3\frac{1}{2}$	**b** $1\frac{3}{4}$	**c** $7\frac{3}{10}$	B:21
	Write these percentages as decimals.			
9	**a** 93%	**b** 7%	**c** 10%	B:22
10	**a** 150%	**b** 113%	**c** 425%	B:22
	Write these decimals as percentages.			
11	**a** 0·07	**b** 0·85	**c** 0·9	B:23
12	**a** 3·15	**b** 1·08	**c** 5·9	B:23



Fun Spot 1:09 | Magic squares

The numbers in every row, column and diagonal have the same total.



1 Find the missing numbers in these magic squares.

a

8		6
3	5	
4	9	

b

	18	
14	10	6
		16

c

6		
	7	9
		8

2 **a** Complete each of the magic squares.
b Show that by putting the nine small magic squares together, you get a large magic square. Make sure that every row, column and diagonal has the same sum.

4	14	12
18		
8	6	16

9	7	17
19	11	3
	15	

3	10	5
	6	
7	2	9

6	7	
1	5	9
	3	4

15	5	7
	9	17
11	13	

10		14
17		9
12	11	16

	8	18
20	12	4
6		14

10		8
5	7	9
6	11	

5	10	
12	8	
7	6	11

Mathematical terms 1

approximate (round off)
- To replace a number with a less accurate one, usually to make it simpler.
  eg A crowd of 88 927 could be approximated as 90 000.

basic numeral
- The simplest way of writing a number.

digit
- Any one of the ten Hindu-Arabic numerals 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.

estimate
- To calculate roughly.
- A good guess or the result of calculating roughly.

expanded notation
- A way of writing a number as the sum of its parts.
  eg $612 = (6 \times 100) + (1 \times 10) + (2 \times 1)$

fraction
- One or more parts of a unit or whole.
  $\frac{7}{8}$ ← numerator
  ← denominator
- Equivalent fractions are equal in size.
  eg $\frac{1}{2} = \frac{2}{4}$
- An improper fraction has a numerator that is bigger than its denominator, eg $\frac{7}{4}$.
- A mixed number has a whole number part and a fraction part, eg $3\frac{2}{5}$.

Hindu-Arabic number system
- A number system based on ten in which a digit's value depends on its position or place value. It is the number system we use in Australia.

numeral
- A symbol used for a number.
  eg 3050, LXXXII, 26 342

operation
- Any process performed on one or more numbers.
- The four basic operations are:
  addition (+) subtraction (–)
  multiplication (×) division (÷)

opposite operations
- One operation undoes what the other does.
  $117 \times 3 \div 3 = 117$.
  addition is the opposite of subtraction.
  multiplication is the opposite of division.

place value
- The value of the column in which a digit is found.

power
- A number formed by repeated multiplication.
  eg $5^3 = 5 \times 5 \times 5$
  $= 125$
  125 is a power of 5.

Diagnostic Test 1: | Beginnings in Number

- Each section of the test has similar items that test a certain type of example.
- Failure in more than one item will identify an area of weakness.
- Each weakness should be treated by going back to the section listed.

	Section
1 Change these Egyptian numerals into our modern numerals. **a**  **b**  **c** 	1:01
2 Change these to Egyptian numerals. **a** 365 **b** 4029 **c** 2 320 000	1:01
3 Change these Roman numerals into our modern numerals. **a** CCCLXIV **b** MMII **c** MCMXCIX	1:01
4 Change these to Roman numerals. **a** 372 **b** 948 **c** 3409	1:01
5 Write the value of the 7 in each case. **a** 374 240 **b** 6075 **c** 7 300 004	1:02
6 Write each as a power. **a** $8 \times 8 \times 8$ **b** $3 \times 3 \times 3 \times 3 \times 3$ **c** $2 \times 2 \times 2 \times 2$	1:06
7 Write the basic numeral for each. **a** 8×10^2 **b** 2×10^4 **c** 7×10^6	1:06
8 Write each as a basic numeral. **a** $(7 \times 10^3) + (2 \times 10^2) + (2 \times 10^1) + (8 \times 1)$ **b** $(5 \times 10^3) + (0 \times 10^2) + (8 \times 10^1) + (0 \times 1)$	1:07
9 Write each in expanded form using powers of ten. **a** 1824 **b** 4307 **c** 2 415 286	1:07
10 Use leading figure estimation to estimate the answer to each. **a** 865 109 + 186 422 **b** 163 241 × 7·6 **c** 39 487 ÷ 4·3	1:08

assignment 1A

Chapter 1 | Revision Assignment

1 Write in words:

a 270 307 b 2 456 325
c 97 005 000 d 294 167 000

2 Write the value of the 7 in each.

a 214 703 b 871 000
c 7 415 216 d 13 701 233

3 Write each as a numeral in its simplest form.

a $(5 \times 10\,000) + (9 \times 1000) + (3 \times 100) + (7 \times 10) + (6 \times 1)$
b $(8 \times 100\,000) + (6 \times 1000) + (5 \times 100) + (4 \times 10) + (5 \times 1)$
c $(3 \times 1\,000\,000) + (2 \times 100\,000) + (1 \times 10\,000) + (5 \times 1000)$

4 Write in expanded notation:

a 8247 b 56 015
c 714 850 d 3 420 000

5 Use leading figure estimation to estimate the answer to:

a 37 145 + 24 810 b 417 816 ÷ 8
c 28 143 × 7

6 Write each in expanded form and as a basic numeral.

a 5^2 b 2^4
c 10^4 d 8×10^3

7 Change these Egyptian numerals into our own numerals.

a 
b 
c 
d 

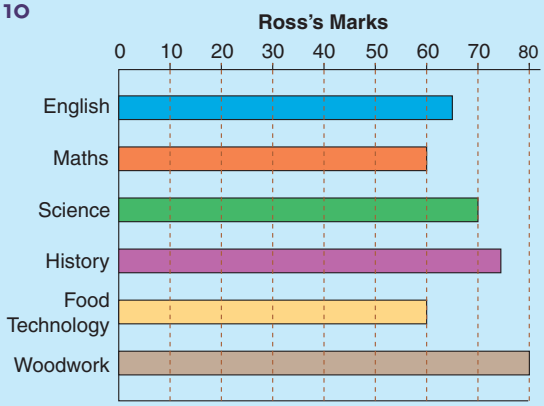
8 Change these Roman numerals into our own numerals.

a CMXXVIII b DCXLIX
c MMDC d $\overline{\text{X}}\,\overline{\text{X}}$DCCX

9 Write as a Roman numeral.

a 327 b 419 c 2555 d 1991

10



a How many subjects did Ross study?
b What mark did Ross get for Maths?
c What mark did Ross get for English?
d What was Ross's best subject?
e In how many subjects did Ross score more than 70 marks?
f Find Ross's total mark for all his subjects.

Drag and Drop CW Interactive
1 Powers of numbers

Review CW Questions

Chapter 1 | Working Mathematically

assignment 1B

id 1

1 Use ID Card 1 on page xiv to give the mathematical term for:

a 1	**b** 3	**c** 4	**d** 5	**e** 9
f 19	**g** 20	**h** 21	**i** 22	**j** 24

2 Use ID Card 2 on page xiv to identify:

a 3	**b** 4	**c** 5	**d** 7	**e** 15
f 16	**g** 19	**h** 20	**i** 23	**j** 24

3 Read each puzzle and see if you can come up with the correct answer.

a If it takes 4 minutes to boil an egg, how long will it take to boil 6 eggs?

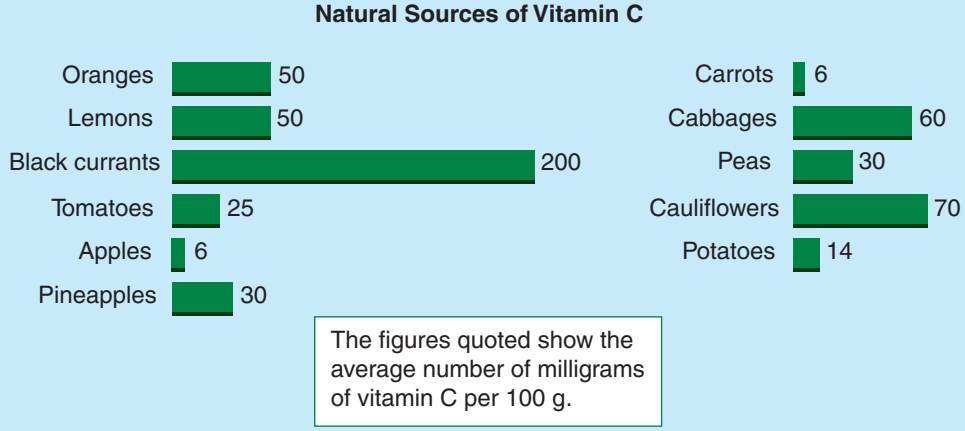
b If 5 cats can catch 5 mice in 5 minutes, how many cats are needed to catch 100 mice in 100 minutes

c If 4 men take 4 hours to dig a hole, how long will 8 men take to dig a similar hole?

d A boy says: 'I have the same number of brothers as I have sisters.' His sister says: 'I have twice as many brothers as sisters.' How many boys and how many girls are in the family?

e A train leaves Munich for Berlin at a speed of 150 km/hour. Another train leaves Berlin for Munich an hour later, at a speed of 120 km/hour. When the two trains meet, which one is closer to Berlin?

4



a Which fruit is the richest source of vitamin C?

b Which vegetable is the richest source of vitamin C?

c How many grams of tomatoes must be eaten to provide the same amount of vitamin C as 100 g of oranges?

d The daily requirement of vitamin C is 32 mg. If I eat an apple weighing 50 g and an orange weighing 50 g, how many mg short of the daily requirement am I?

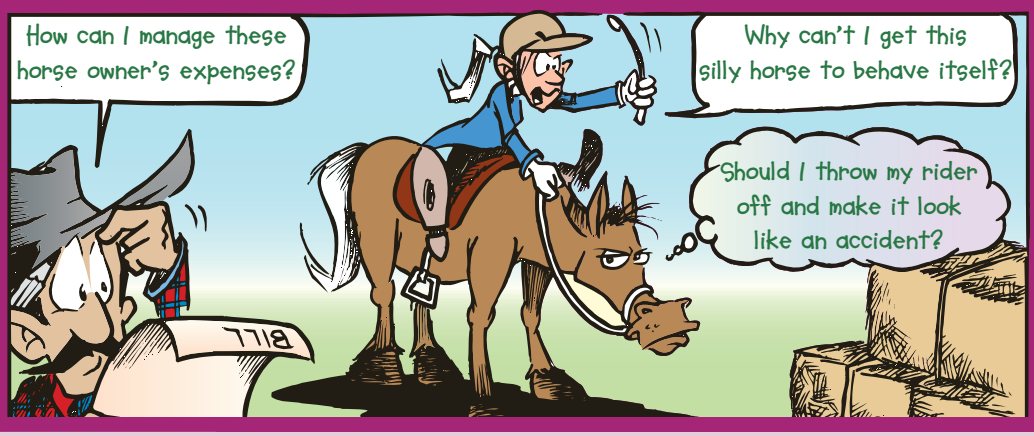
e In one day I eat the following:

100 g of tomato	20 g of peas	50 g of pineapple
50 g of cauliflower	50 g of carrot	100 g of potato

How many mg of vitamin C have I consumed?

2

Working Mathematically: Applying Strategies



Chapter Contents

2:01 Direct computation
Fun Spot: Can you play '31'?
2:02 Strategies for problem solving
Reading Mathematics: A standard pack of cards
2:03 Trial and error
2:04 Make a drawing, diagram or model
Investigation: Catching the crooks
2:05 Make a list, chart, table or tally
2:06 Eliminating possibilities
2:07 Working backwards
2:08 Acting it out
Challenge: Backtracking
2:09 Looking for patterns
2:10 Solving a simpler problem
Reading Mathematics: Let's post a parcel
Revision Assignment, Working Mathematically

Learning Outcomes

Students will:

- Ask questions that could be explored using mathematics in relation to MYP content.
- Analyse a mathematical or real-life situation, solving problems using technology where applicable.
- Use mathematical terminology and notation, algebraic symbols, diagrams, text and tables to communicate mathematical ideas.
- Identify relationships and the strengths and weaknesses of different strategies and solutions, giving reasons.
- Link mathematical ideas and makes connections with and generalisations about existing knowledge and understanding in relation to MYP content.

Areas of Interaction

Approaches to Learning, Homo Faber

2:01 | Direct Computation

fun spot 2:01

Fun Spot 2:01 | Can you play '31'?

This is a game for two players.

- Start with 31 counters.
- The first player chooses a number from 1 to 6 and removes this number of counters from the pile.
- The second player also chooses a number from 1 to 6 and removes that number of counters.
- The players continue in turn to remove up to 6 counters until all counters have been removed.
- The player forced to take the last counter is the loser.

A winning strategy

- Play the game with a partner.
- Can you work out a winning strategy? (The person who goes first should always win.)
- Look to the answers for an explanation of the winning strategy.



In your lifetime you will meet thousands of real problems waiting to be solved: 'What is the quickest way from Heathrow to Northfields?' 'How can I rearrange my working day so that I can attend a meeting at Brooklyn?' 'How can I build a skateboard?' 'How should I arrange the dinner party?'

We can see life as an exciting adventure or as a burden. We each have our own ways of solving problems. In this chapter we will examine some exciting and challenging strategies to help us solve unusual problems.

STEPS FOR SOLVING PROBLEMS

Step 1 **Read the question carefully.**
Step 2 **Decide what you are asked to find.**
Step 3 **Look for information that might be helpful.**
Step 4 **Decide on the method you will use.**
Step 5 **Set out your solution clearly, using solution sentences.**
Step 6 **Make sure that your answer makes sense.**

Always try to show clearly how you have solved the problem so that others can understand and investigate the problem you have attempted.

Setting out	
Find = method = = solution Answer (in words):	Working if needed:



worked example 1

Anna was told that she would be given €15 allowance each week if she was helpful around the home. She decided to bank €2.75 each week and do her best to help around the home. How much would she expect to bank in nine weeks?

Solution

Considering the problem:

(a) What are we asked to find?
What Anna would expect to bank in 9 weeks.

(b) What information is given? What is needed?
Given: Allowance is €15, banks €2.75 per week.
Needed: Banks €2.75 per week.

(c) What method should we use?
Multiply the amount banked weekly by the number of weeks.

(d) What solution sentences will we use?
Amount banked = 9 × €2.75.

Setting out (one-step problem)

Amount banked = 9 × €2.75 = €24.75	€ 2.75 × 9 €24.75 OR

In nine weeks Anna banks €24.75.

worked example 2

Giulio purchased a table for 132 pesos and a radio for 86 pesos. After spending 26.50 pesos on repairs, he sold the two items to a customer for 349.95 pesos. How much profit did he make altogether on the two items?

Solution

Considering the problem:

(a) What are we asked to find?
The profit made altogether on two items.

(b) What are we given?
Cost of items and repairs and selling price.

(c) What method should we use?
Find total cost first. Then subtract this from the selling price of the items.

(d) What solution sentences will we use?
(i) Total cost = cost 1 + cost 2 + repairs.
(ii) Profit on the items = selling price − total cost.

Setting out (two-step problem)

Total cost for the items = ₱132 + ₱86 + ₱26.50 = ₱244.50	₱132.00 ₱ 86.00 ₱ 26.50 ₱244.50 OR
Profit on the items = ₱349.95 − ₱244.50 = ₱105.45	₱349.95 − ₱244.50 ₱105.45
Giulio made ₱105.45 profit on the two items.	

Note: This problem could have been done as a one-step problem:
Profit on the items = selling price − (total costs)
Profit on the items = ₱349.95 − (₱132 + ₱86 + ₱26.50)
Profit on the items = ₱105.45
∴ Giulio made ₱105.45 profit on the two items.

Exercise 2:01

Foundation Worksheet 2:01

Problem solving

1 John is given $3.50 pocket money each week. How much would he be given in 10 weeks?

1 After comparing prices at three stores, Peter found that the cheapest price for mixed canary seed was $3.45 for 5 kg. He bought 15 kg of seed. How much did he pay for each kilogram of seed and how much did he pay altogether?

- **a** How much seed did Peter buy?
- **b** What was the cost of 5 kg of seed?
- **c** To how many stores did Peter go to compare prices?
- **d** What two things are we asked to find?
- **e** Solve the problem, setting out your solution as in the examples.

2 At Greentrees bookshop I bought 3 of the Narnia books at £3.30 each, a Bible at £14.95 and 10 bookmarks at 35 pence each. How much did I spend altogether?

- **a** What am I asked to find?
- **b** How many bookmarks were bought?
- **c** What was the total cost of the bookmarks?
- **d** How many items were bought altogether?
- **e** Solve the problem, setting out your answer as shown in the example.

Solve the following problems, showing your setting out as in the examples.

3 How many eight-page books can be made from 618 pages?

4 Jennifer bought a television set for $875.75 and a DVD player for $963.40. If she had saved $2000 to buy these two items, how much would she have left?

5 Mr and Mrs Lucre were returning home after spending a day at the beach. After travelling for three hours at 60 kilometres per hour, they were still 11 kilometres from home. How far from home were they when they began the trip home?

6 In May, Alan bought 55 different animal magazines for 25 cents each. In August he bought 47 others, all different from those purchased already. If the whole set contains 120 different magazines, how many magazines are missing from Alan's collection?



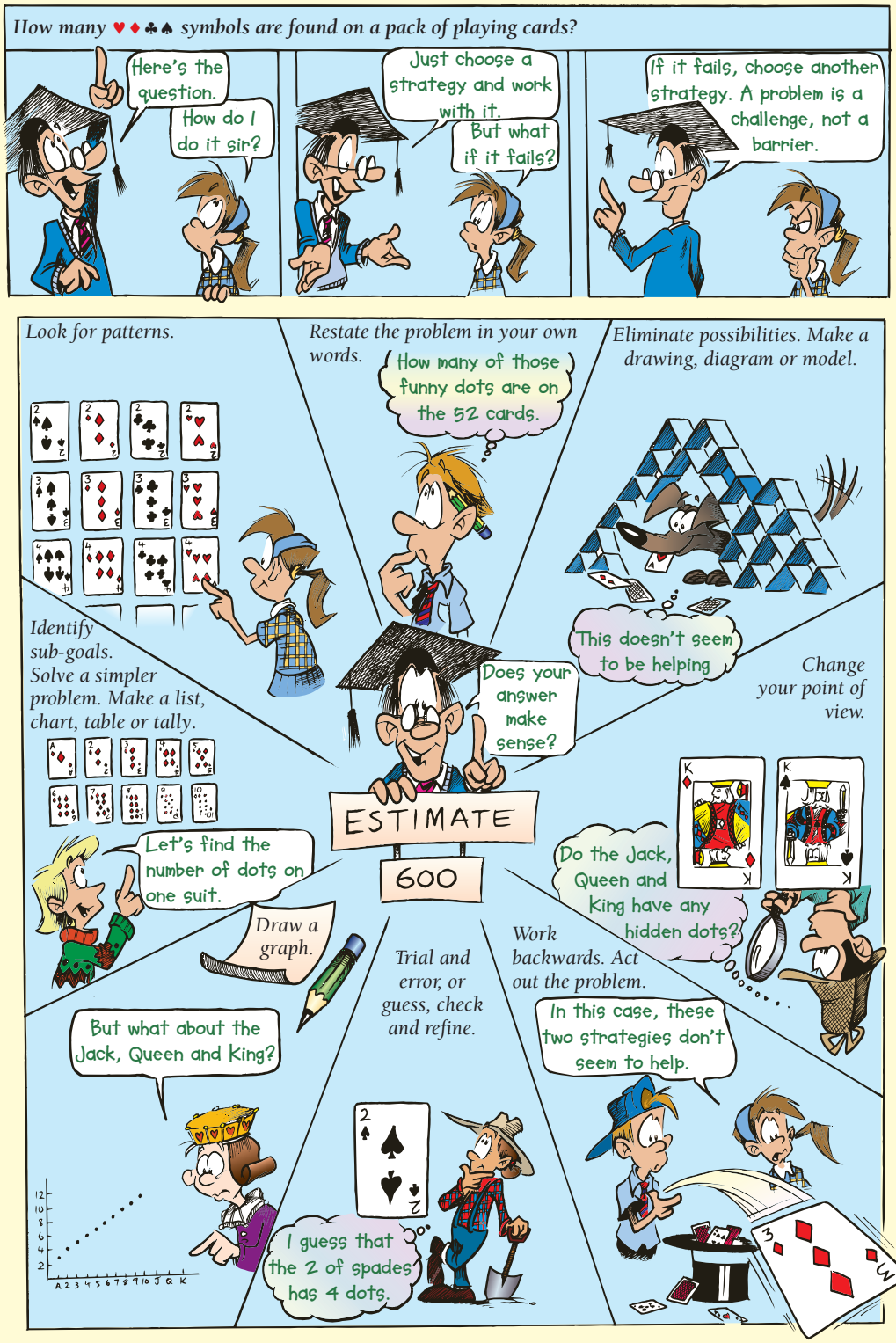
7 Luke has 40 toy cars and 100 marbles. If he shares these with his four sisters, what would his oldest sister be given?

8 Kylie cut an apple into 12 equal parts. If she ate $\frac{1}{4}$ of the apple, how many parts would remain?

9 The total number of votes cast in three electorates was 654 816. If in one of these electorates 211 302 votes were cast and in another 284 965 votes were cast, how many votes were cast in the third electorate?

10 Marie and Adam went to a Chinese restaurant to celebrate their engagement. The dishes they liked were: Sweet and Sour Chicken at $8.50, Steamed Duck with Sweet Corn at $10.95 and Braised Lobster with Seafood at $20.95. Dessert was $3.75 per serving and tea was $1.25 per person. What would be the total cost if Marie chose the lobster, Adam chose the duck and they each had dessert and tea?

2:02 | Strategies for Problem Solving



Working mathematically

Throughout this course you will apply the strategies treated in this chapter.

Along with **applying strategies** you will be developing your skills in **questioning**, **communicating**, **reasoning** and **reflecting**. These are all ways in which we work mathematically.

You now need to practise using these strategies so that you can develop skill in choosing which strategies to use when you are solving a problem. Very often you can use more than one strategy to get an answer, but before you begin you should **estimate the answer**.

To estimate, ask:

- **Will the answer be big or small?**
- **How big? How small?**
- **Will the answer be a whole number?**
- **Does my estimate make sense? Is it reasonable?**

reading mathematics 2:02

Reading Mathematics 2:02 | A standard pack of cards

A standard pack of cards has 4 'suits': *hearts*, *diamonds*, *clubs* and *spades*. Hearts and diamonds are red. Clubs and spades are black.

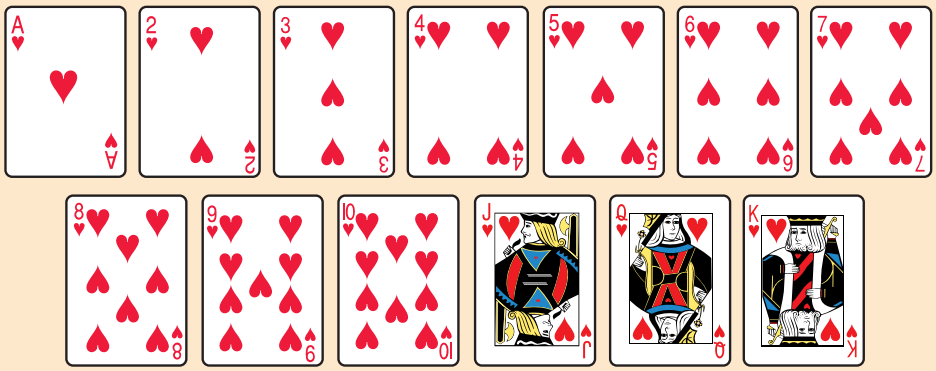

hearts


diamonds


clubs


spades

In each suit there are 13 cards: Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen and King. All the hearts are shown below.



Since there are 4 suits with 13 cards in each suit, the number of cards in a standard pack is 52. (In some games a Joker is also used.)

2:03 | Trial and Error

Another name for *trial and error* is *guess, check and refine*. Trial and error is a useful problem-solving strategy. Its only drawback is that for some problems it is too slow and time-consuming.

worked example 1

Osu is five years older than Erna. Twice Osu's age plus three times Erna's age is equal to 25. What are the ages of the two girls?

Solution

Osu is 5 years older than Erna.

Guess 1: Osu is 10 Erna is 5.

Check 1: $(2 \times 10) + (3 \times 5) = 20 + 15$
$= 35$

This is too large.

Guess 2: Osu is 8, Erna is 3

Check 2: $(2 \times 8) + (3 \times 3) = 16 + 9$
$= 25$

This is correct.

$\therefore$ Osu is 8 years old and Erna is 3 years old.

worked example 2

Sandra's cycle repair shop fixes normal bicycles and tricycles. Sandra had to assemble a number of bicycles and tricycles. She counted 15 handlebars and 36 wheels among the parts she had to put together. How many of each type of cycle were to be assembled?

Solution

Since there are 15 handlebars, there must be 15 cycles.

Guess 1: 7 bicycles and 8 tricycles.

Check 1: Number of wheels $= (7 \times 2) + (8 \times 3)$
$= 14 + 24$
$= 38$

This is too big, so try fewer tricycles.

Guess 2: 9 bicycles and 6 tricycles.

Check 2: Number of wheels $= (9 \times 2) + (6 \times 3)$
$= 18 + 18$
$= 36$

This is the correct number of wheels.

$\therefore$ There are 9 bicycles and 6 tricycles to be assembled.



Exercise 2:03

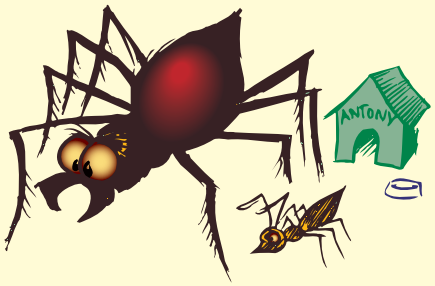
1. Mary is six years older than Jean. The sum of their ages is 20. How old is Mary?
2. Su-Lin is four years older than Mark. Twice Su-Lin's age plus five times Mark's age is equal to 64. What are the ages of Su-Lin and Mark?
3. Derek was 12 years old when his sister was born. Now the sum of their ages is 58. How old is Derek now?

Operation	*Answer is called . . .*
+	sum
−	difference
×	product
÷	quotient

4 The sum of two numbers is 140 and their difference is 6. Find the two numbers.

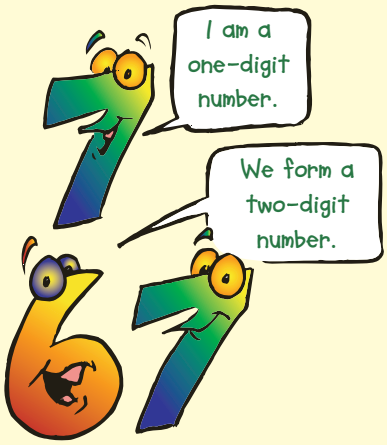
5 The product of two numbers is 72 and their sum is 27. What are the numbers?

6 The product of two numbers is 741 and their sum is 70. What are the numbers?



7 A farmer has chickens and cows. If there are 18 heads and 52 feet on these altogether, how many cows are there?

8 Diane keeps ants and spiders as pets. She counted the legs and heads of these pets and found that there were 136 legs and 21 heads. How many spiders were there?



9 From the digits 3, 4 and 5, two numbers are formed, a two-digit number and a one-digit number. What must the two numbers be if:

- **a** the product is to be as big as possible?
- **b** the product is to be as small as possible?
- **c** the product is to be 170?

10 Two two-digit numbers are made from four of the digits 1, 2, 3, 4 and 5, with no digit being used more than once. What numbers would produce:

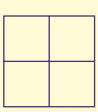
- **a** the smallest product?
- **b** the largest product?
- **c** a product closest to 700?

11 Roland sold bananas for 15 cents and apples for 20 cents. He received $3.50 for a mixture of bananas and apples. If twice as many bananas as apples were sold, how many apples were sold?

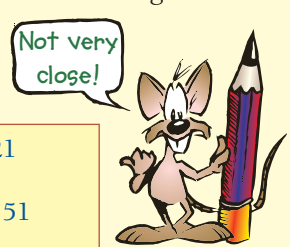
12 Kai-Lin answered all 25 questions on a test in which she received 4 marks for each correct answer but lost one mark for each incorrect answer. If her mark was 65, how many questions did she get right?

13 Find the counting number that would replace the square to make the sentence true.
$16 \times \square - 72 = 56$

14 Louie, Verity and their three children went to the movies. The children were charged half price. If the total cost of the tickets was $52.50, how much did each adult ticket cost?

15  Four non-zero digits are placed in these four squares to make two two-digit numbers across, and two two-digit numbers down. If the sum of these four two-digit numbers is 77, what are they?

16 Place the numbers below into two groups of four, so that the sum of one group is as close as possible to the sum of the other group.
{9, 10, 18, 21, 25, 42, 49, 51}



■ $9 + 10 + 18 + 21 = 58$
$25 + 42 + 49 + 51 = 167$

17 Leonie is 35 years older than her daughter Louise but 3 years younger than her husband Max. If the sum of the three ages is 94, how old is Leonie?

18 Kalmna, April and Lindsay scored 196 marks altogether on a test. If April scored 15 less than Kalmna and 7 less than Lindsay, how many marks did April score?

19 I think of a counting number, add one, multiply the answer by itself, then take away twice the number I first thought of. If my answer is 26, what was the number I first thought of?

20 Twice Alana's age plus three times Luke's age is equal to 25. What are the possible totals of their ages?

2:04 | Making a Drawing, Diagram or Model

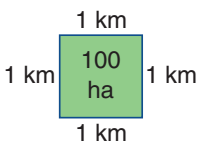
When you draw a picture to represent a problem, you can often see what must be done. It is said that 'a picture is worth a thousand words.'

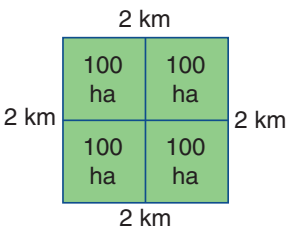
worked example 1

4 km of fencing encloses a square paddock of area 100 hectares. If 8 km of fencing was used to enclose a square paddock, how many hectares would be enclosed?

Solution

A diagram or drawing allows us to see the problem more clearly.





Even though the length of fencing has only been doubled, the area has become 4 times as great.

$\therefore$ 400 hectares would be enclosed by 8 km of fencing.

worked example 2

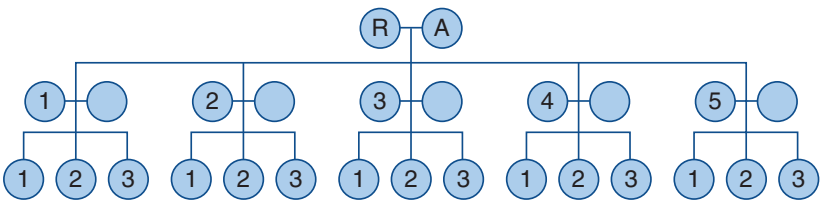
Rhonda and Alan married and had five children. Each child married and had three children. Assuming no-one has died, how many people are now in this extended family altogether?

Solution

Unless you draw a diagram you may miss some of the people.

For example, you might forget to include the people who married the five children.

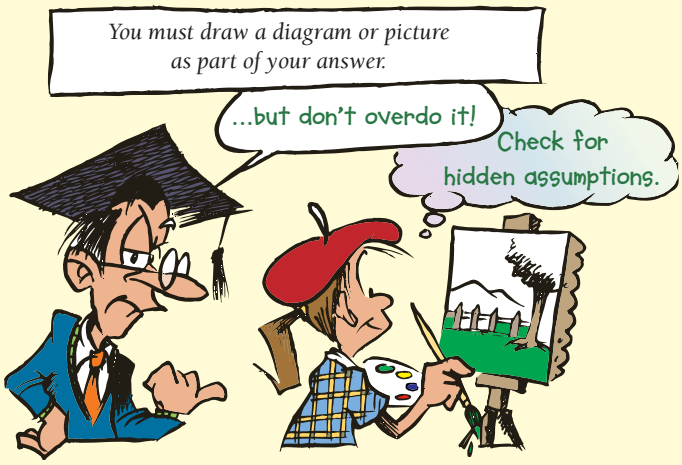
From the diagram you can see that there are 27 people in the family.





Exercise 2:04

1 4 km of fencing encloses a square paddock of area 100 hectares. If 16 km of fencing was used to enclose a square paddock, how many hectares would be enclosed?



2 Rachel had one pair of guinea pigs. These had four baby guinea pigs, all female. Within the following six months, each female guinea pig had produced five baby guinea pigs. If no guinea pigs died, how many did Rachel have at the end of that six-month period?

3 Each child in the Parker family had at least two brothers and one sister. What is the least number of children in the family?

4 **a** I want to make a line of posts 8 m long. If I place the posts 1 m apart, how many posts will I need?
b If I were to use 24 posts and place them 2 m apart, how far would they stretch?
c A line of posts 112 m long is made with posts equally spaced. If 8 posts are used, what is the distance between posts?

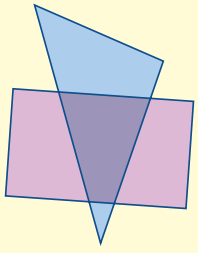
5 How many counting numbers are less than 3 units from the number 4?

6 **a** Eight equally spaced posts are used as uprights in a square pen. How many posts would be placed on each side of the pen?
b A square pen is made using four posts on each side. How many posts were used?

7 If it takes Dave 6 minutes to cut a pipe into two pieces, how long would it take him to cut it into five pieces?

8 It took Luke $1\frac{1}{2}$ seconds to join two pieces of Lego. How long would it take him to join seven pieces into one row of Lego?

9 A triangle and a rectangle are drawn on a page so that they overlap as shown. Inside these figures are placed the digits from 0 to 9. If there are six digits in the triangle and eight digits in the rectangle, how many digits are in the part that belongs to both figures?



10 **a** Nineteen students were waiting in line to be chosen to play in a game. Mr Archbold chose the first student in line and every third one after that. How many students were chosen?

b 99 Roman soldiers who fled from battle were to be punished. The group was lined up and decimated (every 10th one was killed). How many were killed?

c How many of the 99 soldiers would be killed if the second soldier was killed and every tenth soldier after that?

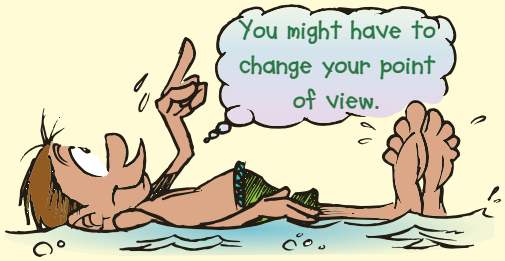
11 A doctor prescribes tablets for June to relieve her pain. They must be taken at least an hour apart and no more than four tablets may be taken in any six-hour period. What is the largest number of tablets she may take in $14\frac{1}{2}$ hours?

12 Two proofreaders are reading the same work. One finds 50 mistakes, the other 63 mistakes. If 40 of the mistakes were found by both readers, how many mistakes were found?

13 At the start of each week, May put $400 into her cheque account. During the week she would then withdraw $300. Assuming that there are no bank or government charges, in which week would she have $1000 in the bank for the first time?

14 The 30 students in 6E study both History and Geography. All students like at least one of these two subjects. If 18 students like History and 7 students like both subjects, how many students like Geography?

15 Jock decided to swim 10 km up his local river without leaving the river. He swam 2 km every hour then floated on his back for 10 minutes. Each time he floated, the current dragged him back 1 km. How long did it take Jock to complete the swim?



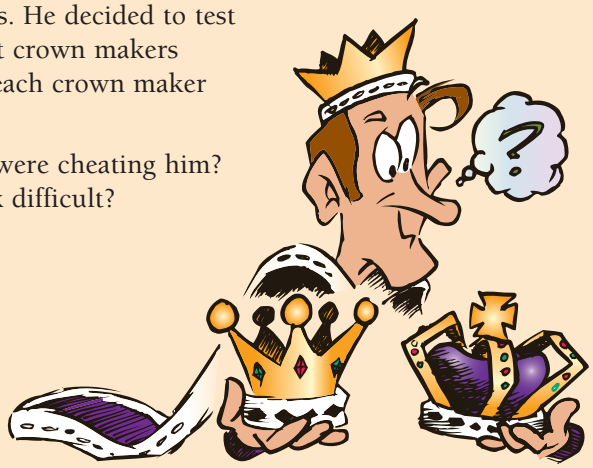
investigation 2:04

Investigation 2:04 | Catching the crooks

Please use the Assessment Grid on the following page to help you understand what is required for this Investigation.

Prince Charming was suspicious of his crown makers. He suspected that they were not using all the gold that he provided to make his crowns. He decided to test the crown makers. He ordered three different crown makers to make identical crowns for him. He made each crown maker think that only one crown was being made.

How could he discover if the crown makers were cheating him? What problems might occur to make his task difficult?



Assessment Grid for Investigation 2:04 | Catching the crooks

The following is a sample assessment grid for this investigation. You should carefully read the criteria *before* beginning the investigation so that you know what is required.

Assessment Criteria (B, C, D)				Achieved ✓
Criterion B Application & Reasoning	a	No systematic approach has been used.	1	
			2	
	b	An organised approach has been used with limited success.	3	
			4	
	c	A systematic approach has been used successfully, but is not well explained.	5	
			6	
	d	The correct result has been obtained by using an organised approach which is reasonably well explained.	7	
			8	
	e	The correct result has been obtained by using an organised approach which is well explained.	9	
			10	
Criterion C Communication	a	No working out or explanation has been given.	1	
			2	
	b	There is some structure to the work making it possible to follow the steps undertaken.	3	
			4	
	c	Results and the steps taken to get them are well explained. The work is well structured.	5	
			6	
Criterion D Reflection & Evaluation	a	There has been some attempt to explain the method used and to check if the result works.	1	
			2	
	b	The method and the majority of the processes used are justified and the reliability of the findings have been checked with some success.	3	
			4	
	c	A reasoned explanation of the method used and the reliability of the findings have been given.	5	
			6	
	d	Problems that could be encountered are discussed and possible solutions have been provided.	7	
			8	

2:05 | Make a List, Chart, Table or Tally

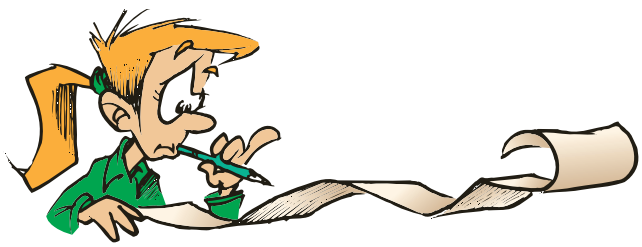
You will often need to make a complete list to solve a problem. It is very important that you select the best way to make your list.

worked example 1

Alana has to list the names of three universities in the order she prefers. The universities she likes are National University of Singapore (NUS), the University of Tokyo (UTK) and Hong Kong University (HKU). How many different ways are there to list these universities in order?

Solution

The way you make your list is very important. It should help you not to leave out any part of the list. We will use abbreviations NUS, UTK and HKU.



Note: (1) Once the first choice has been made there are two left to choose from in making choice 2.
(2) Once the first two choices have been made only one choice is left for choice 3.

Choice 1	Choice 2	Choice 3		Order		
NUS	UTK	HKU	=	NUS	UTK	HKU
	HKU	UTK	=	NUS	HKU	UTK
UTK	NUS	HKU	=	UTK	NUS	HKU
	HKU	NUS	=	UTK	HKU	NUS
HKU	NUS	UTK	=	HKU	NUS	UTK
	UTK	NUS	=	HKU	UTK	NUS

worked example 2

The numerals 1 to 10 are written on ten separate cards, one on each card.
(a) How many pairs of cards have a sum of 8?
(b) How many groups of three cards are there that have a sum of 20?



Solution

(a) The numbers must add to give 8.

1 7, 2 6, 3 5.

We can see that three pairs of cards have a sum of 8.

(b) As 20 is fairly big we will begin with the 10 and see which pairs will go with 10 to give a sum of 20.

10 1 9, 10 2 8, 10 3 7, 10 4 6

Now consider the 9 with other pairs.

9 3 8, 9 4 7, 9 5 6

Note: (1) 9 + 1 + 10 has already been listed.
(2) 9 + 2 + 9 is impossible as the 9 is used twice.

Now consider the 8 with other pairs.

8 5 7 *Note:* (1) 8 + 2 + 10 and 8 + 3 + 9 have already been listed.

Now consider the 7 with other pairs. All that add to give 20 have already been used.
No other groups of three cards will add to give 20.

∴ There are eight groups of three cards which have a sum of 20.

Exercise 2:05

1 **a** A 'Carols by Candlelight' program order must be decided. The items to be put in order are Carols (C), Entertainers (E) and Sermon (S). If the sermon must come either 2nd or 3rd of the items, in how many ways can the program be arranged?

b It is decided that the entertainers Julie Anthony and Kamahl should be considered as two separate items, while the sermon must be placed 3rd or 4th on the program. How many ways are there of arranging the items Carols (C), Anthony (A), Kamahl (K) and Sermon (S)?

2 From five girls, two must be chosen to represent our club at tennis. How many different pairs could be chosen? Call the girls A, B, C, D and E.
(*Note:* AB is the same pair as BA.)



3 The numerals 1 to 7 are written on seven separate cards, one on each card.

a How many pairs of cards add up to 8?

b How many groups of three cards are there that have a sum of 10?

4 How many numerals less than 100 contain the digit 5?

5 **a** If each digit is used only once, how many two-digit numbers can be made using the digits 1, 2, 3, 4? (*Note:* 21 is different from 12.)

b If each digit can be used more than once, how many two-digit numbers can be made using the digits 1, 2, 3, 4?

c If each digit can be used only once, how many two-digit numbers can be made using the digits 0, 1, 2, 3?



6 Five people were waiting for an elevator. When the elevator arrived, there was room for only three people. How many different groups of three people could get in the elevator? (Call the people A, B, C, D, E.)

7 David and Paul own a sailing boat that carries only two people. Four friends, Luke, Naomi, Rachel and Alana, would like to ride in the boat too. Since only David and Paul know how to sail the boat, at least one of these two must go out in the boat whenever it sails. How many pairs could be chosen to go out in the sailing boat?

8 **a** How many different ways can the letters A, E, B and T be arranged so that the T is the 3rd or 4th letter? (AETB and AEBT are two ways.)

b How many of the patterns in (a) are English words?

9 Maurice saw the registration plate of a car involved in a bank robbery. He told the police that the plate had three letters followed by three numbers. He knew that the first letter was E and the other two letters were Z and A but he was unsure of the order of the Z and A. The three numbers were 2, 3 and 8 but he did not know their order. How many registration plate would fit this description?

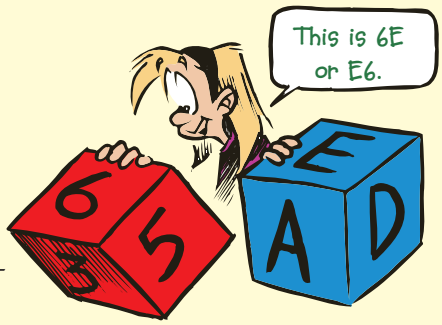


10 A school captain and vice-captain are to be elected from seven candidates. How many different results can occur?

11 Two dice are thrown. One has the digits 1 to 6 written on the faces while the other has the letters A, B, C, D, E and F on the six faces (one on each face of course).

a How many results could contain an A or E (a vowel)?

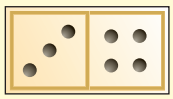
b How many different results are possible?



12 If every letter of the alphabet could be followed by a one-digit number, how many of these pairs could be formed?

13 I own 3 ties, 4 shirts and 2 belts. How many different ways could I wear a tie, a shirt and a belt. (*Hint:* Call the ties T_1, T_2, T_3, the shirts S_1, S_2, S_3, S_4, and the belts B_1 and B_2.)

14 A set of dominoes is a collection of rectangular pieces that have two parts. On each part, one of the numbers 0 to 6 is represented by dots. All possible combinations of numbers are made but no two dominoes are alike.



a How many dominoes are there in a set?

b What is the sum of the numbers shown on a set of dominoes?

15 A number of cards can be shared between 4 people exactly, but when shared between 5 people there are two cards over. When shared between 6 people there are four cards over. If there are fewer than 80 cards, how many cards are there?

16 How many different amounts can be made up using three of the six Australian coins: 5c, 10c, 20c, 50c, $1, $2?

2:06 | Eliminating Possibilities

Sometimes it's possible to find the right answer by eliminating answers that are wrong.

worked example

Three students studying in Vietnam who were born in different countries have last names Sze, Kalra and Poon. Their first names are Sundeep, Christopher and Andy but not necessarily in that order. Sze was born in Indonesia, Poon has never been to the Philippines, Andy was born in Malaysia and Sundeep was born in the Philippines. What is the full name of each student?

Solution

We use tables to eliminate as many possibilities as we can. We hope that what is left will be the answer.

- As Sze was born in Indonesia we can put ✗ ✗ ✓ for Sze.
- Poon has never been to the Philippines so for Poon cross the Philippines box. ✗
- Andy was born in Malaysia so put ✓ ✗ ✗ for Andy.
- Sundeep was born in the Philippines so put ✗ ✓ ✗ for Sundeep.

Now as each person was born in a different country, the fact that neither Sze nor Poon was born in the Philippines means that Kalra must have been. Tick the box Philippines/Kalra. ✓
In this way the other boxes can be filled in.

		Malaysia	*Philippines*	*Indonesia*
Last name	*Sze*	✗	✗	✓
	Kalra	✗	✓	✗
	Poon	✗	✗	✗
First name	*Sundeep*	✗	✓	✗
	Chris	✗	✗	✗
	Andy	✓	✗	✗

From the table we can see that:

Sze and *Chris* were born in Indonesia.
∴ Chris Sze was born in Indonesia.
Kalra and *Sundeep* were born in the Philippines.
∴ Sundeep Kalra was born in the Philippines

Exercise 2:06

1 Find the number between 1 and 10 that is not even, and when divided by 3 gives a remainder of 1.

2	3	4	5
6	7	8	9

2 Find the number less than 50 that has the sum of its digits equal to 6 and the difference of its digits equal to 4.

3 Find the number that is between 0 and 120, is even, is a multiple of 5 and is the result of multiplying one of the counting numbers by itself.

■ A multiple of 5 is a number that can be exactly divided by 5, eg 5, 10, 15, 20 . . .

4 Find the number between 1 and 50 that is not a multiple of 2 or 3 and whose two digits differ by 4.

5 If ties sell for $8.60, $9.20, $10.80 and $12.40, which of these could be the cost of 6 ties?

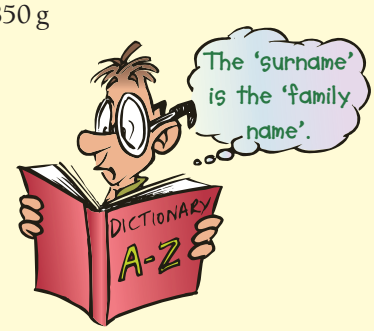
a $51.40 **b** $74.80 **c** $63.30 **d** $62.60

■ Work out the least and greatest possible cost of six ties.

6 Vinh and his wife Thanh owned a grocery store in Hanoi. They received ten sample tins, all of which had a net mass of 225 g, 400 g, 425 g or 450 g. Which of the following could be the total net mass of the ten sample tins?

a 4600 g **b** 3830 g **c** 3625 g **d** 1850 g

7 Three children, Jennifer, Kylie and Bronwyn, are friends. Their surnames are Conway, Collison and Wilkes, but not necessarily in that order. Jennifer lives in a red house, Bronwyn lives in a yellow house, Miss Conway lives in a blue house and Miss Wilkes does not live in a red house. What are the full names of each of the girls?



8 Which fruit did I grow?

My fruit:

- is not red
- is not narrow
- is not green

A



B



C



D



E



F



G



H



2:07 | Working Backwards

For some problems, working backwards gives a solution.

worked example

Mary is 35 years younger than Tom. Fred is half the age of Mary. Judy is 17 years older than Fred. If Judy is 35, how old is Tom?

Solution

When working backwards we often use opposite operations.

We know that Judy is 35.
We know that Judy is 17 years older than Fred.
So Fred's age is $35 - 17$, which is 18.
We know that Fred is half the age of Mary.
So Mary's age is 18×2, which is 36.
We know that Mary is 35 years younger than Tom. So Tom's age is $36 + 35$, which is 71.

$\therefore$ Tom is 71 years old.



Exercise 2:07

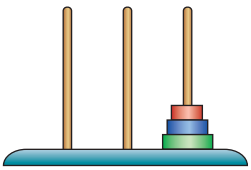
1. Neil is 14 years younger than Hayley. Lisa is 26 years older than Neil. If Lisa is 37, how old is Hayley?
2. Four people weighed themselves. Young Louise was 17 kg lighter than Adele, Gavin was twice as heavy as Louise, and Andrew was 9 kg heavier than Gavin. If Andrew's weight was 73 kg, what was Adele's weight?
3. My age is 60. I was married 23 years ago and graduated 5 years after that. How old was I when I graduated?
4. Hungry Harrie has \$48 now but during the last week he has bought ten hamburgers at \$1.85 each and eight milkshakes at 90 cents each. He also earned \$21.50 mowing lawns during the last week. How much money did he have one week ago?
5. I think of a number, multiply it by 4, then subtract 8 from that answer. I am left with 72. What was the number I first thought of?
6. When Rodney arrived in Belgium he had some money to help him get started. In the first three years this amount grew to 10 times its original size. With this money he started a plant nursery and with a lot of hard work built up his personal wealth by a further €98 000 in a few years. If his personal wealth was now €148 000, how much did he have when he arrived in Belgium?



2:08 | Acting It Out

Sometimes it's possible to act out the problem to find the solution.

worked example

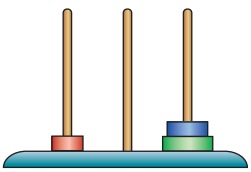


Three circular discs, which have holes through their centres, are to be moved from the right peg to the left peg. The discs are to be moved one at a time and no disc can be placed on top of a smaller one. What is the least number of moves needed to move the discs to the left peg?

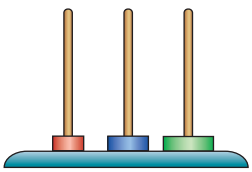
Solution

You could cut out 3 paper circles and act out this problem yourself.

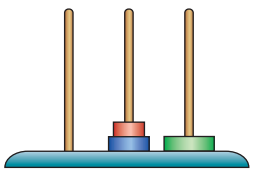
Step 1



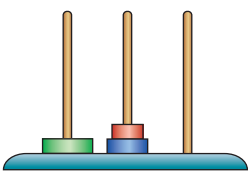
Step 2



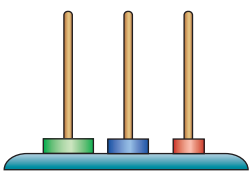
Step 3



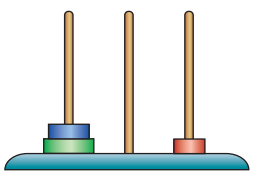
Step 4



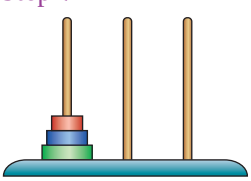
Step 5



Step 6



Step 7

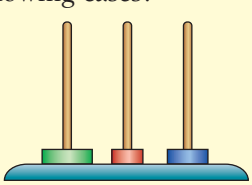


The least number of moves needed to move the discs to the left peg is 7.

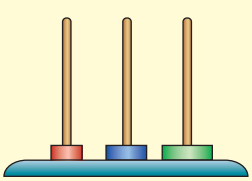
Exercise 2:08

1 Three circular discs, which have holes through their centres, are to be moved to the left peg. The discs are to be moved one at a time and no disc can be placed on top of a smaller one. What is the least number of moves needed to move the discs to the left peg in each of the following cases?

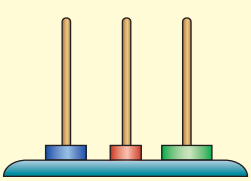
a



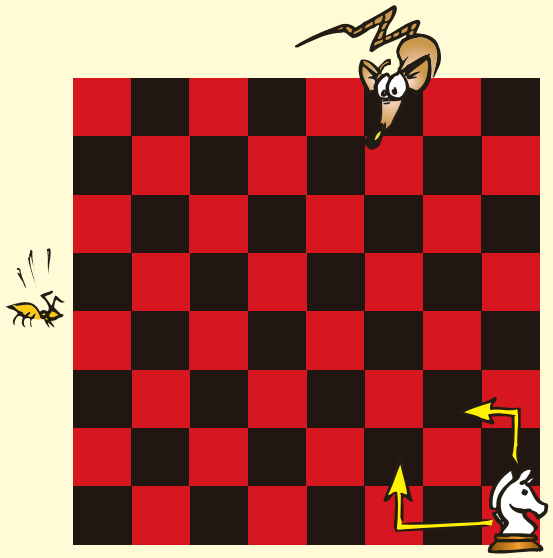
b



c



2 An ant wishes to cross from one side of a draughts board to the other. He feels that he must always keep a black square on his immediate left, and always walks along the lines on the board. If the squares on the board have sides of length 3 cm, what is the least distance he must travel?

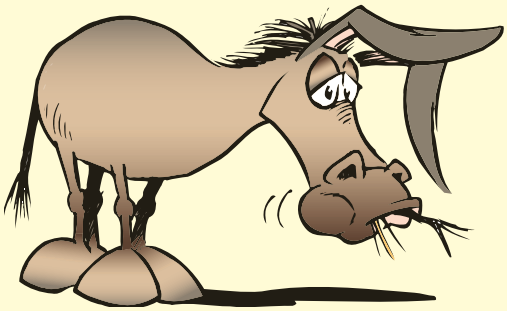


3 A mouse sits on a black square that is on one edge of the board. She decides only to step on black squares, and only to move to other black squares that are touching the black square on which she sits. Onto how many squares must she move to reach each of the other three edges and return to her starting point again?

4 The knight (a chess piece that looks like the head of a horse) moves on the board by going two squares in one direction and then one to the side (or one square in one direction then two to the side). This is counted as one move. What is the least number of moves necessary to get the knight from one corner to the opposite corner?

5 If the mouse steps as in question 3 and starts where she is on the board above, what is the greatest number of black squares that she can step onto in one trip if she must not tread on the same square twice?

6 Walking to church, dressed in their new clothes, Mr and Mrs Ritchie and their two children Andrew and Vanessa came to a very muddy part of the road. Fortunately Mr Ritchie's donkey was grazing nearby. It could, however, only carry one adult or two children at the one time. Explain how the whole family got over the mud using the donkey.



challenge 2:08

Challenge 2:08 | Backtracking

$\square \xrightarrow{+10} 84$ means '$\square$ is changed into 84 by adding ten'.

Find the numbers represented by the frames in each of the examples below.

1 $\square \xrightarrow{+10} 84$

2 $\square \xrightarrow{-8} 37$

3 $\square \xrightarrow{\times 2} 106$

4 $\square \xrightarrow{\div 3} 17$

5 $\square \xrightarrow{\times 2} \bigcirc \xrightarrow{+10} 36$

6 $\square \xrightarrow{\div 3} \bigcirc \xrightarrow{-8} 1$

7 $\square \xrightarrow{\times 7} \bigcirc \xrightarrow{-7} \square \xrightarrow{+4} 67$

8 $\square \xrightarrow{\div 5} \bigcirc \xrightarrow{+6} \square \xrightarrow{-9} 11$

2:09 | Looking for Patterns

If you can find a pattern, it may help you solve the problem.

worked examples

Look for patterns that will help you find the answer to each of these.

a $7-6+7-6+7-6+7-6+7-6+7-6+7-6+7-6+7-6+7-6+7$

b $9+5+2+9+5+2+9+5+2+9+5+2+9+5+2+9+5+2$

c $(1+2+3+4+5+6+7+8+9)+(9+8+7+6+5+4+3+2+1)$

Solutions

a By grouping the numbers in pairs, the answer is quickly found.

$(7-6)+(7-6)+(7-6)+(7-6)+(7-6)+(7-6)+(7-6)+(7-6)+(7-6)+(7-6)+7$

$=$ (ten lots of 1) $+ 7$

$= 17$

b Here we can arrange the numbers in groups of three.

$(9+5+2)+(9+5+2)+(9+5+2)+(9+5+2)+(9+5+2)+(9+5+2)$

$=$ 6 lots of 16

$= 96$

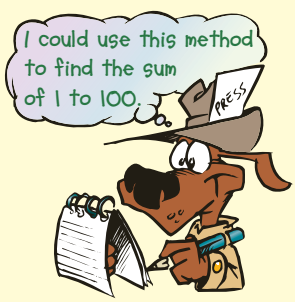
c If we add the first number in each group, we get 10. The second numbers also have a sum of 10, and this pattern continues.

$$\begin{array}{rl} & (1+\ 2+\ 3+\ 4+\ 5+\ 6+\ 7+\ 8+\ 9) \\ + & (9+\ 8+\ 7+\ 6+\ 5+\ 4+\ 3+\ 2+\ 1) \\ \hline \text{Adding} & 10+10+10+10+10+10+10+10+10 \end{array}$$

Even without showing working we could have seen that there were 9 lots of 10.

$\therefore$ The answer is 90.

We can use this idea to find other sums.
To add the counting numbers from 1 to 200, we could use them twice and then divide that answer by 2,



eg

$$\begin{array}{l} (\ 1+\ 2+\ 3+\ 4+\ 5+\ 6+\ldots\ldots+199+200) \\ +(200+199+198+197+196+195+\ldots\ldots+\ 2+\ 1) \\ \hline 201+201+201+201+201+201+\ldots\ldots+201+201 \end{array}$$

Altogether, twice the sum gives us 200 lots of 201.

$\therefore$ The sum of the counting numbers from 1 to 200 $= \frac{1}{2}$ of (200×201)

$= 100 \times 201$

$= 20\,100$

Many problems involving patterns will be met in Chapter 8, 'Patterns and Algebra'.

Exercise 2:09

1 Look for patterns which will help you find the answer to each of these.

a $9-7+9-7+9-7+9-7+9-7+9-7+9-7+9-7+9-7+9-7+9-7+9$

b $1+2+3+4+1+2+3+4+1+2+3+4+1+2+3+4+1+2+3+4$

c $\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}+\frac{1}{4}-\frac{1}{5}+\frac{1}{5}-\frac{1}{6}+\frac{1}{6}-\frac{1}{7}+\frac{1}{7}-\frac{1}{8}+\frac{1}{8}-\frac{1}{9}+\frac{1}{9}-\frac{1}{10}+\frac{1}{10}$

d $29-28+27-26+25-24+23-22+21-20+19-18+17-16+15-14$

e $197-187+177-167+157-147+137-127+117-107+97-87+77-67$

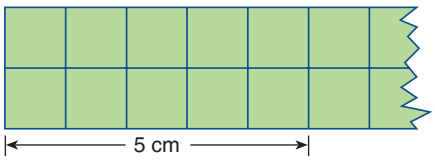
2 Use the method given before this exercise to find the sum of the counting numbers from:

a 1 to 400 **b** 1 to 1000 **c** 1 to 100 **d** 1 to 50

2:10 | Solving a Simpler Problem

If a problem seems complicated, try solving a simpler problem or see if you can solve the problem using easier numbers.

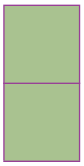
worked example



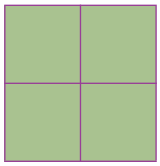
A machine is producing a roll of grid paper 2 cm wide. Within the grid there are squares with sides 1 cm in length and squares with sides 2 cm in length. How many squares can be found if the grid has a length of 100 cm?

Solution

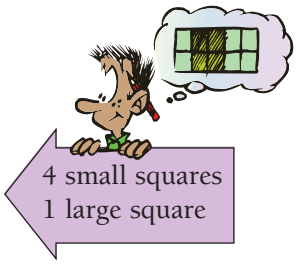
To solve this problem would take a long time so we solve simpler problems and look for a pattern. Try smaller lengths.

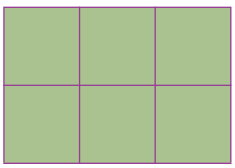


Length	1 cm
Number of squares	2

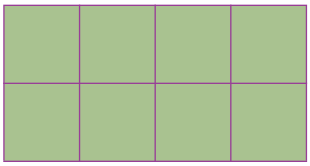


Length	2 cm
Number of squares	4 + 1



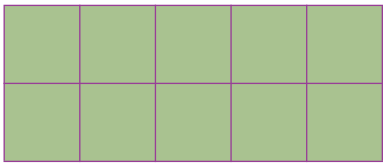


Length	3 cm
Number of squares	6 + 2



Length	4 cm
Number of squares	8 + 3

continued →→→



Length	5 cm
Number of squares	10 + 4

In each case:

Number of 1 cm squares = 2 × length

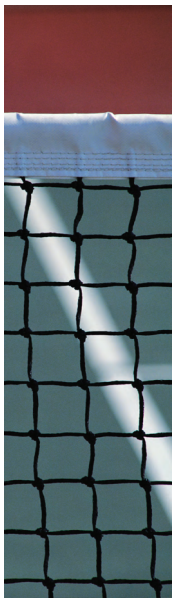
Number of 2 cm squares = 1 × length − 1

∴ Total number of squares = 3 × length − 1

This can be seen in the table below.

Length (cm)	1	2	3	4	5	100
Number of squares	2	5	8	11	14	299

∴ 299 squares can be found in 100 cm of grid paper.



Exercise 2:10

1 The hare challenges the tortoise to a race. The hare can travel 10 m for every 1 m travelled by the tortoise. If they continue to travel at this rate, by how much will the hare beat the tortoise if the race is over a distance of:

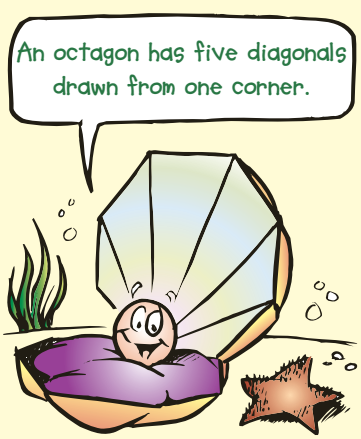
a 10 m? **b** 20 m? **c** 100 m? **d** 1000 m? **e** 2 km?

2 How much start could the hare in question 1 give to the tortoise in a race over 50 m and still expect to beat the tortoise?

3 Blackbeard the pirate was following an unusual treasure map: 'Jump from Dead Man's Rock onto the wet sand and then take 54 steps along the beach.'

a If he obeyed the instructions exactly, how many shoeprints would he have made in the sand?

b If he only had one good leg and a 'pegleg' on the other, how many shoeprints would he have made?



4 How many diagonals can be drawn from one corner of:

a a square? **b** a pentagon (5 sides)?

c a hexagon (6 sides)? **d** a heptagon (7 sides)?

e a figure on a flat surface that has 100 sides?

5 Ten prospectors are working in a gold mine. Every prospector fills a small bag of gold each month. After the first month one prospector disappears, taking with him all the gold dug up in that month. The same thing happens in each of the following months until only one prospector is left. How many bags of gold were stolen?

6 If there had been five honest prospectors among those in question 5, how many bags of gold would have been stolen?

7 Looking again at the example before this set, write down how many lines of length 1 cm are needed to draw a grid of length:

a 1 cm **b** 2 cm **c** 3 cm **d** 4 cm **e** 5 cm

8 In question 7, how many extra 1 cm lengths were needed for each centimetre increase in length? Use this information to find the number of centimetre parts needed to draw a grid of length 100 cm.



9 At a church meeting, each person who arrived shook the hand of every other person present. Upon arrival, how many hands would be shaken by:

a the first person? **b** the second person?
c the third person? **d** the fourth person?
e the fifth person **f** the sixth person?
g the seventh person? **h** the eighth person?

10 If every person at the meeting in question 9 shakes hands with every other person there, how many handshakes will there be altogether if:

a 2 people are present? **b** 3 people are present?
c 4 people are present? **d** 5 people are present?
e 6 people are present? **f** 7 people are present?

11 Use the answers to question 10 to complete this table.

Number of people	2	3	4	5	6	7
Number of handshakes						

Can you see a pattern in the table? What is it?

Extend this pattern to find the number of handshakes needed if 20 people were present.

Note: Other methods, like drawing a picture, making a list or acting it out, can be used to help solve this problem.

Reading Mathematics 2:10 | Let's post a parcel

The amount to be paid for posting a parcel will depend on the destination (where it is going) and the weight of the parcel.

This table assumes that the parcel is being sent from a zip code that is in Zone 1.

Table 1 — Parcel charging zones (Destination)

Zip code	Zone	Zip code	Zone
		2717–2719	Zone 3
0200–0299	Zone 2	2720–2730	Zone 2
1000–2263	Zone 1	2731–2739	Zone 3
2264–2499	Zone 2	2740–2786	Zone 1
2500–2530	Zone 1	2787–2879	Zone 2
2531–2554	Zone 2		
2555–2574	Zone 1	2881–2889	Zone 2
2575–2647	Zone 2	2890	Zone 1
2648	Zone 3	2891–2898	Zone 2
2649–2714	Zone 2		
2715	Zone 3	2900–2999	Zone 2
2716	Zone 2		

Table 2 — Regular parcel charges

Weight	Zone 1	Zone 2	Zone 3
	$	$	$
Up to 250 g	2.75	2.75	2.75
Over 250 to 500 g	3.75	3.75	3.75
Over 500 g to 1 kg	5.20	5.50	7.40
Over 1 kg to 2 kg	5.20	5.80	7.95
Over 2 kg to 3 kg	5.20	6.10	8.50
Over 3 kg to 4 kg	5.20	6.40	9.05
Over 4 kg to 5 kg	5.20	6.70	9.60
Over 5 kg to 6 kg	5.20	7.00	10.15
Over 6 kg to 7 kg	5.20	7.30	10.70
Over 7 kg to 8 kg	5.20	7.60	11.25
Over 8 kg to 9 kg	5.20	7.90	11.80
Over 9 kg to 10 kg	5.20	8.20	12.35
Over 10 kg to 11 kg	5.20	8.50	12.90
Over 11 kg to 12 kg	5.20	8.80	13.45
Over 12 kg to 13 kg	5.20	9.10	14.00
Over 13 kg to 14 kg	5.20	9.40	14.55
Over 14 kg to 15 kg	5.20	9.70	15.10
Over 15 kg to 16 kg	5.20	10.00	15.65

worked example

How much would it cost to send a 10·5 kg parcel to zip code 2650?

Solution

First we use *Table 1* to find that zip code 2650 is in Zone 2.

Then we use *Table 2* to find the charge for 10·5 kg (over 10 kg to 11 kg) in the Zone 2 column.

∴ The cost of sending the parcel is $8.50.



1 How much would it cost to send a parcel to zip code:

a 2041
weight: 15 kg

b 2795
weight: 8·4 kg

c 2640
weight: 260 g

d 2731
weight: 600 g

e 2983
weight: 13·8 kg

f 2530
weight: 200 g?

Chapter 2 | Revision Assignment

assignment 2A

1 What is the value of the 1 in each of the following numerals?
   a 314 b 1608
   c 21 753 d 153 246

2 Simplify:
   a 317 + 72 + 4825 + 196
   b 2176 – 413 c 15 × 12
   d 265 ÷ 5

3 Write these decimals as fractions.
   a 0·7 b 0·01
   c 0·201 d 0·033

4 Write these fractions as decimals.
   a $\frac{3}{10}$ b $\frac{17}{100}$ c $\frac{45}{1000}$ d $1\frac{3}{100}$

5 Simplify the following:
   a 0·5 + 0·21 b 4·2 + 3·17
   c 12 + 1·5 d 7·4 + 10·05 + 3·15

6 Simplify the following:
   a 12 – 3·7 b 4·8 – 1·6
   c 2·1 – 0·85 d 15·17 – 7

7 Simplify the following:
   a 6 × 0·1 b 6 × 0·01
   c 0·6 × 10 d 0·6 × 0·1

8 Simplify the following:
   a 12·6 ÷ 2 b 1·04 ÷ 2
   c 5·5 ÷ 5 d 17·24 ÷ 4

9 Simplify:
   a $1.55 + $3.75 b $15 – $7.90
   c $42.20 × 7 d $24 ÷ 5

10 Simplify:
   a $\frac{3}{5} + \frac{3}{5}$ b $\frac{37}{100} + \frac{42}{100}$
   c $\frac{15}{20} - \frac{7}{20}$ d $\frac{19}{100} - \frac{19}{100}$

1 Percentages
2 Money

Drag and Drop GW Interactive

Chapter 2 | Working Mathematically

assignment 2B

id 2

1 Use ID Card 4 on page xvi to give the mathematical term for:
   a 1 b 2
   c 3 d 4
   e 5 f 6
   g 7 h 8
   i 9 j 13

2 Use the Arithmetic Card on page xx to:
   a multiply column A by column B
   b add column K to column L

3 A girl cuts six lengths of cloth from a roll 20 m long. If each length is 2·4 m, how much cloth remains?

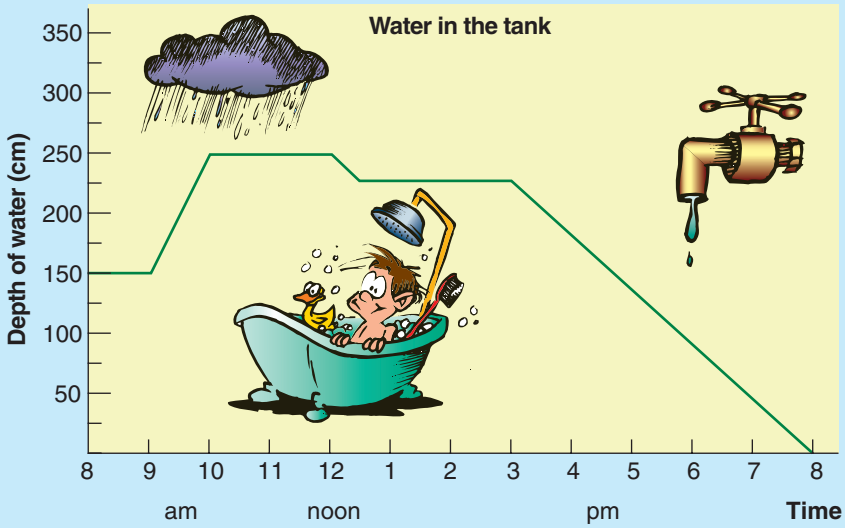
4 A machine caps bottles at a speed of one bottle every 2 seconds. How many bottles does the machine cap in 4 hours?

5 How could you estimate how many households have a phone in your capital city?

6 You are running a basketball competition in which six teams are playing. Work out the draw for the competition if each team has to play each other team once only.

7 A suburban railway line has 10 stations. How many different tickets would be needed by the ticket office to cover any trip from one station to another?

8 Dave depended on the water stored in his tank to meet his everyday needs. This graph shows the depth of water in the tank from 8 am to 8 pm.



During the day it rained, Dave had a bath and Dave's son Brian left the tap running.

a Between what two times did it rain?

b What was the highest level of water in the tank during this 12-hour period?

c When did Brian leave the tap on?

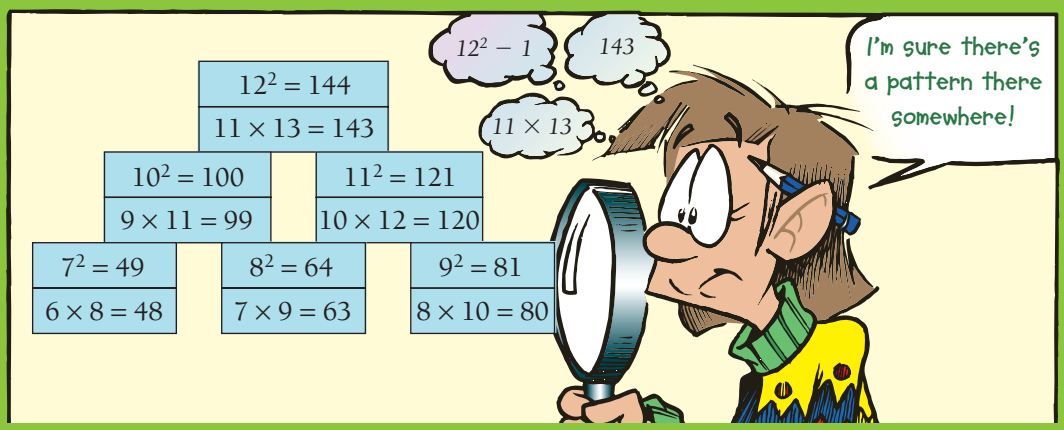
d When was the tank empty?

e How much water was in the tank at 2:30 pm?



3

Number: Its Order and Structure



Chapter Contents

3:01 Order of operations
A Grouping symbols
B Rules for the order of operations
3:02 Using number properties
A Properties involving addition or multiplication
B The distributive property
3:03 Language and symbols used in mathematics
Investigation: Odds and evens
3:04 Special sets of whole numbers
Fun Spot: Fibonacci numbers
Investigation: Pascal's triangle
3:05 Factors and multiples
3:06 Prime and composite numbers
3:07 Divisibility tests
3:08 Square and cube roots
3:09 The binary system (extension)
Fun Spot: Making magic squares
Mathematical Terms, Diagnostic Test, Revision Assignment, Working Mathematically

Learning Outcomes

Students will recognise the properties of special groups of whole numbers and apply a range of strategies to aid computation.

Areas of Interaction

Approaches to Learning, Homo Faber, Environment

3:01 | Order of Operations

3:01A Grouping symbols

Grouping symbols are often used to tell us which operations to perform first. Three commonly used grouping symbols are: parentheses, brackets and braces.

parentheses	brackets	braces
( )	[ ]	{ }

Where grouping symbols occur inside other grouping symbols, deal with the innermost grouping symbols first.

worked examples

1 $7 \times (11 - 6)$
$= 7 \times 5$
$= 35$

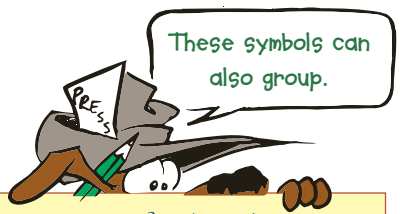
2 $17 - (6 + 10)$
$= 17 - 16$
$= 1$

3 $(6 + 7) \times (10 - 3)$
$= 13 \times 7$
$= 91$

4 $50 - (25 - [3 + 19])$
$= 50 - (25 - 22)$
$= 50 - 3$
$= 47$

5 $6 + 5^2$
$= 6 + (5 \times 5)$
$= 6 + 25$
$= 31$

6 $\dfrac{10 + 30}{10}$
$= (10 + 30) \div 10$
$= 40 \div 10$
$= 4$



- **Powers** eg $5^2 = (5 \times 5)$
- **Fraction bar** eg $\dfrac{5 + 25}{11 - 6} = (5 + 25) \div (11 - 6)$
- **Square root sign** eg $\sqrt{9 + 16} = \sqrt{(9 + 16)}$

Exercise 3:01A

Foundation Worksheet 3:01A

Grouping symbols

1 a $(3 + 2) \times 10$
2 a $(8 - 2) \times 3$
3 a $10 - (4 + 3)$

1 Find the basic numeral for each expression.

a $(4 + 8) \times 10$	**b** $(19 - 9) \div 5$	**c** $25 - (50 - 30)$	
d $8 \times (4 + 5)$	**e** $(4 + 2) \times 7$	**f** $(20 - 14) \times 8$	
g $14 + (36 \div 6)$	**h** $16 - (12 - 3)$	**i** $70 \div (35 \div 5)$	
j $(93 - 12) \div 3$	**k** $40 \div (13 - 8)$	**l** $40 \div (5 \times 4)$	
m $15 + (100 \div 20)$	**n** $(16 - 9) + 23$	**o** $15 + (100 \div 5)$	**p** $100 - (7 \times 5)$
q $20 - (28 - 19)$	**r** $45 - (64 \div 8)$	**s** $(13 - 13) \div 6$	**t** $10 \div (28 - 23)$
u $73 \times (35 - 34)$	**v** $115 - (54 + 29)$	**w** $(0 \div 5) \times 15$	**x** $93 \div (14 + 79)$

2 Simplify:

a $(3 + 8) \times (15 - 9)$ **b** $(9 - 4) \times (7 + 5)$
c $(9 + 16) - (4 + 9)$ **d** $(25 + 7) - (9 + 6)$
e $(10 + 10) \div (7 + 3)$ **f** $(23 + 17) \div (5 + 3)$
g $(3 \times 6) + (4 \times 6)$ **h** $(7 \times 3) + (3 \times 3)$
i $(17 - 3) - (35 - 33)$ **j** $(9 - 3) \times (15 - 5)$
k $(7 - 3) - (9 - 7)$ **l** $(28 - 5) - (62 - 59)$
m $(105 - 5) \div (5 \times 10)$ **n** $(9 \times 7) \div (27 \div 3)$
o $(19 + 13) + (93 + 7)$ **p** $(43 + 17) + (60 + 40)$
q $(15 \times 3) \times (25 \times 4)$ **r** $(7 \times 8) \times (5 \times 2)$
s $(88 - 15) - (7 \times 10)$ **t** $(56 - 37) - (56 - 47)$

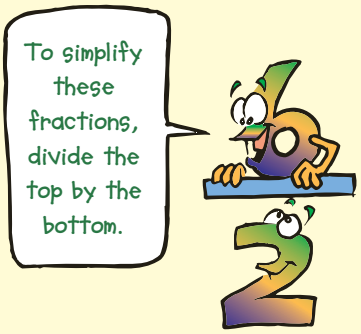


3 Give the simplest answer for:

a $4 + [6 - (7 - 3)]$ **b** $9 + [5 - (9 - 5)]$ **c** $8 + [60 - (4 + 16)]$
d $20 - \{15 + (40 \div 8)\}$ **e** $64 - \{25 - (14 \div 14)\}$ **f** $\{8 \times [5 \times 2]\} - 60$
g $[\{28 + 14\} \div 7] \times 11$ **h** $[(10 + 8) \div 9] \times 10$ **i** $8 \times (35 \div [4 + 3])$
j $[(8 + 4) \times 3] \div 9$ **k** $16 \div [54 - (28 + 18)]$ **l** $(200 \div [31 + 9]) \div 5$
m $[(60 - 14) - (53 - 19)] \times 10$ **n** $[(96 - 10) - (54 - 27)] \times 6$
o $83 - [(8 \times 6) + (4 \times 6)]$ **p** $5015 \div [(60 + 15) \div (3 \times 5)]$

4 Find the basic numeral in each case.

a $3 + 7^2$ **b** $14 - 3^2$
c 5×4^2 **d** $\dfrac{14 + 6}{2}$
e $\dfrac{31 - 10}{10 - 3}$ **f** $\dfrac{100 - 40}{5 \times 2}$
g $\dfrac{8 + 20}{14 \div 2}$ **h** $\dfrac{8 \times 4}{2 + 2}$
i $\dfrac{45 + 939}{8}$



5 Find the basic numeral for each expression.

a $0{\cdot}5 + (3 \times 0{\cdot}1)$ **b** $0{\cdot}44 - (0{\cdot}63 - 0{\cdot}22)$ **c** $3{\cdot}9 - (0{\cdot}2 + 0{\cdot}7)$
d $(0{\cdot}2 + 0{\cdot}3) \times 9$ **e** $(0{\cdot}7 - 0{\cdot}3) \times 4$ **f** $(0{\cdot}5 + 0{\cdot}5) \times 12$
g $9 \div (0{\cdot}5 + 0{\cdot}5)$ **h** $0{\cdot}8 \div (16 \div 8)$ **i** $4{\cdot}8 \div (16 - 12)$
j $(6 \times 0{\cdot}1) + (4 \times 0{\cdot}1)$ **k** $(2{\cdot}6 - 0{\cdot}4) + (1{\cdot}3 - 0{\cdot}8)$ **l** $(1{\cdot}4 + 1{\cdot}6) \times (0{\cdot}8 + 1{\cdot}2)$
m $\$6.50 - (\$10 - \$7)$ **n** $\$10 - (3 \times \$2.50)$ **o** $\$20 \times (153 - 144)$
p $(6 \times \$1.20) + (8 \times \$1.20)$ **q** $4 \times (\$8.32 + \$9.63)$ **r** $7 \times (\$100 - \$9.50)$

6 Insert grouping symbols to make each of the following number sentences true.

a $5 \times 3 + 8 = 23$ **b** $5 \times 3 + 8 = 55$ **c** $10 \times 7 + 5 = 120$
d $6 - 3 \times 2 = 0$ **e** $6 - 3 \times 2 = 6$ **f** $10 \times 7 + 5 = 75$
g $40 \div 4 + 1 = 8$ **h** $40 \div 4 + 1 = 11$ **i** $80 \div 8 + 2 = 8$
j $29 - 15 - 6 = 8$ **k** $29 - 15 - 6 = 20$ **l** $18 - 8 + 2 = 8$
m $40 \div 2 \div 2 = 10$ **n** $40 \div 2 \div 2 = 40$ **o** $144 \div 6 \div 3 = 72$
p $6 + 2 \times 8 + 2 = 80$ **q** $6 + 2 \times 8 + 2 = 24$ **r** $10 \times 3 - 16 - 2 = 16$

3:01B Rules for the order of operations

Mathematicians have agreed on an order of operations so that confusion will not occur.

ORDER OF OPERATIONS

Step 1 **Do operations within grouping symbols.**
Step 2 **Do multiplications and divisions as they appear (from left to right).**
Step 3 **Do additions and subtractions as they appear (from left to right).**

■ *Order of Operations*
1 ( )
2 × and ÷
3 + and –

First, we do any operations that are inside grouping symbols.
Second, we go from left to right doing any multiplications and divisions.
Finally, we go from left to right doing any additions and subtractions.



worked examples

1 $20 - 4 + 8 - 2$
(only + and –, so left to right)
$= 16 + 8 - 2$
$= 24 - 2$
$= 22$

2 $40 \div 5 \times 3 \div 2$
(only × and ÷, so left to right)
$= 8 \times 3 \div 2$
$= 24 \div 2$
$= 12$

3 $20 - 6 \times 3 + 15$
(× before + or –)
$= 20 - 18 + 15$
(only + and –, so left to right)
$= 2 + 15$
$= 17$

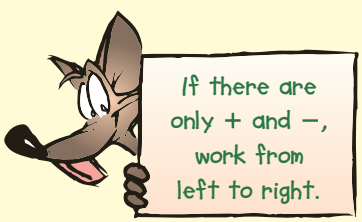
4 $9 + 15 \div (3 + 2)$
(grouping symbols first)
$= 9 + 15 \div 5$
(÷ before +)
$= 9 + 3$
$= 12$

■ The simplest answer is called the *basic numeral*.

Exercise 3:01B

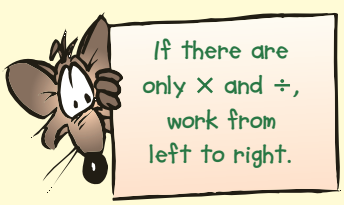
1 Simplify:

a $16 + 4 + 7 + 3$
b $8 + 9 + 3 + 4$
c $20 - 3 - 2 - 1$
d $15 - 4 - 5 - 3$
e $8 + 6 - 3 + 4$
f $9 + 4 - 6 - 3$
g $9 - 8 + 4 - 3$
h $15 - 10 + 6 - 7$
i $16 - 4 - 3 - 5 - 1$
j $18 - 8 - 3 - 5 - 2$
k $14 - 3 - 8 + 5 - 1$
l $12 - 2 - 5 + 4 - 6$



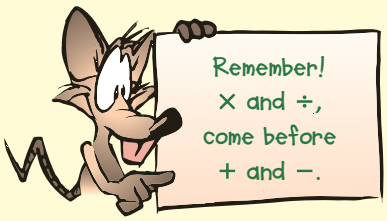
2 Write down the basic numeral for each.

a $10 \times 2 \times 2 \times 2$
b $2 \times 3 \times 3 \times 3$
c $120 \div 6 \div 2 \div 2$
d $40 \div 2 \div 2 \div 2$
e $10 \times 5 \div 2 \times 4$
f $6 \times 4 \div 8 \times 11$
g $20 \div 2 \div 5 \times 8$
h $16 \div 8 \div 2 \times 7$
i $8 \times 7 \div 2 \times 4$
j $10 \div 2 \times 8 \times 2$
k $2 \times 12 \div 3 \times 8$
l $42 \div 7 \times 10 \div 5$



3 Simplify the following.

a $46 - 3 \times 8$
b $40 - 10 \times 3$
c $20 \div 5 - 4$
d $9 - 35 \div 5$
e $6 \times 2 + 6 \times 3$
f $10 \times 4 + 10 \times 5$
g $10 \times 8 - 15 \div 3$
h $6 + 5 \times 6 + 3$
i $20 \div 5 - 4 \div 4$
j $10 - 6 \div 3 + 3$
k $81 - 19 \times 2$
l $15 + 30 \div 5 + 4$



4 Use the rules for the order of operations to find the answers to these.

a $3 \times (4 + 6)$
b $8 + 16 \div 4$
c $(25 - 15) \times 9$
d $(8 - 3) \div 5 + 5$
e $(8 - 3) + 5 \div 5$
f $5 \div 5 + (8 - 3)$
g $8 + (3 \times 4) \div 4$
h $(8 + 3) \times 4 \div 4$
i $(8 + 3 \times 4) \div 4$
j $(10 - 4 \times 2) \div 2$
k $10 - (4 \times 2) \div 2$
l $10 - 4 \times 2 \div 2$
m $40 \div 5 \div (7 - 5)$
n $6 \times (2 + 1) \times 4$
o $(6 + 3) \times 4 + 11$
p $6 \times 5 + 8 \times 3$
q $5 \times 8 + 7 \times 3$
r $6 \times 9 + 4 \times 5$
s $(8 + 8) - 6 \times 2$
t $100 - 20 \times (3 + 1)$
u $(15 - 6) \times 4 \times 25$
v $(6 + 4 \times 5) - 3 \times 2$
w $5 \times 8 - (16 - 4 \div 2)$
x $8 - (13 - 8 - 2) \times 2$

5 Find the basic numeral for each.

a $6 + 8 \times 3 - 25 \div 5 \times 2 + 4$
b $100 - 10 \times 4 - 20 \div 2 \div 5 + 20$
c $11 - (8 - 3) \times 2 + 25 \div 5$
d $44 - 11 \times (9 - 7) + 100 \div 5$
e $8 \times \$5.10 + 10 \times \$5.10 + 2 \times \$5.10$
f $\$3.30 \times 3 + \$5.50 \div 5 + \$8$
g $0{\cdot}8 + 5 \times 0{\cdot}1 - 6 \times 0{\cdot}2$
h $8 + 9 \times (0{\cdot}3 + 0{\cdot}7) - 9$
i $80 - [(3 + 2) \div (10 - 5)] \times 2$
j $96 + [(15 \div 3) + (35 \div 7)] \times 10$
k $16 + 4 \times 2 - (7 \times 3 - 10) \div 11 + 6 - (10 \times 5 - 10 \times 4) - [6 \times (18 - 15)]$



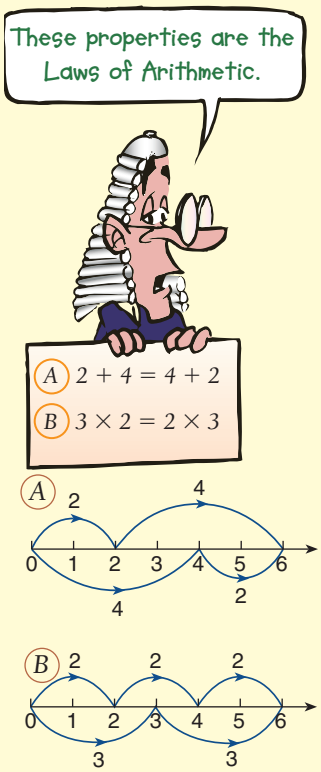
- Estimate the number of terracotta figures.

3:02 | Using Number Properties

3:02A Properties involving addition or multiplication

Knowing these properties will give us a greater understanding of our number system.

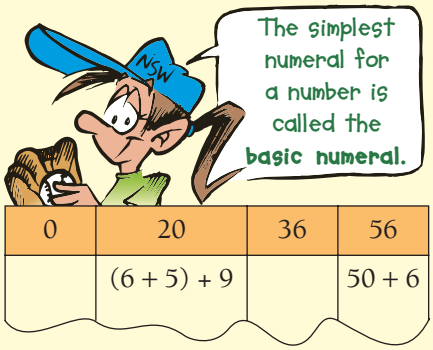
	Property	*Example*
1	Multiplying any number by one leaves it unchanged.	$6583 \times 1 = 6583$
2	Multiplying any number by zero gives the answer zero.	$749 \times 0 = 0$
3	Adding zero to any number leaves it unchanged.	$96 + 0 = 96$
4	When adding two numbers, the order does not change the answer.	$14 + 9 = 9 + 14$
5	When multiplying two numbers, the order does not change the answer.	$9 \times 7 = 7 \times 9$
6	When adding more than two numbers, we may add them in any order.	$41 + 154 + 59 + 6$ $= (41 + 59) + (154 + 6)$ $= 260$
7	When multiplying more than two numbers, we may multiply them in any order.	$4 \times 836 \times 25$ $= (4 \times 25) \times 836$ $= 83\,600$



Exercise 3:02A

1 Draw a table like the one in the cartoon and place each of these under its basic numeral.

$50 + 6$, $(6 + 5) + 9$, 13×0, 8×0, $6 + (5 + 9)$, 4×9, $(2 \times 6) \times 3$, $6 + 50$, 9×4, $2 \times (6 \times 3)$, 20×1, $0 \times (8 \times 11)$, $0 + 36$, 1×56, $(0 \times 8) \times 11$, 56×1, $39 + 16 + 1$, 1×20, $20 + 0$, $39 + 1 + 16$, $3 + 17$, $2 \times 9 \times 2$, 98×0, $17 + 3$, $2 \times 2 \times 9$



0	20	36	56
	$(6 + 5) + 9$		$50 + 6$

2 Work out the value of each side and say whether the number sentence is true or false.

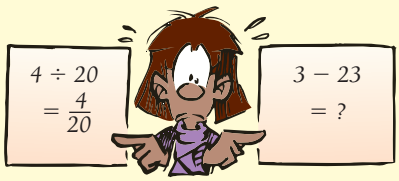
a $84 + 46 = 46 + 84$ **b** $125 \times 8 = 8 \times 125$ **c** $683 \times 1 = 1 \times 683$

d $836 + 0 = 0 + 836$ **e** $989 \times 0 = 0 \times 989$ **f** $384 + 0 = 384 \times 1$

g $4\frac{1}{2} + 2\frac{1}{2} = 2\frac{1}{2} + 4\frac{1}{2}$ **h** $0{\cdot}9 + 0{\cdot}7 = 0{\cdot}7 + 0{\cdot}9$ **i** $6\% + 8\% = 8\% + 6\%$

j $8 \times \frac{1}{4} = \frac{1}{4} \times 8$ **k** $20 \times 0{\cdot}1 = 0{\cdot}1 \times 20$ **l** $5\% \times 4 = 4 \times 5\%$

m $20 \div 4 = 4 \div 20$ **n** $26 \div 13 = 13 \div 26$
o $100 \div 5 = 5 \div 100$ **p** $16 \div 4 = 4 \div 16$
q $23 - 3 = 3 - 23$ **r** $34 - 9 = 9 - 34$
s $18 - 0 = 0 - 18$ **t** $15 - 8 = 8 - 15$

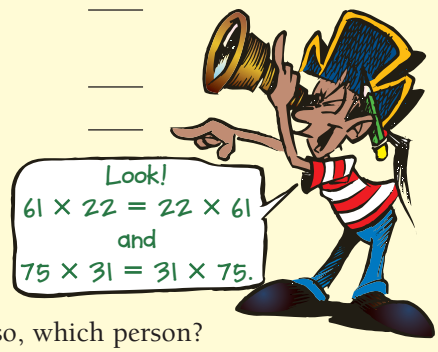


3 Write down a numeral that will replace the square to make each number sentence true.

a $6 \times \square = 0$ **b** $\square \times 1 = 416$ **c** $5800 + \square = 5800$
d $\square + 0 = 365$ **e** $67 \times \square = 67$ **f** $\square \times 0 = 0$
g $9 + 20 = \square + 9$ **h** $18 + 10 = 10 + \square$ **i** $6 + \square = 15 + 6$
j $9 \times 8 = \square \times 9$ **k** $7 \times 4 = 4 \times \square$ **l** $\square \times 11 = 11 \times 9$
m $0{\cdot}4 \times \square = 0$ **n** $1{\cdot}6 + \square = 1{\cdot}6$ **o** $\square \times 1 = 0{\cdot}9$
p $\frac{3}{10} + \frac{4}{10} = \square + \frac{3}{10}$ **q** $6 \times \frac{3}{100} = \frac{3}{100} \times \square$ **r** $\frac{3}{4} \times 0 = \square$

4 Find the answers to each of the following.

a 61 × 22 **b** 75 × 31 **c** 22 × 61 **d** 31 × 75



5 Answer after swapping top and bottom.

a 31 × 658 **b** 21 × 1040 **c** 13 × 6839

6 Does one person contribute more than the other? If so, which person?

a Jan poured 15 four-litre containers of water onto the garden.
Zac poured 4 fifteen-litre containers of water onto the garden.

b Betty scored 12 points in her first basketball game and 16 in her second.
Heather scored 16 points in her first basketball game and 16 in her second.

c André walked for 6 hours at a speed of 3 kilometres per hour.
Rex walked for 3 hours at a speed of 6 kilometres per hour.

d In 5 days Robert gave his father a total of $10.
In 10 days Mary gave her father a total of $5.

e Peter paid for the petrol for our car to travel 50 km at 50 km/h.
Alan paid for the petrol for our car to travel 30 km at 30 km/h.

f Jan goes fishing once and gives her catch of 15 fish to the nursing home.
Gerry gives everything he catches to the nursing home. He went fishing fifteen times and caught no fish each time.

7 True or false?

a $(18 + 6) + 3 = 18 + (6 + 3)$ **b** $(100 + 10) + 5 = 100 + (10 + 5)$
c $(18 \times 6) \times 3 = 18 \times (6 \times 3)$ **d** $(100 \times 10) \times 5 = 100 \times (10 \times 5)$
e $(18 - 6) - 3 = 18 - (6 - 3)$ **f** $(100 - 10) - 5 = 100 - (10 - 5)$
g $(18 \div 6) \div 3 = 18 \div (6 \div 3)$ **h** $(100 \div 10) \div 5 = 100 \div (10 \div 5)$
i $25 \times 37 \times 4 = 100 \times 37$ **j** $99 + 87 + 1 = 100 + 87$

■ Do what is in grouping symbols first.

8 Write down a numeral that will replace each square to form a true sentence.

a $5 \times 83 \times 20 = \square \times 83$
b $198 + 37 + 63 = 198 + \square$
c $90 + 73 + 10 + 7 = 100 + \square$
d $16 \times 45 \times 5 = \square \times 45$
e $960 + 127 + 40 = \square + 127$
f $2 \times 368 \times 5 = \square \times 368$
g $14 \times \frac{13}{100} \times 5 = 70 \times \square$
h $1\frac{1}{2} + \frac{7}{10} + \frac{3}{10} = 1\frac{1}{2} + \square$
i $99{\cdot}9 + 13{\cdot}8 + 0{\cdot}1 = \square + 13{\cdot}8$
j $\$3.80 \times 5 \times 5 \times 4 = \$3.80 \times \square$

9 Use the number properties to find the answers quickly.

a $72 \times 3 \times 8 \times 0$
b $\$0.25 \times 17 \times 4$
c $\$4.50 \times 14 \times 2$
d $1453 + 700 + 300$
e $5 + 476 + 795$
f $466 + 58 + 42$
g $5 \times 18 \times 10 \times 2$
h $4 \times 356 \times 5 \times 5$
i $25 \times 9 \times 2 \times 7 \times 2$
j $52 + 96 + 18 + 4$
k $800 + 496 + 4$
l $25 \times 8 \times 4 \times 6$

10 Use the example as a guide to perform these additions by grouping in tens.

Example

17
32
10 16 10
73
10 94 10
35
2
267

a
44
27
36
83
25
61
—

b
29
35
43
85
21
67
—

c
90
18
63
54
02
57
—



You can check your answers by adding up or adding down.

11 **a** A cyclist travels for 10 hours at an average speed of 8 kilometres per hour. Would he travel the same distance in 8 hours at a speed of 10 kilometres per hour?

b Each time Alana went to the shop she bought 17 packets of crisps, which cost 25 cents per packet. If she went to the shop 4 times, how much did she spend?

c Who would work the greater number of hours: 30 men working for 20 hours or 20 men working for 30 hours?

d In each carton of candles there are 85 boxes. In each box there are 12 candles. How many dozen candles would be in 4 cartons?

12 True or false?

a To multiply a number by 12, you could first multiply by 6 and then double the result.
b To multiply a number by 40, you could first multiply by 4 and then multiply the result by 10.
c To divide by 30, you could first divide by 3 and then divide the result by 10.
d To divide by 27 you could first divide by 3 and then divide the result by 9.

3:02B The distributive property

The **distributive property** involves both multiplication and addition or both multiplication and subtraction.

prep quiz 3:02B

What numeral will replace the square to make each a true sentence?

1 $9 \times 4 = 4 \times \square$ **2** $123 + 7 = \square + 123$ **3** $147 \times \square = 147$

4 $147 \times \square = 0$ **5** $147 + \square = 147$ **6** $25 \times 97 \times 4 = \square$

In each basket of fruit we sold, we placed 15 oranges and 5 apples.
We sold 8 baskets of fruit.

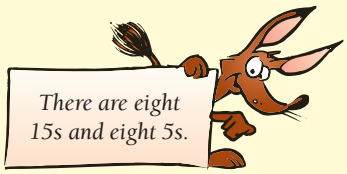
7 How many oranges did we sell?

8 How many apples did we sell?

9 How many pieces of fruit did we sell?

10 Question 9 might have been worked out in two ways:
a $8 \times (15 + 5)$ or **b** $(8 \times 15) + (8 \times 5)$. Is it true that $8 \times (15 + 5) = 8 \times 15 + 8 \times 5$?

In Prep Quiz 3:02B we saw that:
8 lots of (15 + 5) = 8 lots of 15 + 8 lots of 5
ie $8 \times (15 + 5) = 8 \times 15 + 8 \times 5$



worked examples

1 Nine full buses carried spectators to see the German World Cup final. Each bus held 100 people including the driver. How many passengers were there?
Number of passengers
= 9 lots of (100 − 1), ie $9 \times (99)$
= 9 lots of 100 − 9 lots of 1
= 900 − 9
= 891
Note: $9 \times (100 - 1) = 9 \times 100 - 9 \times 1$

2 In last year's ballroom dancing finals, 16 couples competed in each of the twelve events. How many competitors were there altogether?
Number of competitors
= 12 lots of (16 + 16), ie $12 \times (32)$
= 12 lots of 16 + 12 lots of 16
= 192 + 192
= 384
Note: $12 \times 16 + 12 \times 16 = 12 \times (16 + 16)$

In this type of example, multiplication can be *distributed over* addition or subtraction.

3 6×108, ie $6 \times (100 + 8)$
= 6 lots of 100 + 6 lots of 8
= 600 + 48
= 648
$[6 \times 108 = 6 \times 100 + 6 \times 8]$

4 4×98, ie $4 \times (100 - 2)$
= 4 lots of 100 − 4 lots of 2
= 400 − 8
= 392
$[4 \times 98 = 4 \times 100 - 4 \times 2]$

5 $5 \times 432 = 5 \times (400 + 30 + 2)$
$= 5 \times 400 + 5 \times 30 + 5 \times 2$
$= 2160$



Exercise 3:02B

1 **a** Five shepherds were each caring for 100 sheep. If each shepherd lost one sheep, how many sheep were left altogether?

b To each of five classes of 30 students, two extra students were added. How many students are there now altogether?

2 Which is equal to $7 \times (11 + 4)$ [ie 7×15]?

a $7 \times 11 + 4$ **b** $7 \times 11 + 7 \times 4$ **c** $11 + 7 \times 4$

3 True or false? (Try working each side out separately.)

a $6 \times (1 + 7) = 6 \times 1 + 6 \times 7$
b $9 \times (8 + 3) = 9 \times 8 + 9 \times 3$
c $4 \times (5 + 2) = 4 \times 5 + 4 \times 2$
d $6 \times (7 - 1) = 6 \times 7 - 6 \times 1$
e $9 \times (8 - 3) = 9 \times 8 - 9 \times 3$
f $4 \times (5 - 2) = 4 \times 5 - 4 \times 2$

4 What numeral should replace the square?

a $8 \times 6 + 8 \times 3 = 8 \times (6 + \square)$
b $7 \times 3 + 7 \times 5 = 7 \times (\square + 5)$
c $11 \times 9 + 11 \times 6 = \square \times (9 + 6)$
d $5 \times 9 - 5 \times 1 = 5 \times (\square - 1)$
e $4 \times 6 - 4 \times 2 = 4 \times (6 - \square)$
f $6 \times 7 - 6 \times 2 = \square \times (7 - 2)$
g $8 \times 5 + 8 \times 10 = 8 \times \square$
h $5 \times 3 + 10 \times 3 = \square \times 3$
i $17 \times 8 + 3 \times 8 = \square \times 8$
j $7 \times 92 = \square \times 90 + \square \times 2$

5 Use the method given in examples 3 and 4 to simplify:

a 5×99 **b** 7×99 **c** 3×98
d 9×98 **e** 6×95 **f** 8×95
g 12×95 **h** 18×95 **i** 71×101
j 24×102 **k** 8×107 **l** 5×108
m 35×102 **n** 42×101 **o** 15×104
p 8×201 **q** 195×4 **r** 98×5
s 703×6 **t** 980×45



6 We can sometimes use the distributive property to shorten working, eg $97 \times 13 + 3 \times 13 = (97 + 3) \times 13$
$= 1300$

Do these using the same property.

a $40 \times 9 + 60 \times 9$ **b** $7 \times 8 + 3 \times 8$ **c** $4 \times 9 + 7 \times 9$
d $5 \times 6 + 6 \times 6$ **e** $9 \times 14 + 1 \times 14$ **f** $91 \times 8 + 9 \times 8$
g $63 \times 8 - 3 \times 8$ **h** $104 \times 7 - 4 \times 7$ **i** $15 \times 12 - 5 \times 12$
j $23 \times 6 - 19 \times 6$ **k** $67 \times 14 - 65 \times 14$ **l** $96 \times 73 - 94 \times 73$

7 True or false?

a To multiply a number by 13, first multiply by 10 then add 3 times the number.
b To multiply a number by 36, first multiply by 30 then add 6 times the number.
c To multiply 72 by 9, first multiply by 10 and then subtract 72.
d To multiply 13 by 99, first multiply by 100 and then subtract 13.

3:03 | Language and Symbols Used in Mathematics

Use operation symbols to write each of (1) to (24) of ID Card 3 on page xv, and in each case also give the answer.

id 3:03

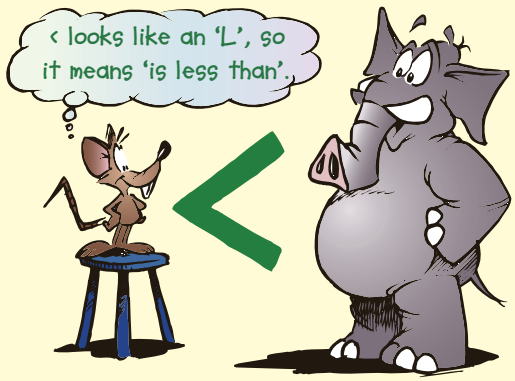
Abbreviations used in mathematics

%	means *per cent*	am	means *before noon*
∴	means *therefore*	pm	means *after noon*
eg	means *for example*	BC	means *before Christ*
ie	means *that is*	AD	means *anno Domini* or *in the year of our Lord*
K	means *thousands*		

■ $70K = $70 000
K comes from the Greek word *khilioi* meaning thousand.

Symbols used in mathematics

=	means *is equal to*
≠	means *is not equal to*
≒ or ≈	means *is approximately equal to*
<	means *is less than*
⩽	means *is less than or equal to*
≮	means *is not less than*
>	means *is greater than*
⩾	means *is greater than or equal to*
≯	means *is not greater than or equal to*
$\sqrt{9}$	means *the square root of 9* or *the number that is multiplied by itself to give 9*, eg $3 \times 3 = 9$ ∴ $\sqrt{9} = 3$
$\sqrt[3]{8}$	means *the cube root of 8* or *the number used in a product 3 times to give 8*, eg $2 \times 2 \times 2 = 8$ ∴ $\sqrt[3]{8} = 2$



examples

1. 1·987 ≒ 2 means 1·987 is approximately equal to 2.
2. 35 000 ≮ 33 999 means 35 000 is not less than 33 999.
3. $99 > $90.50 means $99 is greater than $90.50.

Exercise 3:03

Foundation Worksheet 3:03
Language and symbols
1 True or false?
a $3 = 5 \times 2$ **b** $8 > 5$
2 Use symbols to write:
a 5 *is less than* 6.

1 **a** Find the sum of 15 and 27.
b Find the difference between 91 and 23.
c To the quotient of 18 and 3, add 45.
d From the product of 6 and 8, take away 9.
e Decrease 80 by 6 lots of 9.
f Increase 476 by 12.
g Find the average of 13 and 3.
h Find 6 less than the total of 12, 8 and 3.
i What is 5 less than the square of 4?
j By how much does 362 exceed 285?

2 Rewrite each of these using the symbols introduced above.

a 15 is less than 105.
b 8·8 is not equal to 8·08.
c 0·7 is equal to 70 per cent.
d 5 squared multiplied by 7 cubed.
e 6 times 9, minus 3.
f 12 divided by 4, plus 6.
g Therefore, the square root of 9 is 3.
h That is, the cube root of 8 is 2.
i π is approximately equal to 3·142.
j $\square$ is less than or equal to 10.
k 0·3 is greater than 0·03.
l For example, 5·1 is not less than 5·09.
m The product of 12 and 4 is greater than or equal to the quotient of 12 and 4.

3 Rewrite each as a sentence by replacing the symbols with words.

a $0{\cdot}499 < \frac{1}{2}$
b $3999 \div 4000$
c $61\,000 \neq 61 \times 100$
d $6 + \square \geqslant 9$
e $0{\cdot}1 > 0{\cdot}099$
f $6 \times 199 = 6 \times 200 - 6 \times 1$
g $9{\cdot}1 \not< 9\frac{1}{10}$
h $5{\cdot}1 + 4 = 4 + 5{\cdot}1$
i $\square - 3 \leqslant 6$

4 Are the following true or false?

a $16 > 4$
b $399 \div 400$
c $14 \leqslant 13$
d $3{\cdot}0 = 3$
e $97 \times 5 \times 20 \neq 97 \times 100$
f $800 < 199 \times 4$
g $\frac{12}{2} = 12 \div 2$
h $\frac{1}{2} \not< \frac{2}{4}$
i $\frac{1}{10} < \frac{1}{2}$
j $18 \div 3 \neq 3 \div 18$
k $9 \times 7 \geqslant 7 \times 9$
l $3821 \div 1283$
m $8000 \not< 8 \times 1000$
n $0{\cdot}3 > 0{\cdot}29$
o $1 + 2 + 3 + 4 < 5$

investigation 3:03

Investigation 3:03 | Odds and evens

Please use the Assessment Grid on the following page to help you understand what is required for this Investigation.

Investigate what happens when you start with a number (for example 15) and follow the rules below, continuing to apply the rules over and over again.

If your last answer is:

- **Odd**: multiply by 3 and add 1,
- **Even**: halve the number.

Example

15 ⇒ 46, 23, 70, 35, 106, 53, 160, 80, 40, 20, 10, 5, 16, 8, 4, 2, 1 . . .

- Do all numbers below 20 eventually lead to 1?
- In each case, find the number of steps needed.



- How many ways can an even number be scored using just one dart?

Assessment Grid for Investigation 3:03 | Odds and evens

The following is a sample assessment grid for this investigation. You should carefully read the criteria *before* beginning the investigation so that you know what is required.

Assessment Criteria (B, C, D)				Achieved ✓
Criterion B Application & Reasoning	a	No systematic or organised approach has been used.	1	
			2	
	b	The student has attempted to use a systematic and organised approach and has also attempted to describe what is happening.	3	
			4	
	c	A systematic and organised approach has been used and there is a description of what is happening.	5	
			6	
	d	The given steps are applied without error and an attempt has been made to give a reason for the result.	7	
			8	
	e	The given steps are applied without error and a reason for the result has been given successfully.	9	
			10	
Criterion C Communication	a	There is no working out and the presentation is poor.	1	
			2	
	b	Working out is shown and some of the results have been interpreted in words.	3	
			4	
	c	Working out and the interpretation of results is well communicated using words or symbols.	5	
			6	
Criterion D Reflection & Evaluation	a	An attempt has been made to check the reliability of the results.	1	
			2	
	b	The reliability of the results has been checked with some success.	3	
			4	
	c	A reasoned explanation is given for the results and the reliability of the results has been checked.	5	
			6	
	d	The results are interpreted well to give a full explanation for the results obtained.	7	
			8	

3:04 | Special Sets of Whole Numbers



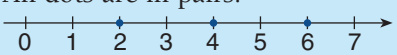
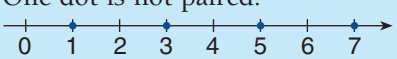
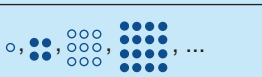
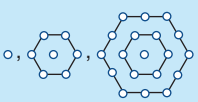
Write the next three terms of each of the number patterns below.

1 2, 4, 6, 8, . . ., . . ., . . .
2 1, 3, 5, 7, . . ., . . ., . . .
3 1, 2, 3, 4, . . ., . . ., . . .
4 1, 4, 9, 16, . . ., . . ., . . .

Answer true or false.

5 The last digit of an even number is always either 0, 2, 4, 6, or 8.
6 The last digit of an odd number is always either 1, 3, 5, 7, or 9.
7 The sum of two even numbers is odd.
8 The sum of two odd numbers is even.
9 The product of an odd and an even number is even.
10 5 squared is equal to 25.

Special Notice:
Zero is a special number and is sometimes included as an even number and as a multiple, but in this chapter we will not include it.

Name of group	*Pattern*	*Diagram or explanation*	*Comment or rule*
Cardinal numbers	0, 1, 2, 3, ...	Zero + counting numbers	Zero is included. 
Counting numbers	1, 2, 3, 4, ...		Zero is not included. 
Even numbers	2, 4, 6, 8, ...		All dots are in pairs. 
Odd numbers	1, 3, 5, 7, ...		One dot is not paired. 
Square numbers	1, 4, 9, 16, ...		A counting number times itself, eg 1×1, 2×2, 3×3, etc
Triangular numbers	1, 3, 6, 10, 15, ...		1, 1 + 2, 1 + 2 + 3, 1 + 2 + 3 + 4, ... The sum of counting numbers
Hexagonal numbers	1, 7, 19, 37, ...		1, $1 + (6 \times 1)$, $1 + (6 \times 1) + (6 \times 2)$, $1 + (6 \times 1) + (6 \times 2) + (6 \times 3)$, ... +6 +12 +18 +24 +30 ie 1, 7, 19, 37, 61, 91, ...
Fibonacci numbers	1, 1, 2, 3, 5, 8, ...	Except for the first two in the pattern, each term is the sum of the two terms before it.	
Palindromic numbers	Examples 929, 7337	Forwards or backwards it is the same number.	'Able was I ere I saw Elba' is a *palindromic sentence.*

Exercise 3:04

1 What is the next odd number:

a after 7006? **b** after 9999? **c** before 975? **d** before 10 340?

2 Write down the odd numbers:

a between 85 and 98 **b** between 992 and 1003

3 What is the even number:

a after 8146? **b** after 111? **c** before 3195? **d** before 10 000?

4 Write down the even numbers:

a between 73 and 83 **b** between 1996 and 2001

5 **a** All odd numbers must end in one of five digits. What are they?

b All even numbers must end in one of five digits. What are they?

6 Complete the following patterns.

a 78, 80, 82, __, __, __, __, __ **b** 193, 195, 197, __, __, __, __, __

c 989, 991, 993, __, __, __, __, __ **d** 1012, 1014, 1016, __, __, __, __, __

7 Say whether the following are odd or even.

a the sum of two odd numbers **b** the product of two odd numbers

c the sum of two even numbers **d** the product of two even numbers

e the sum of an odd and an even number

f the product of an odd and an even number

g the square of an odd number

h the square of an even number

i the difference between two odd numbers

j the difference between two even numbers

8 Complete the following:

Number group	*Term of pattern (eg term 3 = T_3)*														
	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}	T_{11}	T_{12}	T_{13}	T_{14}	T_{15}
Counting numbers	1	2	3	4	5	6	7	8							
Even numbers	2	4	6	8											
Odd numbers	1	3	5	7											
Square numbers	1	4	9	16											
Triangular numbers	1	3	6	10											
Fibonacci numbers	1	1	2	3											

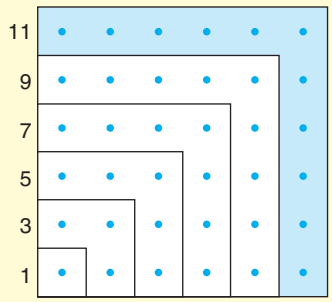


9 Complete the following:

a $1 + 3 =$
b $1 + 3 + 5 =$
c $1 + 3 + 5 + 7 =$
d $1 + 3 + 5 + 7 + 9 =$
e $1 + 3 + 5 + 7 + 9 + 11 =$
f $1 + 3 + 5 + 7 + 9 + 11 + 13 =$
g $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 =$
h $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 =$

10 Use the answers to question 9 to answer true or false for the following statements.

a The sum of the first four odd numbers is equal to 4×4.
b The sum of the first five odd numbers is equal to 5 squared.
c The sum of the first six odd numbers is 6^2.
d The sum of the first eight odd numbers is 8^2.



This diagram shows that:
- the sum of the first five odd numbers is 5^2
- the sum of the first six odd numbers is 6^2.

We can see also that this pattern will continue.

The sum of the first 50 odd numbers is 50^2.
$50^2 = 50 \times 50$

11 Complete the following tables.

a

Question	*Answer as a square*
$0^2 + 1$	$0 + 1 = 1 = 1^2$
$1^2 + 3$	$1 + 3 = 4 = 2^2$
$2^2 + 5$	$4 + 5 = 9 = 3^2$
$3^2 + 7$	$9 + 7 = \ldots = \ldots^2$
$4^2 + 9$	
$5^2 + 11$	

b

Question	*Answer as a square*
$6^2 + 13$	
$7^2 + 15$	
$8^2 + 17$	
$9^2 + 19$	
$10^2 + 21$	
$11^2 + 23$	

12 Which of the numbers in the box are:

a even? **b** odd? **c** square?
d both square and even?

37, 15, 25, 46, 100, 81, 16, 33, 111, 2000, 64, 144, 1, 2, 49, 4, 77, 84, 9, 71, 66, 36, 73, 122

13

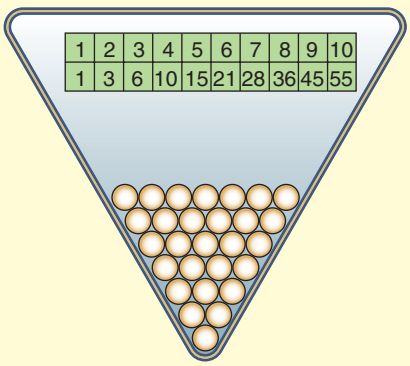


a What triangular number is shown here?
b If the top 5 balls were removed, would the picture still show a triangular number?
c Write the triangular numbers that are less than 10.

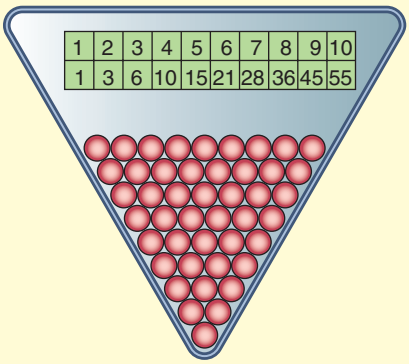
14 Find the number of pills contained in each of the chemist's pill trays below.



a



b



15 **a** How many tenpins are used for indoor bowling?
b Is this a triangular number?

16 2002 is a palindromic number. List all of the palindromic numbers between 1000 and 3000.

17 Which of the numbers here are:
a triangular numbers?
b Fibonacci numbers?
c palindromic numbers?

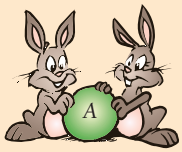
55, 14, 78, 66, 17, 37,
21, 16, 89, 91, 707,
10, 610, 15, 100

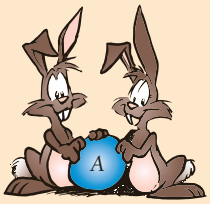
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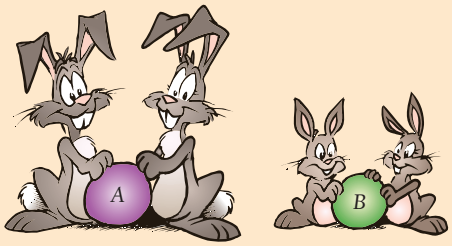


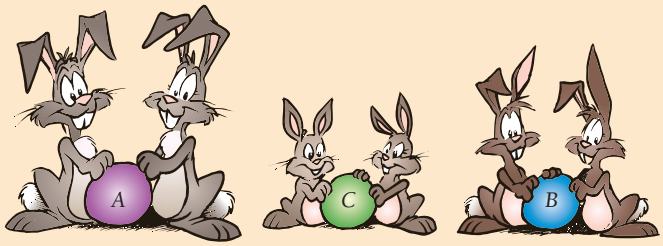
Write as much as you can about the different numbers that might be involved in this tower of cards.

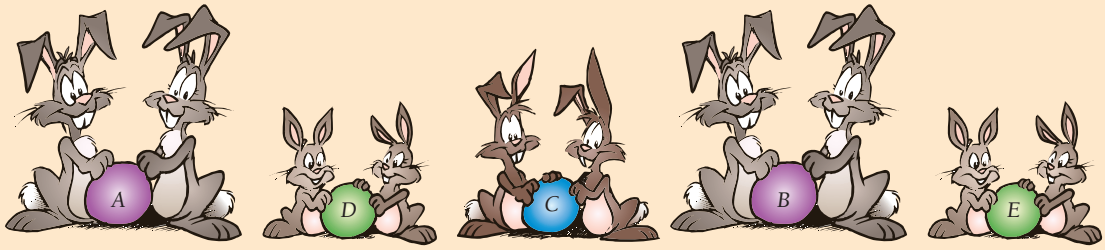
Fun Spot 3:04 | Fibonacci numbers











In these pictures, the smallest rabbits are babies, the middle-sized rabbits are one month old and are too young to have babies. At 2 months of age they are full grown and give birth to a pair of rabbits (one male and one female). Each full grown pair produces a new pair every month. Assuming that no rabbits die, how many pairs will there by on the second day of each month?

Solution

Month 1	1 pair
Month 2	1 pair
Month 3	2 pairs
Month 4	3 pairs
Month 5	5 pairs

1 **a** Draw the rabbits for months 6 and 7, showing their sizes.

b Are the numbers of pairs Fibonacci numbers? (see page 70)

Fibonacci magic: (1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, . . .)

2 Write down the first 15 Fibonacci numbers.

3 Add the first 5 Fibonacci numbers. Is this total one less than the 7th Fibonacci number?

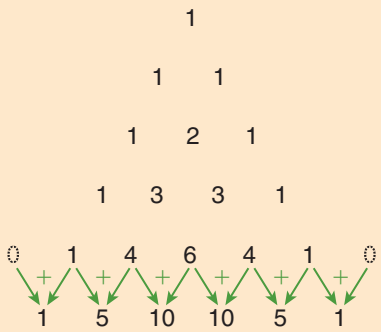
4 Add the first 10 numbers. Is this total one less than the 12th Fibonacci number?

5 Check to see if the pattern in **3** and **4** is true for the rest of the numbers you have written.

6 Challenge your parents to choose any Fibonacci number and add all the Fibonacci numbers up to and including the one chosen. You can give the total first by subtracting 1 from the Fibonacci number which is two places further on.

Investigation 3:04 | Pascal's triangle

Please use the Assessment Grid on the following page to help you understand what is required for this Investigation.



- This pattern of numbers is called **Pascal's triangle**.
- Each number in the triangle is generated by adding together the pair of numbers above it.
- Blaise Pascal, a famous mathematician of the 17th century, used the triangle in developing studies in algebra and probability.

Exercise

1 Copy Pascal's triangle into your book and write the next 4 lines of the triangle.
2 Next to each row of your triangle write the sum of the numbers in that row. What pattern have you made? What is the rule for this pattern?
3 Within the triangle are hidden other patterns of numbers. See how many you can find.



- Estimate the number of windows in this aeroplane.

Assessment Grid for Investigation 3:04 | Pascal's triangle

The following is a sample assessment grid for this investigation. You should carefully read the criteria *before* beginning the investigation so that you know what is required.

Assessment Criteria (B, C, D)				Achieved ✓
Criterion B Application & Reasoning	a	No systematic or organised approach has been used.	1	
			2	
	b	A systematic approach has been attempted and some of the patterns are described in words or symbols.	3	
			4	
	c	A systematic and organised approach has been used. Some simple patterns are described in words or symbols.	5	
			6	
	d	More complex patterns (other than the ones asked for in question 2) have been successfully described in words or symbols.	7	
			8	
	e	All the patterns in Pascal's triangle have been successfully described in words or symbols and some conclusion has been drawn about them.	9	
			10	
Criterion C Communication	a	No working out is shown and there is little use of mathematical symbols.	1	
			2	
	b	Some working out and explanation is given. Presentation is good with some structure to the work.	3	
			4	
	c	Communication and results are interpreted well using symbols or diagrams. Work is well structured and easy to follow.	5	
			6	
Criterion D Reflection & Evaluation	a	Some attempt has been made to explain the method and check the results.	1	
			2	
	b	The methods used are justified and the results have been checked with some success.	3	
			4	
	c	A reasoned explanation of how the results are obtained is given. The results themselves have been checked.	5	
			6	
	d	A concise description of the patterns and method used is given and the significance of the results is discussed.	7	
			8	

3:05 | Factors and Multiples

prep quiz 3:05

Complete these number sentences.

1 $\square \times 6 = 42$ **2** $42 \times \square = 42$ **3** $7 \times \square = 42$ **4** $1 \times \square = 42$

5 List all the counting numbers that divide 42 leaving no remainder.

What is the product of each pair of factors?

6 4 and 9 **7** 3 and 12 **8** 1 and 36 **9** 2 and 18 **10** 6 and 6

A *factor* of a counting number divides it exactly.
A *multiple* of a counting number is found when you multiply it by another counting number, eg $3 \times 8 = 24$.
Here 3 and 8 are factors of 24, and 24 is a multiple of both 3 and 8.

worked example 1

List all the factors of 102.



Solution

1 divides 102 exactly.
$1 \times 102 = 102$
$\therefore$ 1 and 102 are both factors.

2 divides 102 exactly.
$2 \times 51 = 102$
$\therefore$ 2 and 51 are both factors.

3 divides 102 exactly.
$3 \times 34 = 102$
$\therefore$ 3 and 34 are both factors.

4 and 5 do not divide 102 exactly.

6 divides 102 exactly.
$6 \times 17 = 102$
$\therefore$ 6 and 17 are both factors.

7, 8, 9, 10 and 11 do not divide 102 exactly.

Note: $102 \div 11 \fallingdotseq 9{\cdot}27\dot{2}\dot{7}$.
When your answer $(9{\cdot}27\dot{2}\dot{7})$ is smaller than your divisor (11), you have gathered all factors.

The factors of 102 are: 1, 102, 2, 51, 3, 34, 6, 17.

worked example 2

List the first nine multiples of 7.

Solution

$7 \times 1 = 7$, $7 \times 2 = 14$, $7 \times 3 = 21$, $7 \times 4 = 28$, $7 \times 5 = 35$,
$7 \times 6 = 42$, $7 \times 7 = 49$, $7 \times 8 = 56$, $7 \times 9 = 63$.

The first nine multiples of 7 are: 7, 14, 21, 28, 35, 42, 49, 56, 63.

worked example 3 (extension)

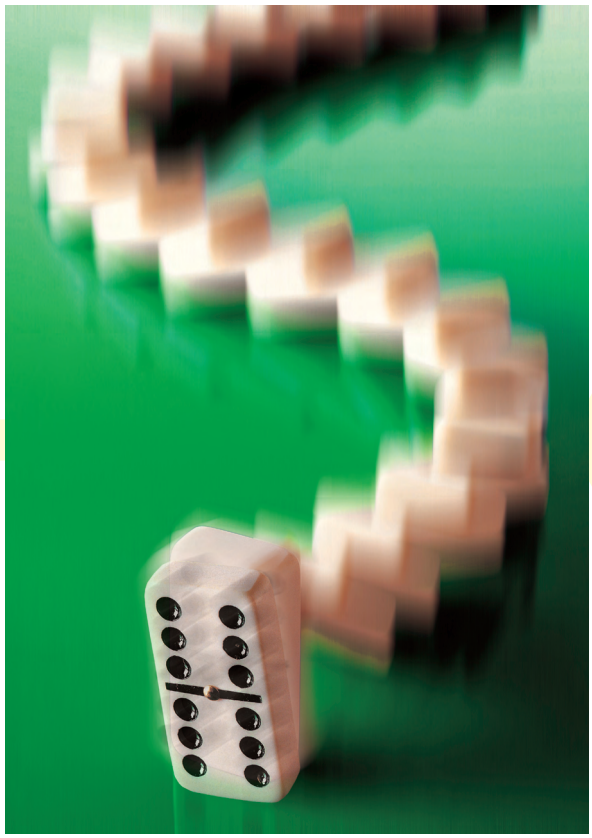
Find the highest common factor (called the HCF) of 102 and 153.

Solution

Number	*Factors*
102	1, 102, 2, (51), 3, 34, 6, 17
153	1, 153, 3, (51), 9, 17

The common factors are in colour.

The highest common factor is 51.



worked example 4 (extension)

Find the lowest common multiple (called the LCM) of 15 and 9.

Solution

Number	*Multiples*
15	15, 30, (45), 60, 75, 90, . . .
9	9, 18, 27, 36, (45), 54, . . .

There would be other common multiples but **the lowest common multiple is seen to be 45.**

Exercise 3:05

1 Complete the table below.

Product	24	24	24	24	20	20	20	12	12	12	18	18	18	25	25
Factor	1	2	3	4	4	2	1	12	6	4	6	9	18	5	1
Factor	24	12													

2 Use the table above to list all factors of:

a 24 **b** 20 **c** 12 **d** 18 **e** 25



3 List all the factors of:

a 10 **b** 8 **c** 15 **d** 9 **e** 7
f 100 **g** 27 **h** 45 **i** 36 **j** 40
k 144 **l** 160 **m** 132 **n** 121 **o** 152

4 Answer true or false.

a 3 is a factor of 313. **b** 99 is a multiple of 9. **e** 210 is a factor of 210.
c 3 is a factor of 300. **d** 2 is a factor of 11 012. **h** 42 is a multiple of 6.
f 210 is a multiple of 210. **g** 35 is a multiple of 5.
i 1 is a factor of all counting numbers. **j** 1 is a multiple of 9.

5 List the first nine multiples of:

a 3 **b** 5 **c** 2 **d** 10 **e** 4
f 11 **g** 6 **h** 9 **i** 7 **j** 8

6 Complete the tables and find the highest common factor (HCF) in each case.

a

Number	*Factors*
36	
24	

The HCF is . . . . .

b

Number	*Factors*
90	
135	

The HCF is . . . . .

c

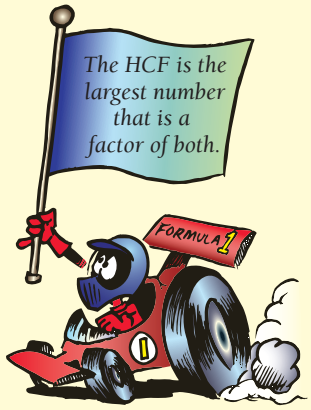
Number	*Factors*
116	
348	

The HCF is . . . . .

d

Number	*Factors*
105	
165	

The HCF is . . . . .



7 Find the highest common factor of:

a 16 and 22 **b** 45 and 60 **c** 80 and 100 **d** 28 and 42
e 102 and 132 **f** 700 and 750 **g** 240 and 144 **h** 64 and 27

8 Complete the tables and find the lowest common multiple (LCM) in each case.

a

Number	*Multiples*
7	
5	

The LCM is . . . . .

b

Number	*Multiples*
10	
12	

The LCM is . . . . .

c

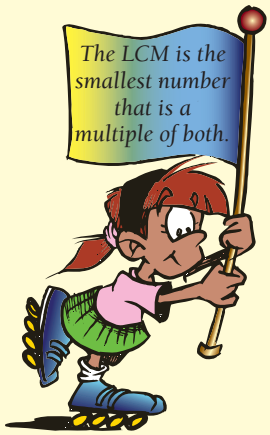
Number	*Multiples*
6	
15	

The LCM is . . . . .

d

Number	*Multiples*
30	
25	

The LCM is . . . . .



9 Find the lowest common multiple of:

a 6 and 4 **b** 25 and 40 **c** 12 and 18 **d** 4 and 14
e 8 and 10 **f** 24 and 30 **g** 20 and 8 **h** 22 and 33

10 **a** Find the multiples of 5 between 60 and 90.
b List all common factors of 160 and 200.
c What are the multiples of 2 between 1 and 20?
d Which of the multiples of 5 that are less than 100 are also multiples of 6?
e Find the multiples of 6 between 20 and 50.

3:06 | Prime and Composite Numbers

prep quiz 3:06

1 6×8 **2** 4×12 **3** 48×1 **4** 24×2

5 List all the factors of 48.

6 Complete: $6 \times 1 = \ldots$, $6 \times 2 = \ldots$, $6 \times 3 = \ldots$, $6 \times 4 = \ldots$, $6 \times 5 = \ldots$

7 List the first five multiples of 6.

8 List the factors of 30.

9 List the factors of 48.

10 The larger of two numbers is 601 and their difference is 20. What is their sum?

A *prime number* is a counting number that has exactly two factors, itself and 1.
For example, 7 has only two factors, 7 and 1, so 7 is a prime number.

A *composite* number has more than two factors.
For example, 25 has three factors, 25, 1 and 5, so 25 is a composite number.

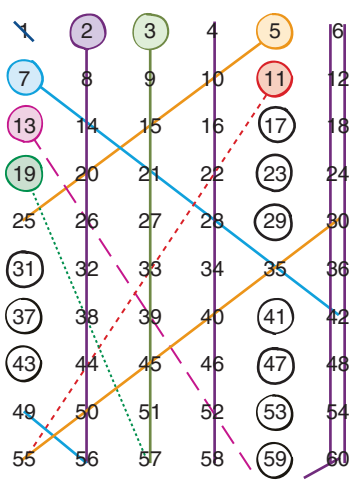
The number 1 has only one factor, 1, so 1 is neither prime nor composite.

worked example 1

Use the **Sieve of Eratosthenes** to list all primes less than 60.

Solution

Eratosthenes was a Greek mathematician who lived from 276 to 194 BC. He invented a way of sorting the prime numbers from the composites.



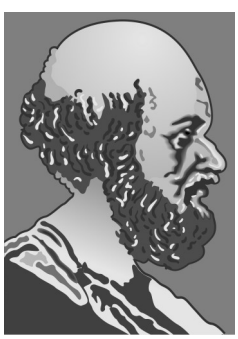
Step 1 Cross out 1 as it is not a prime.

Step 2 Circle the prime number 2 then cross out all multiples of 2.

Step 3 Circle the next number, 3. This is a prime. Now cross out any multiples of 3 not already crossed out.

Step 4 Circle the next number, 5. This is a prime. Now cross out any multiples of 5 not already crossed out.

Step 5 Continue using this method until all composite numbers have been crossed out.



Eratosthenes
(276?–194 BC)
Greek mathematician, historian, astronomer, poet and geographer. Born at Cyrene in northern Africa, he went to Alexandria to take charge of the great library there.

The primes less than 60 are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59.

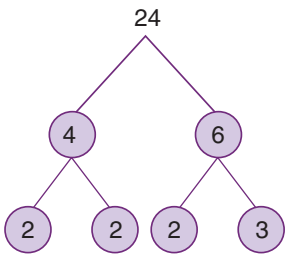
worked example 2

Use factor trees to write as products of prime numbers:

a 24 **b** 252

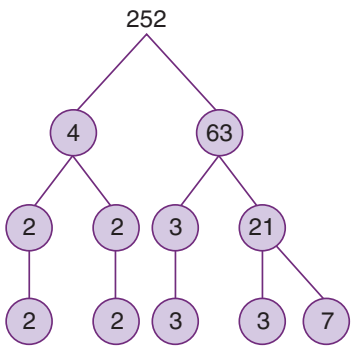
Solutions

a

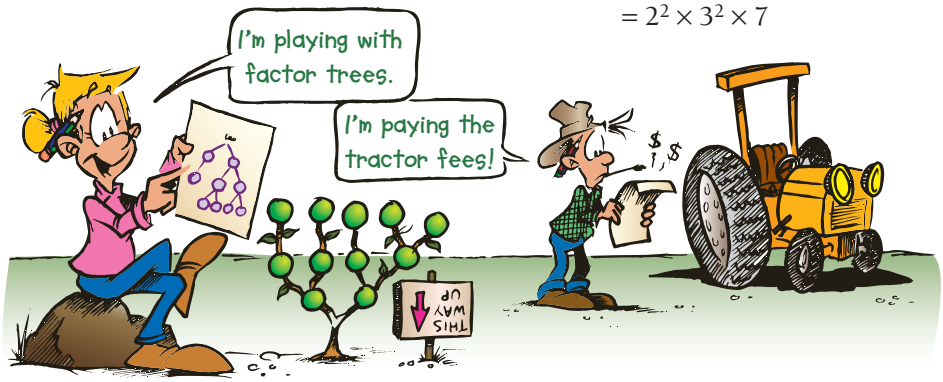


$\therefore\ 24 = 2 \times 2 \times 2 \times 3$
$= 2^3 \times 3$

b



$252 = 2 \times 2 \times 3 \times 3 \times 7$
$= 2^2 \times 3^2 \times 7$



Exercise 3:06

1 Copy and complete the table below, listing all factors of the numbers 1 to 20.

Number	*Factors*	*Number*	*Factors*
1	1	11	
2	1, 2	12	
3	1, 3	13	
4	1, 4, 2	14	
5		15	
6		16	
7		17	
8		18	
9		19	
10		20	1, 20, 2, 10, 4, 5

a List all numbers from 1 to 20 that have only two factors. These are prime numbers.

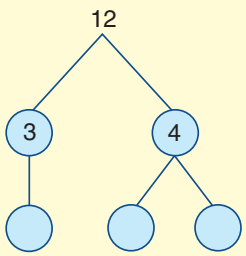
b List all numbers from 1 to 20 that have more than two factors. These are composite numbers.

2 Look back to example 1 and use the Sieve of Eratosthenes to list all prime numbers that are less than 120. Start by listing all the numbers from 1 to 120 in rows of six numbers.

3 **a** Write down all of the even prime numbers.
b How many odd prime numbers are less than 20?
c List all the odd composite numbers that are less than 20.
d How many composite numbers are between 20 and 40?
e Write each of the even numbers from 6 to 24 as the sum of two prime numbers, eg 12 = 5 + 7. (The mathematician Goldbach suggested that every even number greater than 4 is the sum of two odd prime numbers.)

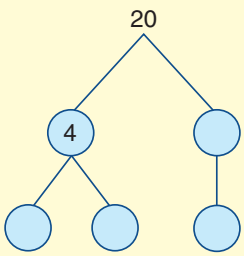
4 Complete these factor trees.

a



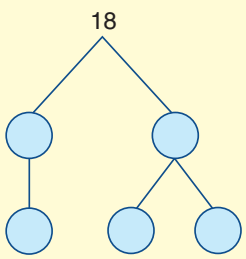
12 = ... × . . . . × . . . .

b



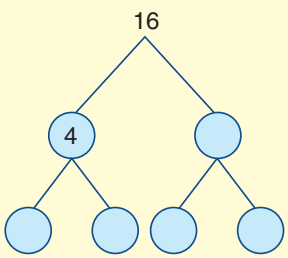
20 = ... × . . . . × . . . .

c



18 = ... × . . . . × . . . .

d



16 = ... × . . . . × . . . . × . . . .

5 Use factor trees to write these numbers as products of their prime factors.

a 30 **b** 36 **c** 100 **d** 144 **e** 96 **f** 225

6 Another way to write a number as a product of its prime factors is to continue to divide by prime factors (starting from the smaller ones) until you get 1 as a result. Use this method, which is shown to the right, to write these numbers as products of prime factors.

a 216 **b** 675 **c** 735 **d** 504
e 1125 **f** 1232 **g** 1323 **h** 3375

Example: 360

2) 360
2) 180
2) 90
3) 45
3) 15
5) 5
1

$360 = 2^3 \times 3^2 \times 5$

6 is called a *perfect number* because the sum of all of its factors, other than 6 itself, is equal to 6, ie 1 + 2 + 3 = 6.

Appendix C
C:01 HCF and LCM by prime factors

Activity CW 3:06 Finding factors game

Challenge worksheet 3:06 HCF and LCM by prime factors

3:07 | Divisibility Tests

prep quiz 3:07

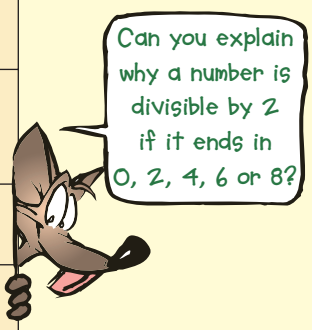
1 List all the factors of 40.
2 Write the next number after 60 that is divisible by 10.
3 Write the next three multiples of 6 after 12.
4 What is the largest even number that is less than 1000?
5 What is the next odd number after 999?

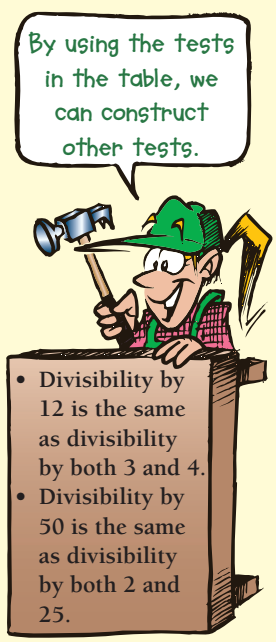
$255 \div 5 = 51$

6 In this number sentence, which number is the quotient?
7 Which is the divisor?
8 Which is the dividend?
9 318 813 ÷ 11
10 102 102 102 ÷ 9

If 4 divides a number and leaves no remainder, then the number is divisible by 4. Here are some simple divisibility tests you should know.

Divisor	*Divisibility test*	*Example*
2	The number must be even, ie it must end in 0, 2, 4, 6 or 8.	4136 is divisible by 2 as it is even.
3	The sum of the digits is divisible by 3.	30 012 is divisible by 3 as (3 + 0 + 0 + 1 + 2) is divisible by 3.
4	The number formed by the last two digits must be divisible by 4.	76 112 is divisible by 4 as 12 is divisible by 4.
5	The last digit must be 5 or 0.	11 225 is divisible by 5 as it ends with a 5.
6	The number must be divisible by both 2 and 3.	40 002 is even and its digit sum is divisible by 3.
8	The number formed by the last three digits is divisible by 8.	963 216 is divisible by 8 as 216 is divisible by 8.
9	The sum of the digits is divisible by 9.	142 128 is divisible by 9 as (1 + 4 + 2 + 1 + 2 + 8) is divisible by 9.
10	The last digit must be 0.	814 710 is divisible by 10 as it ends with 0.
11	The sum of the digits in odd-numbered places will be equal to the sum of the digits in even-numbered places, or will differ by a multiple of 11.	7 081 426 is divisible by 11 as (7 + 8 + 4 + 6) – (0 + 1 + 2) is 22, which is divisible by 11.
25	The last two digits will be 00, 25, 50 or 75.	80 925 is divisible by 25 as it ends in 25.
100	The last two digits will be 00.	81 700 is divisible by 100 as it ends in 00.





Exercise 3:07

Foundation Worksheet 3:07

Divisibility tests

1 Write the first 10 even numbers.

2 Write the first 10 multiples of:
a 2 **b** 3

3 Write the numbers below 30 that are divisible by:
a 2 **b** 3

1 Which of these numbers are divisible by 2?
571, 3842, 5816, 2221, 887, 9000, 374 555, 8774, 8166, 7008

2 Which of these numbers are divisible by 3?
7114, 830, 3006, 7110, 21 441, 30 031, 8145, 60 001, 211 002, 78

3 Which of these numbers are divisible by 4?

1004, 67 814, 7118, 2222, 6124, 8156, 98, 61 852, 934

4 Which of these numbers are divisible by 9?
9994, 31 024, 30 024, 81 810, 41 238, 3333, 727, 411, 765, 936

5 Which of these numbers are divisible by 11?
4115, 8003, 6633, 7228, 860, 74 186, 10 406, 92 180, 999, 61 809

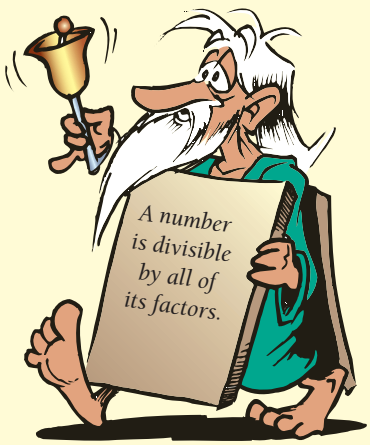
6 Find which of the numbers 3, 4, 5, 6, 8, 9 and 11 will exactly divide:
a 440 **b** 3883 **c** 3916 **d** 1485 **e** 3993

7 Which of the numbers 1078, 7600, 13 476 and 33 885 are divisible by:
a 2? **b** 3? **c** 4? **d** 6? **e** 9?
f 10? **g** 11? **h** 25? **i** 100? **j** 50?

8 What divisibility test could we use for 24?

9 The test for divisibility by 6 is that the number must be divisible by both 2 and 3.
a What divisibility test could we use for 15?
b What divisibility test could we use for 18?
c What divisibility test could we use for 12?

10 Use the divisibility tests where possible to list all of the factors of:
a 720 **b** 346 **c** 270 **d** 1260
e 567 **f** 1575 **g** 729 **h** 5280



11 Find the smallest number that is greater than 1000 and:
a is divisible by 3 **b** is divisible by 8
c is divisible by 9 **d** is divisible by 11
e is a multiple of 25 **f** is a multiple of 100
g is a multiple of 15 **h** is a multiple of 12

12 **a** Find the lowest common multiple of 2, 5, 9, 10 and 11.
b Find the highest common factor of 810, 324 and 1944.
c Find the lowest common multiple of 2, 3, 4, 5, 9 and 100.
d List all factors of $3 \times 5^2 \times 7$. (There are twelve.)
e List all factors of $3^3 \times 11$.

Activity GW 3:07 Multiples patterns

3:08 | Square and Cube Roots

prep quiz 3:08

Give the basic numeral for:

1 5^2 **2** 4^2 **3** 14^2 **4** 2^3 **5** 3^3 **6** 4^3

7 Find the number that has a square of 36.

8 Find the number that has a square of 100.

9 If $1764 = (2 \times 3 \times 7) \times (2 \times 3 \times 7)$, what is the number that has a square of 1764?

10 What number has a square of 10 000?

Foundation Worksheet 3:08

Square and cube roots

1 a If $4 \times 4 = 16$, then $\sqrt{16} = \square$.

2 a If $7^2 = 49$, then $\sqrt{49} = \square$.

3 a If $5 \times 5 \times 5 = 125$ then $\sqrt[3]{125} = \square$.

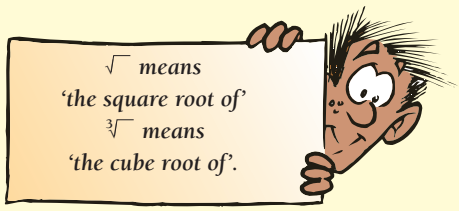
4 a If $6^3 = 216$ then $\sqrt[3]{216} = \square$.

If the square of 15 is 225, then the square root of 225 is 15.

If $15^2 = 225$, then $\sqrt{225} = 15$.

If the cube of 8 is 512, then the cube root of 512 is 8.

If $8^3 = 512$, then $\sqrt[3]{512} = 8$.



The number that gives 225 when squared is called the **square root** of 225.
The number that gives 512 when cubed is called the **cube root** of 512.

Exercise 3:08

1 Complete:

a If $7 \times 7 = 49$, then $\sqrt{49} = \square$

b If $11 \times 11 = 121$, then $\sqrt{121} = \square$

c If $13 \times 13 = 169$, then $\sqrt{169} = \square$

d If $5 \times 5 \times 5 = 125$, then $\sqrt[3]{125} = \square$

e If $9 \times 9 \times 9 = 729$, then $\sqrt[3]{729} = \square$

f If $12 \times 12 \times 12 = 1728$, then $\sqrt[3]{1728} = \square$

g If $16^2 = 256$, then $\sqrt{256} = \square$

h If $21^2 = 441$, then $\sqrt{441} = \square$

i If $14^2 = 196$, then $\sqrt{196} = \square$

j If $7^3 = 343$, then $\sqrt[3]{343} = \square$

k If $4^3 = 64$, then $\sqrt[3]{64} = \square$

l If $9^3 = 729$, then $\sqrt[3]{729} = \square$

m If $(2 \times 9) \times (2 \times 9) = 324$, then $\sqrt{324} = \ldots \times \ldots = \square$

n If $(3 \times 7) \times (3 \times 7) = 441$, then $\sqrt{441} = \ldots \times \ldots = \square$

o If $(9 \times 11) \times (9 \times 11) = 9801$, then $\sqrt{9801} = \ldots \times \ldots = \square$

worked examples

1 i If $3 \times 3 = 9$, then $\sqrt{9} = 3$

ii If $4 \times 4 \times 4 = 64$, then $\sqrt[3]{64} = 4$

iii If $5^2 = 25$, then $\sqrt{25} = 5$

iv If $6^3 = 216$, then $\sqrt[3]{216} = 6$

v If $(2 \times 7) \times (2 \times 7) = 196$ then $\sqrt{196} = 2 \times 7 = 14$

continued ➜➜➜

p If $(2\times3\times5)\times(2\times3\times5)=900$,
then $\sqrt{900}=\ldots\times\ldots\times\ldots$
$=\square$

q If $(7\times11\times13)\times(7\times11\times13)=1\,002\,001$,
then $\sqrt{1\,002\,001}=\ldots\times\ldots\times\ldots$
$=\square$

r If $(5\times5\times11)^3=20\,796\,875$,
then $\sqrt[3]{20\,796\,875}=\ldots\times\ldots\times\ldots$
$=\square$

2 Find the square root of each product.

a $2\times2\times3\times3\times3\times3$
b $3\times3\times5\times5$
c $3\times3\times3\times3\times5\times5$
d $5\times5\times7\times7\times11\times11$
e $2\times2\times2\times2\times5\times5\times7\times7$
f $2\times2\times3\times3\times5\times5$

3 Find the cube root of each product.

a $3\times3\times3\times5\times5\times5$
b $2\times2\times2\times11\times11\times11$
c $2\times2\times2\times3\times3\times3$
d $2\times2\times2\times2\times2\times2$
e $3\times3\times3\times3\times3\times3$
f $2\times2\times2\times5\times5\times5$

4 Find the basic numeral for:

a	$\sqrt{9}$	**b**	$\sqrt{1}$	**c**	$\sqrt{4}$
d	$\sqrt{16}$	**e**	$\sqrt{64}$	**f**	$\sqrt{36}$
g	$\sqrt{0}$	**h**	$\sqrt{100}$	**i**	$\sqrt{81}$
j	$\sqrt{144}$	**k**	$\sqrt{324}$	**l**	$\sqrt{400}$
m	$\sqrt{1296}$	**n**	$\sqrt{1764}$	**o**	$\sqrt{225}$

5 Find the basic numeral for:

a	$\sqrt[3]{8}$	**b**	$\sqrt[3]{27}$	**c**	$\sqrt[3]{1}$
d	$\sqrt[3]{64}$	**e**	$\sqrt[3]{1000}$	**f**	$\sqrt[3]{0}$
g	$\sqrt[3]{216}$	**h**	$\sqrt[3]{512}$	**i**	$\sqrt[3]{1331}$
j	$\sqrt[3]{3375}$	**k**	$\sqrt[3]{4096}$	**l**	$\sqrt[3]{5832}$

6 Use the table of squares below to give the two counting numbers between which each square root lies,

Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Square	1	4	9	16	25	36	49	64	81	100	121	144	169	196

a	$\sqrt{13}$	**b**	$\sqrt{72}$	**c**	$\sqrt{20}$	**d**	$\sqrt{40}$	**e**	$\sqrt{90}$	**f**	$\sqrt{3}$
g	$\sqrt{180}$	**h**	$\sqrt{30}$	**i**	$\sqrt{132}$	**j**	$\sqrt{17}$	**k**	$\sqrt{147}$	**l**	$\sqrt{116}$

vi If $(2\times3\times7)\times(2\times3\times7)=1764$,
then $\sqrt{1764}=2\times3\times7$
$=42$

vii If $(2\times2\times7)^3=21\,952$,
then $\sqrt[3]{21\,952}=2\times2\times7$
$=28$

2 $\sqrt{2\times2\times2\times2\times13\times13}$
$=\sqrt{(2\times2\times13)\times(2\times2\times13)}$
$=2\times2\times13$
$=52$

3 $\sqrt[3]{2\times2\times2\times7\times7\times7}$
$=\sqrt[3]{(2\times7)\times(2\times7)\times(2\times7)}$
$=2\times7$
$=14$

4 **i** $\sqrt{25}=5$,
Since $5\times5=25$

ii Find $\sqrt{576}$
$576=(2\times2\times2\times3)^2$
$\sqrt{576}=24$

5 **i** $\sqrt[3]{125}=5$,
since $5\times5\times5=125$

ii Find $\sqrt[3]{8000}$.
By product of primes,
$8000=(2\times2\times5)^3$
$\sqrt[3]{8000}=20$

6 $\sqrt{135}$ is between 11 and 12,
as 135 is between 11^2 and 12^2.

7 First write an estimate for the square root and then use a calculator to find the answer.

a $\sqrt{256}$ **b** $\sqrt{324}$ **c** $\sqrt{676}$

d $\sqrt{1521}$ **e** $\sqrt{1089}$ **f** $\sqrt{484}$

8 First write an estimate for the cube root and then use a calculator to find the answer.

a $\sqrt[3]{125}$ **b** $\sqrt[3]{729}$ **c** $\sqrt[3]{1331}$

d $\sqrt[3]{8000}$ **e** $\sqrt[3]{4913}$ **f** $\sqrt[3]{1728}$

7 $\sqrt{576}$

$\sqrt{\ }$ 576 = 24

8 $\sqrt[3]{2744}$

$\sqrt[3]{\ }$ 2744 = 14

3:09 | The Binary System: Mathematics for Machines (Extension)

prep quiz 3:09

Write in expanded form:

1 31 627

2 15 038

Write as a basic numeral:

3 $(6 \times 10^4) + (3 \times 10^3) + (7 \times 10^2) + (8 \times 10^1) + (9 \times 1)$

4 $(1 \times 10^4) + (0 \times 10^3) + (5 \times 10^2) + (2 \times 10^1) + (0 \times 1)$

Complete:

5 $2^1 = \ldots$, $2^2 = \ldots$, $2^3 = \ldots$, $2^4 = \ldots$, $2^5 = \ldots$, $2^6 = \ldots$

Simplify:

6 $(1 \times 8) + (1 \times 4) + (0 \times 2) + (1 \times 1)$

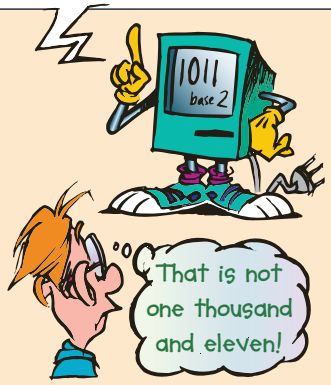
7 $(1 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 1)$

8 $(1 \times 16) + (1 \times 8) + (1 \times 4) + (1 \times 2) + (0 \times 1)$

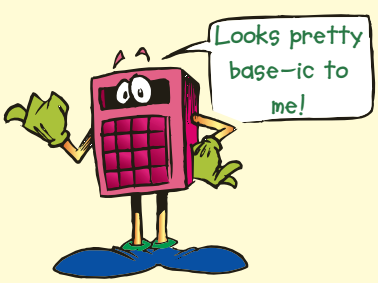
9 $(1 \times 2^4) + (1 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (0 \times 1)$

10 $(1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (0 \times 1)$

1011_{two} is
one-zero-one-one base two,
ie $(1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 1)$
$= 8 + 0 + 2 + 1 = 11$



To use a number system based on ten, we need ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. Machines that work by using electrical currents can only use two digits: 'off' and 'on'. These are their digits for 0 and 1. They use a **base two system that has place value**.



worked examples

1 Change 111011_{two} into a base ten numeral.

2 Write 54_{ten} as a numeral in binary numerals (base two).

continued →→→

Solutions

1

2^5	2^4	2^3	2^2	2^1	1
32s	16s	8s	4s	2s	1s
1	1	1	0	1	1
☀	☀	☀	○	☀	☀

The column values are powers of 2.

$(1 \times 2^5) + (1 \times 2^4) + (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 1)$
$= 32 + 16 + 8 + 0 + 2 + 1$
$= 59$
$\therefore 111011_{two} = 59_{ten}$

2 Starting with the larger column values, distribute 54 across the columns.
54 is made up of 32 and 16 and 4 and 2.
$\therefore 54 = (1 \times 32) + (1 \times 16) + (1 \times 4) + (1 \times 2)$
Put zeros in the other columns.

2^6	2^5	2^4	2^3	2^2	2^1	1
64s	32s	16s	8s	4s	2s	1s
	1	1	0	1	1	0

Method
There are no 64s in 54.
There is one 32 in 54, ie $54 = (1 \times 32) + 22$. Place the one in the 32s column.
There is one 16 in the 22 that remains, ie $22 = (1 \times 16) + 6$. Place the one in the 16s column.
There is one 4 and one 2 in the 6 that remains, ie $6 = (1 \times 4) + (1 \times 2)$.
Place a one in both the 4s column and the 2s column.
$\therefore 54_{ten} = 110110_{two}$

Exercise 3:09

1 Change these base two numerals to numerals in base ten.

a 10_{two} **b** 11_{two} **c** 100_{two} **d** 101_{two} **e** 110_{two}
f 111_{two} **g** 1000_{two} **h** 1001_{two} **i** 1010_{two} **j** 1011_{two}
k 101101_{two} **l** 11000_{two} **m** 100011_{two} **n** 111111_{two} **o** 1101_{two}

2 Write these base ten numerals as binary numerals.

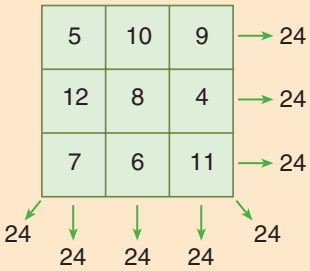
a 17 **b** 20 **c** 64 **d** 35 **e** 40

3 Complete the table below.

a

Numeral (base ten)	2^6	2^5	2^4	2^3	2^2	2^1	1	*Numeral (base two)*
	64s	32s	16s	8s	4s	2s	1s	
								1010001_{two}
								1101011_{two}
	1	0	0	0	1	1	1	
80								
100								
97								
125								

Fun Spot 3:09 | Making magic squares



The rows, columns and diagonals of a magic square have the same sum. In this magic square, each row, column and diagonal has a sum of 24.

To make a 'three-by-three' magic square you need nine numbers. These numbers, when placed in order, must form a pattern with the same difference between each pair of numbers. The numbers used here are:
4, 5, 6, 7, 8, 9, 10, 11 and 12.

Example

2, 4, 6, 8, 10, 12, 14, 16, 18. These have a difference of 2. We will now make a magic square using these numbers.

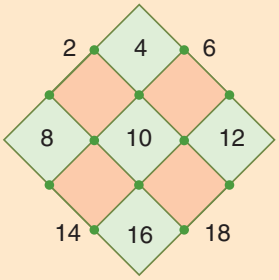
Step 1

Write the numbers in rows of three.

2	4	6
8	10	12
14	16	18

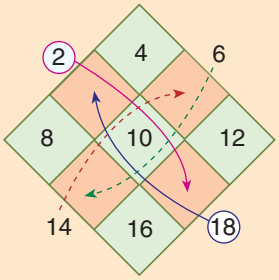
Step 2

Join the half-way points as shown.



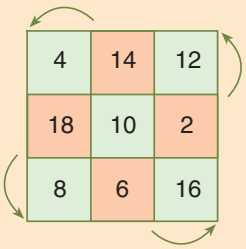
Step 3

Place each number which is outside a square into the *opposite* empty square.



Step 4

Rotate the square anticlockwise to give it the appearance below



Activity

1. Use the method shown to make a magic square using:
   - **a** 3, 4, 5, 6, 7, 8, 9, 10 and 11.
   - **b** 5, 10, 15, 20, 25, 30, 35, 40 and 45.
   - **c** 36, 33, 30, 27, 24, 21, 18, 15 and 12.
2. What is the sum of a column in each magic square of part 1?
3. Use the pattern 1, 2, 4, 8, 16, 32, 64, 128, 256 to make a square using the method above. What is the *product* of numbers in each row, column and diagonal?

Mathematical terms 3

average (arithmetic mean)

- The result of 'evening out' a set of numbers.
- To find the average, divide the sum of the terms by the number of terms.
  eg The average of 3, 5, 3, 7, 10 and 2
  $= (3 + 5 + 3 + 7 + 10 + 2) \div 6$
  $= 30 \div 6$
  $= 5$

basic numeral

- The simplest way of writing a number.
  eg The basic numeral for $(4 + 8) \times 2$ is 24.

cardinal numbers

- Zero and the counting numbers.
  ie {0, 1, 2, 3, . . .}

composite number

- A number that has more than two factors.
  eg 9 is composite as it has three factors, 1, 3 and 9.

counting numbers

- The numbers {1, 2, 3, 4, . . .}.

cube (numbers)

- The answer when a whole number is used 3 times as a product.
  eg $5^3 = 5 \times 5 \times 5 = 125$

cube root ($\sqrt[3]{\ }$)

- To find the cube root of a number, say 8, find the number that needs to be cubed to give 8.
  ie $2^3 = 8$ so $\sqrt[3]{8} = 2$

decrease

- To make smaller.

difference

- The result of subtracting one number from a larger number.

distributive property

- This shows how multiplication (or division) of a number can be done by breaking the number into an addition (or subtraction) of separate parts.
  eg $6 \times 108 = 6 \times (100 + 8)$
  $= (6 \times 100) + (6 \times 8)$
  or $8 \times 99 = 8 \times (100 - 1)$
  $= (8 \times 100) - (8 \times 1)$

divisibility tests

- Rules used to find factors of a number.

even numbers

- Whole numbers that are divisible by 2. They end in 0, 2, 4, 6 or 8.

factor

- A factor of a counting number divides it exactly.
  eg The factors of 6 are 1, 2, 3 and 6.
- common factor
  A number that is a factor of all numbers being considered.
  eg 7 is a common factor of 14, 21 and 70.
- highest common factor, HCF (extension)
  The largest of the common factors.
  eg 18 and 24 have common factors, 2, 3 and 6 but the highest common factor is 6.

Fibonacci numbers

- The set of numbers 1, 1, 2, 3, 5, 8, . . .
  The next number in the pattern is the sum of the two before it.

grouping symbols

- They are used to tell us which operation is to be done first.
- There are 3 types.

parentheses	*brackets*	*braces*
( )	[ ]	{ }

- When one set is within another, do the operation in the innermost grouping symbols frist.

increase

- To make larger.

multiple

- A multiple of a counting number is found by multiplying it by another counting number.
  eg The multiples of 5 are 5, 10, 15, 20, . . .
- common multiple
  A number that is a multiple of all numbers being considered.
  eg 50 is a common multiple of 2 and 5.
- lowest common multiple, LCM (extension)
  The smallest of the common multiples.
  eg 10 is the LCM of 2 and 5.
  20 is the LCM of 2, 5 and 10.

number properties
- Those properties that belong to all numbers.
  eg Multiplying any number by 1 leaves the number unchanged.

odd numbers
- Whole numbers that are not divisible by 2. They end in 1, 3, 5, 7 or 9.

order of operations
- The set of rules that give the order in which the operations are performed.
  ie **1** ( ) **2** × and ÷ **3** + and –

palindromic numbers
- A number that is the same backwards as it is forwards.
  eg 929, 2002, 18 700 781

prime number
- A counting number that has exactly two factors, itself and 1.
  eg 17, 31, 2

product
- The result of multiplying numbers together.

quotient
- The result of dividing one number by another.

square number
- The result of multiplying a counting number by itself.
  eg 1, 16, 25

square root ($\sqrt{\ }$)
- The square root of a number, say 64, is the number that must be squared to give 64. The square root of 64 is 8.
  eg $\sqrt{64} = 8$

sum
- The result of adding numbers together.

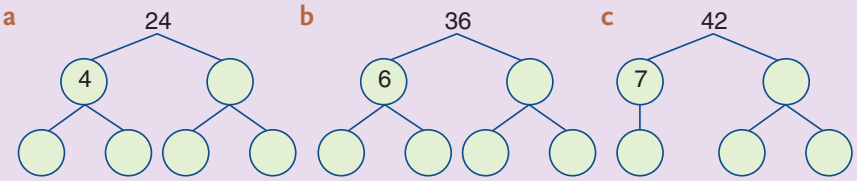
triangular numbers
- Numbers that are the sum of consecutive counting numbers beginning from 1.
  eg 1, 3 (that is 1 + 2),
  6 (that is 1 + 2 + 3),
  10 (that is 1 + 2 + 3 + 4), 15 etc.

Diagnostic Test 3: | Number: Its Order and Structure

diagnostic test 3

- Each section of the test has similar items that test a certain type of example.
- Failure in more than one item will identify an area of weakness.
- Each weakness should be treated by going back to the section listed.

	Section
1 Write in words. **a** 2 456 325 **b** 97 005 000 **c** 294 167 000	1:02
2 Write the value of the 7 in each. **a** 214 703 **b** 871 000 **c** 73 415 216	1:02
3 Write each as a numeral in its simplest form. **a** (5 × 1000) + (4 × 100) + (9 × 10) + (6 × 1) **b** (8 × 10 000) + (6 × 100) + (5 × 10) + (9 × 1) **c** (3 × 1 000 000) + (2 × 100 000) + (1 × 10 000)	1:06
4 Write in expanded notation. **a** 56 015 **b** 714 850 **c** 3 420 000	1:06

	Section
5 Write the basic numeral for each. **a** 5^2 **b** 3^3 **c** 10^5	1:05
6 Write as a power. **a** $8 \times 8 \times 8$ **b** $10 \times 10 \times 10 \times 10$ **c** 9×9	1:05
7 Write the basic numeral for each. **a** 7×10^2 **b** 4×10^4 **c** 5×10^3	1:05
8 Simplify: **a** $(6 + 8) \times (11 - 9)$ **b** $33 - (16 - [4 + 10])$ **c** $\frac{10 + 40}{5}$	3:01
9 Simplify: **a** $18 - 3 \times 4 + 5$ **b** $12 + 18 \div (10 - 4)$ **c** $120 - 80 \div 8 - 6 \times 10$	3:01
10 Copy and complete to make each number sentence true. **a** $6 \times (900 + 8) = \square \times 900 + \square \times 8$ **b** $7 \times (600 - 3) = \square \times 600 - \square \times 3$ **c** $4 \times 398 = 4 \times 400 - 4 \times \square$	3:02
11 **a** List the first three square numbers. **b** List the first three triangular numbers. **c** List the first three odd numbers.	3:04
12 List all the factors of: **a** 24 **b** 63 **c** 100	3:05
13 List the first five multiples of: **a** 4 **b** 8 **c** 11	3:05
14 Find the highest common factor of: **a** 36 and 48 **b** 60 and 75 **c** 70 and 98	3:05
15 Find the lowest common multiple of: **a** 12 and 9 **b** 6 and 8 **c** 20 and 14	3:05
16 **a** Write down all prime numbers that are less than ten. **b** Write down all composite numbers that are less than ten. **c** Which counting number is neither prime nor composite?	3:06
17 Complete these factor trees. **a** 24 (4) **b** 36 (6) **c** 42 (7) 	3:06
18 Find the highest common factor and the lowest common multiple of: **a** $2 \times 2 \times 2 \times 3$ and $2 \times 2 \times 5$ **b** $3 \times 3 \times 3 \times 5 \times 7$ and $3 \times 5 \times 5 \times 7$ **c** $2 \times 3 \times 5 \times 7$ and $3 \times 3 \times 5$	3:07
Find the simplest answer for:	
19 **a** $\sqrt{25}$ **b** $\sqrt{(7 \times 11) \times (7 \times 11)}$ **c** $\sqrt{2 \times 2 \times 3 \times 2 \times 2 \times 3}$	3:09
20 **a** $\sqrt[3]{27}$ **b** $\sqrt[3]{11 \times 11 \times 11}$ **c** $\sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3}$	3:09

Chapter 3 | Revision Assignment

assignment 3A

1 Write the basic numeral in each case.
- **a** $12 \div (28 - 24)$
- **b** $84 - 10 \times 8$
- **c** $100 - [50 - (20 - 7)]$
- **d** $23 + 5 - 2 + 3$
- **e** $6 \times 5 + 6 \times 3$
- **f** $3 + 7(0{\cdot}7 + 1{\cdot}3)$

2 We sell buckets of blocks. In each bucket there are 157 blocks. We sold 93 buckets of blocks on Monday and 7 on Tuesday. How many blocks did we sell on those two days altogether?

3 Given that $12 \times 1256 = 15\,072$, what is the value of:
- **a** 1256×12?
- **b** 13×1256?
- **c** 11×1256?

4 True or false?
- **a** $2 \times 3186 \times 5 = 31\,860$
- **b** $9 \times 888 = 8880 - 888$
- **c** $555 \times 12 = (555 \times 6) \times 2$
- **d** $18\,158 \div 20 = (18\,158 \div 2) \div 10$
- **e** $4 \times 186 \times 25 = (4 \times 25) \times 186$
- **f** $16 \times 100 - 1 = 16 \times 99$

5 Write as a basic numeral.
- **a** 1846×1
- **b** $0 \times 26\,040$
- **c** $8145 + 0$
- **d** $446 - 0$
- **e** $0 \div 186$
- **f** $4186 \div 1$
- **g** $23 \times 11 - 22 \times 11$
- **h** $31 \times 98 + 69 \times 98$
- **i** $98 \times 37 + 2 \times 37$

6 Write true or false for:
- **a** $8 \times 3 \neq 3 \times 8$
- **b** $563 < 560$
- **c** $213 \times 0 \geqslant 0$
- **d** $\sqrt{16} = 8$
- **e** $16 \times 5 \not\leqslant 16 \times 4$
- **f** $5 \times 816 > 0$
- **g** $816 \div 1 \not< 816 \times 1$
- **h** $999 < 100 - 0$
- **i** $\sqrt[3]{1} = 1$

7 Which of the numbers 25, 27, 41 214 and 13 are:
- **a** odd?
- **b** even?
- **c** square?
- **d** cube?
- **e** Fibonacci?
- **f** palindromic?

8
- **a** List the factors of 100.
- **b** List the factors of 125.
- **c** List all the common factors of 100 and 125.

9
- **a** List the first 10 multiples of 8.
- **b** List the first 10 multiples of 12.
- **c** What is the lowest common multiple of 8 and 12?

10 From the set 2, 5, 7, 12, 15, 36, 41 write:
- **a** the prime numbers
- **b** the composite numbers

1 Order of operations
2 Number puzzles

Drag and Drop CW Interactive

Teacher's Resource TR Chapter Test

Review CW Questions

assignment 3B

id 3

Chapter 3 | Working Mathematically

1 Use ID Card 4 on page xvi to give the mathematical term for:

a 14	**b** 15	**c** 16	**d** 17
e 18	**f** 19	**g** 20	**h** 21
i 22	**j** 23		

2 Use the Arithmetic Card on page xix to square the numbers in column B.

3 A bookshelf has six shelves with 45 books on each shelf. How many books are in the bookshelf?

4 A bus can hold 58 people seated and 35 standing. How many of these buses would be needed to take a school of 750 children and 30 teachers on a school outing?

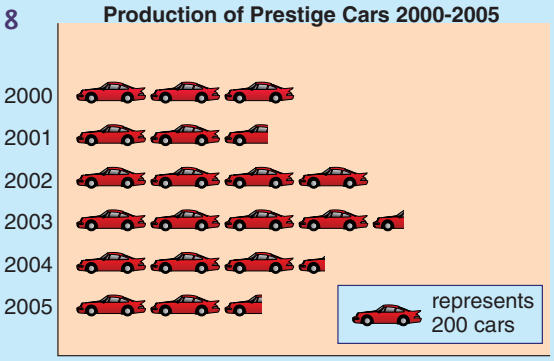


5 A grocer bought 50 kg of tomatoes for $30. He sold 20 kg at $1.20 per kg and the remainder, which were overripe, he sold for 80c per kg. How much did he get for selling the tomatoes? How much did he make?

6 In Rugby Union, a team scores 5 points for a try, 7 points for a converted try and 3 points for a penalty goal. In how many ways could a team score 22 points?

7 A school offers the following sports choices:
Summer: cricket, water polo, basketball, volleyball, tennis
Winter: football, hockey, soccer, squash
How many different combinations are available if one summer sport and one winter sport are to be selected?

8



a In what years were:
- **i** the greatest number of cars produced?
- **ii** the least number of cars produced?

b In what year were 600 cars produced?

c How many cars were produced in 2002?

d How many cars do you think were made in 2001? Can you be sure?

e Are the same symbols used to represent a fraction of 200 cars in 2001 and 2004?

f Do you think these graphs are intended to convey accurate figures? Give a reason for your answer.

4 Decimals



Chapter Contents

4:01 Review of decimals
4:02 Addition and subtraction of decimals
4:03 Multiplying a decimal by a whole number
4:04 Dividing a decimal by a whole number
4:05 Using decimals
4:06 Multiplying a decimal by a decimal
4:07 Dividing by a decimal
Fun Spot: Why did the orange stop in the middle of the road?
4:08 Changing fractions to decimals
4:09 Rounding off
4:10 Applications of decimals
Fun Spot: World championship diving
Investigation: Applying decimals
Mathematical Terms, Diagnostic Test, Revision Assignment, Working Mathematically

Learning Outcomes

Students will operate with fractions, decimals, percentages, ratios and rates.

Areas of Interaction

Approaches to Learning, Homo Faber, Environment, Community

4:01 | Review of Decimals

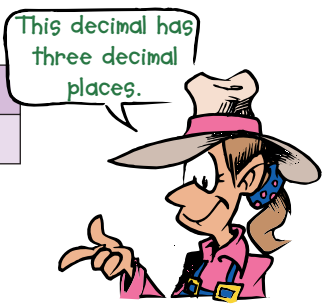
see 1:09C

With our world's increasing dependence on the calculator and the computer, decimals have become more important. These machines use decimals in their displays and output. Where parts of a whole number are to be shown, a decimal point is used.

worked example

Write 734·625 in expanded notation.

Hundreds	*Tens*	*Units*		*Tenths*	*Hundredths*	*Thousandths*
7	3	4	•	6	2	5



Solution

In expanded form this is:

$(7 \times 100) + (3 \times 10) + (4 \times 1) + (6 \times \frac{1}{10}) + (2 \times \frac{1}{100}) + (5 \times \frac{1}{1000})$

Exercise 4:01

Foundation Worksheet 4:01

Review of decimals

1 Write as a decimal:
a $\frac{3}{10}$ b $\frac{7}{100}$

2 Write as a fraction:
a 0·7 b 0·03

2 Which is larger:
a 0·7 or 0·08?

1 Answer true or false for each.

a 0·3 means 3 of 10 equal parts.
b 0·4 is the same 0·40.
c 0·8 is equal to $\frac{8}{10}$.
d $2 \times \frac{1}{100} = \frac{2}{100}$
e 0·14 has two decimal places.
f The value of the 7 in 6·174 is $\frac{7}{100}$.
g 0·7 is the same as 0·007.
h 0·35 is the same as $\frac{35}{100}$.
i 7 = 7·00
j Ten lots of 0·1 gives 1.

worked example

$\frac{7}{100} + \frac{5}{1000}$

$= 0{\cdot}075$

1		$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
0	•	0	7	5



2 Write each as a decimal.

a $\frac{6}{10} + \frac{4}{100}$
b $\frac{1}{10} + \frac{7}{100}$
c $\frac{2}{10} + \frac{3}{100} + \frac{8}{1000}$
d $\frac{5}{100} + \frac{1}{1000}$
e $\frac{3}{10} + \frac{9}{1000}$
f $\frac{6}{100} + \frac{6}{1000}$
g $2 + \frac{3}{10} + \frac{5}{100}$
h $9 + \frac{4}{10}$
i $15 + \frac{3}{10} + \frac{6}{100}$
j $11 + \frac{2}{100}$
k $6 + \frac{8}{100} + \frac{1}{1000}$
l $2 + \frac{5}{10} + \frac{3}{1000}$

3 Write these decimals in expanded form.

a 7·342
b 3·483
c 4·215
d 45·03
e 30·75
f 40·07

g 256·04 **h** 24·125 **i** 69·345
j 246·379 **k** 482·375 **l** 238·667

4 Which is smaller?

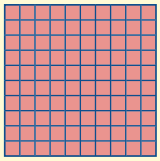
a 0·7 *or* 1 **b** 0·05 *or* 2 **c** 3 *or* 1·2
d 0·4 *or* 0·05 **e** 0·7 *or* 0·69 **f** 0·03 *or* 0·22
g 0·2 *or* 0·1 **h** 2·8 *or* 4·1 **i** 0·99 *or* 0·612
j 0·5 *or* 0·77 **k** 0·05 *or* 0·049 **l** 0·333 *or* 0·29

5 Arrange in ascending order.

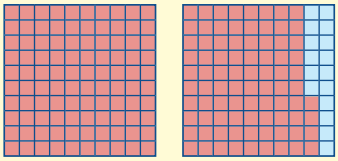
a 0·3, 0·8, 0·6 **b** 3·85, 1·2, 7·5
c 0·8, 1, 0·55 **d** 3, 0·3, 0·03
e 0·3, 0·33, 0·03 **f** 0·39, 0·41, 0·4
g 0·85, 0·9, 0·792 **h** 0·1, 0·09, 0·8

6 Each 'hundred square' represents one whole.
What decimal is represented in each example?

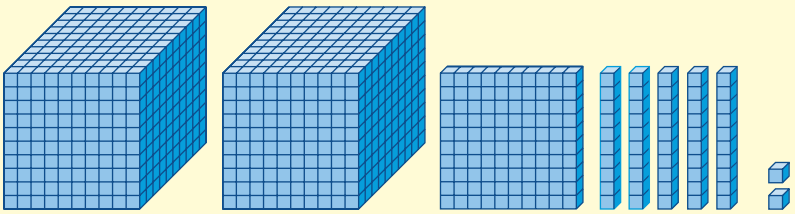
a



b



7 If the 'large cube' represents one whole, the 'flat' represents 0·1, the 'long' represents 0·01 and the 'short' represents 0·001, what number is represented by the following?



8 How many decimal places has each of these?

a 8·14 **b** 23·123 **c** 963·5
d 0·6 **e** 0·06 **f** 0·066
g 5·068 **h** 15·03 **i** 900·1
j 463·05 **k** 6·004 **l** 0·7



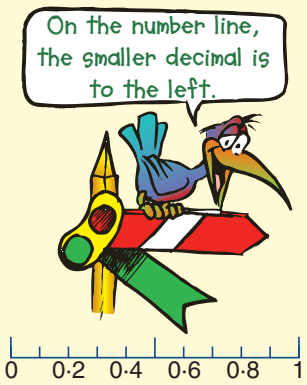
9 Write as decimals.

a $\frac{23}{100}$ **b** $\frac{12}{100}$ **c** $\frac{7}{1000}$ **d** $\frac{125}{1000}$
e $\frac{15}{1000}$ **f** $\frac{604}{1000}$ **g** $2\frac{93}{100}$ **h** $15\frac{103}{1000}$
i $10\frac{9}{100}$ **j** $7\frac{999}{1000}$ **k** $11\frac{33}{1000}$ **l** $8\frac{5}{1000}$

■ To compare decimals, give them the same number of decimal places, eg

0·3	0·300
0·33	0·330
0·303	0·303

'Ascending order' means 'smallest to largest'.



10 Write as fractions.

a 0·7	**b** 0·9	**c** 0·3
d 0·21	**e** 0·77	**f** 0·91
g 0·447	**h** 0·601	**i** 0·853
j 2·99	**k** 8·3	**l** 9·007

11 Write each as a decimal.

a $\frac{14}{10}$	**b** $\frac{86}{10}$	**c** $\frac{73}{10}$
d $\frac{724}{100}$	**e** $\frac{806}{100}$	**f** $\frac{903}{100}$
g $\frac{1186}{1000}$	**h** $\frac{7406}{1000}$	**i** $\frac{8014}{1000}$
j $\frac{4504}{100}$	**k** $\frac{6504}{10}$	**l** $\frac{842}{1000}$

12 Write in simplest fraction form.

a 0·30	**b** 0·3	**c** 0·300
d 0·7	**e** 0·70	**f** 0·700
g 0·01	**h** 0·010	**i** 0·0100
j 0·090	**k** 0·030	**l** 0·0010



■ 0·3 = 0·30 = 0·300
Zeros at the end of a decimal do not change its value.

13 What is the value of the 5 in each of these?

a 24 560·7	**b** 28·53	**c** 0·125	**d** 6·25
e 0·005	**f** 7·152	**g** 4·6125	**h** 0·054

14 True or false?

a $14 + \frac{6}{100} = 14{\cdot}6$	**b** $0{\cdot}97 > 1$	**c** $5 + 0{\cdot}007 < 5 + 0{\cdot}07$
d $0{\cdot}606 < 0{\cdot}66$	**e** $0{\cdot}125 = 0{\cdot}1 + 0{\cdot}02 + 0{\cdot}005$	**f** $0{\cdot}083 = 0{\cdot}08 + 0{\cdot}003$
g 10 lots of 0·01 equals 0·1.	**h** 10 lots of 0·001 equals 0·01.	

4:02 | Addition and Subtraction of Decimals

- The column in which a figure is placed determines its size.
- The decimal point separates the whole numbers from the parts of a whole, so the decimal points must be under one another in question and answer.

To add or subtract decimals use the PUP rule: place Points Under Points.
An empty space may be filled by a zero.

worked examples

1 14·6 + 3·15 + 9
2 97·3 − 8·95

Solutions

1
$$\begin{array}{r} 14{\cdot}60 \\ 3{\cdot}15 \\ 9{\cdot}00 \\ \hline 26{\cdot}75 \end{array}$$

2
$$\begin{array}{r} 97{\cdot}30 \\ -\ 8{\cdot}95 \\ \hline 88{\cdot}35 \end{array}$$



Exercise 4:02

Foundation Worksheet 4:02
+ and − of decimals
1 a 6·4 + 0·2
2 a 4·7 − 1·1
3 a 6·2 + 0·1

see Appendix B:13 & B:14

1 **a** 3·85 + 1·6 + 0·41 **b** 6 + 5·8 + 4·25 **c** 9·175 + 4·7 + 9·36

d 26 459·6 + 9 006·8 **e** 54 000 + 12 673·5 **f** 3·1486 + 9·852

g 9·08 + 12·16 **h** 4·018 + 0·582
i 6·9 + 3·1 **j** 12 + 0·12
k 6·3 + 8·05 **l** 150·86 + 9·1
m 0·9 + 15 + 3·125 **n** 8·6 + 9·3 + 0·75
o 8·25 + 6·85 + 9·125 **p** 0·185 + 0·25 + 0·03

We place tenths under tenths, hundredths under hundredths, and thousandths under thousandths.



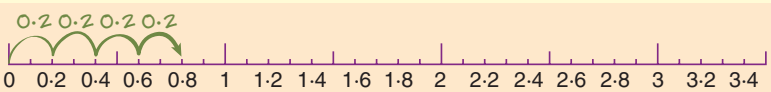
2 **a** 12·30 − 9·85 **b** 15·4 − 6·75 **c** 8·35 − 1·148

d 18·000 − 6·375 **e** 25 − 19·625 **f** 9 − 0·025

g 9·68 − 2·90 **h** 40·125 − 9·04 **i** 37·889 − 7·8

j 9·3 − 1·6 **k** 6·75 − 0·45 **l** 8·875 − 6·125
m 9 − 0·366 **n** 8·4 − 0·11 **o** 23·8 − 0·08
p 8·799 − 3·89 **q** 1·086 − 0·7 **r** 3·467 − 0·35
s 689·5 − 18·25 **t** 634·8 − 115·85 **u** 4725 − 6·845

4:03 | Multiplying a Decimal by a Whole Number



Use the number line above to find:

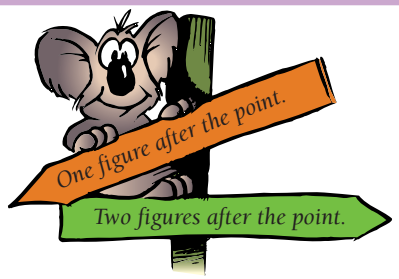
1 4 lots of 0·2 **2** 5 lots of 0·4 **3** 6 lots of 0·4
4 3 lots of 0·6 **5** 4 lots of 0·8 **6** 5 lots of 0·5
7 4 × 0·3 **8** 5 × 0·6 **9** 3 × 0·7
10 (7 lots of 0·2) + (3 lots of 0·6)

prep quiz 4:03

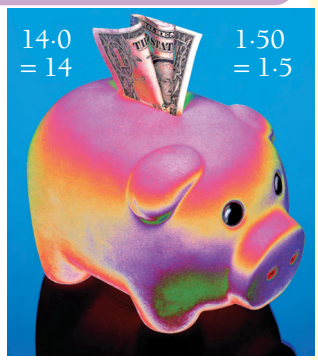
When multiplying decimals by a whole number, the number of figures after the decimal points in the question and answer is the same.

worked examples

1 $2{\cdot}8 \times 5$



2 $\$0{\cdot}85 \times 7$



Solutions

1
$$\begin{array}{r} 2{\cdot}8 \\ \times\ \ 5 \\ \hline 14{\cdot}0 \end{array}$$

2
$$\begin{array}{r} \$0{\cdot}85 \\ \times\ \ \ 7 \\ \hline \$5{\cdot}85 \end{array}$$

Multiplying by 10, 100 and 1000

3 $0{\cdot}81 \times 10$
$= 8{\cdot}10$
$= 8{\cdot}1$

4 $0{\cdot}031 \times 100$
$= 3{\cdot}100$
$= 3{\cdot}1$

5 $1{\cdot}015 \times 1000$
$= 1015{\cdot}000$
$= 1015$

6 $0{\cdot}6 \times 1000$
$= 600{\cdot}0$
$= 600$

From the examples above we can see an easier way to get the answer.

When we multiply by 10, 100 or 1000, we move the decimal point 1, 2, or 3 places to the right.

That is,

3 $0{\cdot}8\,1 \times 10$
$= 8{\cdot}1$

4 $0{\cdot}0\,3\,1 \times 100$
$= 3{\cdot}1$

5 $1{\cdot}0\,1\,5 \times 1000$
$= 1015$

6 $0{\cdot}6\,0\,0 \times 1000$
$= 600$

Exercise 4:03

Foundation Worksheet 4:03
Multiplying a decimal
1 a $0{\cdot}3 \times 2$
2 a $3 \times 0{\cdot}4$
3 a $0{\cdot}6 \times 10$

1 **a** $3 \times 0{\cdot}2$ **b** $2 \times 1{\cdot}2$ **c** $4 \times 1{\cdot}1$
d $5 \times 0{\cdot}08$ **e** $7 \times 0{\cdot}03$ **f** $8 \times 0{\cdot}06$
g $0{\cdot}11 \times 6$ **h** $0{\cdot}7 \times 3$ **i** $0{\cdot}04 \times 9$

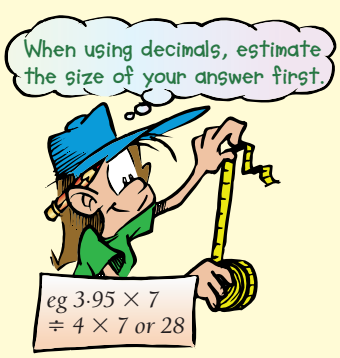
2 **a**
$$\begin{array}{r} 2{\cdot}4 \\ \times\ \ 6 \\ \hline \end{array}$$
b
$$\begin{array}{r} 1{\cdot}9 \\ \times\ \ 4 \\ \hline \end{array}$$
c
$$\begin{array}{r} 0{\cdot}25 \\ \times\ \ 4 \\ \hline \end{array}$$
d
$$\begin{array}{r} 3{\cdot}8 \\ \times\ \ 5 \\ \hline \end{array}$$
e
$$\begin{array}{r} 4{\cdot}62 \\ \times\ \ 5 \\ \hline \end{array}$$
f
$$\begin{array}{r} 2{\cdot}4 \\ \times\ \ 5 \\ \hline \end{array}$$
g
$$\begin{array}{r} 0{\cdot}63 \\ \times\ \ 3 \\ \hline \end{array}$$
h
$$\begin{array}{r} 2{\cdot}318 \\ \times\ \ 4 \\ \hline \end{array}$$
i
$$\begin{array}{r} 17{\cdot}2 \\ \times\ \ 8 \\ \hline \end{array}$$

worked example

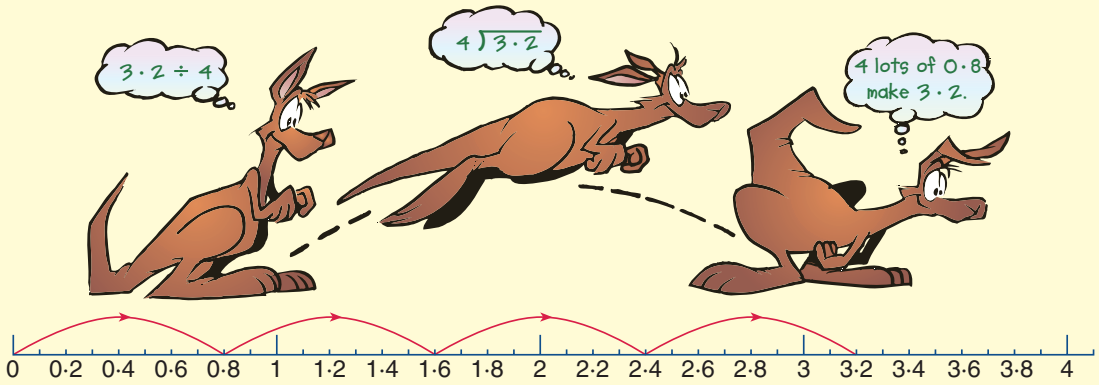
$$\begin{array}{r} 0{\cdot}125 \\ \times\ \ 8 \\ \hline 1{\cdot}000 \end{array}$$
$\therefore\ 0{\cdot}125 \times 8 = 1$

3 **a** $3{\cdot}8 \times 2$ **b** $4{\cdot}11 \times 3$ **c** $3{\cdot}08 \times 3$
d $0{\cdot}375 \times 8$ **e** $5{\cdot}11 \times 9$ **f** $6{\cdot}07 \times 5$
g $16{\cdot}43 \times 7$ **h** $26{\cdot}08 \times 6$ **i** $78{\cdot}46 \times 4$

4 **a** $1{\cdot}25 \times 10$ **b** $9{\cdot}125 \times 10$ **c** $84{\cdot}35 \times 10$
d $6{\cdot}5 \times 10$ **e** $13{\cdot}8 \times 10$ **f** $3{\cdot}7 \times 10$
g $2{\cdot}315 \times 100$ **h** $6{\cdot}85 \times 100$ **i** $0{\cdot}12 \times 100$
j $0{\cdot}3 \times 100$ **k** $8{\cdot}1 \times 100$ **l** $0{\cdot}125 \times 100$
m $0{\cdot}1111 \times 1000$ **n** $0{\cdot}015 \times 1000$ **o** $8{\cdot}1 \times 1000$
p $0{\cdot}75 \times 1000$ **q** $30{\cdot}05 \times 1000$ **r** $6{\cdot}75 \times 1000$



4:04 | Dividing a Decimal by a Whole Number



When dividing a decimal by a whole number, place the *point in the answer above* the *point in the question.*

worked examples

1 $1{\cdot}624 \div 8$ **2** $62{\cdot}1 \div 6$ **3** $0{\cdot}0136 \div 4$

Solutions

1
$$8\overline{)1\cdot_{1}6\,2_{2}4}\quad 0\cdot 2\,0\,3$$

2
$$6\overline{)6\,2\cdot_{2}1_{3}0}\quad 1\,0\cdot 3\,5$$

3
$$4\overline{)0\cdot 0\,1_{1}3_{1}6}\quad 0\cdot 0\,0\,3\,4$$

Dividing by 10, 100 and 1000

4
$$10\overline{)8\,1\cdot_{1}3_{3}0}\quad 8\cdot 1\,3$$

5
$$100\overline{)7\,5\cdot_{75}8_{58}4_{84}0_{40}0}\quad 0\cdot\ 7\ 5\ 8\ 4$$



From examples **4** and **5** above, we can see an easier way to get the answer.

When we divide by 10, 100 or 1000, we move the decimal 1, 2 or 3 places to the

That is,

4 $8\overset{\curvearrowleft}{1}{\cdot}3 \div 10$
$= 8{\cdot}13$

5 $7\overset{\curvearrowleft}{5}{\cdot}84 \div 100$
$= 0{\cdot}7584$

6 $0\overset{\curvearrowleft}{0}\overset{\curvearrowleft}{0}{\cdot}6 \div 1000$
$= 0{\cdot}0006$

Exercise 4:04

1 **a** $4\overline{)0{\cdot}84}$ **b** $3\overline{)6{\cdot}3}$ **c** $2\overline{)0{\cdot}008}$

d $5\overline{)30{\cdot}15}$ **e** $4\overline{)12{\cdot}12}$ **f** $3\overline{)9{\cdot}27}$

g $9\overline{)82{\cdot}8}$ **h** $7\overline{)0{\cdot}014}$ **i** $6\overline{)7{\cdot}308}$

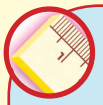
Foundation Worksheet 4:04
Dividing a decimal
1 a $2\overline{)0{\cdot}8}$
2 a $3\overline{)0{\cdot}69}$
3 a $5\overline{)2{\cdot}5}$
4 a $7\overline{)7{\cdot}14}$

2 **a** $0{\cdot}35 \div 5$ **b** $9{\cdot}8 \div 2$ **c** $5{\cdot}4 \div 6$
d $0{\cdot}11 \div 11$ **e** $0{\cdot}125 \div 5$ **f** $0{\cdot}027 \div 9$
g $8{\cdot}82 \div 7$ **h** $243{\cdot}2 \div 8$ **i** $1{\cdot}008 \div 8$

3 **a** $2\overline{)0{\cdot}5}$ **b** $2\overline{)0{\cdot}7}$ **c** $4\overline{)0{\cdot}6}$
d $5\overline{)1{\cdot}3}$ **e** $8\overline{)2{\cdot}8}$ **f** $4\overline{)1{\cdot}34}$
g $6\overline{)2{\cdot}19}$ **h** $8\overline{)1{\cdot}1}$ **i** $6\overline{)14{\cdot}1}$
j $4\overline{)3{\cdot}}$ **k** $6\overline{)9{\cdot}}$ **l** $8\overline{)5{\cdot}}$

4 **a** $34{\cdot}1 \div 10$ **b** $75{\cdot}5 \div 10$ **c** $64{\cdot}8 \div 10$
d $0{\cdot}6 \div 10$ **e** $0{\cdot}15 \div 10$ **f** $0{\cdot}05 \div 10$
g $751{\cdot}6 \div 100$ **h** $0{\cdot}38 \div 100$ **i** $6784{\cdot}5 \div 100$
j $0{\cdot}04 \div 100$ **k** $0{\cdot}35 \div 100$ **l** $35 \div 100$
m $876{\cdot}4 \div 1000$ **n** $3156{\cdot}4 \div 1000$ **o** $3437 \div 1000$
p $34{\cdot}6 \div 1000$ **q** $8{\cdot}75 \div 1000$ **r** $0{\cdot}8 \div 1000$

5 **a** $1846 \div 1000$ **b** $6{\cdot}75 \div 10$ **c** $87{\cdot}3 \div 100$
d $976{\cdot}5 \div 10$ **e** $346{\cdot}8 \div 100$ **f** $0{\cdot}8 \div 10$
g $34{\cdot}6 \div 1000$ **h** $4 \div 100$ **i** $75{\cdot}8 \div 10$



- **When dividing a decimal, you may add extra zeros to the decimal.**
- **You can place extra zeros before the whole number part too.**

worked example

$0{\cdot}31 \div 100$
$= 0\,0{\cdot}31 \div 100$
$= 0{\cdot}0031$

4:05 | Using Decimals

Decimals are used in almost every walk of life. Measurement, money and almost every area where numbers are used will demand an understanding of decimals.

Exercise 4:05

Foundation Worksheet 4:05
Using decimals
1 a $2 \times 0{\cdot}3$ kg
2 a 0·8 m + 0·3 m
3 a $3\overline{)1{\cdot}8}$ L

1 **a** $\$3.45 + \$15.52$ **b** $\$18 + \$9.65$ **c** $\$0.95 + \$3.15$
d $\$8.95 - \$3.58$ **e** $\$20 - \$6.72$ **f** $\$50.00 - \$0.35$
g $\$7.45 \times 6$ **h** $\$0.94 \times 10$ **i** $\$1.30 \times 9$
j $\$8.45 \div 5$ **k** $\$7.60 \div 10$ **l** $\$15.40 \div 7$

2 Find short cuts in answering these by using properties of numbers.
a $3{\cdot}125 \times 1$ **b** $17{\cdot}94 \times 0$ **c** $3{\cdot}008 + 0$
d $0{\cdot}9 + 1{\cdot}37 + 0{\cdot}1$ **e** $9{\cdot}83 \times 5 \times 0{\cdot}2$ **f** $(0{\cdot}9 \times 6) + (0{\cdot}9 \times 4)$
g $1 \times 36{\cdot}08$ **h** $8{\cdot}6 \times 39 \times 0$ **i** $9{\cdot}8 - 7{\cdot}9 + 7{\cdot}9$

■ Order of Operations
1 ( )
2 × and ÷
3 + and −

3 True or false?
a $0{\cdot}91 + 0{\cdot}89 = 0{\cdot}89 + 0{\cdot}91$ **b** $0{\cdot}8 \times 6 = 6 \times 0{\cdot}8$
c $15 - 7{\cdot}218 = 7{\cdot}218 - 15$ **d** $8{\cdot}4 \div 2 = 2 \div 8{\cdot}4$
e $3{\cdot}45 \times 30 = (3{\cdot}45 \times 10) \times 3$ **f** $0{\cdot}86 \times 200 = (0{\cdot}86 \times 100) \times 2$
g $0{\cdot}9 \times 8 + 0{\cdot}1 \times 8 = (0{\cdot}9 + 0{\cdot}1) \times 8$ **h** $10 \times (1{\cdot}3 + 0{\cdot}7) = 10 \times 1{\cdot}3 + 10 \times 0{\cdot}7$

4 Simplify.
a $4 + 5 \times 0{\cdot}8$ **b** $6{\cdot}1 - 0{\cdot}9 + 0{\cdot}6$ **c** $0{\cdot}3 \times 800 - 15$
d $15 - 3{\cdot}5 \times 3$ **e** $1{\cdot}6 - 8{\cdot}8 \div 8$ **f** $160 \div (1{\cdot}8 + 8{\cdot}2)$
g $5 \times 0{\cdot}3 + 0{\cdot}7 \times 4$ **h** $10 - 0{\cdot}6 \times 5 + 7$ **i** $8{\cdot}1 + 3{\cdot}95 \div 5 + 0{\cdot}9$

■ To multiply by 800, put down two zeros and multiply by 8.

5 Dermot kept a record of the distance he travelled to football grounds during one weekend. The trips were 68·4 km, 8·9 km, 42·4 km and 9 km.

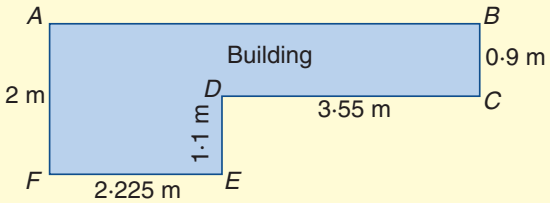
- **a** What was the total distance Dermot travelled?
- **b** What was the difference between the longest and shortest trips?
- **c** Two of the trips were to the same ground. Which two do you think they were, and why would the numbers be different?

6 A duplicating machine takes 0·8 seconds to produce each copy. How long would it take the machine to produce 100 copies?

7 It took 16·8 litres of paint to paint 4 identical rooms.

- **a** How much paint would be needed to paint one room?
- **b** How much paint would be needed to paint ten rooms?

8 $BC = 0{\cdot}9$ m, $DC = 3{\cdot}55$ m, $DE = 1{\cdot}1$ m, $FE = 2{\cdot}225$ m, $AF = 2$ m.



- **a** What two lengths have a sum equal to AB?
- **b** What is the length of AB?
- **c** What is the perimeter of the building?
- **d** What is the difference in length between FE and DC?

4:06 | Multiplying a Decimal by a Decimal

Complete:

1 $8 \times 0{\cdot}3$	**2** $0{\cdot}5 \times 6$	**3** $2{\cdot}35 \times 3$	**4** $2 \times 0{\cdot}076$
5 $0{\cdot}35 \times 10$	**6** $10 \times 7{\cdot}1$	**7** $0{\cdot}01 \times 8$	**8** $15 \times 0{\cdot}1$
9 $3{\cdot}105 \times 4$	**10** $0{\cdot}3 \times 70$		

prep quiz 4:06

We need to find a rule for multiplying a decimal by a decimal.

Let the large square here be 1 square unit of area.

$\therefore$ 1 small square $= \frac{1}{100}$ of the large square
$= 0{\cdot}01$ of the large square
$= 0{\cdot}01$ square units

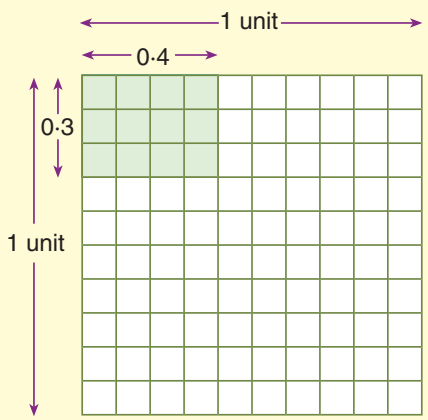
Shade a rectangle 0·4 units long and 0·3 units wide.

Area of shaded rectangle = length × breadth
$= 0{\cdot}4 \times 0{\cdot}3$ square units

But the shaded rectangle has 12 small squares.

Area of shaded rectangle $= \frac{12}{100}$ of large square
$= 0{\cdot}12$ square units

$\therefore\ 0{\cdot}4 \times 0{\cdot}3 = 0{\cdot}12$

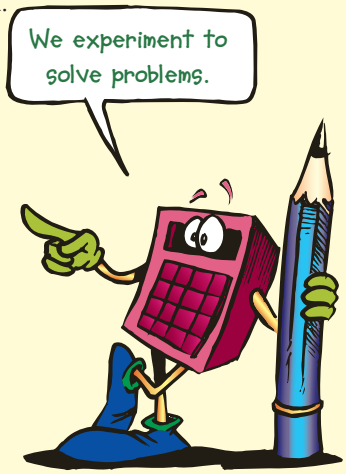


It looks as though we must have the same number of figures after the decimal point in the answer as we had altogether after decimal points in the question.

Let's use a calculator to test this idea.

Copy the table into your workbook and use a calculator to complete it.

Question	*Number of decimal places in question*	*Calculator answer*	*Number of decimal places in answer*
$0{\cdot}7 \times 6$	1		
$0{\cdot}3 \times 0{\cdot}2$	2		
$0{\cdot}15 \times 0{\cdot}3$	3		
$0{\cdot}92 \times 0{\cdot}11$	4		
$0{\cdot}123 \times 0{\cdot}1$	4		



Your solution should show that what we suspected was true.

When multiplying decimals, the number of figures after the decimal point in the answer must be the same as the total number of figures that come after the decimal points in the question.

worked examples

1 $0{\cdot}008 \times 0{\cdot}5$
$= 0{\cdot}0040$
$= 0{\cdot}004$

You can drop 'trailing' zeros like this, but *only* after you have made sure your decimal multiplication is correct.

2 $5{\cdot}5 \times 1{\cdot}7$
$= 9{\cdot}35$

3 $(0{\cdot}03)^2$
$= 0{\cdot}03 \times 0{\cdot}03$
$= 0{\cdot}0009$



Exercise 4:06

Foundation Worksheet 4:06
Multiplying decimals
1 a $0{\cdot}41 \times 2$
2 a $0{\cdot}4 \times 0{\cdot}7$
3 a $7{\cdot}2 \times 0{\cdot}3$

1 How many decimal places has each numeral?

a 9·35	**g** 0·7	**c** 0·014
d 0·0004	**e** 0·15	**f** 721·5
g 156·84	**h** 927·1	**i** 3·861

2

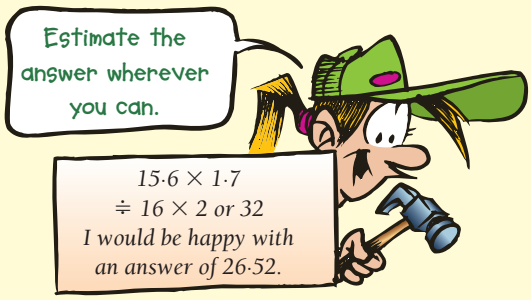
a $0{\cdot}4 \times 0{\cdot}6$	**b** $0{\cdot}11 \times 0{\cdot}8$	**c** $0{\cdot}7 \times 0{\cdot}1$	**d** $3{\cdot}1 \times 0{\cdot}2$
e $6 \times 0{\cdot}7$	**f** $0{\cdot}3 \times 0{\cdot}3$	**g** $0{\cdot}04 \times 0{\cdot}7$	**h** $0{\cdot}8 \times 0{\cdot}03$
i $0{\cdot}07 \times 0{\cdot}07$	**j** $0{\cdot}8 \times 0{\cdot}5$	**k** $0{\cdot}05 \times 0{\cdot}6$	**l** $0{\cdot}15 \times 4$
m $0{\cdot}09 \times 6$	**n** $0{\cdot}08 \times 0{\cdot}7$	**o** $0{\cdot}4 \times 0{\cdot}4$	

3

a $0{\cdot}125 \times 0{\cdot}3$	**b** $2{\cdot}14 \times 0{\cdot}6$	**c** $6{\cdot}7 \times 0{\cdot}4$
d $3{\cdot}825 \times 700$	**e** $0{\cdot}426 \times 80$	**f** $7{\cdot}6 \times 900$
g $8{\cdot}125 \times 0{\cdot}8$	**h** $0{\cdot}048 \times 0{\cdot}5$	**i** $620 \times 0{\cdot}07$
j $6800 \times 0{\cdot}002$	**k** $312\,000 \times 0{\cdot}09$	**l** $540 \times 0{\cdot}5$
m $0{\cdot}01 \times 346{\cdot}7$	**n** $0{\cdot}6 \times 87{\cdot}2$	**o** $0{\cdot}11 \times 73{\cdot}1$

Multiply as though there were no decimal points, then use the rule to place the point in your answer.

■ 15·6 ← 1 decimal place
× 1·7 ← 1 decimal place
109 2
156 0
26·52 ← 2 decimal places



4 **a** 9·6 × 3·1 **b** 0·412 × 2·4

c 1·33 × 8·3 **d** 36·51 × 0·14

e 15.6×1.7 **f** 23.8×0.12 **g** 654×0.18 **h** 0.16×0.16

i 3.5×3.5 **j** 0.25×0.25 **k** 365.1×1.4 **l** 365.1×0.14

5 **a** $(0.2)^2$ **b** $(0.03)^2$ **c** $(0.6)^2$

d $(0.8)^2$ **e** $(1.1)^2$ **f** $(0.05)^2$

g $(0.021)^2$ **h** $(1.5)^2$ **i** $(0.17)^2$

■ $0.03^2 = 0.03 \times 0.03$

4:07 | Dividing by a Decimal

prep quiz

1 $0.3 + 0.14$ **2** $6 - 0.05$ **3** $6 + 0.04 + 0.3$ **4** 2.5×6 **5** $(0.2)^2$

6 $0.3 + 0.6 \times 4$ **7** $2\overline{)2.08}$ **8** $2\overline{)2.09}$ **9** $7\overline{)38.5}$ **10** $4\overline{)0.0136}$

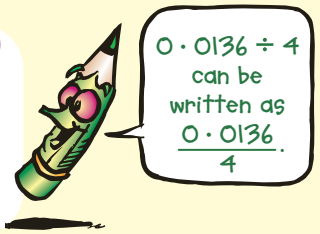
4:07

worked example 1

$0.0136 \div 4$

$= 4\overline{)0.01_{1}3_{1}6}$ $\quad 0.00\,3\,4$

When dividing by a whole number, place the point in the answer above the point in the



To divide by a decimal, we change the question so that we are dividing by a whole number.

worked example 2

$0.0136 \div 0.04$

$= \dfrac{0.0136}{0.04}$

$= \dfrac{0.0136 \times 100}{0.04 \times 100}$

$= \dfrac{1.36}{4}$

$= 0.34$

■ To multiply by 100, move the point two places right.

$\dfrac{1.36}{4} = 4\overline{)1.3_{1}6}$ $\quad 0.\,3\,4$

From this example we can see that as long as the point is moved the same number of places in each number, we will get the right answer.

$\therefore 0.0136 \div 0.04$

$= 1.36 \div 4$

further examples

1 $7{\cdot}8\,16 \div 0{\cdot}4$
$= 78{\cdot}16 \div 4$

2 $8{\cdot}6\,0\,0 \div 0{\cdot}0\,0\,2$
$= 8600 \div 2$

3 $\dfrac{875}{0{\cdot}05} = \dfrac{875{\cdot}0\,0}{0{\cdot}0\,5} = \dfrac{87500}{5}$

Exercise 4:07

1 Change each of these divisions into divisions by whole numbers.

a $8{\cdot}35 \div 0{\cdot}5$	**b** $0{\cdot}344 \div 0{\cdot}08$	**c** $0{\cdot}651 \div 0{\cdot}003$
d $0{\cdot}049 \div 0{\cdot}7$	**e** $0{\cdot}8 \div 0{\cdot}2$	**f** $7{\cdot}125 \div 0{\cdot}05$
g $38 \div 0{\cdot}04$	**h** $18 \div 0{\cdot}9$	**i** $8 \div 0{\cdot}008$
j $2{\cdot}7 \div 0{\cdot}03$	**k** $4{\cdot}72 \div 0{\cdot}8$	**l** $2{\cdot}2 \div 0{\cdot}11$

2 Find the answers to each of the parts in question 1.

3

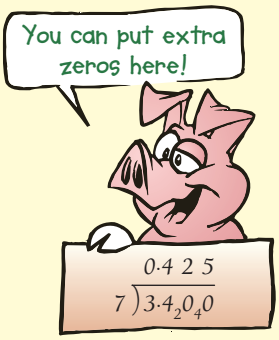
a $0{\cdot}9 \div 0{\cdot}3$	**b** $3{\cdot}6 \div 0{\cdot}9$	**c** $0{\cdot}24 \div 0{\cdot}6$
d $0{\cdot}08 \div 0{\cdot}008$	**e** $0{\cdot}8 \div 0{\cdot}04$	**f** $42 \div 0{\cdot}007$
g $365 \div 0{\cdot}04$	**h** $14 \div 0{\cdot}07$	**i** $15 \div 0{\cdot}3$
j $49 \div 0{\cdot}1$	**k** $16 \div 0{\cdot}01$	**l** $2 \div 0{\cdot}001$

4

a $5{\cdot}75 \div 2{\cdot}5$	**b** $11{\cdot}11 \div 1{\cdot}1$	**c** $4{\cdot}5 \div 0{\cdot}15$
d $7 \div 3{\cdot}5$	**e** $48 \div 2{\cdot}4$	**f** $8{\cdot}76 \div 0{\cdot}12$
g $0{\cdot}322 \div 0{\cdot}04$	**h** $0{\cdot}081 \div 0{\cdot}05$	**i** $34{\cdot}96 \div 2{\cdot}3$
j $7{\cdot}23 \div 0{\cdot}6$	**k** $5{\cdot}58 \div 0{\cdot}18$	**l** $0{\cdot}47 \div 0{\cdot}04$
m $\dfrac{25{\cdot}5}{0{\cdot}5}$	**n** $\dfrac{31{\cdot}71}{0{\cdot}04}$	**o** $\dfrac{2{\cdot}718}{0{\cdot}03}$

When moving the point to the right, you may need to add zeros.

$8{\cdot}600$

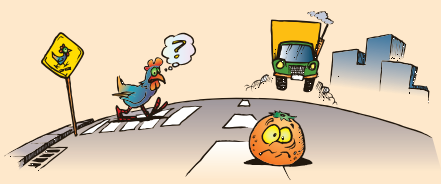


fun spot 4:07

Fun Spot 4:07 | Why did the orange stop in the middle of the road?

Work out the answer to each part and put the letter for that part in the box above the correct answer.

A $1{\cdot}58 + 2{\cdot}3$	**W** $3 + 8{\cdot}6$	**T** $3{\cdot}91 - 0{\cdot}01$
Y $8{\cdot}6 - 6$	**P** $0{\cdot}7 \times 6$	**A** $(0{\cdot}3)^2$
E $5{\cdot}2 \times 0{\cdot}2$	**H** $0{\cdot}02 \times 15$	**O** $4{\cdot}06 \div 2$
N $12{\cdot}1 \div 5$	**U** $0{\cdot}3 \div 4$	**D** $0{\cdot}24 \div 8$
T $0{\cdot}3 + 0{\cdot}008$	**E** $0{\cdot}25 \times 4$	**S** $0{\cdot}08 \times 1000$
L $0{\cdot}5 \div 100$	**S** $11{\cdot}11 \div 11$	**Q** $0{\cdot}7 + 0{\cdot}2 \times 3$



H Write $\frac{3}{10} + \frac{7}{100} + \frac{5}{1000}$ as a decimal.

A Which is the smallest: 0·38, 0·378, 0·4?

0·3	1·04	11·6	3·88	2·42	3·9	1	0·03	0·308	2·03	4·2	0·005	0·09	2·6	1·01	1·3	0·075	0·378	80	0·375

4:08 | Changing Fractions to Decimals

Write as decimals:

1 $\frac{3}{10}$ **2** $\frac{7}{100}$ **3** $\frac{196}{100}$ **4** $\frac{5}{1000}$ **5** $\frac{37}{1000}$

Simplify:

6 $\frac{16}{2}$ **7** $\frac{6}{3}$ **8** $\frac{120}{10}$ **9** $\frac{148}{4}$ **10** $\frac{8315}{5}$

$\frac{3}{10}$ ← numerator
← denominator

prep quiz 4:08

The Prep Quiz introduces two methods that we can use to change fractions to decimals.

METHOD 1 **If the fraction is written as 10th, 100ths, 1000ths, etc, put the last digit of the numerator in the matching column.**

worked examples

Change these fractions to decimals.

1 $\frac{37}{1000}$

1		$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
0	•	0	3	7

$\therefore \frac{37}{1000} = 0{\cdot}037$

2 $\frac{451}{100}$

1		$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
4	•	5	1	

$\therefore \frac{451}{100} = 4{\cdot}51$

3 $\frac{16}{5} = \frac{32}{10}$

1		$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
3	•	2		

$\therefore \frac{16}{5} = 3{\cdot}2$

METHOD 2 **To change a fraction to a decimal, divide the numerator by the denominator (divide the top by the bottom),**
eg $\frac{3}{4} = 3 \div 4$ or $4\overline{)3{\cdot}00}$.

worked examples

Change these fractions to decimals.

1 $\frac{3}{4} = 3 \div 4$

$$4\overline{)3{\cdot}_{3}0_{2}0} \quad 0{\cdot}\,7\,5$$

$\therefore \frac{3}{4} = 0{\cdot}75$

2 $1\frac{5}{8} = 1 + (5 \div 8)$

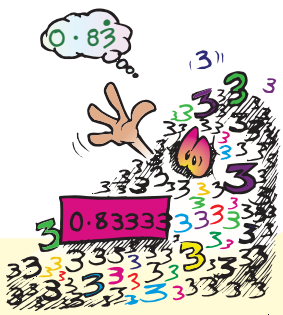
$$8\overline{)5{\cdot}_{5}0_{2}0_{4}0} \quad 0{\cdot}\,6\,2\,5$$

$\therefore 1\frac{5}{8} = 1{\cdot}625$

3 $\frac{5}{6} = 5 \div 6$

$$6\overline{)5{\cdot}_{5}0_{2}0_{2}0_{2}0_{2}0_{2}\ldots} \quad 0{\cdot}\,8\,3\,3\,3\,3\,\ldots$$

$\therefore \frac{5}{6} = 0{\cdot}8\dot{3}$



Fractions can be written as either **terminating** or **repeating** decimals.

1 A **terminating** decimal comes to a definite end. It has a definite number of decimal places; eg 3·6, 0·000125, 0·75.

2 A **repeating** or **recurring** decimal does not come to an end, but forms a pattern that is repeated indefinitely (ie forever). To show that a set of digits is repeated, we place a dot over the first and last of the repeating digits:

eg $0{\cdot}666\ldots = 0{\cdot}\dot{6}$, $1{\cdot}272727\ldots = 1{\cdot}\dot{2}\dot{7}$, $32{\cdot}4608608\ldots = 32{\cdot}4\dot{6}0\dot{8}$

Exercise 4:08

1 Is each decimal terminating or repeating?

a 0·65	**b** 0·333 . . .	**c** 21·9	**d** $8.\dot{7}$
e 0·777	**f** 0·777 . . .	**g** $0.\dot{7}$	**h** 7·7
i $14.3\dot{3}\dot{5}$	**j** 4·0408	**k** 9·105 105 . . .	**l** $2.1\dot{6}5\dot{}$

2 Write these repeating decimals in the shorter way.

a 0·999 . . .	**b** 3·1444 . . .	**c** 0·181818 . . .	**d** 0·1666 . . .
e 0·909090 . . .	**f** 6·745745 . . .	**g** 66·666 . . .	**h** 0·142857142857 . . .

3 Write each as a decimal.

a $\frac{3}{10}$	**b** $\frac{6}{100}$	**c** $\frac{9}{1000}$
d $\frac{19}{1000}$	**e** $\frac{55}{100}$	**f** $1\frac{1}{10}$
g $3\frac{77}{100}$	**h** $\frac{709}{1000}$	**i** $6\frac{2}{100}$
j $\frac{1}{2}$	**k** $\frac{44}{50}$	**l** $\frac{175}{500}$
m $\frac{7}{20}$	**n** $\frac{1}{4}$	**o** $\frac{3}{250}$

If you can change the denominator to 10, 100 or 1000, you can still use method 1,

eg $\frac{11}{20} \times \frac{5}{5} = \frac{55}{100} = 0{\cdot}55$

If using a calculator, you would use method 2.

4 Use method 2 to change these to decimals.

a $\frac{4}{5}$	**b** $\frac{1}{8}$	**c** $\frac{1}{4}$
d $\frac{3}{8}$	**e** $\frac{5}{8}$	**f** $\frac{7}{8}$
g $\frac{7}{20}$	**h** $\frac{33}{50}$	**i** $\frac{3}{4}$
j $\frac{1}{3}$	**k** $\frac{2}{3}$	**l** $\frac{1}{9}$
m $\frac{2}{9}$	**n** $\frac{4}{9}$	**o** $\frac{7}{9}$
p $\frac{5}{6}$	**q** $\frac{1}{6}$	**r** $\frac{5}{9}$



5 Change these to decimals.

a $3\frac{8}{9}$	**b** $2\frac{3}{5}$	**c** $6\frac{2}{3}$
d $\frac{3}{11}$	**e** $\frac{5}{11}$	**f** $\frac{10}{11}$
g $2\frac{5}{12}$	**h** $\frac{11}{15}$	**i** $1\frac{3}{16}$
j $\frac{4}{7}$	**k** $\frac{3}{7}$	**l** $\frac{1}{14}$

6 **a** Convert $6\frac{7}{8}$ to a decimal.

b Express $\frac{9}{11}$ as a decimal.

c Change $12\frac{1}{4}$ to a decimal.

d How would you write $\frac{5}{16}$ as a decimal?

e Write $8\frac{5}{6}$ as a decimal.

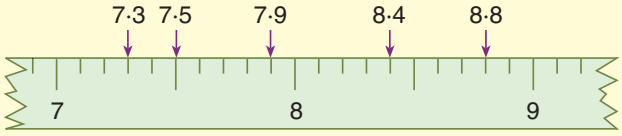
f Convert $2\frac{124}{200}$ to a decimal.

4:09 | Rounding Off

prep quiz 4:09

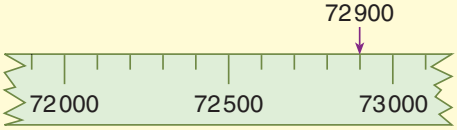
1 Is 16 closer to 10 *or* 20?
2 Is 153 closer to 150 *or* 160?
3 Is 153 closer to 100 *or* 200?
4 Is 4·1 closer to 4 *or* 5?
5 Is 4·7 closer to 4 *or* 5?
6 Is $7.66 closer to $7.60 *or* $7.70?
7 Is 7·66 closer to 7·6 *or* 7·7?
8 Is 3·111 closer to 3·11 *or* 3·12?
9 Which two numbers does 14·63 lie between: 14·5 and 14·6 *or* 14·6 and 14·7?
10 What is the decimal halfway between 6·7 and 6·8?

To **round off** a decimal to the **nearest whole number**, we write down the whole number closest to it.



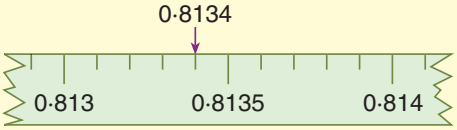
7·3 is closer to 7	7·9 is closer to 8	8·4 is closer to 8	8·8 is closer to 9
7·5 is exactly halfway between 7 and 8. In cases like this it is common to round up. We say 7·5 = 8, correct to the nearest whole number.			

To round off 72 900 to the **nearest thousand**, we write the thousand closest to it.



72 900 is closer to 73 000 than to 72 000.
∴ 72 900 = 73 000, correct to the nearest thousand.

To round off 0·8134 to the **nearest thousandth**, we write the thousandth closest to it.



0·8134 is closer to 0·813 than to 0·814.
∴ 0·8134 = 0·813, correct to the nearest thousandth.

To round off (or approximate) a number correct to a given place, we round up if the next figure is 5 or more, and round down if the next figure is less than 5.

worked examples

Round off: 1 56 700 000 to the nearest million
2 0·0851 to the nearest hundredth
3 86·149 to one decimal place
4 0·66666 to four decimal places

Solutions

1 56 700 000 has a 6 in the millions column. The number after the 6 is 5 or more (ie 7).
∴ 56 700 000 = 57 000 000, correct to the nearest million.

2 0·0851 has an 8 in the hundredths column. The number after the 8 is 5 or more (ie 5).
∴ 0·0851 = 0·09, correct to the nearest hundredth.

3 86·149 has a 1 in the first decimal place. The number after the 1 is less than 5 (ie 4).
$\therefore$ 86·149 = 86·1, correct to one decimal place.

4 0·66666 has a 6 in the fourth decimal place. The number after the 6 is 5 or more (ie 6).
$\therefore$ 0·66666 = 0·6667, correct to four decimal places.

Exercise 4:09

Foundation Worksheet 4:09
Rounding off
1 Round off to the nearest cent.
a 84·6c
2 Round off to the nearest dollar.
a $7.63

1 Round off these numbers to the nearest hundred.

a 7923	**b** 1099	**c** 67 314	**d** 853·461
e 609·99	**f** 350	**g** 74 932	**h** 7850

2 Round off these numbers to the nearest whole number.

a 9·3	**b** 79·5	**c** 45·1	**d** 2·7
e 2·314	**f** 17·81	**g** 236·502	**h** 99·5



3 Round off these numbers to the nearest hundredth.

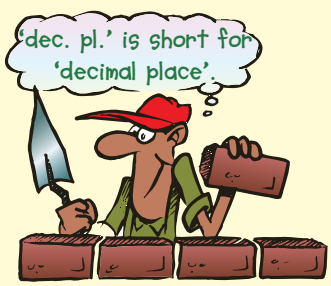
a 243·128	**b** 79·664	**c** 91·351	**d** 9·807
e 0·3046	**f** 0·0852	**g** 0·097	**h** 1·991

4 Round off these numbers to one decimal place.

a 6·70	**b** 8·45	**c** 2·119	**d** 6·092
e 0·05	**f** 246·739	**g** 29·88	**h** 9·99

5 The cost of petrol is 89·3 cents per litre. Angelika bought 12 litres of petrol. How much did this cost, correct to the nearest cent?

6 Leonie bought 12 doors at $59.30 each. What is the cost of the doors, rounded off to the nearest dollar?

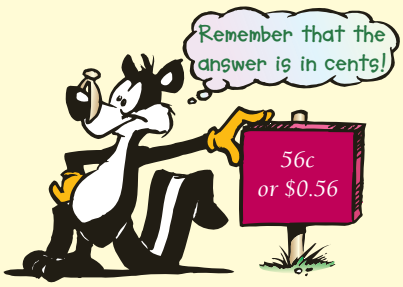


7 To change $\frac{4}{9}$ to a decimal, Mel divided 9 into 4·000 . . . Her answer was $0\cdot\dot{4}$. Give this answer correct to:

a 1 dec. pl.	**b** 2 dec. pl.
c 3 dec. pl.	**d** 4 dec. pl.

8 Peter needed to divide a 3 metre length into 11 equal parts. After dividing he got the answer $0\cdot\dot{2}\dot{7}$. Give this answer correct to:

a 1 dec. pl.	**b** 2 dec. pl.
c 3 dec. pl.	**d** 4 dec. pl.



9 These calculator displays represent an answer in cents. Round the answers off to the nearest cent.

a 1362.836	**b** 1941.341853
c 6118.91	**d** 509.5008
e 549.9342	**f** 1768049.5

10 Round off the answers shown on the displays in question 9 correct to the nearest dollar.

4:10 | Applications of Decimals

prep quiz 4:10

1 10 lots of 10c
2 $10 \times \$0.10$
3 $10 \times 0{\cdot}1$
4 I have eleven coins in my pocket. There are three dollar coins, seven ten-cent coins and one five-cent coin. How much do I have?
5 $0{\cdot}97 + 0{\cdot}033$
6 $\$20 - \$1.67$
7 $\$10 - \$0.32$
8 $\$3.45 \times 6$
9 $0{\cdot}3 \times 0{\cdot}2$
10 $0{\cdot}84 \div 0{\cdot}4$

Exercise 4:10

Foundation Worksheet 4:10
Application of decimals
1 Round off to the nearest dollar.
a $105.56
2 Round off to the nearest 5 cents.
a 73c
2 Round off to the nearest 5 cents.
a $3.98

1 Round off each amount to the nearest 5 cents.

a $34.27	b $8.52	c $0.99
d $17.11	e $85.44	f $9.55
g $103.33	h $11.68	i $27.16
j $277.37	k $108.08	l $654.72

1 cent and 2 cent pieces were withdrawn from circulation in 1990.
The final bill is rounded to the nearest five cents.

2 Petrol is sold for 87·9 cents per litre. Find the price to be paid to the nearest 5 cents for buying:

a 8 L	b 3 L	c 6 L	d 7 L
e 10 L	f 30 L	g 20 L	h 100 L
i 27 L	j 34 L	k 52 L	l 41 L
m 25 L	n 19 L	o 8·5 L	p 0·5 L

3 Coola Soda Drinks sends out their bottles in containers that hold 1000 bottles, 100 bottles or 10 bottles. 8360 bottles are to be sent out.
a What is the least number of full containers that could be used?
b What is the greatest number of full containers that could be used?



4 Katherine Collison owns a dress shop. She can purchase pink ribbon on rolls that contain 0·1 km, 0·01 km or 0·001 km of ribbon. She needs to purchase 0·836 km of pink ribbon.
a If she wanted to purchase as few rolls as possible, how many of each type of roll would she purchase? How many rolls is this altogether?
b What is the greatest number of rolls that she could purchase in order to buy this ribbon?

5 These are the winning times for the women's 100 m sprint in the Olympic Games from 1964 to 1980. 1964: 11·4 s, 1968: 11·0 s, 1972: 11·07 s, 1976: 11·08 s, 1980: 11·06 s. Arrange these times in order from least time taken to most time taken.

6 In the World Championship pairs skating finals, Jane and Christopher were given the scores 9·9, 9·8, 10, 10, 10, 10, 10, 10, 9·9, 9·1, 10 and 9·4 by the twelve judges. Two of the scores are not counted, the lowest and one of the equal highest scores.

a What is the total of the remaining ten scores?

b To get the average score we divide the total found in **a** by 10. Write their average score as a decimal.

7 When building his house, Peter drove 5 nails into a wall. The nails were in line, 0·75 metres apart. What was the distance from the first nail to the last? (*Hint:* Draw a diagram.)

8 A box is to be filled with 100 pencils, each having a mass of 9·6 grams. If the box has a mass of 54·5 grams, what will be the mass of the box full of pencils?

9 Helen bought 6·6 litres of petrol at 91·3 cents per litre. How much did she pay?

10 A carton contains 0·6 litres of milk. How many 0·25 litre glasses can be filled from this carton?

11 In the 400-metres event, the runner who came second ran the distance in 44·37 seconds. The race was won in a time of 43·79 seconds.

a What was the time difference between 1st and 2nd?

b If the winner broke the previous record by 0·07 seconds, what was the previous record?

12 A container holds less than 0·8 litres of water. Which of the following answers could be the amount in four of these containers.

A 5 litres **B** 4·2 litres **C** 3·2 litres **D** 2·4 litres

13 Each tin of paint covers an area of 100 square metres. Joan wishes to cover an area of 460 square metres.

a How many tins of paint will she need?

b What is the cost of the paint if each tin costs $27.55?

14 Joel has a salary of $32 640 per year. He divided this amount by 365·25 to find his salary per day, then multiplied by 7 to find his weekly salary. He calculated his weekly salary to be $625.544. He was paid only $625.54 each week.

a How much short of his correct weekly salary was he paid?

b If there are 52 pays this year, how much will he have lost in payments?

c If 1000 employees lost an average of $0.005 per week, how much would be lost by them altogether in 52 weeks?

d Who could profit by the loss (which would probably go unnoticed by the employees)?



Appendix E E:01 Changing repeating decimals to fractions

Fun Spot 4:10 | World championship diving

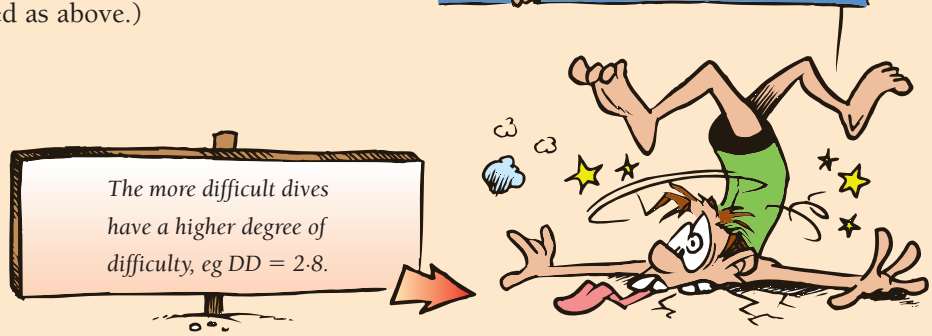
At the championships there were seven diving judges.

6	5·5	7	7	7·5	8	7·5
Degree of difficulty 2·2						

Scoring:

- Eliminate the highest and lowest points awarded and find the total of the rest.
- Multiply this total by the degree of difficulty.
- Divide the score by 5 and multiply by 3. (This converts the dive back to what it would have been if only five judges had been used with two rejected as above.)





worked example (seven judges)

Points	*Total × DD (DD = degree of difficulty)*	*÷ 5 then × 3*	*Score*
~~8~~, 7·5, 7·5, 7, 7, 6, ~~5·5~~	35 × 2·2 = 77	77 ÷ 5 × 3 = 46·2	46
~~9~~, 9, 8·5, 8·5, 8, 8, ~~7·5~~	42 × 2·8 = 117·6	117·6 ÷ 5 × 3 = 70·56	71

Calculate the score for each set of points.

1	8, 7, 7, 7, 7, 7, 6·5	DD = 2·0	**2**	6, 6, 6, 5·5, 5, 5, 4	DD = 2·4
3	4, 3·5, 3, 3, 3, 2, 2·5	DD = 2·8	**4**	9, 9, 8, 8, 8, 7·5, 7	DD = 1·8
5	6, 6, 6, 6, 6, 6, 6	DD = 2·2	**6**	7, 6·5, 6·5, 6, 6, 5, 4·5	DD = 2·6
7	8, 8, 8, 7, 6·5, 6·5, 6	DD = 2·4	**8**	2, 2, 1·5, 1·5, 1, 1, 0	DD = 2·8
9	9·5, 9·5, 9, 9, 9, 9, 9	DD = 2·6	**10**	8, 7, 6·5, 6·5, 6·5, 6, 6	DD = 2·0

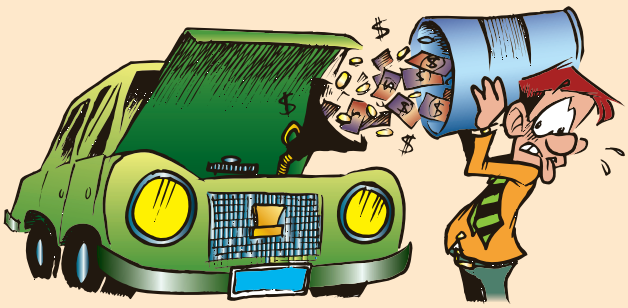
4:10A Decimal magic squares
4:10B Currency conversion
Activity GW

Investigation 4:10 | Applying decimals

Please use the Assessment Grid on the following page to help you understand what is required for this Investigation.

Calculate the cost of running the family car for one year.
You will need to estimate or discover the costs involved.

- distance travelled
- petrol costs
- tyres
- insurance
- petrol consumption
- oil and service costs
- repairs
- registration



Mathematical terms 4

ascending order
- From smallest to largest.

descending order
- From largest to smallest.

decimal (or decimal fraction)
- A way of writing a common fraction using place value.
  eg $\frac{3}{8} = 0{\cdot}375$

decimal point
- The point that is used in a decimal to separate the whole number part from the fraction part.

decimal places
- The number of digits used in a decimal that are to the right of the decimal place.

recurring (or repeating) decimal
- A decimal whose digits form a pattern that continues to repeat.
  eg $3{\cdot}7777\ldots = 3{\cdot}\dot{7}$
  $0{\cdot}5454\ldots = 0{\cdot}\dot{5}\dot{4}$

terminating decimal
- A decimal that has a definite number of decimal places.
  eg 0·375

round off (or approximate)
- To write in a simpler (less accurate) way.
  eg $0{\cdot}94999 \div 0{\cdot}95$

estimate
- To make a close guess or approximation.

Assessment Grid for Investigation 4:10 | Applying decimals

The following is a sample assessment grid for this investigation. You should carefully read the criteria *before* beginning the investigation so that you know what is required.

		Assessment Criteria (B, C, D)		Achieved ✓
Criterion B Application & Reasoning	a	No systematic or organised approach has been used to solve the problem.	1	
			2	
	b	An organised approach has been used and the student has attempted to find out the necessary data.	3	
			4	
	c	An organised approach has been used and there is some explanation of how the results were obtained.	5	
			6	
	d	There is a full explanation of the how the results were obtained.	7	
			8	
	e	Full explanations are given and all the answers are reasonable.	9	
			10	
Criterion C Communication	a	There is no working out and no explanations have been given.	1	
			2	
	b	Working out is shown and presentation is good and easy to follow.	3	
			4	
	c	Working out and explanations are well communicated using symbols or words.	5	
			6	
Criterion D Reflection & Evaluation	a	Some attempt has been made to explain the method used.	1	
			2	
	b	The method and majority of processes are justified and the answers have been checked for reasonableness with some success.	3	
			4	
	c	Reasoned explanations are given for the processes used and all answers have been checked for reasonableness.	5	
			6	
	d	Very concise and exact justifications are given for the processes used and answers have been evaluated.	7	
			8	

diagnostic test

4

Diagnostic Test 4: | Decimals

- Each section of the test has similar items that test a certain type of question.
- Failure in more than one item will identify an area of weakness.
- Each weakness should be treated by going back to the section listed.

				Section
1	Write each decimal in expanded notation.			4:01
	a 36·152	**b** 0·3333	**c** 7·039	
2	Arrange in order, smallest to largest.			4:01
	a 0·3, 0·9, 0·6	**b** 0·4, 0·11, 0·6	**c** 0·7, 0·77, 0·077	
3	**a** 0·45 + 12·3 + 4	**b** 0·008 + 0·15 + 0·9	**c** 0·95 + 0·005 + 0·5	4:02
4	**a** 23·587 − 4·35	**b** 880·6 − 77·03	**c** 50 − 6·9	4:02
5	**a** 0·015 × 8	**b** 3·9 × 6	**c** 4·72 × 4	4:03
6	**a** 5·35 ÷ 5	**b** 6·2 ÷ 5	**c** 11·031 ÷ 6	4:04
7	**a** 0·4 × 100	**b** 3·375 × 10	**c** 1·56 × 1000	4:03
8	**a** 134·65 ÷ 100	**b** 9·135 ÷ 1000	**c** 0·045 ÷ 10	4:04
9	**a** 0·008 × 0·5	**b** 11·2 × 0·11	**c** $(0{\cdot}03)^2$	4:06
10	**a** 0·0136 ÷ 0·04	**b** 8·6 ÷ 0·002	**c** $\frac{875}{0{\cdot}05}$	4:07
11	Is each decimal terminating or repeating?			4:08
	a 0·75	**b** 1·272727 . . .	**c** $1{\cdot}\dot{2}\dot{7}$	
12	Write each as a decimal.			4:08
	a $3\frac{77}{100}$	**b** $3 + \frac{7}{10} + \frac{7}{100}$	**c** $6\frac{7}{1000}$	
13	Write each as a decimal.			4:08
	a $\frac{5}{8}$	**b** $\frac{1}{6}$	**c** $1\frac{10}{11}$	
14	Round off these decimals correct to two decimal places.			4:09
	a 78·484	**b** 0·6666 . . .	**c** 9·295	

Chapter 4 | Revision Assignment

assignment 4A

1 **a** Write the numeral for one million, three hundred and twenty-six thousand, seven hundred and sixty-three.
b Write in words: 16 020.
c What is the value of the 1 in 716 250?
d Write 42 375 in expanded notation.

2 Simplify:
a $(4 + 3) \times (8 - 2)$
b $17 - (12 - [2 + 10])$
c $\dfrac{25 + 35}{5}$ **d** $\dfrac{12}{(1 + 3)}$

3 Simplify:
a $16 + 2 \times 4 - 5$
b $18 + 24 \div (10 - 4)$
c $200 - 120 \div 4 - 3 \times 5$
d $18 \div [54 - (24 \times 2)]$

4 Are the following true or false?
a $6 \times 96 > 6 \times 100$ **b** $0{\cdot}3 < 0{\cdot}0333$
c $\frac{1}{10} < \frac{1}{2}$ **d** $\frac{12}{2} = \frac{2}{12}$

5 Copy and complete these number sentences to make each sentence true.
a $6 \times (300 + 6) = \square \times 300 + \square \times 6$
b $73 \times 10 = 10 \times \square$
c $6 \times 199 = 6 \times 200 - 6 \times \square$
d $98 + 47 + 2 = 98 + \square + 47$

6 Change these Roman numerals into our own numerals.
a LXXXVII **b** CCXCIII
c MDCLXIV **d** MCMXC

7 Write each in expanded form and as a basic numeral.
a 4^3 **b** 3^3 **c** 10^6
d 2×10^4 **e** $3^2 \times 2^4$

assignment 4B

id 4

Chapter 4 | Working Mathematically

1 Use ID Card 5 on page xvii to identify:

a 1 b 2 c 3 d 4
e 5 f 6 g 7 h 8

2 In an election one candidate received 9873 votes, another received 7828 votes, and the third 2314. If 215 voting papers were informal (not filled out properly) and there were 20 609 voters in the electorate, how many did not vote?



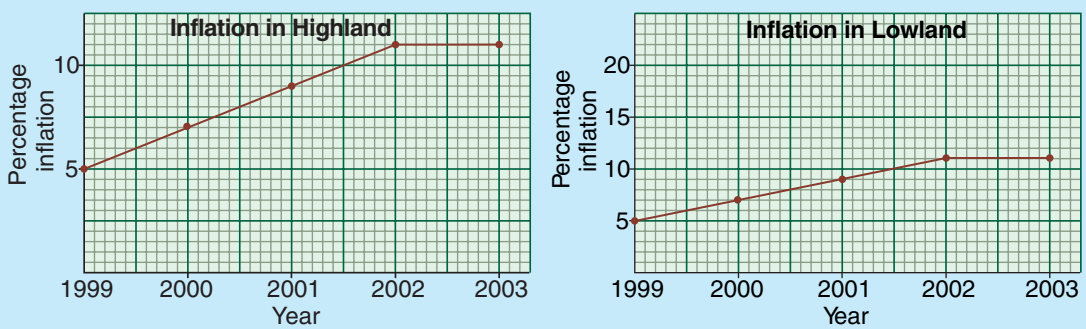
3 A school hall has 24 rows of chairs with 20 chairs in each row. If the chairs are reset in rows of 16, how many rows will there be?

4 A publishing company wants to send some advertising material to 650 schools. If an envelope costs 6 cents, the advertising pamphlet costs 38 cents to produce and the stamp for the letter costs 55 cents, find how much it costs to send the advertising material to the schools.

5 A tennis tournament has 64 players. If only the winner of each match continues in the competition, how many matches must be played to obtain the winner of the tournament?

6 A group of units has four different buildings. Each building has five floors with six apartments on the first four floors, and four apartments on the fifth floor. How many apartments are there in the group of units?

7



a What was Highland's inflation rate in 1999?
b What was Lowland's inflation rate in 2001?
c What was Highland's increase in inflation rate from 1999 to 2002?
d Which place appears to have the higher inflation rate? What is the real situation?
e In which year did Lowland first have an inflation rate of 11%?

Drag and Drop CW Interactive 1 Writing fractions as decimals

Teacher's Resource TR Chapter Test

Review CW Questions

5

Directed Numbers and the Number Plane



Chapter Contents

5:01 Graphing points on the number line
5:02 Reading a street directory
5:03 The number plane
Mastery test: The number plane
5:04 Directed numbers
5:05 Adventure in the jungle
Investigation: Directed numbers
5:06 Addition and subtraction of directed numbers
5:07 Subtracting a negative number
ID Card
5:08 The number plane extended
5:09 Multiplication of directed numbers
Investigation: Multiplying directed numbers
Challenge: Using pronumerals
5:10 Division of directed numbers
5:11 Using directed numbers
5:12 Directed numbers and the calculator
Fun Spot: Try this quick quiz!
Mathematical Terms, Diagnostic Test, Revision Assignment, Working Mathematically

Learning Outcomes

Students will:

- Compare, order and calculate with integers.
- Create, record, analyse and generalise number patterns using words and algebraic symbols in a variety of ways.
- Graph and interpret linear relationships on the number plane.

Areas of Interaction

Approaches to Learning, Homo Faber

5:01 | Graphing Points on the Number Line

prep quiz 5:01

Write down the number sentence represented by each number line.

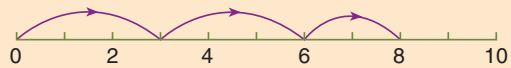
1


. . . + . . . = 10

2


. . . + . . . = 9

3


4


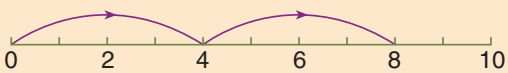
5


6


7


8


9


10


The number line can be very useful. It can help us to perform additions, subtractions, multiplications and even divisions. The number line can also be used to graph sets of numbers.

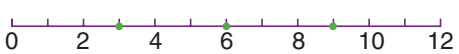
To graph a number on the number line, place a large dot at the position of that number on the number line.

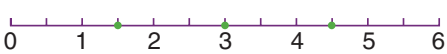
worked examples

Graph the following sets of points:

1 {2, 4, 5}


2 {0, 1, 2, 3, 4}


3 {3, 6, 9}


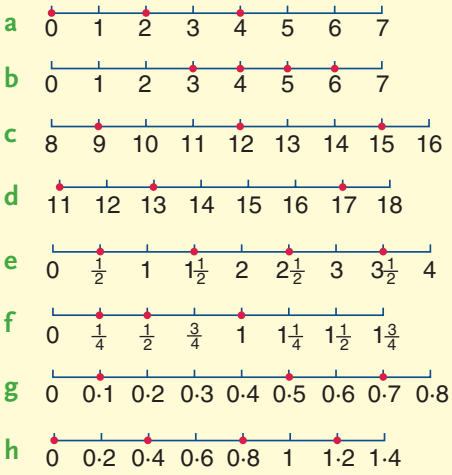
4 $\{1\frac{1}{2}, 3, 4\frac{1}{2}\}$


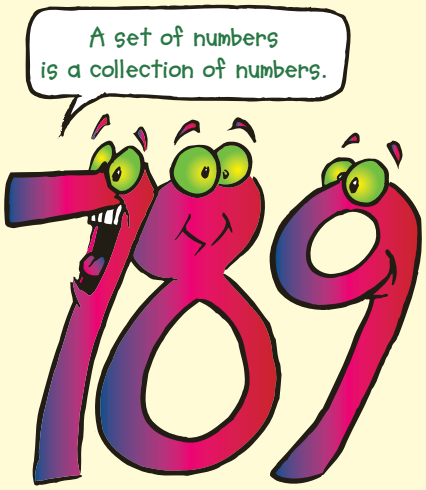
5 {0·2, 0·5, 1·1}




Exercise 5:01

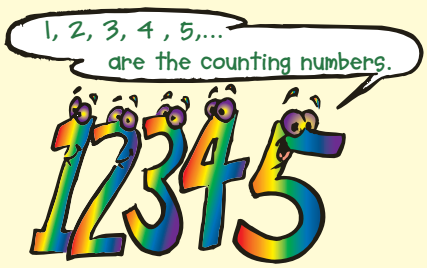
1 Use braces to show the set of numbers graphed on each number line.





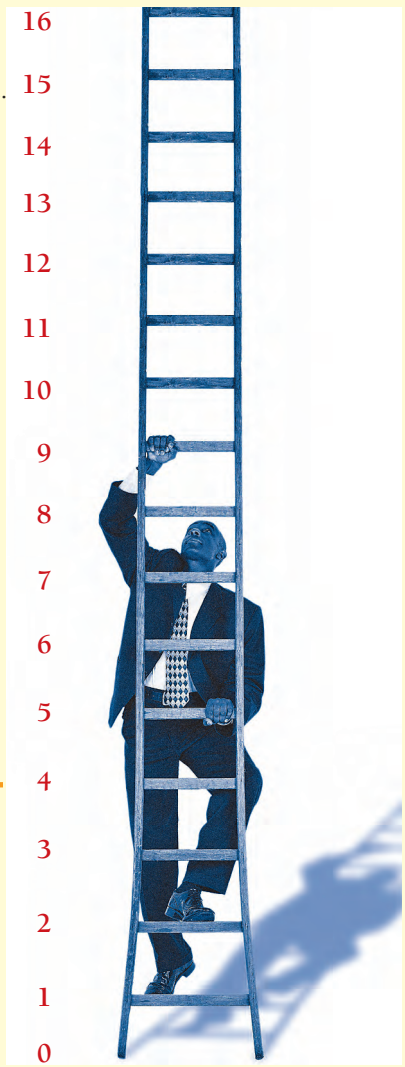
2 Graph each set of numbers below on a separate number line.

a {1, 3, 5}	**b** {0, 2, 4, 6}	**c** {2, 3, 5, 7}
d {0}	**e** {1, 2, 3, 4, 5}	**f** {10, 12, 14}
g {0·3, 0·5, 0·6}	**h** $\{\frac{1}{4}, \frac{3}{4}, 1\}$	**i** {0, 0·5, 1, 1·5}



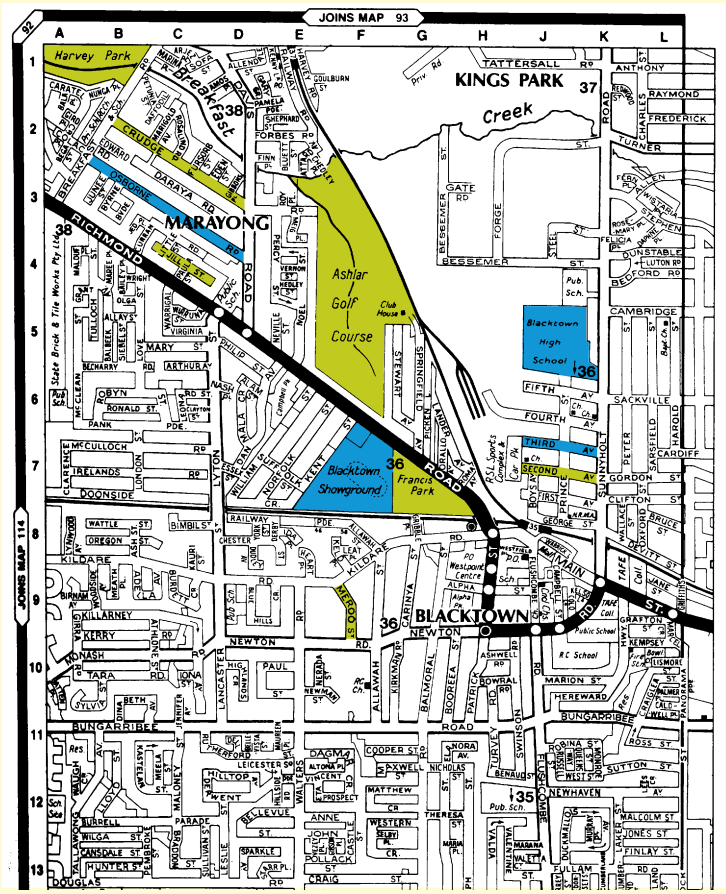
3 Graph each set on a separate number line.

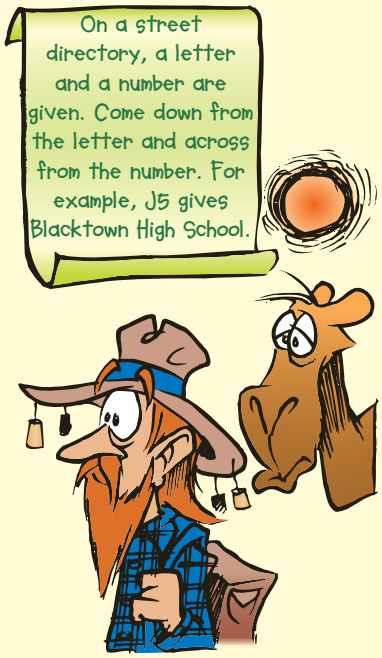
a the counting numbers between 1 and 6
b the numbers two away from 5
c the odd numbers less than 8
d the counting numbers less than 4
e the even numbers between 11 and 17
f the even numbers between 8 and 12



- Number lines can be vertical.

5:02 | Reading a Street Directory





Exercise 5:02

1 Name the places or roads closest to the centre of the following reference coordinates.

a H4	**b** K3	**c** B3	**d** E5	**e** F8	**f** H1
g A6	**h** D9	**i** C5	**j** C2	**k** B5	**l** F9
m E2	**n** J5	**o** D8	**p** A3	**q** E4	**r** D5

2 Which reference coordinates could be used to show where to find:

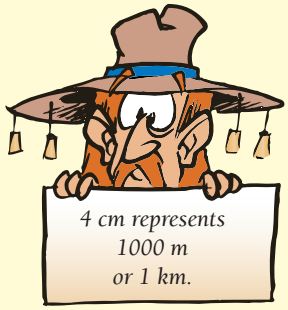
a Ashlar Golf Course?
b Harvey Park (top left)?
c Francis Park?
d Crudge Road (top left)?
e Blacktown Showground?
f Meroo Street (centre)?
g Jill Street (near Richmond Road)?
h Second Avenue (off Sunnyholt Road)?

3 It is not always easy to decide on the best reference coordinate for a street.

- **a** What three reference coordinates might be given for Osborne Road, which is on the top left of the map?
- **b** Name the two roads that meet at D4.
- **c** What reference coordinates would be used for Third Avenue?

4 On this map 1 cm represents 250 m.

- **a** Find the approximate length of Crudge Rd (B2).
- **b** Find the approximate length of Newton Rd (D10).
- **c** Find the approximate length of Ashlar Golf Course.
- **d** Find (approximately) the shortest distance by road from the intersection of Osborne and Davis Roads (D4) to Blacktown High School.

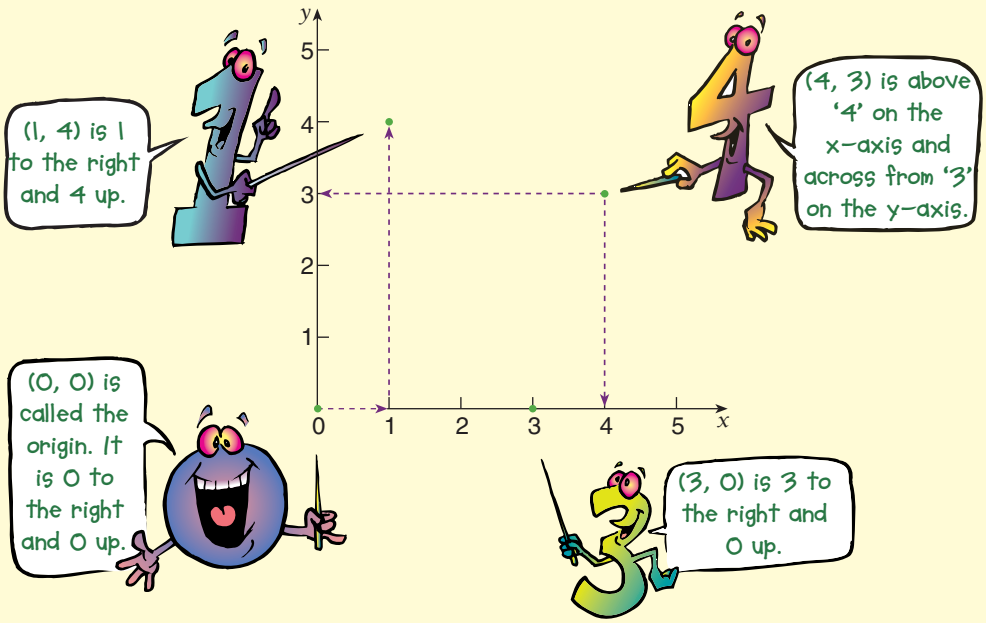


5:03 | The Number Plane

On the street directory, a letter and a number were used to name a position. On the **number plane**, two numbers are used. These are called **coordinates**, and are written in parentheses and separated by a comma. The two axes used are called the ***x*-axis** (horizontal) and the ***y*-axis** (vertical).
The ***x*-coordinate** is always written before the ***y*-coordinate**.

(2, 5) is called a *number pair* or the *coordinates of a point*.

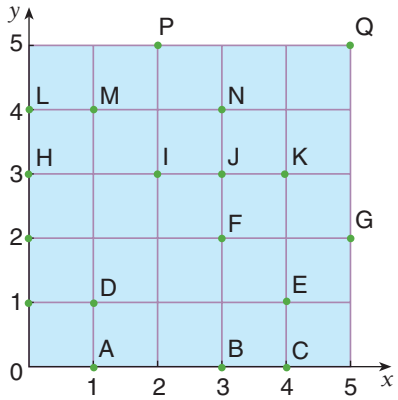
- **The first number is the reading on the *x*-axis (how far right).**
- **The second number is the reading on the *y*-axis (how far up).**
- **The point (0, 0) is called the *origin*. ('Origin' means 'starting point' or 'beginning'.)**



worked examples

1 Write down the letter naming the points:
a (3, 4) **b** (2, 3) **c** (1, 0)

2 What are the coordinates of:
a G? **b** L? **c** P?



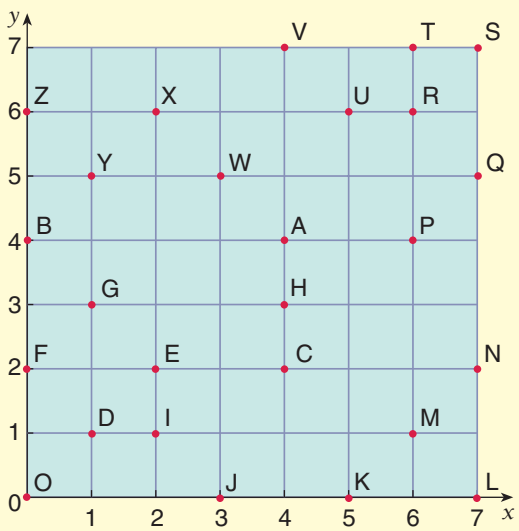
Solutions

1 To find each point, start at the origin.
a (3, 4) is 3 to the right and 4 up.
∴ (3, 4) is N.
b (2, 3) is 2 to the right and 3 up.
∴ (2, 3) is I.
c (1, 0) is 1 to the right and 0 up.
∴ (1, 0) is A.

2 **a** G is above 5, and to the right of 2.
∴ G is (5, 2).
b L is above 0, and on 4.
∴ L is (0, 4).
c P is above 2, and to the right of 5.
∴ P is (2, 5).

Exercise 5:03

1 Write down the letter used to name each of these points.



a (1, 3) **b** (2, 2)
c (5, 6) **d** (4, 7)
e (0, 0) **f** (7, 5)
g (3, 5) **h** (6, 6)
i (0, 6) **j** (0, 2)
k (3, 0) **l** (5, 0)
m (4, 3) **n** (6, 4)
o (7, 2) **p** (1, 5)
q (2, 1) **r** (4, 4)
s (4, 2) **t** (7, 7)
u (0, 4) **v** (6, 1)
w (7, 0) **x** (6, 7)
y (1, 1) **z** (2, 6)



2 Use the number plane in question 1 to write down the coordinates of these points.

a A **b** B **c** C **d** D **e** E **f** F **g** G
h H **i** I **j** J **k** K **l** L **m** M **n** N
o O **p** P **q** Q **r** R **s** S **t** T **u** U

3 Make a grid that has 10 equal units on the *x*-axis and 7 equal units on the *y*-axis. You may draw your own or use graph paper. Plot the points below on your number plane. What shape is formed when each set of points is joined in the given order?

a (7, 1) (7, 3) (9, 3) (9, 1) (7, 1)
b (4, 1) (4, 4) (5, 4) (5, 1) (4, 1)
c (6, 0) (6, 3) (7, 0) (6, 0)
d (0, 4) (2, 6) (5, 6) (3, 4) (0, 4)
e (1, 0) (0, 1) (0, 2) (1, 3) (2, 3) (3, 2) (3, 1) (2, 0) (1, 0)

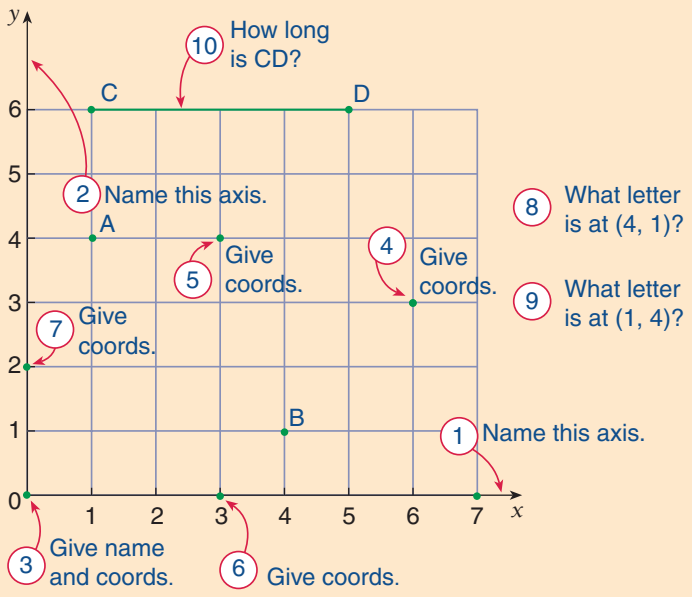
4 This time, draw a grid with 10 units on the *x*-axis and 16 units on the *y*-axis. All units must be the same size. Draw each shape on this number plane as you did in question 3.

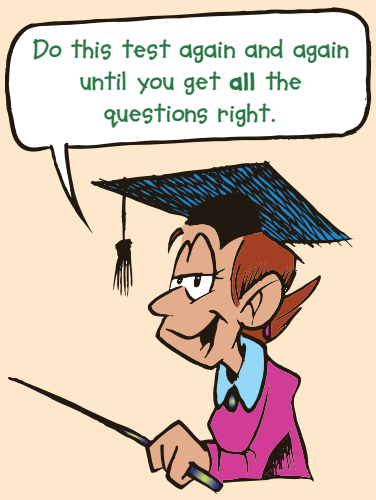
a (2, 5) (2, 3) (1, 1) (1, 0) (2, 0) (3, 1) (4, 4)
b (8, 14) (7, 14) (7, 12) (8, 11) (8, 9) (9, 9)
c (2, 11) (3, 11) (4, 12) (5, 12) (5, 10) (4, 10) (1, 8) (0, 9) (0, 10) (1, 12) (5, 16) (6, 16) (9, 13) (10, 10) (9, 9) (9, 7) (7, 4) (7, 3) (8, 1) (8, 0) (7, 0) (6, 1) (5, 4) (4, 4) (2, 5) (1, 7) (1, 8)
d Put a big nose on your picture by starting from (5, 14) and ending at (6, 14).
e At (5, 15) and (6, 15) draw the pupils of Fred's eyes. Draw the eyes around these.
f Draw in a small mouth and the fingers. Colour your picture.

5 Use 5 millimetre grid paper with 30 units on the *x*-axis and 24 units on the *y*-axis. (All units must be the same size.) Draw each shape indicated by the coordinates as you have done above. When you join the points, join them with curved lines.

a (6, 0) (8, 4) (10, 6) (12, 8) (10, 10) (8, 11) (2, 10) (4, 12) (8, 13) (12, 14) (6, 14) (2, 12) (2, 10) (1, 12) (2, 16) (4, 18) (6, 18) (8, 20) (12, 22) (16, 23) (20, 23) (24, 22) (28, 20) (26, 20) (29, 16) (28, 16) (29, 12) (28, 12) (29, 8) (28, 8) (30, 4) (29, 4) (30, 0)
b (8, 13) (6, 14)
c (11, 17) (11, 19) (12, $19\frac{1}{2}$) (13, 19) (13, 17) (12, $16\frac{1}{2}$) (11, 17)
d Colour the inside of the mouth to the right of (8, 13) and above it.
e Draw a large black dot in the eye.
f Add feathers and colour the eagle.

Mastery test 5:03 | The number plane





5:04 | Directed Numbers

Many of the numbers we use represent situations that have direction as well as size. We call these numbers **directed numbers**.

Once a direction is chosen to be **positive** (+), the **opposite** direction is taken to be **negative** (–).

Choosing directions for numbers
- If north is positive, then south is negative.
- If east is positive, then west is negative.
- If above zero degrees is positive, then below zero degrees is negative.
- If to the right is positive, then to the left is negative.



examples

1. Mount Everest is 8000 metres above sea level, so its height could be written as +8000 m or just 8000 m.
2. The Dead Sea is 100 metres below sea level, so its height could be written as −100 m.
3. In cold countries the temperature is often below the freezing point of water (0°C). The temperature on a cold day could be −5°C.

−20 is read 'negative 20' or 'minus 20'.
+20 is read 'positive 20' or just '20'.
***Note:* +20 can be written more simply as 20.**

Exercise 5:04

1 **a** Use a directed number to represent each of the following.

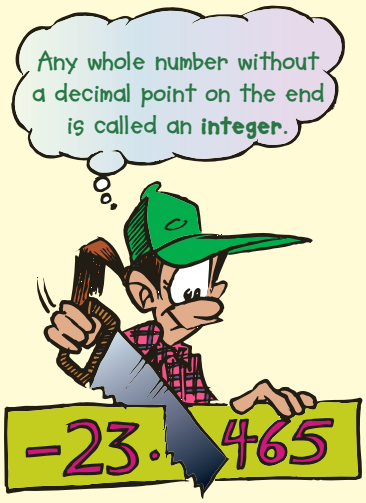
(A) a profit of $50	(B) 50 km south	(C) a fall of 50 points
(D) a gain of 50 kg	(E) $50 deposit	(F) 50 m above sea level
(G) 50° below zero	(H) 3 floors above ground	(I) an increase of 50
(J) 50 cm to the right	(K) 50 s after takeoff	(L) 50 km east
(M) 50 s before takeoff	(N) 50° above zero	(O) 50 cm to the left
(P) a rise of 50 points	(Q) a loss of $50	(R) a loss of 50 kg
(S) 50 km west	(T) 50 m below sea level	(U) a decrease of 50
(V) $50 withdrawal	(W) 3 floors below ground	(X) 50 km north

b Match up each of A to L in part **a** with its opposite chosen from M to X.

Directed numbers have both size and direction.

2 Write a directed number to show the size and direction of the number in each sentence.

- **a** Jan's Mathematics mark was 8 marks higher this time.
- **b** Today, the temperature is 4°C below yesterday's temperature.
- **c** John took fourteen steps backwards.
- **d** After playing marbles, Melissa had lost 15 marbles.
- **e** The price of Microsoft shares rose by twelve cents last week.
- **f** The level of water in the reservoir has dropped two metres.
- **g** The price of fish has not changed this week.
- **h** My profits have increased by $1800 this year.
- **i** In the last hour we have travelled 60 km south.
- **j** I have just discovered that my bank balance is overdrawn by $135.



3 Write two different sentences that might be represented by the directed number −3.

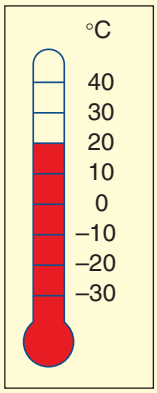
4 Write two different sentences that might be represented by the directed number +7.

5 Write a directed number to show the change described in each sentence.

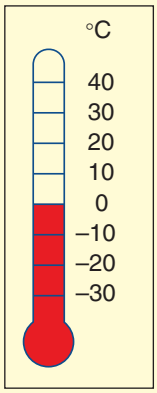
- **a** The temperature fell from 37°C to 26°C.
- **b** I had ten golf balls when I arrived. Now I have only 3.
- **c** I was 4 km east of home. Now I am 2 km west of home.
- **d** I bought shares for $23 per share and sold them for $19 per share.
- **e** The diver was 2·4 m below the surface. She is now 4·4 m below the surface.
- **f** The frog was 2 cm behind the line. After the jump it was 5 cm in front of the line.
- **g** We entered the lift on the tenth floor. We left the lift one floor below ground level.

6 What temperature is shown on each thermometer below?

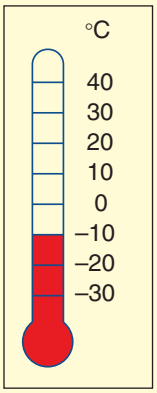
a



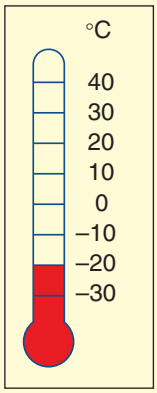
b



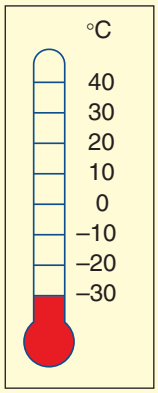
c



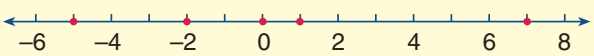
d



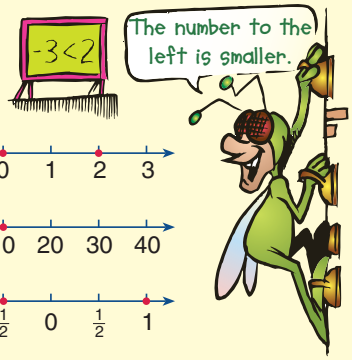
e

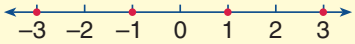


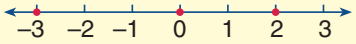
7 We can extend our number line to the left of zero to include negative numbers. Use braces to show the set of numbers graphed on each number line,

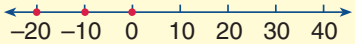
eg 

This set of numbers is {−5, −2, 0, 1, 7}.

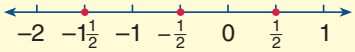


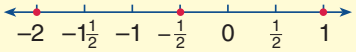
a 

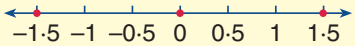
b 

c 

d 

e 

f 

g 

h 

8 Graph each set of numbers below on a separate number line.

a {−2, −1, 0} **b** {−4, −2, 0, 2} **c** {−1, 0, 1}
d {−1·5, −0·5, 0·5} **e** {−2, −0·5, 1} **f** {−0·8, −0·6, −0·2}

9 Which is the lower temperature?

a −2°C or 2°C **b** −5°C or 0°C **c** −5°C or 2°C
d −5°C or −2°C **e** −5°C or −10°C **f** 7°C or −7°C

10 Arrange these directed numbers in order from smallest to largest.

a 1, 5, −2, 4 **b** 0, −4, 1, −1 **c** −1, −2, −3, −4
d 2, −2, 3, −3 **e** −2, 4, −3, 0 **f** 7, −8, 9, −10

An integer is a directed whole number. . . . , −3, −2, −1, 0, 1, 2, 3, . . .

When playing snakes and ladders:
- going up a ladder would be a move in a positive direction
- going down a snake would be a move in a negative direction.



Activity CW™ **5:04A Plotting points**
5:04B Coordinate noughts and crosses

5:05 | Adventure in the Jungle

Miss Travers decided to play a game with her Grade 6 class.

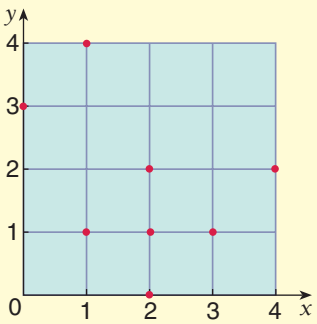
She drew a number plane on the board and for each part of her favourite adventure she chose coordinates. She then decided on a positive or negative score for each part of the adventure. So each part of the adventure was represented by a point on the number plane.

Keeping secret the coordinates she had chosen, she asked the members of two teams to take turns to select coordinates on the number plane. Two class members kept a total score for each team on the board.

The game continued until each team had made ten choices. The team with the highest score won the game.



Part of the adventure	*Score (points)*	*Miss Travers' secret coordinates*
1 Photograph a zebra	1	(0, 2)
2 Photograph a gnu	2	(1, 3)
3 Photograph a lion	3	(2, 4)
4 Photograph a tiger	4	(1, 1)
5 Photograph a rhinoceros	5	(4, 0)
6 Meet friendly natives	6	(3, 2)
7 Helped by Tarzan	7	(2, 0)
8 Discover a lost city	8	(0, 0)
9 Bridge destroyed	−3	(1, 0)
10 Lose your compass	−4	(0, 4)
11 Lost in the jungle	−5	(3, 1)
12 Shoot yourself in foot	−6	(2, 2)
13 Bitten by snake	−7	(3, 4)
14 Fall into quicksand	−8	(4, 4)
15 Selecting a coordinate already chosen	−2	These change for each new game.
16 All other coordinates	−1	



As each selection is made, that point is graphed on this number plane.

Team 1	*Team 2*
−1	7
−6	6
−7	5
−3	

worked examples

Each team hopes to guess the secret coordinates that have positive values. The table on the previous page records the scores of the choices made by the two teams below.

TEAM 1			
Coordinates chosen	*Result*	*Score*	*Total so far*
(1, 4)	Miss	−1	−1
(3, 1)	Lost in the jungle	−5	−6
(4, 2)	Miss	−1	−7
(1, 1)	Photograph tiger	4	−3

TEAM 2			
Coordinates chosen	*Result*	*Score*	*Total so far*
(2, 0)	Helped by Tarzan	7	7
(2, 1)	Miss	−1	6
(0, 3)	Miss	−1	5
(2, 2)	Shoot yourself in foot	−6	−1



Exercise 5:05

1 Copy and complete the following using Miss Travers' secret coordinates from page 129.

a

Coordinates chosen	*Result*	*Score*	*Total so far*
(0, 0)			
(0, 4)			
(2, 1)			
(1, 0)			
(1, 3)			
(3, 4)			

b

Coordinates chosen	*Result*	*Score*	*Total so far*
(3, 1)			
(3, 2)			
(0, 2)			
(2, 2)			
(4, 1)			
(1, 1)			

2 Copy and complete the following:

a

Coordinates chosen	*Score*	*Total so far*
(0,4)		
(1, 4)		
(2, 4)		
(3, 4)		
(4, 4)		

b

Coordinates chosen	*Score*	*Total so far*
(4, 4)		
(3, 3)		
(2, 2)		
(1, 1)		
(0, 0)		

c

Coordinates chosen	*Score*	*Total so far*
(3, 0)		
(3, 1)		
(3, 2)		
(3, 3)		
(3, 4)		

3 Copy and complete the following:

a

Score	Total so far
−1	
4	
−2	
1	
−5	

b

Score	Total so far
1	
−6	
−4	
−3	
5	

c

Score	Total so far
−1	
−1	
−1	
−1	
−1	

d

Score	Total so far
−8	
1	
2	
3	
4	



4 Combine the following scores to find the total so far.

a a loss of 6, a gain of 3, a loss of 1
b a gain of 5, a loss of 3, a gain of 6
c a gain of 2, a loss of 1, a loss of 2
d a loss of 1, a gain of 3, a loss of 7
e a loss of 5, a loss of 4, a loss of 8
f a loss of 7, a gain of 2, a gain of 4
g −3, −4
h −2, 5
i 1, −4
j 6, −6
k −8, −7
l −6, 4
m −3, −2
n 8, 6
o −1, 8
p −7, −6
q −1, −2
r −8, −1
s −3, 4, −1
t 2, −5, 1
u 7, −5, −3
v 4, −7, 3

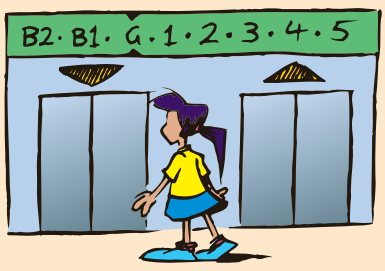
Investigation 5:05 | Directed numbers

investigation 5:05

Please use the Assessment Grid on the following page to help you understand what is required for this Investigation.

Collect magazine or newspaper articles (including graphs) that use directed number quantities and explain in each case how the directed numbers were used.





Assessment Grid for Investigation 5:05 | Directed numbers

The following is a sample assessment grid for this investigation. You should carefully read the criteria *before* beginning the investigation so that you know what is required.

Assessment Criteria (B, C, D)				Achieved ✓
Criterion B Application & Reasoning	a	There has been no organised effort to collect material.	1	
			2	
	b	An organised approach has been attempted to find the material.	3	
			4	
	c	An organised approach has been used and there is some discussion of how numbers are used.	5	
			6	
	d	There is a full discussion of the how the numbers are obtained.	7	
			8	
	e	There is a full and reasonable discussion on how the numbers are used.	9	
			10	
Criterion C Communication	a	There is no setting out of work or discussion given.	1	
			2	
	b	Presentation is good and the discussion is easy to follow.	3	
			4	
	c	Ideas are well communicated using examples and words.	5	
			6	
Criterion D Reflection & Evaluation	a	Some attempt has been made to explain how numbers are used.	1	
			2	
	b	The examples given are used to justify the discussion.	3	
			4	
	c	Reasoned explanations are given for why the numbers are used the way they are, using the examples.	5	
			6	
	d	Very concise and exact justifications are given for the way in which the numbers are used.	7	
			8	

5:06 | Addition and Subtraction of Directed Numbers

In the game of indoor cricket, 1 point is gained for every run made and 5 points are lost every time a player gets out. So 1 run and 1 player out gives a total score of (1) + (−5) = −4. Find the total score for each of these.

1 6 runs and 1 player out
2 2 runs and 1 player out
3 5 runs and 1 player out
4 4 runs and 1 player out
5 1 player out and 10 runs
6 1 player out and 3 runs
7 1 player out and 5 runs
8 1 player out and no runs
9 2 players out and 0 runs
10 8 runs and 2 players out



prep quiz 5:06

worked examples

1 Adding directed numbers:
In Section 8:05 and in the Prep Quiz we have been adding directed numbers.

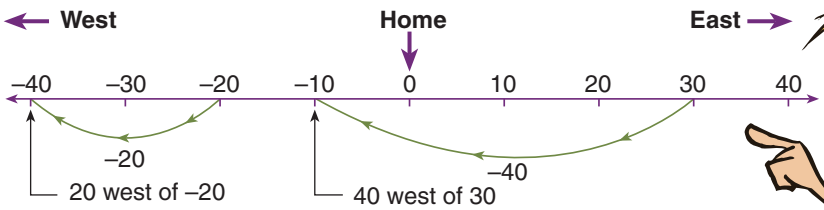
a −5 + 3 = −2 or (−5) + (3) = −2
A loss of 5 and a gain of 3 gives a loss of 2.

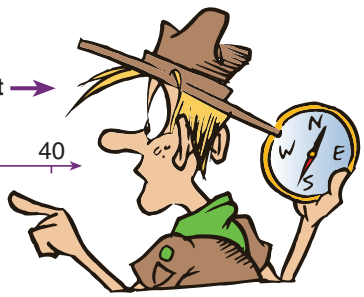
b −3 + −5 = −8 or (−3) + (−5) = −8
A loss of 3 and a loss of 5 gives a loss of 8.

c 1 + −10 = −9 or (1) + (−10) = −9
A gain of 1 and a loss of 10 gives a loss of 9.

■ A negative number can represent a loss; a positive number can represent a gain.

2 Using the east–west road to subtract positive numbers:

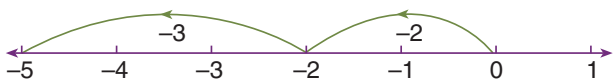




a −20 − 20 = −40
−20 take away 20 gives −40.

b 30 − 40 = −10
30 take away 40 gives −10.

3 Using the number line for addition and subtraction:



The number line could be used to show that:
(−2) + (−3) = −5 or −2 − 3 = −5.
A loss of 2 and a loss of 3 gives a loss of 5.

Adding the negative of a number is the same as subtracting the positive number, eg −2 + −3 = −2 − 3.

Exercise 5:06

1

a $-8+4$	**b** $-3+5$	**c** $-6+6$	**d** $-10+3$
e $-1+12$	**f** $-6+8$	**g** $-11+8$	**h** $-3+7$
i $(-2)+(-3)$	**j** $(-6)+(-1)$	**k** $(-7)+(-10)$	**l** $(-4)+(-4)$
m $-10+-3$	**n** $-8+-1$	**o** $-3+-5$	**p** $-8+-2$
q $(4)+(-3)$	**r** $(9)+(-15)$	**s** $(2)+(-1)$	**t** $(100)+(-7)$
u $9+-1$	**v** $8+-8$	**w** $6+-11$	**x** $1+-7$

2 Use the east–west road on page 220 to write a number sentence, giving the answer for each as a directed number.



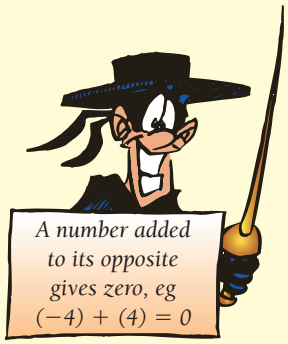
a 30 west of 10	**b** 20 east of −10
c 30 east of 10	**d** 20 west of −10
e 50 west of 5	**f** 40 east of −20
g 20 east of −53	**h** 100 west of 33
i 33 east of −4	**j** 33 east of −6

3

a $2-0$	**b** $2-1$	**c** $2-2$	**d** $2-3$
e $2-4$	**f** $2-5$	**g** $0-2$	**h** $-1-2$
i $-6-1$	**j** $1-4$	**k** $3-5$	**l** $5-15$
m $-1-1$	**n** $-3-4$	**o** $-4-2$	**p** $-5-3$
q $2-12$	**r** $5-9$	**s** $3-13$	**t** $1-19$
u $30-90$	**v** $-20-30$	**w** $-15-45$	**x** $9-30$

4

a $-3+3$	**b** $-6+6$	**c** $-5+5$
d $(4)+(-4)$	**e** $(9)+(-9)$	**f** $(1)+(-1)$
g $6+-6$	**h** $2+-2$	**i** $7+-7$
j $-8+8$	**k** $-11+11$	**l** $-9+9$
m $0{\cdot}5+-0{\cdot}5$	**n** $(-1{\cdot}4)+(1{\cdot}4)$	**o** $(-0)+(0)$
p $(-\frac{3}{4})+(\frac{3}{4})$	**q** $(\frac{3}{10})+(-\frac{3}{10})$	**r** $(-1\frac{1}{2})+(1\frac{1}{2})$



5

a $(6)+(-3)$	**b** $(1)+(-3)$	**c** $(12)+(-5)$
d $6-3$	**e** $1-3$	**f** $12-5$
g $4+-2$	**h** $1+-5$	**i** $7-+2$
j $4-2$	**k** $1-5$	**l** $7-2$

The answers to question 5 suggest that 'when an addition and a subtraction sign are side by side (with no numeral between them), the two unlike signs give a minus'.
eg $7+-3=7-3$, $(2)+(-8)=2-8$, $(6)-(+4)=6-4$

6

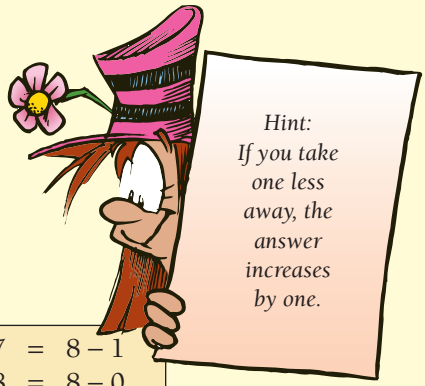
a $-5+-7$	**b** $6+-3$	**c** $8+-10$	**d** $-2+-3$
e $6-(-3+5)$	**f** $-2-(-8+9)$	**g** $-5-(-3+6)$	**h** $4-(7-2)$
i $(3-11)+2$	**j** $(-1-5)-1$	**k** $(2+8)-12$	**l** $9+(-3-4)$
m $-8+(4\times2)$	**n** $-5+(3\times2)$	**o** $-16+(7\times2)$	**p** $(8\times2)-20$

5:07 | Subtracting a Negative Number

Discussion

Complete the two patterns below.

$5 - 4 = 1$	$8 - 2 = 6$
$5 - 3 = 2$	$8 - 1 =$
$5 - 2 =$	$8 - 0 =$
$5 - 1 =$	$8 - -1 =$
$5 - 0 =$	$8 - -2 =$
$5 - (-1) =$	$8 - -3 =$
$5 - (-2) =$	$8 - -4 =$



Investigation	$5 - 3 = 2 = 5 - 3$ $5 - 2 = 3 = 5 - 2$ $5 - 1 = 4 = 5 - 1$ $5 - 0 = 5 = 5 - 0$ $5 - (-1) = 6 = 5 + 1$ $5 - (-2) = 7 = 5 + 2$	$8 - 1 = 7 = 8 - 1$ $8 - 0 = 8 = 8 - 0$ $8 - -1 = 9 = 8 + 1$ $8 - -2 = 10 = 8 + 2$ $8 - -3 = 11 = 8 + 3$ $8 - -4 = 12 = 8 + 4$
Pattern and discovery	The answers were increasing one at a time. To take away a negative, add its opposite. eg $5 - (-3) = 5 + 3$ $\quad 8 - -5 = 8 + 5$ $5 - (-4) = 5 + 4$ $\quad 8 - -6 = 8 + 6$	
Rule	When two minus signs are side by side (with no numeral between them), the two like signs give a plus.	



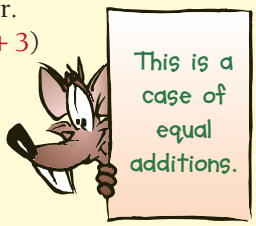
Using opposites

$6 - -3 = (6) - (-3)$

If we add 3 to both numbers, it won't change the answer.

$= (6 + 3) - (-3 + 3)$
$= (6 + 3) - (0)$
$= 6 + 3$

$\therefore\ 6 - -3 = 6 + 3$



Using reasoning

If in 'Adventure in the Jungle' (Section 8:05), our total so far is 9 and we lose 3, what is the new total?

$9 + (-3) = 6$

If we were told to lose 3 by mistake, we would have to take that score away in order to get back to our previous score.

$\therefore\ 6 - (-3) = 9$ OR $6 + 3 = 9$.

The two minus signs became a plus.

To take away −3, add its opposite.

Exercise 5:07

1 **a** $3 - -4$ **b** $8 - -1$ **c** $0 - -6$ **d** $6 - -10$
e $5 - (-3)$ **f** $4 - (-8)$ **g** $9 - (-2)$ **h** $16 - (-7)$
i $-9 - -8$ **j** $-6 - -10$ **k** $-5 - -12$ **l** $-10 - -9$
m $-3 - (-6)$ **n** $-9 - (-4)$ **o** $-2 - (-7)$ **p** $-8 - (-5)$
q $(-10) - (-20)$ **r** $(-7) - (-1)$ **s** $(-2) - (-8)$ **t** $(-60) - (-4)$
u $8 - -7$ **v** $-8 - -7$ **w** $-9 - -9$ **x** $-11 - -11$

2 We have seen that $--3$ or $-(-3)$ can mean 'the opposite of -3'. Simplify:

a $-(-5)$	**b** $-(-15)$	**c** $-(-2)$	**d** $-(-8)$
e $--34$	**f** $--7$	**g** $--11$	**h** $--9$

3 Simplify each of these mixed expressions.

a $8--9$	**b** $6-8$	**c** $-3+4$	**d** $16-9$
e $-4-13$	**f** $17--1$	**g** $2-23$	**h** $-4-6$
i $-5-(-10)$	**j** $-2--12$	**k** $14+(-4)$	**l** $-9+5$
m $6-(-7+3)$	**n** $3-(6-10)$	**o** $5-(-1-8)$	**p** $-10-(6-7)$
q $--10$	**r** $0-(-10)$	**s** $0--13$	**t** $-(-13)$
u $6-(-3+8)$	**v** $12-(6-2)$	**w** $-5-(-1+10)$	**x** $-3-(-8+9)$

4 Simplify:

a $0{\cdot}2--0{\cdot}3$	**b** $2{\cdot}2--0{\cdot}7$	**c** $-1{\cdot}4--0{\cdot}4$
d $-0{\cdot}11-0{\cdot}43$	**e** $0{\cdot}8-0{\cdot}9$	**f** $0{\cdot}1-0{\cdot}8$
g $\frac{3}{10}-\frac{9}{10}$	**h** $-\frac{3}{4}--\frac{1}{4}$	**i** $-\frac{2}{10}-\frac{5}{10}$
j $-(-\frac{3}{8})$	**k** $0--\frac{2}{3}$	**l** $(-\frac{3}{5})+(-\frac{2}{5})$

■ The same methods can be used with decimals and fractions.

id

Write down the question and its answer for each of (1) to (24) on ID Card 3 on page xv.

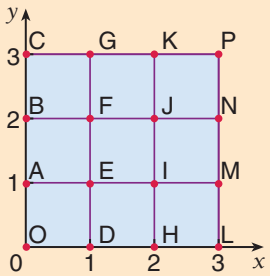
5:07





- We have produced 843 coconuts from our farm in Hawaii and 1104 from our farm in Sri Lanka. We calculated that the number left after filling an order for 2000 coconuts will be -53. What does this answer mean?

5:08 | The Number Plane Extended



1 What are the coordinates of the origin?

Give the letter used to name these points:

2	(1, 2)	3	(3, 3)	4	(2, 3)
5	(0, 3)	6	(1, 0)	7	(3, 1)
8	(0, 0)	9	(2, 2)	10	(0, 2)





Just as we extended the number line, we can now extend the number plane.

worked examples

1 (−3, 2) is 3 to the left and 2 up.

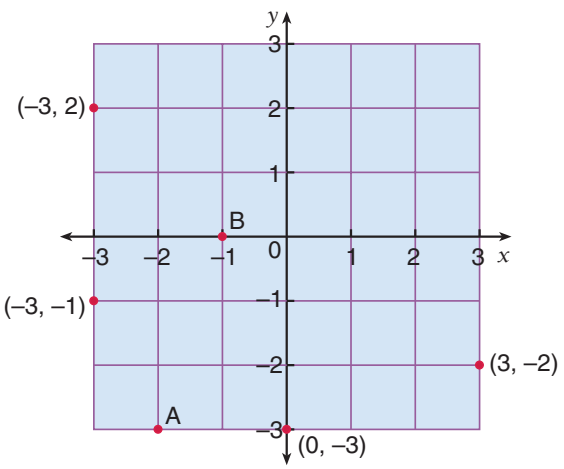
2 (−3, −1) is 3 to the left and 1 down.

3 (0, −3) is 0 to the right and 3 down.

4 (3, −2) is 3 to the right and 2 down.

5 A is below −2 and cross from −3,
∴ A is (−2, −3).

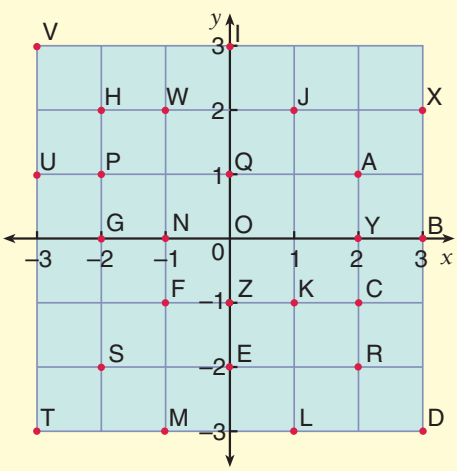
6 B is on −1 and across from 0,
∴ B is (−1, 0).



Exercise 5:08

1 Write down the letter used to name each of these points.

a	(3, 2)	b	(1, −1)	c	(0, 0)
d	(−2, −2)	e	(−1, 2)	f	(0, 3)
g	(−2, 0)	h	(1, −3)	i	(−3, 1)
j	(−1, −3)	k	(1, 2)	l	(−2, 1)
m	(0, 1)	n	(0, −1)	o	(2, −1)
p	(−3, −3)	q	(2, 1)	r	(3, 0)
s	(−1, −1)	t	(3, −3)	u	(−2, 2)
v	(2, −2)	w	(2, 0)	x	(−3, 3)
y	(−1, 0)	z	(0, −2)		



2 Use the number plane in question 1 to write down the coordinates of these points.

a A	**b** B	**c** C	**d** D	**e** E	**f** F	**g** G
h H	**i** I	**j** J	**k** K	**l** L	**m** M	**n** N
o O	**p** P	**q** Q	**r** R	**s** S	**t** T	**u** U
v V	**w** W	**x** X	**y** Y	**z** Z		

3 Use the letters on the number plane in question 1 to decode these messages.

a (−2, −2) (0, −2) (0, −2) (1, −1) . . . (−1, 2) (0, 3) (−2, −2) (3, −3) (0, 0) (−1, −3) . . . (−2, −2) (0, −2) (0, −2) (1, −1) . . . (1, −1) (−1, 0) (0, 0) (−1, 2) (1, −3) (0, −2) (3, −3) (−2, 0) (0, −2)

b (3, −3) (0, 3) (−2, −2) (2, −1) (0, 3) (−2, 1) (1, −3) (0, 3) (−1, 0) (0, −2) . . . (2, 0) (0, 0) (−3, 1) (2, −2) . . . (2, −1) (−2, 2) (0, 3) (1, −3) (3, −3) (2, −2) (0, −2) (−1, 0) . . . (−1, 2) (−2, 2) (0, 3) (1, −3) (0, −2) . . . (−3, −3) (−2, 2) (0, −2) (2, 0) . . . (2, 1) (2, −2) (0, −2) . . . (2, 0) (0, 0) (−3, 1) (−1, 0) (−2, 0) . . . (0, −2) (−1, 0) (0, 0) (−3, 1) (−2, 0) (−2, 2) . . . (−3, −3) (0, 0) . . . (1, −3) (0, −2) (2, 1) (2, −2) (−1, 0)

4 Use a grid that has values on the *x*-axis from −16 to 20 and values on the *y*-axis from −20 to 16 (5 mm graph paper is ideal). On this number plane, graph the points listed below, joining them in the order given. Colour the picture you have made.

a (−1, −6) (−1, −4) (1, −1) (7, 1) (10, 8) (13, 11) (19, 8) (17, 7) (17, 6) (18, 6) (16, 4)

b (15, 10) (13, 8) (13, 7) (15, 6) (16, 4) (15, 2) (15, −2) (13, −7) (6, −11) (7, −16) (8, −18) (6, −17) (4, −18) (6, −16) (5, −11)

c (5, −11) (4, −12) (3, −12) (4, −9) (3, −12) (2, −16) (3, −18) (1, −17) (−1, −18) (1, −16) (2, −12) (3, −9) (2, −12) (1, −11) (−1, −6)

d (−1, −6) (−3, −2) (−1, −10) (−6, −1) (−6, −3) (−7, −1) (−8, −7) (−9, 2) (−12, −7) (−11, 4) (−14, −2) (−11, 9) (−13, 9) (−9, 11) (−4, 11) (2, 7) (1, −1)

e (12, 10) (9, 13) (10, 14) (11, 13) (12, 15) (13, 14) (14, 15) (15, 15) (16, 14) (18, 13) (19, 11) (17, 9)

f (15, 8) (15, 9) (16, 9) (16, 8) (15, 8)



5:09 | Multiplication of Directed Numbers

prep quiz

5:09

Complete:

1 6 + 6 + 6 + 6 = four lots of 6
= . . . × . . .

2 (−7) + (−7) + (−7) = three lots of (−7)
= . . . × . . .

3 8 + 8 + 8 = three lots of 8
= . . . × . . .

4 (−2) + (−2) + (−2) = three lots of (−2)
= . . . × . . .

Evaluate:

5 3 lots of −10

6 6 lots of −3

7 10 lots of −7

Complete the patterns.

8 6, 4, 2, 0, . . ., . . .

9 9, 6, 3, 0, . . ., . . .

10 −9 −6, −3, 0, . . ., . . .

We can discover the rules for multiplication by extending number patterns.

Investigation	$3 \times 3 = 9$ $3 \times 2 = 6$ $3 \times 1 = 3$ $3 \times 0 = 0$ $3 \times -1 = -3$ $3 \times -2 = -6$	$3 \times 3 = 9$ $2 \times 3 = 6$ $1 \times 3 = 3$ $0 \times 3 = 0$ $-1 \times 3 = -3$ $-2 \times 3 = -6$	3 lots of $-3 = -9$ $2 \times -3 = -6$ $1 \times -3 = -3$ $0 \times -3 = 0$ $-1 \times -3 = 3$ $-2 \times -3 = 6$
Pattern and discovery	The answers above were decreasing by 3 each time.		The answers above were increasing by 3 each time.
	plus × minus = minus minus × plus = minus		minus × minus = plus
Rule	Multiplying two unlike signs gives a minus. $-8 \times 7 = -56$		Multiplying two like signs gives a plus. $-8 \times -7 = 56$



worked examples

1 $-3 \times -8 \times -5$
$= (-3 \times -8) \times -5$
$= 24 \times -5$
$= -120$

2 $(-6)^2$
$= -6 \times -6$
$= 36$

Exercise 5:09

1 **a** $1 \times (-1)$ **b** 4×-2 **c** $6 \times (-2)$ **d** 5×-5
e -8×1 **f** -3×4 **g** -11×4 **h** -2×8
i $(-2) \times (-5)$ **j** -3×-3 **k** -2×-4 **l** $(-1) \times (-1)$
m -8×0 **n** 0×-7 **o** -12×0 **p** 0×-9
q -3×7 **r** 3×-2 **s** -2×-6 **t** -4×3
u 4×-5 **v** -1×-7 **w** -8×2 **x** -7×-2

2 **a** 7×-5 **b** 4×-7 **c** -3×9 **d** 9×-5
e -4×-4 **f** -8×-4 **g** -7×-6 **h** -8×6
i $(-6) \times (-4)$ **j** 9×-9 **k** 5×-8 **l** 4×-9
m -8×7 **n** $7 \times (-7)$ **o** -7×-4 **p** $(-8) \times 9$
q -6×-6 **r** 9×-8 **s** -9×-7 **t** -4×6
u 11×11 **v** -9×-6 **w** -8×8 **x** -7×-9

3 **a** $(-1)^2$ **b** $(-2)^2$ **c** $(-3)^2$ **d** $(-4)^2$
e $(-5)^2$ **f** $(-6)^2$ **g** $(-7)^2$ **h** $(-8)^2$
i $(-9)^2$ **j** $(-10)^2$ **k** $(-11)^2$ **l** $(-12)^2$

4 **a** -4×80 **b** -5×-30 **c** -9×10 **d** -80×3
e $6 \times -0{\cdot}3$ **f** $8 \times (-0{\cdot}5)$ **g** $-0{\cdot}2 \times 0{\cdot}2$ **h** $-0{\cdot}3 \times -0{\cdot}6$
i $0{\cdot}7 \times -0{\cdot}1$ **j** $-0{\cdot}11 \times 0{\cdot}7$ **k** $0{\cdot}06 \times -8$ **l** $-0{\cdot}7 \times -0{\cdot}8$

5 **a** $-3 \times -3 \times -3$ **b** $-2 \times -2 \times -2$ **c** $-5 \times -5 \times -5$
d $-5 \times -2 \times 5$ **e** $-1 \times -3 \times 6$ **f** $-4 \times -8 \times 2$
g $-19 \times 3 \times 5$ **h** $4 \times -2 \times 5$ **i** $3 \times 4 \times -3$
j $2 \times 6 \times -1$ **k** $6 \times -3 \times -1$ **l** $-3 \times -8 \times -1$

investigation 5:09

Investigation 5:09 | Multiplying directed numbers

Please use the Assessment Grid on the following page to help you understand what is required for this Investigation.

- Imagine a video showing a person walking forwards, towards a tree. If the video is played backwards, in which direction (forwards or backwards) does the person appear to be walking?
- Think of a football umpire running backwards after awarding a free kick. When the video is played backwards, in which direction does he appear to be running?
- On a videotape an umpire is shown running both forwards (+) and backwards (–). The tape can be played forwards (+) or in reverse (–). Complete the table to demonstrate the rules for multiplication of directed numbers.

Tape forwards (+) or backwards (–)	*Umpire's movement forwards (+) or backwards (–)*	*Apparent result on screen*	*Number sentence*
Forwards for 3 seconds (+3)	Forwards at 4 steps/sec (+4)	**a** 12 steps forwards (+12)	**a** $3 \times 4 = 12$
Forwards for 3 seconds (+3)	Backwards at 4 steps/sec (–4)	**b**	**b**
Backwards for 3 seconds (–3)	Forwards at 4 steps/sec (+4)	**c**	**c**
Backwards for 3 seconds (–3)	Backwards at 4 steps/sec (–4)	**d**	**d**



challenge 5:09

Challenge 5:09 | Using pronumerals

1 Complete each table using the rule given.

a $y = 1 - x$

x	1	2	3	4
y				

b $y = -4 \times x$

x	0	2	4	6
y				

c $y = -2 \times x + 3$

x	–3	–1	1	3
y				

2 If $x = -7$, find the value of y in each case.

a $y = 4 - x$ **b** $y = x \times -4$ **c** $y = x^2$ **d** $y = \sqrt{23 + x}$

3 What values of the pronumeral will make each true?

a $5 \times x = -15$ **b** $x - 3 = -2$ **c** $x \div 2 = -7$ **d** $x \times x = 9$

Assessment Grid for Investigation 5:09 | Multiplying directed numbers

The following is a sample assessment grid for this investigation. You should carefully read the criteria *before* beginning the investigation so that you know what is required.

		Assessment Criteria (B, C, D)		Achieved ✓
Criterion B Application & Reasoning	a	An organised approach has not been used to answer the questions.	1	
			2	
	b	An organised approach has been used and an attempt has been made to work out the answers.	3	
			4	
	c	An organised approach has been used and there is some explanation of how the results were obtained.	5	
			6	
	d	There is a full explanation of the how the results were obtained.	7	
			8	
	e	Full explanations are given and all the answers are reasonable.	9	
			10	
Criterion C Communication	a	There is no working out or explanations given.	1	
			2	
	b	Working out is shown and presentation is good and easy to follow.	3	
			4	
	c	Working out and explanations are well communicated using symbols or words.	5	
			6	
Criterion D Reflection & Evaluation	a	Some attempt has been made to explain how the answers were obtained.	1	
			2	
	b	The method and majority of processes are justified and the answers have been checked for reasonableness with some success.	3	
			4	
	c	Reasoned explanations are given for the processes used and all answers have been checked for reasonableness.	5	
			6	
	d	Very concise and exact justifications are given for the processes used and answers are evaluated.	7	
			8	

5:10 | Division of Directed Numbers

prep quiz

5:10

Find the value of the square in each if the sentence is true.

1 $6 \times 5 = \square$, $\therefore \square \div 5 = 6$ **2** $6 \times -5 = \square$, $\therefore \square \div -5 = 6$

3 $-6 \times -5 = \square$, $\therefore \square \div -5 = -6$ **4** $-6 \times 5 = \square$, $\therefore \square \div 5 = -6$

Copy and complete:

5 $6 \times \ldots = 30$ **6** $6 \times \ldots = -30$

7 $-6 \times \ldots = 30$ **8** $-6 \times \ldots = -30$

9 How many losses of 2 are needed to make a loss of 10? (ie $-10 \div -2 = \ldots$)

10 What would you multiply 5 by to get -10? (ie $-10 \div 5 = \ldots$)

From the Prep Quiz questions **1** to **4** we see that:

1 $30 \div 5 = 6$
2 $-30 \div -5 = 6$ ⇨ Division involving two like signs gives a plus.

3 $30 \div -5 = -6$
4 $-30 \div 5 = -6$ ⇨ Division involving two unlike signs gives a minus.

■ If $-6 \times -5 = 30$
then
$30 \div -6 = -5$
and
$30 \div -5 = -6$

For multiplication and division:
- **two *like* signs give a *plus***
- **two *unlike* signs give a *minus***

Exercise 5:10

Foundation Worksheet 5:10

Division of directed numbers

1 a $-8 \div 2$ **b** $-12 \div 3$

2 a $60 \div (-10)$ **b** $-4 \div (-2)$

3 a $\frac{-15}{3}$ **b** $\frac{-10}{-2}$

1 **a** $18 \div 3$ **b** $-18 \div 3$ **c** $18 \div -3$

d $-8 \div -2$ **e** $16 \div -4$ **f** $-20 \div 2$

g $9 \div (-1)$ **h** $(-6) \div (-1)$ **i** $(-10) \div 2$

j $-28 \div 4$ **k** $-24 \div -3$ **l** $60 \div -6$

m $-35 \div -7$ **n** $20 \div -5$ **o** $-30 \div 10$

p $-16 \div 2$ **q** $-27 \div -3$ **r** $20 \div -4$

s $6 \div (-6)$ **t** $(-9) \div 9$ **u** $(-7) \div (-7)$

2 **a** $\frac{-30}{6}$ **b** $\frac{6}{-2}$ **c** $\frac{-21}{-3}$

d $\frac{42}{-6}$ **e** $\frac{-21}{-7}$ **f** $\frac{30}{-5}$

g $\frac{-32}{8}$ **h** $\frac{-56}{-7}$ **i** $\frac{45}{-9}$

j $\frac{49}{-7}$ **k** $\frac{-48}{-8}$ **l** $\frac{36}{6}$

m $\frac{-630}{-9}$ **n** $\frac{810}{-9}$ **o** $\frac{-300}{-10}$



5:11 | Using Directed Numbers

prep quiz 5:11

Give the basic (simplest) numeral for:

1 $8 - 15$ **2** $-9 - 4$ **3** $-8 + 6$ **4** $6 - -3$

5 $5 + (-7)$ **6** $- -6$ **7** -8×5 **8** $(-400) \div (-5)$

9 $\frac{32}{-4}$ **10** $(-8)^2$

The directed numbers . . ., −4, −3, −2, −1, 0, 1, 2, 3, 4, . . . are called the integers.

Order of operations

The order of operations introduced in 3:01 will, of course, apply to directed numbers.

■ $(-3)^2 = -3 \times -3$
$= 9$

worked examples

1 $8 - 5 \times 3 = 8 - 15$
$= -7$

2 $-6 + (-3)^2 - (-1 - 3) = -6 + 9 - (-4)$
$= -6 + 9 + 4$
$= 7$

Exercise 5:11

Foundation Worksheet 5:11
Using directed numbers
1 a $2 \times 2 - 6$ **b** $5 - 3 - 3$
2 a $(-3)^2 + 1$ **b** $-1 + 2 \times 3$
3 Complete these patterns.
a 3, 2, 1, 0, −1, . . ., . . ., . . .

1 **a** $8 - 5 \times 2$ **b** $15 - 4 \times 4$ **c** $6 + 5 - 25$
d $11 - 15 + 8$ **e** $6 \times -4 \div 2$ **f** $-50 \div 10 \times 2$
g $(-3 - 7) \times -9$ **h** $3 \div (-3 + 6)$ **i** $9 \div (3 \times -3)$
j $(-3 - 3) \div (1 - 7)$ **k** $6 - 3 \times 4 - 3$ **l** $10 - 12 \div 4 - 7$
m $-1 \times [-8 \div (-4 + 6)]$ **n** $6 - [15 - (-3 - 2)]$

■ *Order of Operations*
1 ( )
2 × and ÷
3 + and −

2 **a** $-3{\cdot}8 + 9$ **b** $-4{\cdot}7 - 5{\cdot}4$ **c** $8{\cdot}6 - 10{\cdot}4$
d $4{\cdot}11 - -0{\cdot}11$ **e** $-100 + 9{\cdot}56$ **f** $14{\cdot}65 - 20$
g $6{\cdot}35 \times -6$ **h** $-0{\cdot}5 \times -0{\cdot}5$ **i** $(-0{\cdot}3)^2$
j $-103 \div 10$ **k** $-6{\cdot}24 \div -6$ **l** $0{\cdot}05 \div (-5)$
m $-100 \times (0{\cdot}1)^2$ **n** $-1{\cdot}3 + 0{\cdot}3$ **o** $(-2)^3 \times 0{\cdot}2$



3 Complete these number patterns.
a 9, 7, 5, 3, . . ., . . ., . . .
b −10, −8, −6, −4, . . ., . . ., . . .
c −2, −4, −8, −16, . . ., . . ., . . .
d 2, −4, 8, −16, . . ., . . ., . . .
e −6, 6, −6, 6, . . ., . . ., . . . **f** 64, −32, 16, −8, . . ., . . ., . . .

4 **a** $\frac{-6 + 10}{-2}$ **b** $\frac{-32 - 16}{4}$ **c** $\frac{-15 + 65}{-10}$
d $\frac{-10 \times -4}{8}$ **e** $\frac{-6 - 4}{-2 + 7}$ **f** $\frac{3 - 48}{1 - 10}$

■ The fraction bar groups both the top and bottom.

5 Travelling in a lift 3 floors up, then 4 floors down could be shown as $3 - 4 = -1$. Make up stories of your own for:

a $-4 - 7 = \square$ **b** $-2 + 10 = \square$ **c** $6 - 10 = \square$

d $6 \times -3 = \square$ **e** $-60 \div 4 = \square$ **f** $-8 + 2 = \square$



6 True or false?

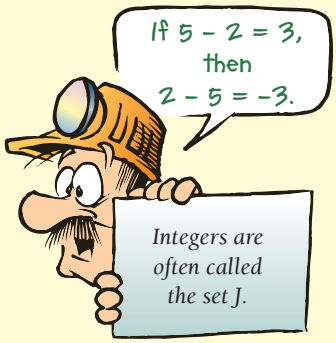
a $-6 \times 0 = -6$ **b** $-8 \times 1 = -8$

c $3 - 5 = 5 - 3$ **d** $3 - 5 = -5 + 3$

e $-3 - 5 = -5 - 3$ **f** $-6 \times -3 = -3 \times -6$ **g** $5 \times (-6 + 3) = 5 \times -6 + 5 \times 3$

h $(-6 - 7) - 8 = -6 + (-7 - 8)$ **i** $(-3 \times -4) \times -5 = -3 \times (-4 \times -5)$

7 Which of the integers -10, 7, 3, and -2 is closest to zero on the number line?



8 From the integers -7, -5, -3, 1, 4, 7, find the three integers whose sum is:

a -2 **b** 2 **c** 0

9 Find the value of the square if:

a $-8 \times \square = 304$ **b** $6 \times \square = -6{\cdot}12$ **c** $6 + \square = -17$

10 One more than -4 is added to the product of 2 and four less than 2. What is the result?

11 Start with the integer 2. Subtract 7. From your answer subtract 1, then multiply your last answer by -3. Now subtract 20 from the last answer. What is your final answer?

12 On a very cold day the temperature at 6 am was -4°C. By 12 noon the temperature had risen by 6°C but in the following 8 hours it fell by 11°C. What was the temperature at 8 pm?

13 Yachts *Southern Cross* and *Northern Lights* raced over a course that had six sections. *Northern Lights* started with a 10-second advantage. Over the six sections that followed the start, *Northern Lights* gained 20 seconds, then lost 1 minute, then gained 15 seconds, then lost 27 seconds, then gained 41 seconds and then lost 6 seconds. Who won the race and by how much?

14 Local time in Vladivostok is always 3 hours ahead of Singapore time, and Singapore is always 5 hours behind Wellington, which is 12 hours ahead of London. What is the time difference between Vladivostok and London?

15 **a** Two numbers have a product of 12 but their sum is -7. What are the numbers?

b The sum of two numbers is 1 but their product is -30. What are they?

16 At a Weightwatchers Club since the last meeting, the most weight lost by a member was 2·4 kg; the most gained was 0·7 kg. Five members worked out the total of their changes since the last meeting. Which of the following could be their answer?

A a gain of 4 kg **B** a loss of 0·1 kg

C a loss of 15 kg **D** a loss of 144 kg

5:12 | Directed Numbers and the Calculator

prep quiz 5:12

1 $-3+1$	**2** $-7-3$	**3** $2-10$	**4** $-4-8+3$	**5** $0-7$
6 $-10\div 2$	**7** -3×8	**8** $(-4)\times(-3)$	**9** $(-4)^2$	**10** $\frac{-12}{-3}$

The +/− key on a calculator

Modern calculators allow you to enter negative numbers using the +/− key. The +/− is pressed after the second part of the number.

worked examples

1 $16-(-7)$

16 [−] 7 [+/−] [=]

Answer = 23

2 $(-11)^2$

11 [+/−] [x^2]

Answer = 121

3 $-8-3\times-11$

8 [+/−] [−] 3 [×] 11 [+/−] [=]

Answer = 25

Exercise 5:12

1 Complete the first column without a calculator and the second column with a calculator.

Question	*Without calculator*	*With calculator*
$-3-8$	-11	
$-9-2$		
$-7+2$		
$-15+6$		
$-7+23$		
$-2+15$		
$-2--3$		
$8--2$		

Question	*Without calculator*	*With calculator*
-3×4		
-6×3		
-7×-2		
-9×-5		
4×-7		
10×-6		
$20\div(-5)$		
$-18\div-3$		

Do the answers you found with a calculator agree with your answers found without a calculator? If not, do those questions again.

Use a calculator to find the answers.

2 **a** -17×34 **b** $-123-479$ **c** $-78+372$
d $1567-3842$ **e** $-2160\div 16$ **f** $797\,544\div-318$
g $-841--793$ **h** $14-2107$ **i** $-14-2107$

3 **a** $(-407)^2$ **b** $(-2{\cdot}7)^2$ **c** $(-0{\cdot}13)^2$

4 **a** $-6-7\times-11$ **b** $-23\times-6+91$ **c** $8-6{\cdot}1\times-8{\cdot}4$
d $0{\cdot}5--1{\cdot}86\times 0{\cdot}7$ **e** $-105-23\times-2$ **f** $-5\times-8\times-3\times-9$

fun spot

5:12

Fun Spot 5:12 | Try this quick quiz!

You can practise working with integers by completing these tables, starting with the number given and then performing the operations in order. Examine the first one and check to see that you can get each answer.

Operation	Start	+ 3	− 10	× 2	÷ (−4)	− 1
Answer	5	8	− 2	− 4	1	0

FINAL ANSWER



Operation	Start	+ 6	× (−4)	+ 8	÷ (−4)
Answer	−3				1

FINAL ANSWER

In each case complete the table.

1

Operation	Start	× (−2)	+ 10	÷ 5	− 3	+ 6
Answer	10					

2

Operation	Start	− 10	× 3	+ 8	÷ 2	+ 2
Answer	6					

3

Operation	Start	+ 3	× (−5)	− 5	÷ 2	+ 10
Answer	−2					

4

Operation	Start	− 3	÷ 4	+ 4	× (−3)	+ 7
Answer	−5					

5

Operation	Start	− 10	× 5	+ 8	÷ 2	− 4
Answer	8					

6

Operation	Start	− 6	÷ 3	+ 5	× (−1)	− 2
Answer	0					

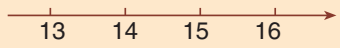


You could make up more tables for practice. Perhaps make them longer, and check your answer with a calculator. (Push = after each step.)

Mathematical terms 5

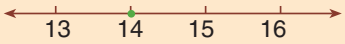
number line

- A line with a scale showing the relative positions of numbers.

eg 

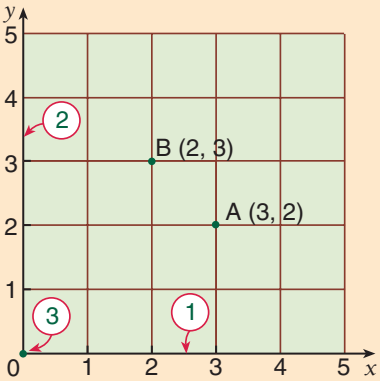
graph (a number)

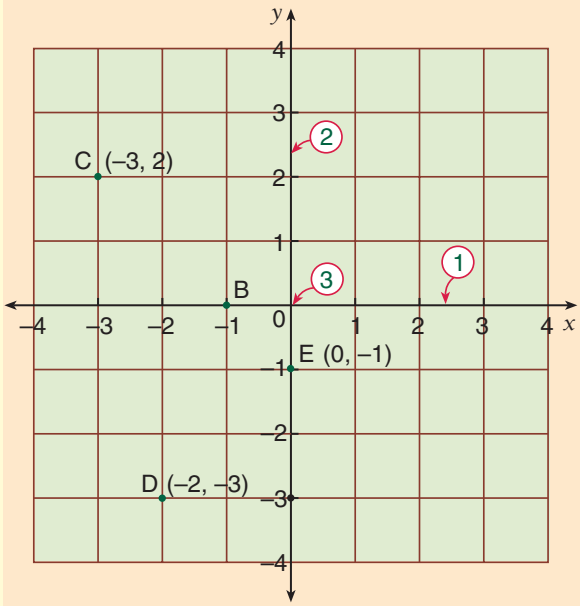
- To mark the position of a number on a number line we use a dot.
  eg The number 14 has been graphed on the number line.



number plane (or number lattice)

- A rectangular grid that allows the position of points in a plane to be identified by an ordered pair of numbers.





x-axis

- The horizontal number line in a number plane.
  See (1) under number plane.

y-axis

- The vertical number line in a number plane.
  See (2) under number plane.

origin

- The point where the *x*-axis and the *y*-axis intersect, (0, 0).
  See (3) under number plane.

coordinates

- A pair of numbers that gives the position of a point in a number plane relative to the origin.
  See (3, 2), (2, 3), (−3, 2), (−2, −3) and (0, −1) under number plane.

***x*-coordinate**

- The first of the coordinates. It tells how far right (or left) the point is from the origin.

***y*-coordinate**

- The second of the coordinates. It tells how far above (or below) the origin the point is.

directed number

- A number that has both a size and a direction.

positive

- Greater than zero.

negative

- Less than zero.

integer

- Any whole number.

sign (of a number)

- Whether a number is positive or negative.

opposite

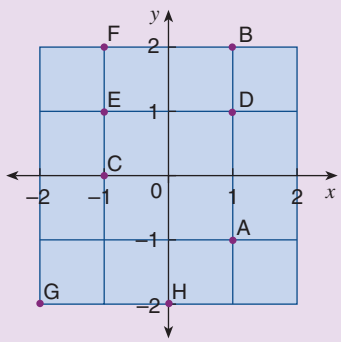
- The number of the same size but opposite sign.
  eg The opposite of 3 is −3.

diagnostic test

5

Diagnostic Test 5: | Directed Numbers and the Number Plane

- Each section of the test has similar items that test a certain type of question.
- Failure in more than one item will identify an area of weakness.
- Each weakness should be treated by going back to the section listed.

	Section
1 On a separate number line, graph each set. **a** {0, 3, 5} **b** {−2, 1, 3} **c** {−5, −4, −3, 0}	5:01
 **2** Use this number plane to write down the coordinates of the points: **a** A **b** B **c** C	5:08
3 Which points have coordinates: **a** (−2, −2)? **b** (1, 1)? **c** (−1, 2)? **d** (0, −2)?	5:08
4 **a** $-5+3$ **b** $(-8)+12$ **c** $-17+2$	5:06
5 **a** $(-6)+(-3)$ **b** $-8+-2$ **c** $9+(-10)$	5:06
6 **a** $-5-3$ **b** $4-9$ **c** $0-7$	5:06
7 **a** $10--3$ **b** $-6--4$ **c** $(-4)-(-3)$	5:07
8 **a** -8×7 **b** -5×-8 **c** 6×-4	5:09
9 **a** $(-6)^2$ **b** $(-1)^2$ **c** $(-10)^2$	5:09
10 **a** $-30\div 3$ **b** $(-42)\div(-6)$ **c** $77\div -11$	5:10
11 **a** $\frac{-15}{-3}$ **b** $\frac{36}{-6}$ **c** $\frac{-16}{2}$	5:10
12 **a** $10-7\times 2$ **b** $-4+8\div 2$ **c** $(3-17)\div(-10+8)$	5:11
13 Complete these number patterns. **a** 12, 9, 6, 3, . . ., . . ., . . . **b** −40, −30, −20, −10, . . ., . . ., . . . **c** 1, −3, 9, −27, . . ., . . ., . . . **d** −64, −32, −16, −8, . . ., . . ., . . .	5:11
14 **a** $\frac{-3+8}{5}$ **b** $\frac{4-12}{6-4}$ **c** $\frac{-9+12}{2-5}$	5:11
15 **a** One more than −4. **b** One less than −10.	5:11

Chapter 5 | Revision Assignment

assignment 5A

1 Simplify:

a $(6 + 3) \times 2$ **b** $(7 - 2) \times (7 + 3)$

c $20 - 3 \times 2 + 5$ **d** $4 - 3 + 2 \times 4$

2 Change the following Roman numerals to Hindu–Arabic numerals.

a XIV **b** DCXXV

c MCMLXXXVII **d** CDLXXIX

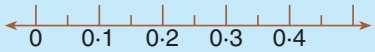
3 What is the value of the 5 in each of the following?

a 0·15 **b** 1·52

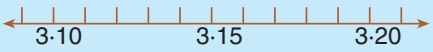
c 5·16 **d** 12·715

4 Copy these number lines and place a number under each mark.

a



b



5 Write each numeral in simplest form.

a $(7 \times 1000) + (3 \times 100) + (2 \times 10)$

b $(6 \times 1000) + (4 \times 10) + (8 \times 1)$

c $(6 \times 10\,000) + (7 \times 1000) + (4 \times 100)$

d $(3 \times 1\,000\,000) + (5 \times 10\,000)$

6 Complete the tables, carrying out each operation on the last answer found.

a

Operation	Start	− 7	× 3	+ 11	− 35	÷ 6
Answer	5					

b

Operation	Start	× 3	− 45	÷ 9	+ 5	× 96
Answer	−3					

7 How much do I have in dollars and cents if I have:

a 30 twenty-cent pieces?

b 100 five-cent pieces?

c 200 ten-cent pieces?

d 500 twenty-cent pieces?

1 Addition and subtraction of directed numbers
2 Multiplication and division of directed numbers
3 Plotting coordinates

Drag and Drop CW Interactive

Review CW Questions

assignment 5B

id 5

Chapter 5 | Working Mathematically

1 Use ID Card 7 on page xix to identify (6) to (16).

2 Write down the different combinations that will produce a total of seven when a green dice and a red dice are rolled.

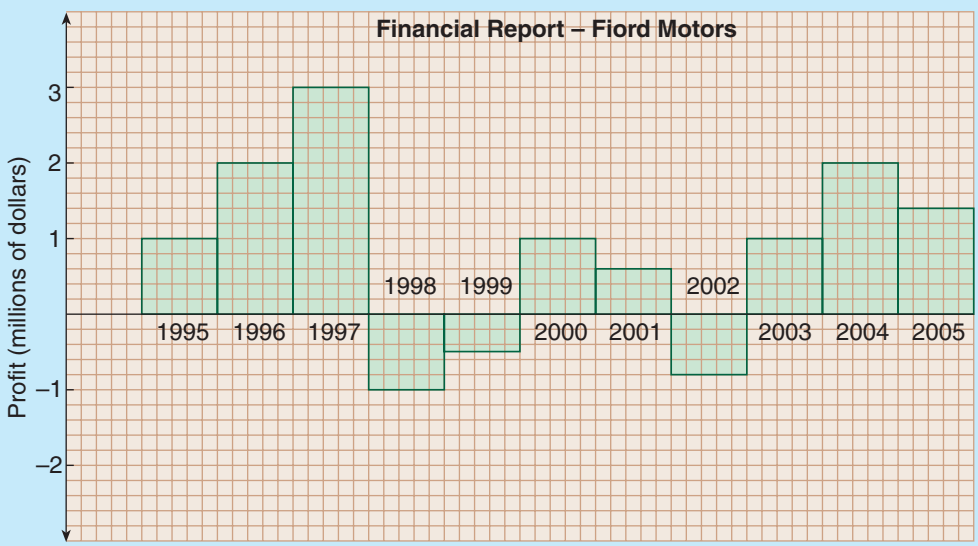
3 A woman buys 300 mining shares at $4.50 each and 1000 oil shares at $1.80 each. After one year she sells the oil shares for $1.95 and the mining shares for $4.10. How much money has she made or lost on the deal?

4 A builder hires three wheelbarrows. Each wheelbarrow costs $6 a day to hire or $24 a week. If he hires them from 27th October to 16th November (both days inclusive), how much will he pay?

5 In a chess match, each player scores one point for a win, $\frac{1}{2}$ a point for a draw and no point for a loss. If the match score is presently $3\frac{1}{2}$ to $1\frac{1}{2}$, how many games have been played? How could this score have come about?

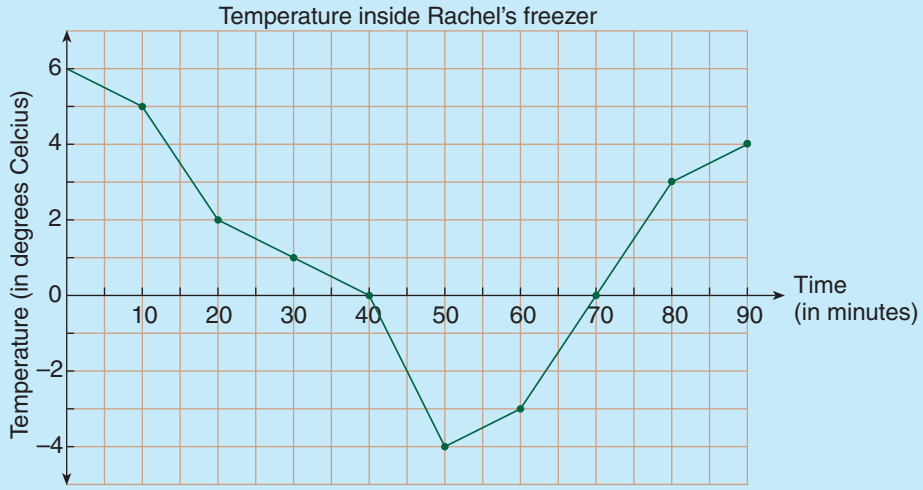
6 A ferry service operates a scenic cruise to and from the same wharf. The cruise takes 1 hour and it takes 20 minutes to load and 10 minutes to unload passengers. If the first cruise leaves the wharf at 9 am and the last one must be back by 5 pm, work out a timetable for the ferry, allowing at least half an hour for lunch.

7



a In what year was the greatest profit made?
b In what year was the first loss made?
c In how many years was a loss made?
d In how many years was the profit over $1 000 000?
e What was the profit in 2001?
f What was the loss in 2002?

8 Rachel recorded the temperature inside her freezer every 10 minutes. She then drew the graph below.



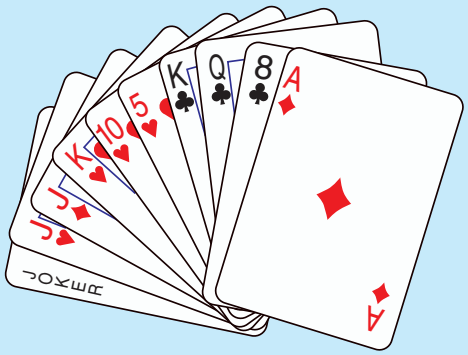
a What was the temperature after:
i 20 min? **ii** 40 min? **iii** 50 min? **iv** 60 min? **v** 90 min?

b At what time was the temperature lowest?

c At what times was the temperature 0°C?

d What do you think the temperature would have been after 45 minutes?

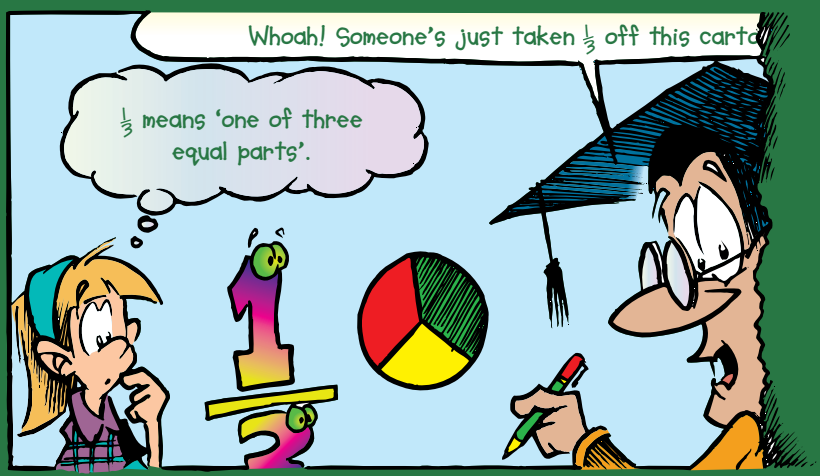
e Between which two times was the change in temperature greatest? What was this change?



- I am playing the card game called 500.
- My score is 170 and I have called '8 hearts'.
- If I succeed in making my call, my score will increase by 300. If I fail it will decrease by 300.
- After the hand is played, what two possible scores might I have?

6

Fractions, Percentages and Probability



Chapter Contents

6:01 Exploring fractions
6:02 Comparing fractions
6:03 Review of fractions
Investigation: Fractions ID Card
6:04 Addition and subtraction of fractions
6:05 Addition and subtraction of mixed numbers
6:06 Multiplication of fractions
6:07 Division involving fractions
6:08 Fractions of quantities
6:09 Review of percentages
6:10 Changing fractions and decimals to percentages
6:11 Changing percentages to fractions and decimals (extension)
6:12 Finding a percentage of a quantity
6:13 One quantity as a percentage of another
6:14 Applications of fractions and percentages
6:15 What chance of survival?
Fun Spot: Where should the numbers go?
6:16 Probability
Mathematical Terms, Diagnostic Test, Revision Assignment, Working Mathematically

Learning Outcomes

Students will:

- Operate with fractions, decimals, percentages, ratios and rates.
- Compare, order and calculate with decimals, simple fractions and simple percentages.
- Solve probability problems involving simple events.

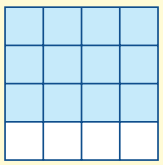
Areas of Interaction

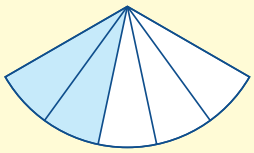
Approaches to Learning, Homo Faber

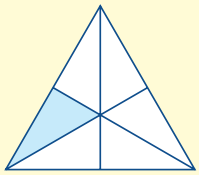
6:01 | Exploring Fractions

Exercise 6:01

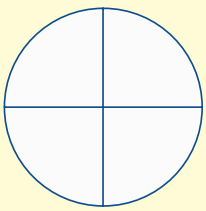
1 What fraction of each figure has been shaded blue?

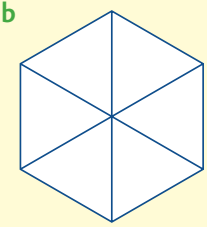
a 

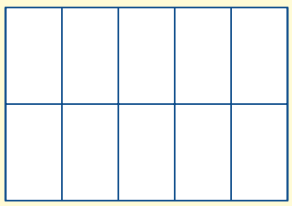
b 

c 

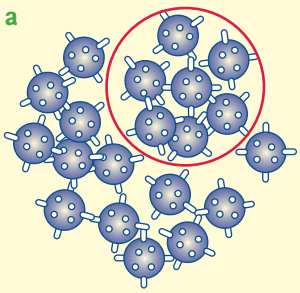
2 Copy the figure and shade the fraction indicated.

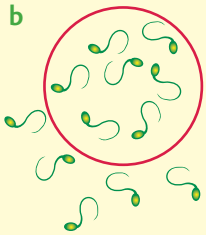
a  $\frac{3}{4}$

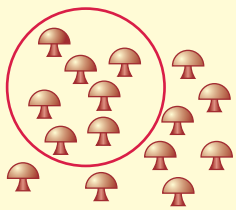
b  $\frac{6}{6}$

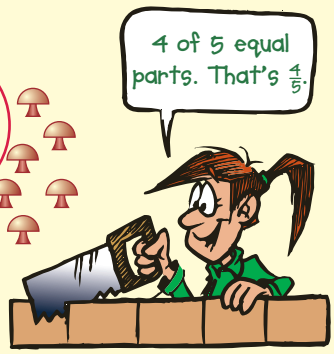
c  $\frac{7}{10}$

3 What fraction of each group has been circled?

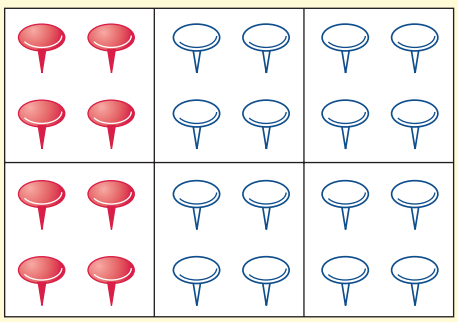
a 

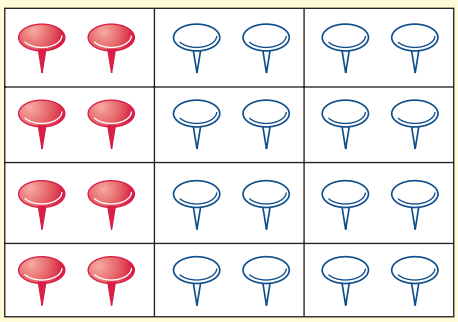
b 

c 



4

A 

B 

The same fraction has been coloured in both A and B. Use the diagrams to complete:

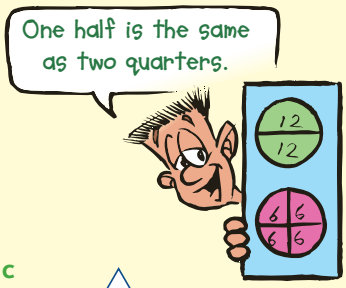
$$\frac{8}{24} = \frac{\ldots}{12} = \frac{\ldots}{6} = \frac{\ldots}{3}$$

5 Use the diagrams in question 4 to find:

a $\frac{1}{2}$ of 24 **b** $\frac{1}{3}$ of 24 **c** $\frac{1}{4}$ of 24

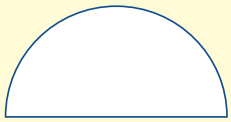
d $\frac{1}{6}$ of 24 **e** $\frac{1}{8}$ of 24 **f** $\frac{2}{3}$ of 24

g $\frac{3}{4}$ of 24 **h** $\frac{5}{6}$ of 24 **i** $\frac{3}{8}$ of 24

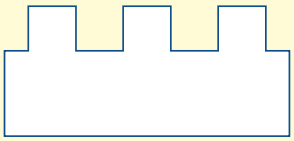


6 Copy and shade $\frac{2}{3}$ of each figure.

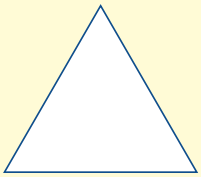
a



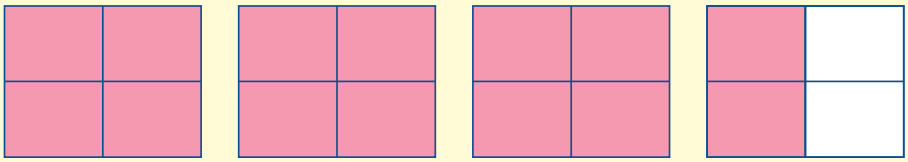
b



c



7



a How many quarters have been shaded?

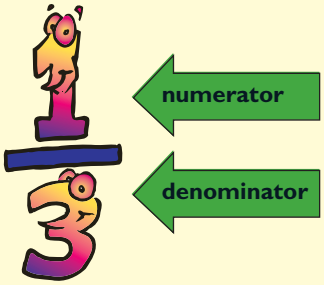
b How many quarters are in one whole?

c How many quarters are in $3\frac{1}{2}$?

d What is 8 lots of $\frac{1}{4}$? **e** What is 10 lots of $\frac{1}{4}$?

f $12 \times \frac{1}{4}$ **g** $6 \times \frac{1}{4}$

h $16 \times \frac{1}{4}$ **i** $14 \times \frac{1}{4}$



8 **a** If $\frac{1}{4}$ of my money is \$3.15, how much do I have?

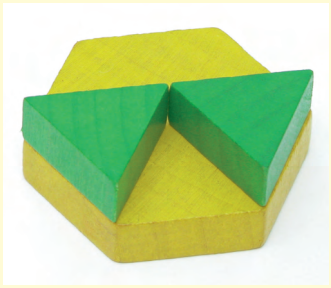
b If $\frac{1}{10}$ of my swap cards is 23 cards, how many do I have?

c If $\frac{1}{3}$ of Australia's population is 6 500 000 people, what is Australia's population?

9 When a water tank is $\frac{3}{4}$ full, it holds 3390 litres of water.

a How much would the tank hold when it is $\frac{1}{4}$ full?

b How much would the tank hold when it is full?



- What fraction of the yellow block is covered?

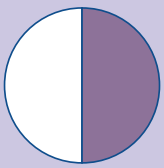
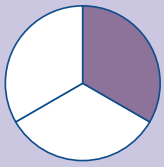
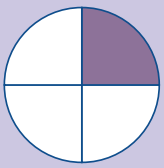
10 **a** How many people could share a cake if each received $\frac{1}{10}$?

b How many people could share a cake if each received $\frac{2}{100}$?

6:02 | Comparing Fractions

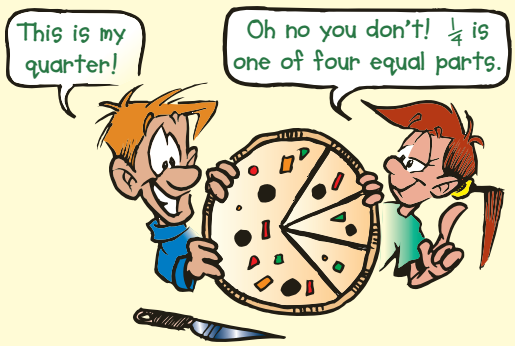
Exercise 6:02

1

Figure			
Fraction shaded	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$

a Which fraction has the smallest denominator?
b Which fraction has the largest shaded part?
c Which fraction has the largest denominator?
d Which fraction has the smallest shaded part?
e Complete the following:

'As the denominator increases, the value of the fraction . . . . . .'



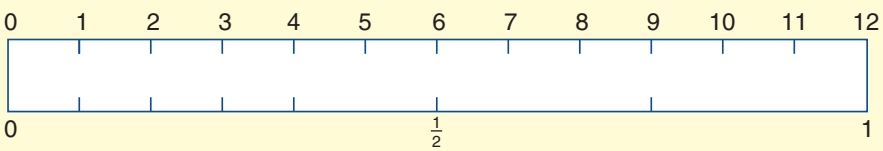
2 Which is bigger?

a $\frac{1}{4}$ or $\frac{1}{2}$ **b** $\frac{1}{3}$ or $\frac{1}{10}$ **c** $\frac{1}{6}$ or $\frac{1}{8}$

3 **a** $\frac{1}{2}$ of 12 **b** $\frac{1}{4}$ of 12 **c** $\frac{1}{3}$ of 12

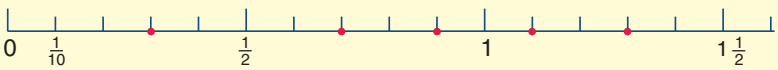
d $\frac{1}{6}$ of 12 **e** $\frac{1}{12}$ of 12 **f** $\frac{3}{4}$ of 12

4



Copy the diagram above into your book.
Use the answers to question 3 to place the fractions, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{12}$ and $\frac{3}{4}$ in their correct places along the lower edge.

5 List the fractions that have been graphed on the number line below.

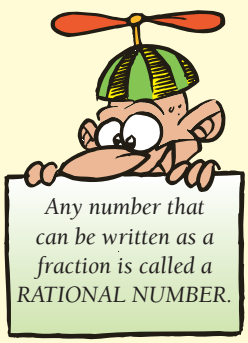


6 Graph each set of fractions on a separate number line.

a $\{\frac{1}{4}, \frac{1}{2}, \frac{3}{4}\}$ **b** $\{\frac{2}{10}, \frac{7}{10}, 1\frac{1}{10}\}$ **c** $\{\frac{5}{12}, \frac{1}{3}, \frac{3}{4}\}$

7 By referring to the number line in question 5, place either < or > between each pair of fractions.

a $\frac{1}{2}$ $\frac{3}{10}$ **b** $\frac{9}{10}$ $1\frac{1}{10}$ **c** $1\frac{1}{2}$ $\frac{13}{10}$



Any number that can be written as a fraction is called a RATIONAL NUMBER.

■ < means 'is less than'.

8 Place each set of fractions in ascending order.

a $\{\frac{3}{10}, \frac{1}{2}, \frac{1}{4}\}$ **b** $\{\frac{1}{3}, \frac{3}{4}, \frac{1}{5}\}$ **c** $\{\frac{7}{10}, \frac{1}{2}, \frac{3}{4}\}$

■ *Note:* $\frac{2}{0}$ is not real. You cannot have 2 of zero equal parts.

9 Lachlan used Cuisenaire material to demonstrate equivalent fractions. Complete his findings.

Orange									
$\frac{1}{2}$					$\frac{1}{2}$				
$\frac{1}{5}$		$\frac{1}{5}$		$\frac{1}{5}$		$\frac{1}{5}$		$\frac{1}{5}$	
$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

← 1 whole

← 2 halves

← 5 fifths

← 10 tenths

a $\frac{1}{1} = \frac{\cdots}{2} = \frac{\cdots}{5} = \frac{\cdots}{10}$

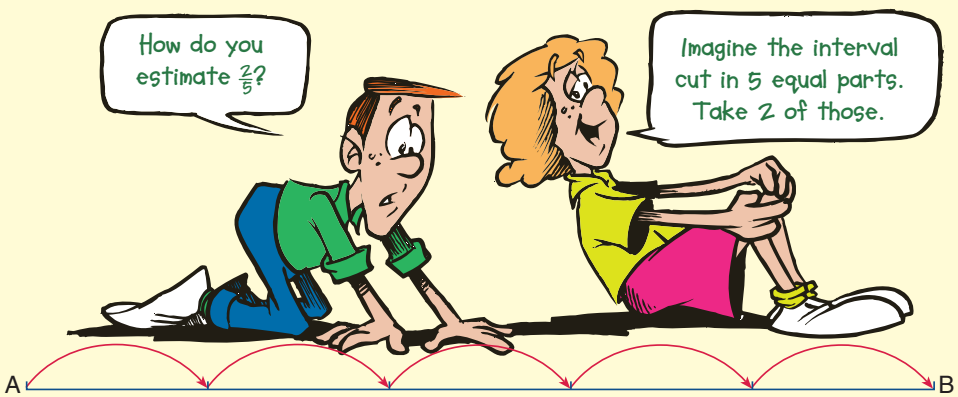
b $\frac{1}{2} = \frac{\cdots}{10}$ **c** $\frac{1}{5} = \frac{\cdots}{10}$

d $\frac{2}{5} = \frac{\cdots}{10}$ **e** $\frac{3}{5} = \frac{\cdots}{10}$

f $\frac{4}{5} = \frac{\cdots}{10}$

10 Draw an interval 10 centimetres long. Let this represent one unit. Using only a pair of compasses, a ruler and a pencil, mark and label the positions on the interval that would represent $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{8}$ and $\frac{5}{8}$.

11



Draw four intervals, each 120 mm long, like the one above.

a On the first interval, estimate the position that is $\frac{1}{2}$ of the length from A to B.
What is $\frac{1}{2}$ of 120 mm? Measure the distance from A to your estimate and calculate your error.

b What is $\frac{1}{3}$ of 120 mm? Estimate this position on your second interval and calculate your error.

c What is $\frac{2}{3}$ of 120 mm? Estimate this position on your third interval and calculate your error.

d What is $\frac{3}{5}$ of 120 mm? Estimate this position on your fourth interval and calculate your error.

12



Consider the interval *XY*. Which point shows:

a $\frac{1}{2}$ **b** $\frac{1}{6}$ **c** $\frac{1}{4}$ **d** $\frac{1}{3}$ **e** $\frac{3}{4}$ **f** $\frac{2}{3}$ **g** $\frac{9}{10}$

6:03 | Review of Fractions

If you have difficulty with this review try Checkup 1:09B on page 22 and get help from Appendix B if necessary.

see 1:08B

Summary of important ideas

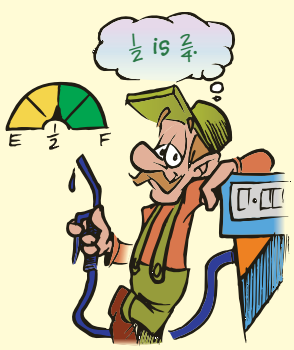
- **The size of a fraction is unchanged if both the numerator and the denominator are multiplied or divided by the same number.**
  eg $\frac{3}{8} = \frac{3\times5}{8\times5} = \frac{15}{40}$ … $\frac{3}{8}$ **and** $\frac{15}{40}$ **are equivalent fractions.**
- **To compare fractions, make their denominators the same. Then compare the numerators.**
  eg $\frac{3}{8} < \frac{4}{8}$
- **An *improper fraction* has a numerator that is greater than its denominator.**
  eg $\frac{5}{2}, \frac{13}{10}$
- **A *mixed number* is one that has a whole number part and a fraction part.**
  eg $2\frac{1}{2}, 1\frac{3}{10}$
- **When fractions have the *same denominator*, we can add them by adding numerators.**
  eg $\frac{3}{10} + \frac{4}{10} = \frac{7}{10}$
  We can subtract them by subtracting numerators.
  eg $\frac{7}{8} - \frac{2}{8} = \frac{5}{8}$
- **$5 \times \frac{3}{4}$ means '5 lots of $\frac{3}{4}$',**
  or $\frac{5\times3}{4} = \frac{15}{4}$ **or** $3\frac{3}{4}$
- **To find $\frac{7}{8}$ of a number, find $\frac{1}{8}$ of the number and then multiply by 7.**
  (To find $\frac{1}{8}$ of a number, divide it by 8.)

Exercise 6:03

1 Complete each set of equivalent fractions.

a $\frac{1}{2} = \frac{\cdots}{4} = \frac{\cdots}{10}$ **b** $\frac{1}{4} = \frac{\cdots}{8} = \frac{\cdots}{100}$ **c** $\frac{1}{5} = \frac{\cdots}{10} = \frac{\cdots}{100}$

d $\frac{3}{2} = \frac{\cdots}{4} = \frac{\cdots}{10}$ **e** $\frac{3}{4} = \frac{\cdots}{8} = \frac{\cdots}{100}$ **f** $\frac{2}{5} = \frac{\cdots}{10} = \frac{\cdots}{100}$



2 In each case find the value of n that makes the sentence true.

a $\frac{4}{5} = \frac{n}{10}$ **b** $\frac{3}{10} = \frac{n}{100}$ **c** $\frac{3}{5} = \frac{n}{20}$

d $\frac{n}{20} = \frac{3}{2}$ **e** $\frac{n}{18} = \frac{2}{3}$ **f** $\frac{n}{100} = \frac{7}{20}$

3 Find the value of x if the fractions in each pair are equivalent.

a $\frac{3}{10}, \frac{x}{40}$ **b** $\frac{3}{8}, \frac{x}{24}$ **c** $\frac{4}{5}, \frac{x}{20}$

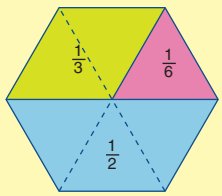
d $\frac{x}{100}, \frac{8}{25}$ **e** $\frac{x}{3}, \frac{12}{18}$ **f** $\frac{x}{48}, \frac{3}{4}$

4 Simplify:

a $\frac{6}{10}$ **b** $\frac{15}{20}$ **c** $\frac{12}{36}$

d $\frac{50}{80}$ **e** $\frac{35}{100}$ **f** $\frac{14}{42}$

g $\frac{24}{72}$ **h** $\frac{35}{49}$ **i** $\frac{24}{40}$



The pattern blocks show:

$\frac{1}{3} = \frac{2}{6}$

$\frac{1}{2} = \frac{3}{6}$

5 What fraction is:

a 60° of 180°? **b** 10° of 180°? **c** 90° of 180°?

d 120° of 180°? **e** 70° of 180°? **f** 135° of 180°?

g 5° of 360°? **h** 40° of 360°? **i** 220° of 360°?

6 By writing each fraction with a common denominator, arrange these fractions in ascending order.

a $\frac{7}{10}, \frac{1}{2}, \frac{1}{5}$ **b** $\frac{5}{8}, \frac{1}{2}, \frac{3}{4}$ **c** $\frac{11}{24}, \frac{1}{2}, \frac{5}{8}$

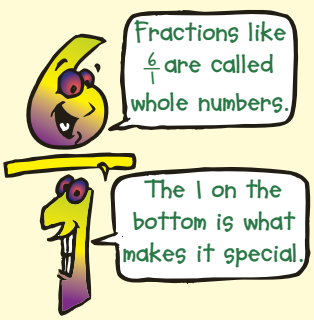
d $\frac{2}{3}, \frac{5}{12}, \frac{3}{4}$ **e** $\frac{3}{4}, \frac{4}{5}, \frac{73}{100}$ **f** $\frac{3}{10}, \frac{11}{40}, \frac{1}{8}$

'Common' means 'belonging to both'.

7 Which of the signs <, = or > should be placed in each circle?

a $\frac{7}{10} \bigcirc \frac{3}{4}$ **b** $\frac{5}{8} \bigcirc \frac{15}{24}$ **c** $\frac{18}{24} \bigcirc \frac{14}{20}$

d $\frac{32}{100} \bigcirc \frac{1}{3}$ **e** $\frac{4}{5} \bigcirc \frac{5}{7}$ **f** $\frac{8}{10} \bigcirc \frac{6}{7}$



8 Write each improper fraction as a mixed number.

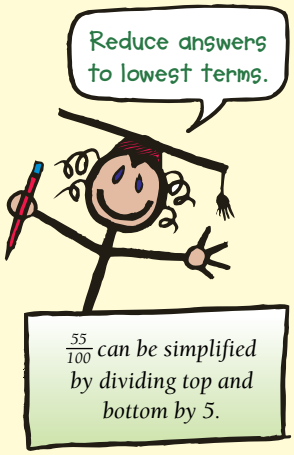
a $\frac{7}{4}$ **b** $\frac{11}{8}$ **c** $\frac{3}{2}$

d $\frac{8}{5}$ **e** $\frac{13}{10}$ **f** $\frac{723}{100}$

9 Write each mixed number as an improper fraction.

a $3\frac{1}{2}$ **b** $2\frac{3}{4}$ **c** $1\frac{1}{3}$

d $10\frac{2}{3}$ **e** $9\frac{3}{5}$ **f** $76\frac{7}{10}$



10 **a** $\frac{7}{10} + \frac{1}{10}$ **b** $\frac{5}{10} + \frac{3}{10}$ **c** $\frac{37}{100} + \frac{18}{100}$

d $\frac{3}{4} + \frac{3}{4}$ **e** $\frac{7}{10} + \frac{3}{10}$ **f** $\frac{13}{20} + \frac{8}{20}$

g $\frac{5}{8} + \frac{1}{8}$ **h** $\frac{4}{5} + \frac{3}{5}$ **i** $\frac{2}{3} + \frac{2}{3}$

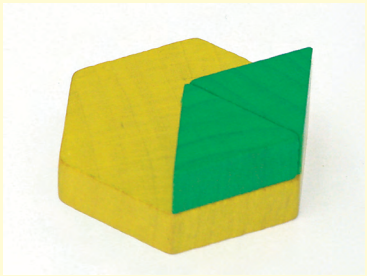
11 a $\frac{9}{10} - \frac{3}{10}$ b $\frac{7}{10} - \frac{1}{10}$ c $\frac{8}{10} - \frac{3}{10}$

d $\frac{7}{4} - \frac{3}{4}$ e $\frac{12}{5} - \frac{7}{5}$ f $\frac{13}{20} - \frac{4}{20}$

g $\frac{78}{100} - \frac{31}{100}$ h $\frac{11}{24} - \frac{3}{24}$ i $\frac{16}{16} - \frac{3}{16}$

12 a $1 + \frac{3}{4}$ b $4 + \frac{3}{10}$ c $6 + \frac{2}{3}$

d $1 - \frac{3}{4}$ e $1 - \frac{3}{10}$ f $1 - \frac{2}{3}$



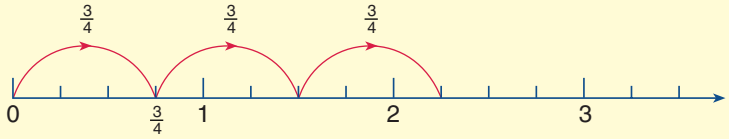
- What is $1 - \frac{2}{6}$?

13 Use this number line to find:

a two lots of $\frac{3}{4}$

b three lots of $\frac{3}{4}$

c four lots of $\frac{3}{4}$



d $3 \times \frac{3}{4}$ e $2 \times \frac{3}{4}$ f $4 \times \frac{3}{4}$

14 a $2 \times \frac{2}{5}$ b $5 \times \frac{2}{10}$ c $7 \times \frac{2}{3}$

d $3 \times \frac{3}{10}$ e $6 \times \frac{9}{10}$ f $8 \times \frac{2}{5}$

g $9 \times \frac{13}{100}$ h $4 \times \frac{3}{8}$ i $19 \times \frac{1}{2}$

worked example

$$9 \times \frac{11}{20} = \frac{9 \times 11}{20}$$
$$= \frac{99}{20}$$
$$= 4\frac{19}{20}$$

15 a $\frac{1}{4}$ of 20 b $\frac{1}{5}$ of 100 c $\frac{1}{8}$ of 24

d $\frac{3}{4}$ of 20 e $\frac{4}{5}$ of 100 f $\frac{7}{8}$ of 24

g $\frac{7}{10}$ of 150 h $\frac{3}{8}$ of 800 i $\frac{11}{100}$ of 9000

16 a $\frac{3}{4} + 0$ b $\frac{5}{8} \times 1$ c $\frac{7}{10} - 0$ d $\frac{2}{3} \times 0$

e $\frac{17}{20}$ of 0 f $0 \times \frac{91}{100}$ g $0 + \frac{9}{10}$ h $1 \times \frac{7}{8}$

i $\frac{4}{5} + \frac{3}{5} = \frac{3}{5} + \frac{4}{5}$. True or false? j $5 \times \frac{2}{5} = \frac{2}{5} \times 5$. True or false?

k Are $\frac{3}{5}$ and $\frac{21}{35}$ equivalent? l Are $\frac{75}{85}$ and $\frac{7}{8}$ equivalent?

m $6 \times \frac{3}{5} + 6 \times \frac{2}{5}$ n $6 \times (\frac{3}{5} + \frac{2}{5})$

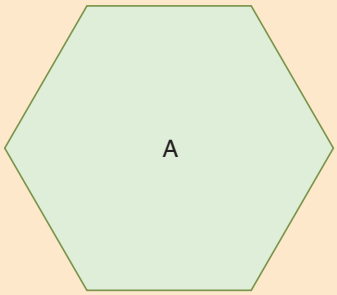
o Is $6 \times \frac{3}{5} + 6 \times \frac{2}{5}$ equal to $6 \times (\frac{3}{5} + \frac{2}{5})$?

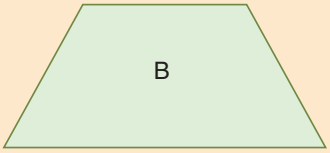
p Is $10 \times 1\frac{1}{2} + 10 \times \frac{1}{2}$ equal to $10 \times (1\frac{1}{2} + \frac{1}{2})$?

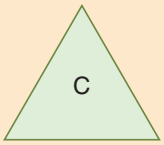
investigation 6:03

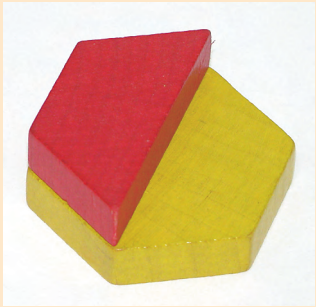
Investigation 6:03 | Fractions

Please use the Assessment Grid on the following page to help you understand what is required for this Investigation.





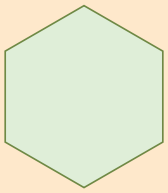
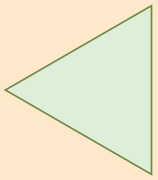




These blocks are called pattern blocks.

- How many blocks like B are needed to cover A?
- How many blocks like C are needed to cover A?
- What fraction of A is block C?

Complete the table below, writing in each white square the fraction that the left figure is of the top figure.

What fraction?				
				
				

Extension: What fractions should go in the blue squares?

id 6:03

Refer to ID Card 4 on page xvi. Identify figures (1) to (24).
Learn the terms you do not know.

Assessment Grid for Investigation 6:03 | Fractions

The following is a sample assessment grid for this investigation. You should carefully read the criteria *before* beginning the investigation so that you know what is required.

		Assessment Criteria (B, C, D)		Achieved ✓
Criterion B Application & Reasoning	a	No systematic or organised approach has been used to solve the problem.	1	
			2	
	b	An attempt was made at an organised approach, but this did not really help solve the problem.	3	
			4	
	c	A systematic approach was made and helped arrive at reasonable answers.	5	
			6	
	d	A systematic and organised approach was made and the majority of the answers were reasonable.	7	
			8	
	e	All the answers obtained were within reason, well explained and demonstrated the ability to adopt a systematic approach.	9	
			10	
Criterion C Communication	a	Not many explanations were given and the steps undertaken are hard to follow.	1	
			2	
	b	Some working out was shown and an attempt was made to use both words and diagrams.	3	
			4	
	c	Working out was shown and is good and easy to follow. The steps were given in an organised manner.	5	
			6	
Criterion D Reflection & Evaluation	a	Some attempt has been made to explain the steps taken.	1	
			2	
	b	The steps undertaken were explained with some success and some answers were checked for reasonableness.	3	
			4	
	c	All the steps were explained successfully and all answers checked for their reasonableness.	5	
			6	
	d	All steps were explained and answers checked. Alternatives may have been discussed.	7	
			8	

6:04 | Addition and Subtraction of Fractions

prep quiz 6:04

Complete:

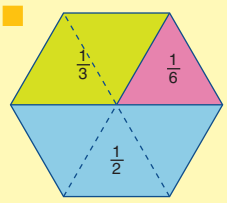
1 $\frac{3}{4} = \frac{\dots}{12}$ 2 $\frac{1}{3} = \frac{\dots}{12}$ 3 $\frac{9}{12} + \frac{4}{12}$ 4 $\frac{9}{12} - \frac{4}{12}$

5 $\frac{4}{5} = \frac{\dots}{15}$ 6 $\frac{2}{3} = \frac{\dots}{15}$ 7 $\frac{12}{15} + \frac{10}{15}$ 8 $\frac{12}{15} - \frac{10}{15}$

9 $\frac{1}{2} = \frac{\dots}{4} = \frac{\dots}{6} = \frac{\dots}{8} = \frac{\dots}{10} = \frac{\dots}{12}$

10 $\frac{1}{5} = \frac{\dots}{10} = \frac{\dots}{15} = \frac{\dots}{20} = \frac{\dots}{25} = \frac{\dots}{30}$

We are reminded in the Prep Quiz that we must have fractions with the same denominator if we wish to add or subtract the numerators.



The pattern blocks show:

$\frac{1}{2} + \frac{1}{3}$

$= \frac{3}{6} + \frac{2}{6}$

$= \frac{5}{6}$

Before adding or subtracting fractions, we must express each fraction with the same denominator. This is called a *common denominator*. We always try to use the *lowest common denominator* (the LCD).

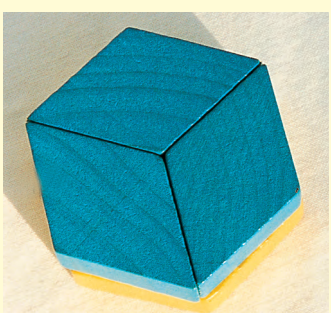
eg $\frac{4}{5} = \frac{12}{15}$, $\frac{2}{3} = \frac{10}{15}$ $\therefore \frac{4}{5} + \frac{2}{3} = \frac{12}{15} + \frac{10}{15} = \frac{22}{15} = 1\frac{7}{15}$

Each rational number can be represented by an infinite number of equivalent fractions, eg

three-tenths: $\frac{3}{10}, \frac{6}{20}, \frac{9}{30}, \frac{12}{40}, \frac{15}{50}, \dots$

five-eighths: $\frac{5}{8}, \frac{10}{16}, \frac{15}{24}, \frac{20}{32}, \frac{25}{40}, \dots$

To add $\frac{3}{10}$ and $\frac{5}{8}$, we look for a common denominator in each row.



• $1 = \frac{3}{3}$.

worked examples

1 $\frac{3}{10} + \frac{5}{8}$

10 and 8 could each be changed to 40 or to 80. The LCD is 40.

$\frac{3}{10} + \frac{5}{8} = \frac{3}{10} \times \frac{4}{4} + \frac{5}{8} \times \frac{5}{5}$

$= \frac{12}{40} + \frac{25}{40}$

$= \frac{37}{40}$

2 $\frac{3}{4} - \frac{2}{5}$

4 and 5 each go into 20 exactly. 20 is the LCD.

$\frac{3}{4} - \frac{2}{5} = \frac{3}{4} \times \frac{5}{5} - \frac{2}{5} \times \frac{4}{4}$

$= \frac{15}{20} - \frac{8}{20}$

$= \frac{7}{20}$

Exercise 6:04

1 a $\frac{7}{10} + \frac{2}{10}$ b $\frac{1}{6} + \frac{4}{6}$ c $\frac{5}{8} + \frac{2}{8}$

d $\frac{3}{20} + \frac{8}{20}$ e $\frac{2}{10} + \frac{5}{10}$ f $\frac{1}{12} + \frac{4}{12}$

g $\frac{5}{15} + \frac{2}{15}$ h $\frac{17}{30} + \frac{2}{30}$ i $\frac{40}{100} + \frac{17}{100}$

2 a $\frac{9}{10} - \frac{2}{10}$ b $\frac{11}{15} - \frac{4}{15}$ c $\frac{7}{8} - \frac{6}{8}$

d $\frac{29}{30} - \frac{6}{30}$ e $\frac{3}{4} - \frac{2}{4}$ f $\frac{14}{20} - \frac{7}{20}$

g $\frac{20}{20} - \frac{11}{20}$ h $\frac{100}{100} - \frac{47}{100}$ i $\frac{10}{10} - \frac{7}{10}$

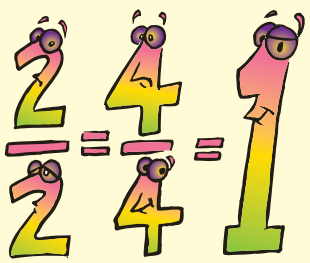
3 Write the simplest answer for each.

a $\frac{3}{10} + \frac{1}{10}$ b $\frac{9}{15} - \frac{4}{15}$ c $\frac{3}{8} + \frac{1}{8}$

d $\frac{20}{20} - \frac{14}{20}$ e $\frac{7}{8} + \frac{1}{8}$ f $\frac{11}{12} - \frac{8}{12}$

g $\frac{3}{5} + \frac{3}{5}$ h $\frac{7}{10} + \frac{3}{10}$ i $\frac{3}{8} - \frac{3}{8}$

j $\frac{11}{100} - \frac{6}{100}$ k $\frac{9}{15} - \frac{9}{15}$ l $\frac{14}{15} + \frac{16}{15}$



4 Simplify:

a $\frac{7}{10} + \frac{1}{5}$ b $\frac{7}{10} + \frac{3}{5}$ c $\frac{1}{10} + \frac{4}{5}$

d $\frac{1}{2} + \frac{1}{10}$ e $\frac{1}{2} + \frac{3}{10}$ f $\frac{1}{2} + \frac{7}{10}$

g $\frac{3}{4} + \frac{1}{3}$ h $\frac{1}{4} + \frac{2}{3}$ i $\frac{3}{4} + \frac{2}{3}$

j $\frac{1}{3} + \frac{1}{2}$ k $\frac{2}{3} + \frac{1}{2}$ l $\frac{4}{3} + \frac{1}{2}$

m $\frac{3}{5} + \frac{1}{4}$ n $\frac{2}{5} + \frac{3}{4}$ o $\frac{4}{5} + \frac{3}{4}$

p $\frac{3}{4} + \frac{7}{10}$ q $\frac{2}{3} + \frac{3}{5}$ r $\frac{3}{4} + \frac{1}{2}$

s $\frac{3}{5} + \frac{1}{2}$ t $\frac{1}{3} + \frac{3}{10}$ u $\frac{3}{10} + \frac{1}{2}$



Foundation Worksheet 6:04

Addition and subtraction of fractions

1 a $\frac{2}{10} + \frac{3}{10} = \ldots$

2 a 0, $\frac{4}{8}$, 1, $1\frac{4}{8}$

$\frac{3}{8} + \frac{2}{8} = \ldots$

3 a $\frac{3}{5} - \frac{1}{5} = \ldots$

4 a $\frac{2}{8} + \frac{1}{8}$ b $\frac{7}{8} - \frac{6}{8}$

5 Simplify:

a $\frac{9}{10} - \frac{1}{2}$ **b** $\frac{7}{10} - \frac{1}{2}$ **c** $\frac{8}{10} - \frac{1}{2}$ **d** $\frac{3}{8} - \frac{1}{5}$

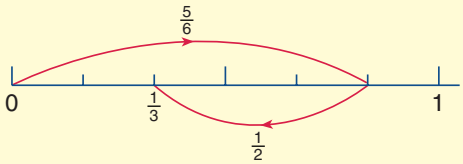
e $\frac{5}{8} - \frac{2}{5}$ **f** $\frac{7}{8} - \frac{4}{5}$ **g** $\frac{3}{5} - \frac{3}{10}$ **h** $\frac{3}{4} - \frac{2}{3}$

i $\frac{1}{2} - \frac{1}{3}$ **j** $\frac{3}{4} - \frac{1}{5}$ **k** $\frac{3}{4} - \frac{7}{10}$ **l** $\frac{1}{4} - \frac{1}{5}$

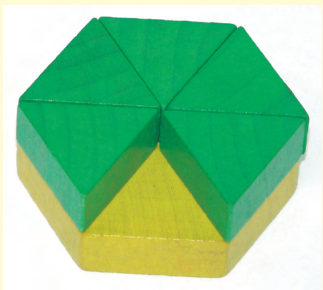
m $\frac{1}{4} - \frac{1}{10}$ **n** $\frac{2}{3} - \frac{2}{5}$ **o** $\frac{3}{5} - \frac{1}{2}$

p $\frac{2}{3} - \frac{3}{10}$ **q** $\frac{9}{10} - \frac{1}{2}$ **r** $\frac{97}{100} - \frac{3}{4}$

s $\frac{7}{8} - \frac{2}{3}$ **t** $\frac{9}{20} - \frac{7}{10}$ **u** $\frac{79}{100} - \frac{3}{5}$



■ $\frac{5}{6} - \frac{1}{2} = \frac{1}{3}$



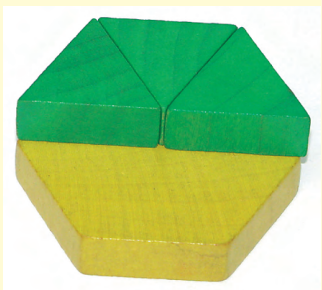
6 Simplify:

a $\frac{3}{4} + \frac{1}{6}$ **b** $\frac{7}{18} - \frac{1}{6}$ **c** $\frac{5}{12} + \frac{3}{8}$

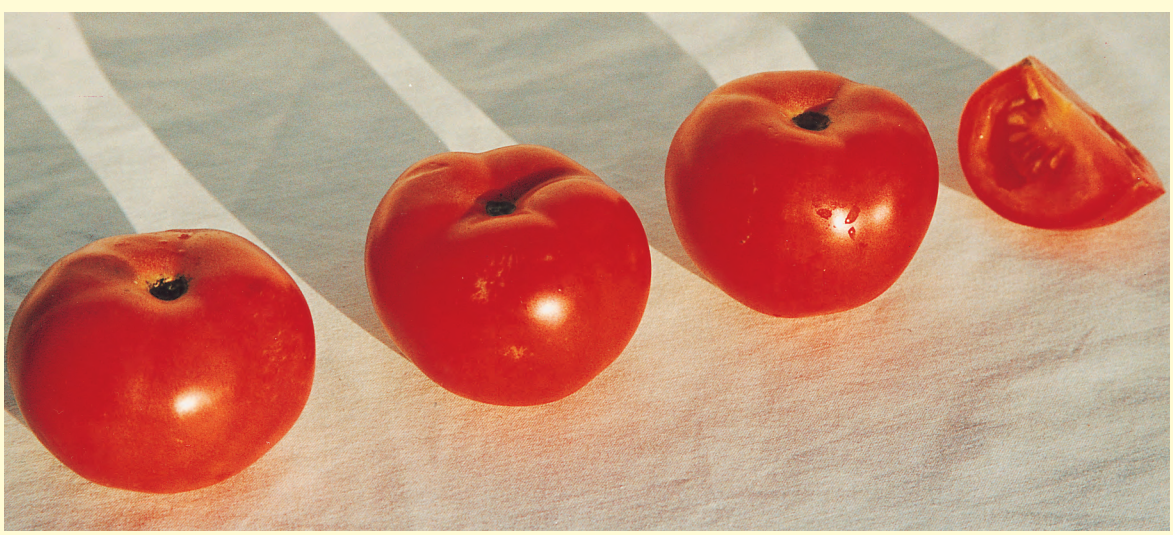
d $\frac{1}{2} - \frac{7}{25}$ **e** $\frac{3}{25} + \frac{1}{4}$ **f** $\frac{5}{6} - \frac{3}{8}$

g $\frac{2}{3} - \frac{3}{8}$ **h** $\frac{3}{50} + \frac{11}{20}$ **i** $\frac{3}{4} - \frac{1}{12}$

j $\frac{3}{40} + \frac{2}{3}$ **k** $\frac{3}{4} - \frac{2}{7}$ **l** $\frac{4}{9} - \frac{1}{4}$



• $\frac{5}{6} - \frac{1}{2} = \ldots$



• How many quarters in $3\frac{1}{4}$?

6:05 | Addition and Subtraction of Mixed Numbers

prep quiz 6:05

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
5	10	15	20	25	30	35	40	45	50

2-stick

5-stick

Heather wrote the answers to multiplication tables along some ice-block sticks. She had spaced the answers 1 cm apart. When she placed one stick under another she was surprised to see a whole list of equivalent fractions.

Use the sticks above to complete these.

1 $\frac{1}{3} = \frac{\cdots}{24}$ **2** $\frac{1}{3} = \frac{\cdots}{21}$ **3** $\frac{9}{27} = \frac{\cdots}{30}$ **4** $\frac{4}{12} = \frac{\cdots}{18}$

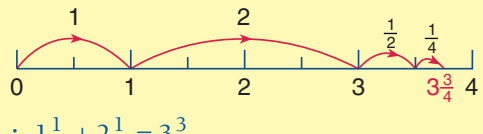
5 $\frac{2}{5} = \frac{\cdots}{35}$ **6** $\frac{2}{5} = \frac{\cdots}{50}$ **7** $\frac{8}{20} = \frac{\cdots}{30}$ **8** $\frac{18}{45} = \frac{\cdots}{25}$

What is the LCD (lowest common denominator) of:

9 $\frac{1}{3}$ and $\frac{2}{5}$ **10** $\frac{3}{5}$ and $\frac{1}{2}$

Addition of mixed numbers

■ $1\frac{1}{2} + 2\frac{1}{4} = (1 + 2) + (\frac{1}{2} + \frac{1}{4})$



$\therefore 1\frac{1}{2} + 2\frac{1}{4} = 3\frac{3}{4}$

worked example

$3\frac{2}{5} + 1\frac{3}{4} = (3 + 1) + (\frac{2}{5} + \frac{3}{4})$

$= 4 + (\frac{2}{5} \times \frac{4}{4} + \frac{3}{4} \times \frac{5}{5})$

$= 4 + (\frac{8}{20} + \frac{15}{20})$

$= 4\frac{23}{20}$ (ie $4 + 1\frac{3}{20}$)

$= 5\frac{3}{20}$

■ $\frac{20}{20} = 1$



■ The average family has $1\frac{1}{2}$ girls and $1\frac{1}{4}$ boys.

Subtraction of mixed numbers

worked examples

1 $5 - 2\frac{3}{5} = (5 - 2) - \frac{3}{5}$

$= 3 - \frac{3}{5}$

$= 2 + (1 - \frac{3}{5})$

$= 2\frac{2}{5}$

2 $6\frac{3}{4} - 2\frac{1}{10} = (6 - 2) + (\frac{3}{4} - \frac{1}{10})$

$= 4 + (\frac{3}{4} \times \frac{5}{5} - \frac{1}{10} \times \frac{2}{2})$

$= 4 + (\frac{15}{20} - \frac{2}{20})$

$= 4\frac{13}{20}$

continued →→→

3 *Harder type:* Method 1

$$13\tfrac{1}{5} - 1\tfrac{1}{2} = (13 - 1) + (\tfrac{1}{5} - \tfrac{1}{2})$$
$$= 12 + (\tfrac{1}{5} - \tfrac{1}{2})$$
[$\frac{1}{2}$ is bigger than $\frac{1}{5}$]
$$= 11 + (1\tfrac{1}{5} - \tfrac{1}{2})$$
$$= 11 + (\tfrac{6}{5} - \tfrac{1}{2})$$
$$= 11 + (\tfrac{12}{10} - \tfrac{5}{10})$$
$$= 11 + \tfrac{7}{10}$$
$$= 11\tfrac{7}{10}$$

4 *Harder type:* Method 2

$$13\tfrac{1}{5} - 1\tfrac{1}{2} = (13 - 1) + (\tfrac{1}{5} - \tfrac{1}{2})$$
$$= 12 + (\tfrac{2}{10} - \tfrac{5}{10})$$
$$= 12 + (-\tfrac{3}{10})$$ [using directed numbers]
$$= 12 - \tfrac{3}{10}$$
$$= 11\tfrac{7}{10}$$

Note: $1 - \frac{3}{10} = \frac{7}{10}$

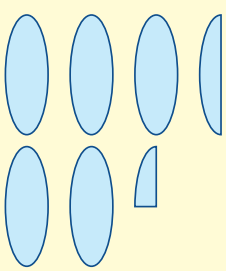
Exercise 6:05

Foundation Worksheet 6:05
Addition and subtraction of mixed numbers
1 a $1 + \frac{3}{4}$ b $5 + \frac{1}{10}$
2 a $2 + 1\frac{1}{2}$ b $2\frac{1}{4} + 3$
3 a $1 - \frac{1}{2}$ b $2 - \frac{1}{4}$
4 a $3\frac{1}{2} - 1$ b $4\frac{3}{4} - 2$

1 a $1 + \frac{3}{4}$ b $3 + \frac{1}{5}$ c $4\frac{1}{10} + 3$
d $7\frac{1}{2} + 6$ e $2 + 3\frac{1}{3}$ f $8 + 1\frac{7}{8}$
g $6 + 4\frac{1}{4}$

2 a $3\frac{3}{10} + 2\frac{4}{10}$ b $6\frac{3}{8} + \frac{4}{8}$ c $1\frac{2}{4} + 1\frac{1}{4}$
d $1\frac{3}{5} + 4\frac{2}{5}$ e $6\frac{1}{4} + 3\frac{3}{4}$ f $2\frac{7}{8} + 1\frac{1}{8}$
g $4\frac{1}{4} + 4\frac{1}{4}$ h $9\frac{3}{10} + 4\frac{5}{10}$ i $1\frac{1}{8} + 6\frac{3}{8}$

3 a $1\frac{3}{4} + \frac{1}{2}$ b $3\frac{3}{10} + \frac{8}{10}$ c $2\frac{7}{8} + \frac{5}{8}$
d $2\frac{3}{10} + 1\frac{1}{2}$ e $6\frac{7}{10} + 7\frac{1}{5}$ f $8\frac{3}{4} + 2\frac{1}{2}$
g $4\frac{3}{8} + 2\frac{1}{3}$ h $6\frac{1}{5} + 1\frac{1}{4}$ i $2\frac{5}{8} + 1\frac{1}{5}$
j $1\frac{7}{8} + 2\frac{3}{5}$ k $6\frac{1}{2} + 5\frac{2}{3}$ l $4\frac{7}{10} + 1\frac{2}{3}$
m $6\frac{2}{3} + 2\frac{4}{5}$ n $3\frac{3}{20} + 2\frac{3}{4}$ o $10\frac{7}{50} + 2\frac{19}{100}$
p $4\frac{1}{6} + 3\frac{2}{5}$ q $6\frac{3}{4} + 1\frac{5}{6}$ r $4\frac{8}{15} + 2\frac{5}{8}$



$3\frac{1}{2} + 2\frac{1}{4}$

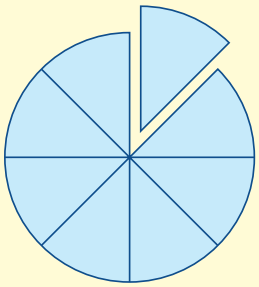
4 a $1 - \frac{1}{10}$ b $1 - \frac{7}{10}$ c $1 - \frac{3}{4}$
d $1 - \frac{5}{8}$ e $1 - \frac{9}{30}$ f $1 - \frac{17}{20}$
g $1 - \frac{93}{100}$ h $1 - \frac{3}{12}$ i $1 - \frac{4}{15}$

$$1 - \tfrac{17}{20} = \tfrac{1}{1} - \tfrac{17}{20}$$
$$= \tfrac{20}{20} - \tfrac{17}{20}$$
$$= \tfrac{3}{20}$$

5 **a** $2 - \frac{1}{10}$ **b** $5 - \frac{3}{10}$ **c** $7 - \frac{3}{8}$

d $4 - \frac{7}{12}$ **e** $9 - \frac{1}{30}$ **f** $3 - \frac{2}{15}$

g $10 - \frac{21}{30}$ **h** $1 - \frac{5}{24}$ **i** $1 - \frac{31}{100}$



$1 - \frac{1}{8}$

6 **a** $8\frac{3}{4} - 6$ **b** $14\frac{1}{2} - 9$ **c** $8\frac{2}{5} - 3$

d $6\frac{7}{10} - 2\frac{6}{10}$ **e** $7\frac{9}{12} - 1\frac{4}{12}$ **f** $4\frac{80}{100} - 3\frac{9}{100}$

g $5\frac{7}{8} - 5\frac{4}{8}$ **h** $9\frac{3}{10} - 5\frac{3}{10}$ **i** $2\frac{17}{20} - 2\frac{7}{20}$

7 **a** $6\frac{8}{10} - 2\frac{1}{2}$ **b** $4\frac{7}{8} - 1\frac{1}{4}$ **c** $5\frac{9}{10} - 2\frac{3}{5}$

d $5\frac{2}{3} - 3\frac{1}{4}$ **e** $10\frac{7}{10} - 1\frac{2}{3}$ **f** $7\frac{3}{8} - 5\frac{1}{10}$

g $2\frac{11}{20} - 2\frac{2}{5}$ **h** $12\frac{11}{16} - 12\frac{3}{8}$ **i** $9\frac{2}{3} - 9\frac{1}{2}$

8 **a** $4\frac{1}{2} - \frac{7}{8}$ **b** $6\frac{3}{5} - 1\frac{9}{10}$ **c** $7\frac{1}{4} - 2\frac{7}{12}$

d $7\frac{1}{2} - 2\frac{2}{3}$ **e** $12\frac{3}{10} - 4\frac{2}{3}$ **f** $9\frac{3}{8} - 2\frac{4}{5}$

g $6\frac{1}{100} - 3\frac{1}{8}$ **h** $2\frac{7}{100} - 1\frac{3}{40}$ **i** $15\frac{7}{15} - 8\frac{11}{12}$



6:06 | Multiplication of Fractions

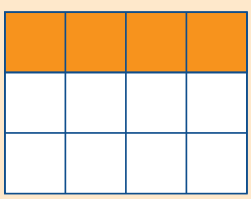
1 7 lots of $\frac{3}{8}$ **2** $4 \times \frac{7}{10}$ **3** $\frac{1}{3}$ of 6 fifths **4** $\frac{1}{2}$ of 4 fifths

Change these mixed numbers to improper fractions.

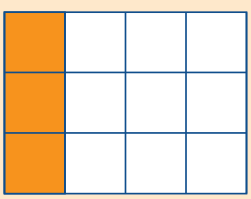
5 $2\frac{1}{4}$ **6** $3\frac{2}{3}$ **7** $1\frac{4}{5}$

What fraction of each rectangle has been shaded?

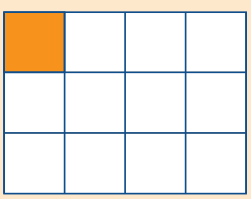
8



9



10

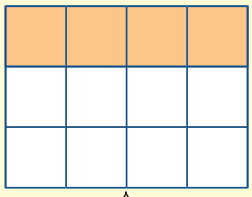


prep quiz 6:06

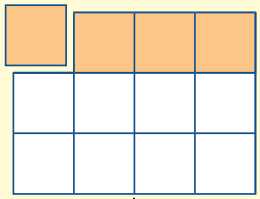
see Appendix B:09

Discussion: multiplying a fraction by a fraction

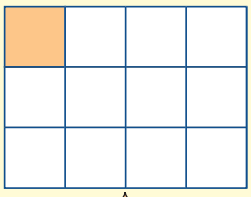
1



$\frac{1}{3}$ has been shaded.



$\frac{1}{4}$ of the shaded part has been cut out.



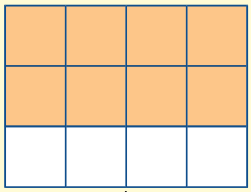
($\frac{1}{4}$ of $\frac{1}{3}$) or ($\frac{1}{4} \times \frac{1}{3}$) is $\frac{1}{12}$.



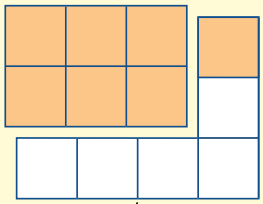
Conclusion

$$\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$$

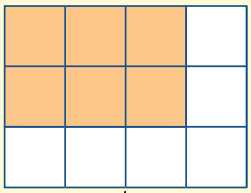
2



$\frac{2}{3}$ has been shaded.



$\frac{3}{4}$ of the shaded part has been cut out.



($\frac{3}{4}$ of $\frac{2}{3}$) or ($\frac{3}{4} \times \frac{2}{3}$) is $\frac{6}{12}$.



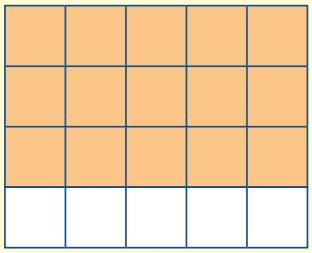
Conclusion

$$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$$

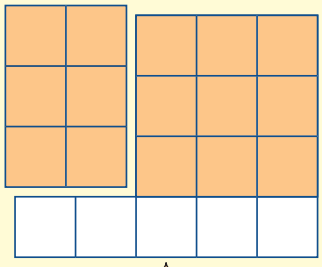
Note:

$$\frac{\cancel{6}^{1}}{\cancel{12}_{2}} = \frac{1}{2}$$

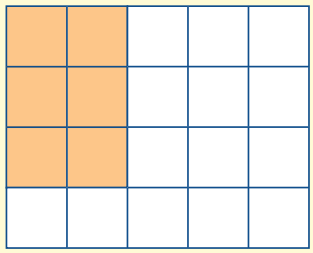
3



$\frac{3}{4}$ has been shaded.



$\frac{2}{5}$ of the shaded part has been cut out.



($\frac{2}{5}$ of $\frac{3}{4}$) or ($\frac{2}{5} \times \frac{3}{4}$) is $\frac{6}{20}$.



Conclusion

$\frac{2}{5} \times \frac{3}{4} = \frac{6}{20}$

Note:

$\frac{\cancel{6}^{3}}{\cancel{20}_{10}} = \frac{3}{10}$

In the discussion above we have discovered how to multiply a fraction by a fraction.

To multiply two fractions, multiply the numerators and multiply the denominators.

Answer = $\dfrac{\text{numerator 1} \times \text{numerator 2}}{\text{denominator 1} \times \text{denominator 2}}$

worked examples

1 $\frac{1}{4} \times \frac{1}{3} = \frac{1 \times 1}{4 \times 3} = \frac{1}{12}$

2 $\frac{3}{4} \times \frac{2}{3} = \frac{3 \times 2}{4 \times 3} = \frac{6}{12}$ or $\frac{1}{2}$

3 $\frac{2}{5} \times \frac{3}{4} = \frac{2 \times 3}{5 \times 4} = \frac{6}{20}$ or $\frac{3}{10}$

Simplifying the working when multiplying fractions

If there is a number that will divide exactly into both a numerator and a denominator, this division should be done. It is easier to do it before multiplying rather than when we reduce the answer.

worked examples

1 $\frac{7}{8} \times \frac{9}{10}$

(In these two examples there is no number that will exactly divide a top and a bottom.)

$= \frac{7 \times 9}{8 \times 10}$

$= \frac{63}{80}$

2 $\frac{3}{10} \times \frac{7}{10}$

$= \frac{3 \times 7}{10 \times 10}$

$= \frac{21}{100}$

3 $\frac{3}{5} \times \frac{2}{3}$

$= \frac{\overset{1}{\cancel{3}}}{5} \times \frac{2}{\cancel{3}_{1}}$

$= \frac{1 \times 2}{5 \times 1}$

$= \frac{2}{5}$

4 $\frac{9}{10} \times \frac{15}{16}$

$= \frac{9}{\cancel{10}_{2}} \times \frac{\cancel{15}^{3}}{16}$

$= \frac{9 \times 3}{2 \times 16}$

$= \frac{27}{32}$

continued ➜➜➜

5 $4 \times \frac{3}{5}$

$= \frac{4}{1} \times \frac{3}{5}$

$= \frac{12}{5}$

$= 2\frac{2}{5}$

6 $1\frac{7}{8} \times 3$

$= \frac{15}{8} \times \frac{3}{1}$

$= \frac{45}{8}$

$= 5\frac{5}{8}$

7 $3\frac{3}{4} \times 1\frac{1}{3}$

$= \frac{\cancel{15}^{5}}{\cancel{4}_{1}} \times \frac{\cancel{4}^{1}}{\cancel{3}_{1}}$

$= \frac{5}{1}$

$= 5$

■ To multiply mixed numerals, write them as improper fractions.

Exercise 6:06

Foundation Worksheet 6:06

Multiplication of fractions

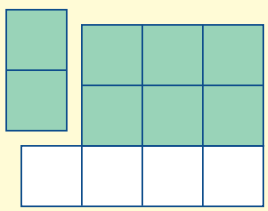
1 a $2 \times \frac{1}{4}$ **b** $\frac{2}{1} \times \frac{1}{4}$ **c** $5 \times \frac{11}{100}$ **d** $\frac{5}{1} \times \frac{11}{100}$

2 a $\frac{1}{2} \times \frac{4}{8}$ **b** $\frac{1}{2} \times \frac{12}{15}$ **c** $\frac{1}{2} \times \frac{10}{20}$ **d** $\frac{1}{2} \times \frac{6}{10}$

3 a $\frac{1}{2} \times \frac{1}{3}$ **b** $\frac{1}{4} \times \frac{1}{3}$

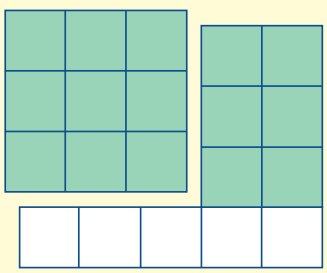
1 Use the diagrams to find the products below them.

a



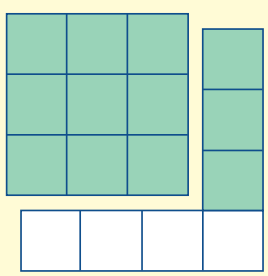
$\frac{1}{4} \times \frac{2}{3} =$

b



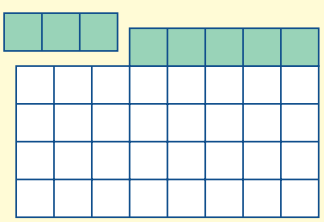
$\frac{3}{5} \times \frac{3}{4} =$

c

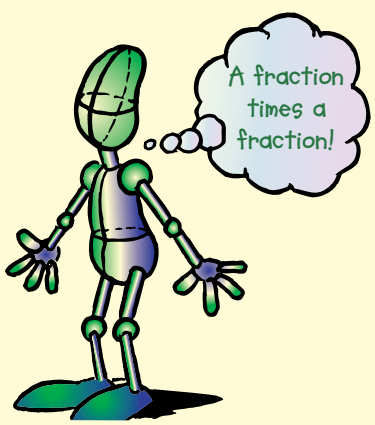


$\frac{3}{4} \times \frac{3}{4} =$

d



$\frac{3}{8} \times \frac{1}{5} =$



2 **a** $\frac{1}{10} \times \frac{1}{10}$ **b** $\frac{3}{10} \times \frac{7}{10}$ **c** $\frac{7}{10} \times \frac{1}{10}$

d $\frac{7}{10} \times \frac{9}{10}$ **e** $\frac{1}{10} \times \frac{3}{10}$ **f** $\frac{9}{10} \times \frac{3}{10}$

g $\frac{11}{100} \times \frac{7}{10}$ **h** $\frac{3}{100} \times \frac{1}{10}$ **i** $\frac{9}{10} \times \frac{7}{100}$

3 **a** $\frac{1}{2} \times \frac{1}{2}$ **b** $\frac{1}{4} \times \frac{1}{2}$ **c** $\frac{1}{2} \times \frac{1}{3}$

d $\frac{1}{2} \times \frac{3}{10}$ **e** $\frac{1}{4} \times \frac{9}{10}$ **f** $\frac{1}{4} \times \frac{3}{4}$

g $\frac{2}{5} \times \frac{3}{10}$ **h** $\frac{7}{8} \times \frac{5}{6}$ **i** $\frac{9}{10} \times \frac{7}{8}$

■ $\frac{3}{10} \times \frac{3}{10} = \frac{9}{100}$

$= 0{\cdot}09$

OR

$\frac{3}{10} \times \frac{3}{10} = 0{\cdot}3 \times 0{\cdot}3$

$\therefore 0{\cdot}3 \times 0{\cdot}3 = 0{\cdot}09$

The rules for fractions and decimals agree.

4 Give answers as basic fractions.

a $\frac{1}{2}$ of $\frac{4}{5}$ **b** $\frac{2}{3}$ of $\frac{3}{10}$ **c** $\frac{3}{5}$ of $\frac{5}{8}$

d $\frac{1}{2} \times \frac{2}{5}$ **e** $\frac{2}{3} \times \frac{3}{8}$ **f** $\frac{3}{5} \times \frac{1}{6}$

g $\frac{5}{6} \times \frac{3}{4}$ **h** $\frac{1}{2} \times \frac{2}{3}$ **i** $\frac{6}{15} \times \frac{7}{8}$

j $\frac{3}{4} \times \frac{4}{5}$ **k** $\frac{3}{4} \times \frac{9}{10}$ **l** $\frac{2}{3} \times \frac{1}{4}$

m $\frac{15}{20} \times \frac{3}{4}$ **n** $\frac{27}{100} \times \frac{5}{6}$ **o** $\frac{18}{25} \times \frac{3}{4}$

p $\frac{3}{10} \times \frac{15}{16}$ **q** $\frac{2}{3} \times \frac{3}{4}$ **r** $\frac{2}{3} \times \frac{9}{16}$



■ $\frac{5}{6} \times \frac{3}{4} = \frac{\cancel{15}^{5}}{\cancel{24}_{8}} = \frac{5}{8}$

OR

$\frac{5}{\cancel{6}_{2}} \times \frac{\cancel{3}^{1}}{4} = \frac{5}{8}$

You may reduce fractions before or after you multiply.

5 **a** $\frac{3}{4} \times \frac{4}{3}$ **b** $\frac{2}{3} \times \frac{3}{2}$ **c** $\frac{5}{8} \times \frac{8}{5}$

d $\frac{3}{10} \times \frac{10}{3}$ **e** $\frac{1}{3} \times \frac{3}{1}$ **f** $\frac{6}{5} \times \frac{5}{6}$

g $\frac{8}{3} \times \frac{3}{8}$ **h** $\frac{4}{5} \times \frac{5}{4}$ **i** $\frac{1}{10} \times \frac{10}{1}$

If two numbers multiply to give 1, they are called *reciprocals* of one another.

6 Give the simplest answer for each.

a $1\frac{1}{2} \times \frac{3}{4}$ **b** $2\frac{1}{2} \times \frac{2}{3}$

c $1\frac{2}{3} \times \frac{3}{4}$ **d** $\frac{5}{8} \times 2\frac{3}{5}$

e $\frac{7}{10} \times 1\frac{1}{3}$ **f** $\frac{3}{4} \times 2\frac{4}{5}$

g $\frac{7}{8} \times 5$ **h** $\frac{3}{5} \times 7$

i $\frac{7}{10} \times 4$ **j** $1\frac{3}{4} \times 1\frac{1}{2}$

k $2\frac{1}{4} \times 1\frac{4}{5}$ **l** $1\frac{2}{3} \times 2\frac{1}{2}$

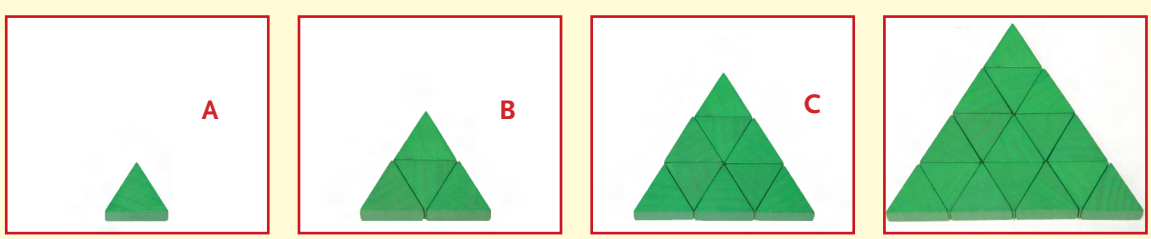
m $3\frac{1}{5} \times 5\frac{3}{4}$ **n** $2\frac{2}{5} \times 1\frac{3}{4}$ **o** $1\frac{9}{10} \times 1\frac{3}{5}$

p $3 \times 2\frac{2}{3}$ **q** $8 \times 1\frac{4}{5}$ **r** $6 \times 2\frac{3}{8}$

s $\frac{5}{8} \times 1\frac{3}{5}$ **t** $1\frac{1}{3} \times \frac{3}{4}$ **u** $\frac{3}{10} \times 3\frac{1}{3}$



■ $4\frac{3}{10} = \frac{(4 \times 10)}{10} + \frac{3}{10} = \frac{43}{10}$



What fraction is:

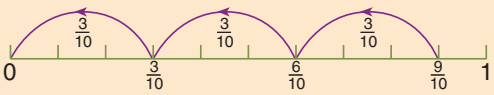
- A of B?
- B of C?
- A of C?
- B of D?
- A of D?
- C of D?

6:07 | Division Involving Fractions

prep quiz

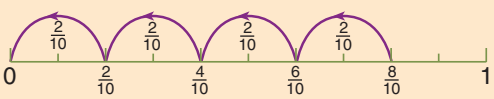
6:07

1



How many lots of $\frac{3}{10}$ in $\frac{9}{10}$? (ie $\frac{9}{10} \div \frac{3}{10}$)

2



How many lots of $\frac{2}{10}$ in $\frac{8}{10}$? (ie $\frac{8}{10} \div \frac{2}{10}$)



- How many $\frac{1}{5}$s are in $\frac{4}{5}$?

3 If $11 \times 4 = 44$, then $44 \div 4 = ?$

4 If $5 \times \frac{3}{4} = \frac{15}{4}$, then $\frac{15}{4} \div \frac{3}{4} = ?$

5 If $3 \times \frac{4}{5} = \frac{12}{5}$, then $\frac{12}{5} \div \frac{4}{5} = ?$

6 Is $\frac{40}{2}$ equal to 20?

7 $\frac{40}{2} \div \frac{10}{2}$ (or $20 \div 5$)

8 $\frac{60}{3} \div \frac{6}{3}$ (or $20 \div 2$)

9 $\frac{27}{100} \div \frac{3}{100}$

10 $\frac{99}{100} \div \frac{9}{100}$

worked examples

Method 1

1 $\frac{9}{10} \div \frac{1}{2} = \frac{9}{10} \div \frac{1}{2} \times 5 \times 5$

Make each denominator the same.

How many groups of $\frac{5}{10}$ can we get from $\frac{9}{10}$?

$\frac{9}{10} \div \frac{5}{10} = 9 \div 5$

$= \frac{9}{5}$

$= 1\frac{4}{5}$

Method 2

2 $\frac{9}{10} \div \frac{1}{2} = \frac{\frac{9}{10} \times \frac{2}{1}}{\frac{1}{2} \times \frac{2}{1}}$

Make the denominator equal 1.

$\frac{9}{10} \div \frac{1}{2} = \frac{\frac{9}{10} \times \frac{2}{1}}{1}$

$= \frac{9}{\cancel{10}_{5}} \times \frac{\cancel{2}^{1}}{1}$

$= \frac{9}{5}$ or $1\frac{4}{5}$

■ $25 \div 5 = \frac{25}{5}$ so $\frac{9}{10} \div \frac{1}{2} = \frac{\frac{9}{10}}{\frac{1}{2}}$

When fractions have the same denominator, you can divide the numerators to get the answer.

eg $\frac{3}{4} \div \frac{5}{8} = \frac{6}{8} \div \frac{5}{8} = 6 \div 5$

To divide by a fraction, invert the fraction and multiply (ie multiply by the reciprocal).

eg $\frac{3}{4} \div \frac{2}{5} = \frac{3}{4} \times \frac{5}{2}$

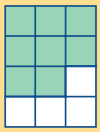
■ "Invert" means turn upside-down.

Exercise 6:07

Foundation Worksheet 6:07

Division involving fractions

1



a How many $\frac{1}{12}$ are in $\frac{8}{12}$?

$\frac{8}{12} \div \frac{1}{12} = \ldots$

b How many $\frac{2}{12}$ are in $\frac{8}{12}$?

$\frac{8}{12} \div \frac{2}{12} = \ldots$

1

a $\frac{4}{5} \div \frac{2}{5}$ **b** $\frac{8}{10} \div \frac{4}{10}$ **c** $\frac{33}{100} \div \frac{11}{100}$

d $\frac{9}{10} \div \frac{5}{10}$ **e** $\frac{7}{8} \div \frac{3}{8}$ **f** $\frac{4}{5} \div \frac{1}{5}$

g $\frac{1}{5} \div \frac{4}{5}$ **h** $\frac{3}{8} \div \frac{5}{8}$ **i** $\frac{7}{10} \div \frac{8}{10}$

2 Use method 2 to complete these number sentences.

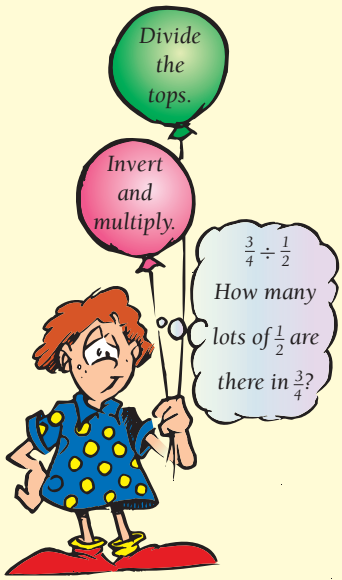
a $\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \times \square$ **b** $\frac{5}{8} \div \frac{1}{3} = \frac{5}{8} \times \square$

c $\frac{7}{10} \div \frac{1}{4} = \frac{7}{10} \times \square$ **d** $\frac{2}{5} \div \frac{5}{8} = \frac{2}{5} \times \square$

e $\frac{3}{10} \div \frac{7}{8} = \frac{3}{10} \times \square$ **f** $\frac{1}{3} \div \frac{1}{10} = \frac{1}{3} \times \square$

g $\frac{3}{8} \div \frac{2}{3} = \frac{3}{8} \times \square$ **h** $\frac{1}{5} \div \frac{4}{5} = \frac{1}{5} \times \square$

i $\frac{7}{100} \div \frac{49}{100} = \frac{7}{100} \times \square$



3 Use either method to find the basic fraction.

a $\frac{3}{4} \div \frac{1}{2}$ **b** $\frac{5}{8} \div \frac{1}{3}$ **c** $\frac{7}{10} \div \frac{1}{4}$

d $\frac{2}{5} \div \frac{5}{8}$ **e** $\frac{3}{10} \div \frac{7}{8}$ **f** $\frac{1}{3} \div \frac{1}{10}$

g $\frac{3}{8} \div \frac{2}{3}$ **h** $\frac{1}{5} \div \frac{4}{5}$ **i** $\frac{7}{100} \div \frac{49}{100}$

j $\frac{5}{8} \div \frac{1}{2}$ **k** $\frac{3}{4} \div \frac{2}{3}$ **l** $\frac{7}{10} \div \frac{2}{3}$

m $\frac{1}{2} \div \frac{3}{5}$ **n** $\frac{1}{2} \div \frac{1}{10}$ **o** $\frac{1}{2} \div \frac{3}{100}$

p $\frac{49}{100} \div \frac{7}{8}$ **q** $\frac{15}{16} \div \frac{3}{10}$ **r** $\frac{18}{25} \div \frac{3}{5}$

4 Simplify by first changing mixed numerals to improper fractions.

a $5 \div \frac{1}{4}$ **b** $2 \div \frac{5}{8}$ **c** $1 \div \frac{2}{3}$

d $1 \div \frac{4}{10}$ **e** $3 \div \frac{4}{5}$ **f** $2 \div \frac{3}{8}$

g $6 \div \frac{2}{3}$ **h** $5 \div \frac{5}{8}$ **i** $7 \div \frac{3}{4}$

j $1\frac{3}{8} \div \frac{3}{4}$ **k** $2\frac{1}{2} \div \frac{1}{3}$ **l** $4\frac{4}{5} \div \frac{3}{4}$

m $3\frac{1}{2} \div 6$ **n** $5\frac{3}{8} \div 4$ **o** $6\frac{3}{4} \div 5$

p $1\frac{3}{4} \div 1\frac{1}{2}$ **q** $3\frac{1}{3} \div 1\frac{1}{4}$ **r** $6\frac{3}{5} \div 4\frac{1}{10}$

s $3\frac{3}{8} \div 1\frac{1}{4}$ **t** $10\frac{1}{2} \div 1\frac{3}{8}$ **u** $7\frac{1}{4} \div 3\frac{4}{5}$

worked examples

1 $4 \div \frac{3}{5} = \frac{4}{1} \div \frac{3}{5}$

$= \frac{4}{1} \times \frac{5}{3}$

$= \frac{20}{3}$

$= 6\frac{2}{3}$

2 $1\frac{1}{2} \div 3\frac{3}{4} = \frac{3}{2} \div \frac{15}{4}$

$= \frac{\cancel{3}^{1}}{\cancel{2}_{1}} \times \frac{\cancel{4}^{2}}{\cancel{15}_{5}}$

$= \frac{2}{5}$

6:08 | Fractions of Quantities

prep quiz 6:08

Write as mixed numerals: **1** $\frac{11}{4}$ **2** $\frac{22}{7}$

Write as improper fractions: **3** $3\frac{1}{2}$ **4** $7\frac{3}{10}$

Complete: **5** $\frac{3}{4} = \frac{\square}{24}$ **6** $\frac{9}{10} = \frac{\square}{100}$

Simplify: **7** $\frac{30}{80}$ **8** $\frac{72}{108}$ **9** $\frac{9}{10} + \frac{3}{10}$ **10** $1 - \frac{7}{10}$

worked examples

1 Find $\frac{3}{5}$ of 2 km.

2 What fraction of 2 m is 40 cm?

Solutions

1 $\frac{1}{5}$ of 2 km $= \frac{1}{5}$ of 2000 m
$= 2000 \div 5$ m
$= 400$ m
$\therefore \frac{3}{5}$ of 2 km $= 3 \times 400$ m
$= 1200$ m or 1·2 km

2 Fraction $= \frac{40 \text{ cm}}{200 \text{ cm}}$
$= \frac{4}{20}$
$= \frac{1}{5}$

Exercise 6:08

1 Find:

a $\frac{1}{5}$ of 1 m **b** $\frac{1}{4}$ of 1 hour **c** $\frac{1}{8}$ of 1 kg
d $\frac{1}{10}$ of 3 m **e** $\frac{1}{3}$ of 90° **f** $\frac{1}{6}$ of 2 h
g $\frac{1}{100}$ of 200 g **h** $\frac{1}{100}$ of 1 kg **i** $\frac{1}{100}$ of 2 L

2 Use your answers to question 1 to find:

a $\frac{4}{5}$ of 1 m **b** $\frac{3}{4}$ of 1 hour **c** $\frac{5}{8}$ of 1 kg
d $\frac{9}{10}$ of 3 m **e** $\frac{2}{3}$ of 90° **f** $\frac{5}{6}$ of 2 h
g $\frac{13}{100}$ of 200 g **h** $\frac{21}{100}$ of 1 kg **i** $\frac{99}{100}$ of 2 L

3 What fraction is:

a 50c of $1? **b** 20 g of 40 g? **c** 30 m of 100 m?
d 400 of 500? **e** 3 g of 12 g? **f** 4 m of 100 m?
g $25 of $100? **h** $30 of $70? **i** $4 of $200?

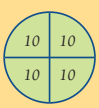
4 Find:

a $\frac{3}{4}$ of 6000 L **b** $\frac{5}{8}$ of $48 000 **c** $\frac{3}{10}$ of $1 600 000
d $\frac{3}{5}$ of 3 kg **e** $\frac{9}{10}$ of 7 kg **f** $\frac{7}{8}$ of 9 km
g $\frac{2}{3}$ of $6.30 **h** $\frac{3}{4}$ of 7·56 kg **i** $\frac{3}{10}$ of 8·6 m

Foundation Worksheet 6:08
Fractions of quantities

1 a $\frac{1}{2}$ of 20 min
b $\frac{1}{2}$ of 6 kg

2 a



$\frac{1}{4}$ of 40
$\frac{2}{4}$ of 40
$\frac{3}{4}$ of 40



5 What fraction is:

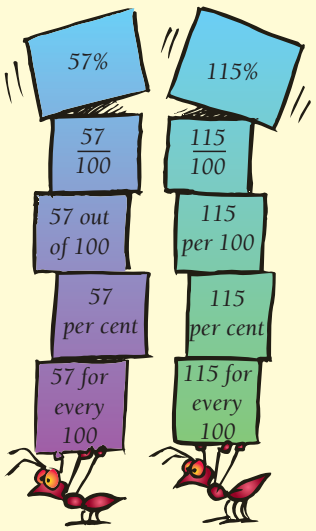
a 60 cm of 2 m?	**b** 200 mL of 1 L?	**c** 50 g of 3 kg?
d 1c of $10?	**e** 8 min of 3 h?	**f** 90° of 1 revolution?
g 6·6 m of 13·2 m?	**h** 13·6 cm of 4 m?	**i** 14·7 cm of 4·9 m?

6:09 | Review of Percentages

If you have difficulty with this review, try Checkup 1:09D on page 24 and get help from Appendix B if necessary.

Summary of important ideas

- **A percentage is a convenient way of writing fractions that have a denominator of 100. 'Per cent', written %, means 'per 100' or 'for every 100'.**
  eg $7\% = \frac{7}{100}$
- **To write a percentage as a fraction or mixed number, first write it as a fraction with denominator 100, then simplify.**
  eg $125\% = \frac{125}{100} = 1\frac{1}{4}$
- **To change fractions to percentages, first change the denominator of the fraction to 100.**
  eg $\frac{3}{20} = \frac{15}{100} = 15\%$
- **To change a percentage to a decimal, we can write it first as hundredths and then as a decimal.**
  eg $93\% = \frac{93}{100} = 0{\cdot}93$
- **To change a decimal to a percentage, we can write it as a fraction first, then change it to a decimal, OR we can multiply the decimal by 100%.**
  eg $0{\cdot}93 = 0{\cdot}93 \times 100\% = 93\%$



Exercise 6:09

1 Change these percentages to fractions.

a 9%	**b** 7%	**c** 1%
d 37%	**e** 99%	**f** 97%
g 11%	**h** 13%	**i** 33%

2 Change each to its equivalent basic fraction.

a 50%	**b** 25%	**c** 80%
d 10%	**e** 5%	**f** 26%
g 44%	**h** 35%	**i** 2%

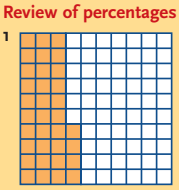
3 Change each percentage to a whole or mixed number.

a 100%	**b** 150%	**c** 225%
d 310%	**e** 175%	**f** 400%
g 203%	**h** 112%	**i** 109%

Foundation Worksheet 6:09

Review of percentages

1



a What percentage is coloured?
b What percentage is not coloured?
c What fraction is coloured?
d What fraction is not coloured?

2 Write as a fraction and as a decimal.
a 3%

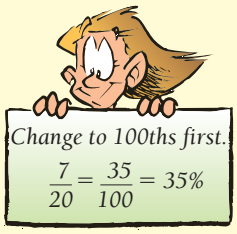
3 Write as a percentage.
a $\frac{7}{100}$ **b** 0·07

4 Change each to a percentage.

a $\frac{3}{100}$ **b** $\frac{7}{100}$ **c** $\frac{9}{100}$

d $\frac{18}{100}$ **e** $\frac{27}{100}$ **f** $\frac{69}{100}$

g $\frac{50}{100}$ **h** $\frac{44}{100}$ **i** $\frac{90}{100}$



5 Express each as a percentage.

a $\frac{1}{20}$ **b** $\frac{3}{4}$ **c** $\frac{7}{10}$

d $\frac{4}{5}$ **e** $\frac{3}{10}$ **f** $\frac{13}{50}$

g 2 **h** $1\frac{1}{2}$ **i** $3\frac{1}{10}$

6 Write each percentage as a decimal.

a 14% **b** 37% **c** 81%

d 7% **e** 2% **f** 1%

g 60% **h** 30% **i** 90%

j 100% **k** 126% **l** 215%

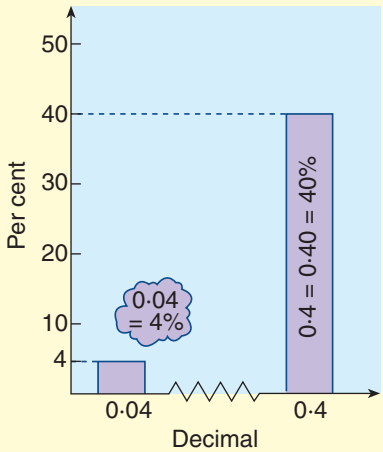
7 Write each decimal as a percentage.

a 0·35 **b** 0·76 **c** 0·88

d 0·04 **e** 0·06 **f** 0·05

g 0·4 **h** 0·6 **i** 0·5

j 1·87 **k** 2·6 **l** 1·02



8 **a** $7 \times 10\%$ **b** $8 \times 9\%$ **c** $5 \times 35\%$

d $12\% \div 3$ **e** $95\% \div 5$ **f** $100\% \div 20$

g $31\% + 9\%$ **h** $8\% + 11\%$ **i** $51\% + 49\%$

j $9\% - 5\%$ **k** $17\% - 7\%$ **l** $100\% - 37\%$

m If I spend 20% of my money, what percentage of it is left.

n I have lost 3% of the worms from my worm farm. What percentage remains?

o Three friends were each given 25% of a pie. What percentage were they given altogether?

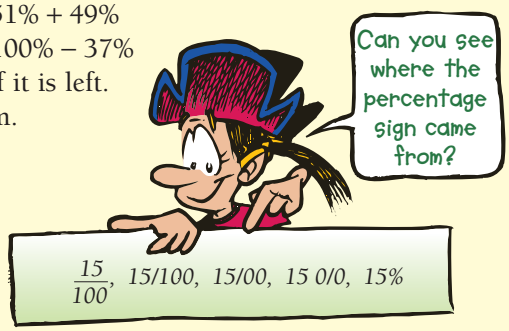
p 50% of an inheritance is to be shared among four sisters. What percentage would each receive?

q 20% of my money is \$12. What is 10% of my money?

r 30% of my mass is 21 kg. What is 10% of my mass?

s 100% of my money is \$400. What amount is:

i 50%? **ii** 10%? **iii** 25%? **iv** 20%?



6:10 | Changing Fractions and Decimals to Percentages

prep quiz

1 Write 39% as a fraction.
2 Change $\frac{3}{100}$ to a percentage.
3 Write $\frac{7}{20}$ as a percentage.
4 Write 16% as a decimal.
5 Write 0·13 as a percentage.
6 Write 100% as a whole number.
7 Write the number 1 as a percentage.
8 $\frac{3}{20} \times \frac{100}{1}$
9 $0{\cdot}07 \times 100$
10 $0{\cdot}325 \times 100$

6:10

see

6:09, B:21, B:23

From Chapter 1 we learned that:

$\frac{62}{100} = 0{\cdot}62 = 62\%$

However, $\frac{62}{100} \times 100 = 62$ and $0{\cdot}62 \times 100 = 62$.

It looks as though multiplying a fraction or decimal by 100 will give the number per cent.

worked examples

1 Change $\frac{5}{8}$ to a percentage.

2 Change 0·625 to a percentage.

Solutions

1 $\frac{5}{8} = \frac{5}{8} \times 1$

$= \frac{5}{8} \times 100\%$

$= \frac{5}{\cancel{8}_2} \times \frac{\cancel{100}^{25}}{1}\%$

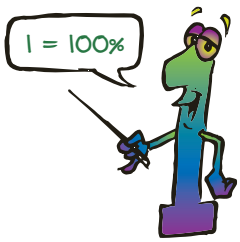
$= \frac{125}{2}\%$

$= 62{\cdot}5\%$

2 $0{\cdot}625 = 0{\cdot}625 \times 1$

$= 0{\cdot}6\overset{\curvearrowright}{2}5 \times 100\%$

$= 62{\cdot}5\%$



- **To express a fraction as a percentage, multiply by $\frac{100}{1}$ %.**
- **To express a decimal as a percentage, multiply by 100%.**

Exercise 6:10

1 Write these fractions as percentages by multiplying by $\frac{100}{1}$ %.

a $\frac{3}{5}$	b $\frac{9}{10}$	c $\frac{7}{50}$	d $\frac{17}{20}$	e $\frac{1}{4}$
f $\frac{73}{100}$	g $\frac{1}{2}$	h $\frac{8}{25}$	i $\frac{3}{20}$	j $\frac{3}{4}$
k $\frac{4}{5}$	l $\frac{9}{100}$	m $\frac{2}{3}$	n $\frac{1}{6}$	o $\frac{3}{8}$
p $\frac{5}{8}$	q $\frac{133}{200}$	r $\frac{9}{500}$	s $\frac{27}{400}$	t $\frac{61}{300}$
u $1\frac{1}{2}$	v $2\frac{1}{4}$	w $3\frac{1}{3}$	x $3\frac{1}{7}$	

Foundation Worksheet 6:10

Changing fractions and decimals to percentages

1 Write as percentages:
a $\frac{15}{100}$ b $\frac{2}{100}$

2 Write as percentages:
a 0·14 b 0·07

■ $3\frac{1}{7} = \frac{22}{7}$

$\frac{22}{7} = \frac{22}{7} \times \frac{100}{1}\%$

2 Write these decimals as percentages by multiplying by 100%.

a 0·37	**b** 0·71	**c** 0·95	**d** 0·04
e 0·7	**f** 0·2	**g** 0·1	**h** 0·6
i 0·345	**j** 0·125	**k** 0·375	**l** 0·825
m 0·3	**n** 0·045	**o** 0·85	**p** 0·02
q 1·8	**r** 1·1	**s** 2·5	**t** 3·6
u 1	**v** 3	**w** 2	**x** 5

3 Use the table at the right to find:

a 0·01 as a percentage

b 5% as a fraction

c $\frac{1}{3}$ as a percentage

d $\frac{1}{20}$ as a decimal

e 10% as a decimal

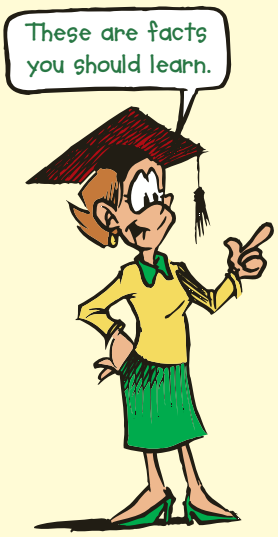
f 0·125 as a fraction

g 20% as a decimal

h $\frac{1}{8}$ as a percentage

i $0\dot{3}$ as a fraction

j $12\frac{1}{2}\%$ as a decimal.



Percentage	*Decimal*	*Fraction*
1%	0·01	$\frac{1}{100}$
5%	0·05	$\frac{1}{20}$
10%	0·1	$\frac{1}{10}$
$12\frac{1}{2}\%$	0·125	$\frac{1}{8}$
20%	0·2	$\frac{1}{5}$
25%	0·25	$\frac{1}{4}$
$33\frac{1}{3}\%$	$0\dot{3}$	$\frac{1}{3}$
50%	0·5	$\frac{1}{2}$
100%	1	1

4 Use the table in question 3 and the method at the right of this question to complete this table.

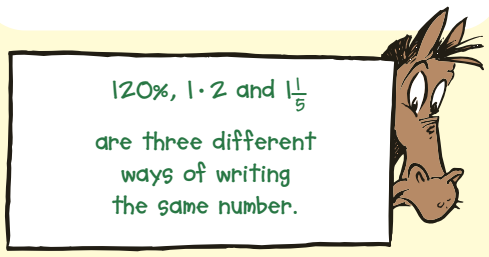
	Percentage	*Decimal*	*Fraction*
a	3%		
b			$\frac{3}{20}$
c		0·9	
d		1·5	
e	60%		
f			$\frac{2}{3}$
g			$\frac{9}{10}$
h		0·09	
i	75%		
j			$\frac{3}{8}$

worked examples

1 $10\% = 0{\cdot}1 = \frac{1}{10}$
Multiply each by 6.
$60\% = 0{\cdot}6 = \frac{6}{10}$

2 $5\% = 0{\cdot}05 = \frac{1}{20}$
Multiply each by 7.
$35\% = 0{\cdot}35 = \frac{7}{20}$

3 $20\% = 0{\cdot}2 = \frac{1}{5}$
Add 1 or 100% to each.
$120\% = 1{\cdot}2 = 1\frac{1}{5}$



6:11 | Changing Percentages to Fractions and Decimals (Extension)

These are harder examples. For easier types see B:19, B:20 and B:22, in the appendix on the CD.



B:19, B:20, B:22

worked examples

1 Write $3\frac{1}{4}\%$ as a fraction and as a decimal.

2 Convert 18·6% to a fraction and to a decimal.

Solutions

1 $3\frac{1}{4}\% = \dfrac{3\frac{1}{4}}{100}$

$= \dfrac{\frac{13}{4}}{100} \begin{matrix}\times 4\\ \times 4\end{matrix}$

$= \dfrac{13}{400}$

$3\frac{1}{4}\% = 3{\cdot}25\%$

$= \dfrac{3{\cdot}25}{100}$

$= 3{\cdot}25 \div 100$

$= 0{\cdot}0325$

2 $18{\cdot}6\% = \dfrac{18{\cdot}6}{100}$

$= \dfrac{18{\cdot}6}{100} \begin{matrix}\times 10\\ \times 10\end{matrix}$

$= \dfrac{186}{1000}$

$= \dfrac{93}{500}$

$18{\cdot}6\% = \dfrac{18{\cdot}6}{100}$

$= 18{\cdot}6 \div 100$

$= 0{\cdot}186$

Exercise 6:11

1 Change these percentages to fractions.

a $2\frac{1}{4}\%$	**b** $1\frac{3}{4}\%$	**c** $4\frac{1}{2}\%$	**d** $6\frac{3}{4}\%$	**e** $12\frac{3}{4}\%$	**f** $16\frac{1}{2}\%$
g $14\frac{3}{4}\%$	**h** $19\frac{1}{2}\%$	**i** 19·6%	**j** 14·3%	**k** $106\frac{1}{2}\%$	**l** $108\frac{3}{4}\%$

2 Write these percentages as decimals.

a 4·3%	**b** 18·6%	**c** 7·7%	**d** 11·2%	**e** $21\frac{1}{2}\%$	**f** $17\frac{1}{4}\%$
g $13\frac{3}{4}\%$	**h** $4\frac{1}{2}\%$	**i** 97·3%	**j** $37\frac{1}{2}\%$	**k** $112\frac{1}{2}\%$	**l** 253·8%

3 **a** $87\frac{1}{2}\%$ of children in Clareville were vaccinated against infectious diseases by the time they were 6 years old. What fraction were vaccinated?

b June invested $87 000 in a savings account at an interest rate of $16\frac{1}{4}\%$ per annum (which means 'each year'). What fraction of the invested amount does she earn as interest in the first year?

c Three authors, Alan, Rob and Steve, receive $3\frac{1}{3}\%$ each of the recommended retail price of their textbooks as royalties for writing the book. What fraction of the recommended retail price does each receive?

6:12 | Finding a Percentage of a Quantity

To find a percentage of a quantity, write the percentage as a decimal (or fraction) and multiply by the quantity. eg 11% of 800 = 0·11 × 800 = 88

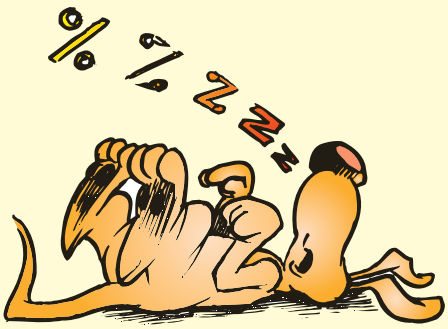
Using decimals

worked examples

1 8% of 450 g
$= 0{\cdot}08 \times 450$
$= 36{\cdot}00$
$= 36$ g

2 90% of 870
$= 0{\cdot}90 \times 870$
$= 783{\cdot}00$
$= 783$

3 11% of \$800
$= 0{\cdot}11 \times 800$
$= \$88.00$
$= \$88$



Using fractions

worked examples

4 8% of 450 g

$= \frac{8}{100} \times 450$

$= \frac{\cancel{8}^{4}}{\cancel{100}_{\cancel{2}_{1}}} \times \frac{\cancel{450}^{9}}{1}$

$= 36$ g

5 90% of 870

$= \frac{9}{\cancel{10}_{1}} \times \frac{\cancel{870}^{87}}{1}$

$= 9 \times 87$

$= 783$

6 11% of \$800

$= \frac{11}{\cancel{100}_{1}} \times \frac{\cancel{800}^{8}}{1}$

$= 11 \times 8$

$= \$88$

Exercise 6:12

Foundation Worksheet 6:12
Finding a percentage of a quantity
1 a 50% of \$30 **b** 10% of 80 g
2 a 20% of \$20 **b** 70% of \$50

1 Find the simplest answer for:

a 10% of 880	**b** 33% of 300	**c** 6% of 1000 h
d 3% of 800 mg	**e** 16% of 50 km	**f** 24% of 25 g
g 25% of 440 ha	**h** 75% of 24 min	**i** 40% of 760 m
j 80% of 700 L	**k** 80% of 140	**l** 5% of 660 m

2 Give answers correct to the nearest cent.

a 15% of \$400	**b** 8% of \$300	**c** 2% of \$900	**d** 10% of \$470
e 6% of \$850	**f** 4% of \$11 000	**g** 40% of \$3.00	**h** 7% of \$5.00
i 12% of \$8.50	**j** 9% of \$348	**k** 11% of \$333	**l** 5% of \$438.60

3 **a** 30% of our football team is injured. How many people are injured if there are 20 in the team?
b Ben decided to give 5% of his money to charity. How much does he give if he has \$1600?
c Heather has 30 birds. Of these, 60% are doves. How many doves does Heather have?
d There are 6000 library books in our library. 85% of these are in the fiction section. How many books are in this section? How many of the books are not in this section?
e Of 4000 worms in our worm farm, 3% escaped. How many did we have left?
f A housing development was built on 600 ha of bushland. Of this, 8% was made into parks. What was the area of parks made?

6:13 | One Quantity as a Percentage of Another

prep quiz 6:13

Write the first quantity as a fraction of the second.

1 90°, 180°	**2** 2 m, 7 m	**3** 5 s, 20 s	**4** 50c, $1
5 15 min, 1 h	**6** $13, $50	**7** 20 cm, 1 m	**8** 10 g, 50 g
9 $1.50, $4.50	**10** 750 cm, 800 cm		

The fractions obtained in the Prep Quiz could be changed to percentages by multiplying by $\frac{100}{1}$.

eg $\frac{5\text{ s}}{20\text{ s}} = \frac{1}{4}$

$= \frac{1}{\cancel{4}_1} \times \frac{\cancel{100}^{25}}{1}\%$

$= 25\%$



- Estimate what percentage is white.

To write one quantity as a percentage of another:
- **Make sure that the two quantities are in the same units.**
- **Write the first as a fraction of the second.**
- **Change the fraction to a percentage by multiplying by $\frac{100}{1}$.**

worked examples

1 I have 800 Australian stamps. My whole collection has 6000 stamps. What percentage of my stamps are Australian?

2 For every kilogram of sugar bought, 200 g will be wasted. What percentage is wasted?

Solutions

1 Fraction $= \frac{800 \text{ stamps}}{6000 \text{ stamps}} = \frac{800}{6000} = \frac{8}{60} = \frac{2}{15}$

Percentage $= \frac{2}{\cancel{15}_3} \times \frac{\cancel{100}^{20}}{1}\% = \frac{40}{3}\%$

$= 13\frac{1}{3}\%$

$\therefore 13\frac{1}{3}\%$ of my stamps are Australian.



2 Fraction $= \frac{200\text{ g}}{1\text{ kg}} = \frac{200\text{ g}}{1000\text{ g}} = \frac{200}{1000} = \frac{1}{5}$

Percentage $= \frac{1}{\cancel{5}_1} \times \frac{\cancel{100}^{20}}{1}$

$= 20\%$

$\therefore$ 20% of the sugar is wasted.

Exercise 6:13

Foundation Worksheet 6:13

One quantity as a percentage of another

1 What fraction is:
a 10 g of 20 g **b** 20 m of 100 m

2 What percentage is:
a 10 g of 20 g **b** 20 m of 100 m
c $7 of $100

1 **a** Express 10 cents as a percentage of $1.
b What percentage of 400 g is 20 g?
c What percentage of 200 m is 160 m?
d Express 70 points as a percentage of 100 points.
e Express 36 g as a percentage of 300 g.

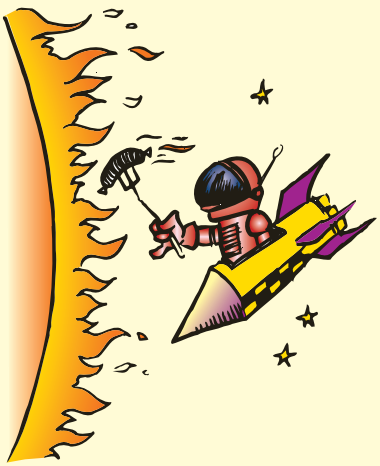
2 What percentage is:
a 500 g of 1 kg?
b 15 min of 1 h?
c 180° of 1 revolution?
d 7500 m^2 of 1 ha?
e 60 mm of 8 cm?
f 40 year of 1 century?
g 13 of 200?
h 420 of 8000?

3 **a** I have 308 sheets of blue paper. Altogether I have 1400 sheets of paper. What percentage of the paper is blue?
b 12 of my 30 birds are gouldian finches. What percentage are *not* gouldian?
c I sold my computer for $500. What percentage is this of my cost price if I bought the computer for $4000?
d Out of $200 000 I received in one year, I paid $85 000 in wages. What percentage of the sum received did I pay as wages?
e Of the 40 books Alan wrote, 5 are now out of print. What percentage of the books are out of print?



4 At the school concert there were 8 teachers, 30 boys, 40 girls and 82 parents. What percentage of those at the concert were:
a girls?
b teachers?
c boys?
d parents?
e not girls?
f boys or girls?

5 **a** The time I took to finish the ski course was 48 s. My previous best time was 50 s. What percentage was this better than my previous best?
b To get to school, 13 students travel by bus, 10 walk and 7 come by car. What percentage of these students walk to school?
c The Davis Cup is an international tennis competition held each year. From 1947 to 1986 Australia won 19 times. What was the percentage of times that Australia won during that period?
d A spaceship travelling to the sun from Earth has travelled 120 000 000 km. If the sun is 150 000 000 km from Earth, what percentage of the journey has the spaceship still to complete?
e The lead from a propelling pencil was 6 cm long before use. What percentage of the lead has been used when its length is 57 mm?
f A container holds 10 L of juice. What percentage of the juice remains after seven 230 mL cups have been filled?



6:14 | Applications of Fractions and Percentages

worked examples

1 Mary scored 47 out of 48 in her Mathematics test. What percentage score was this, to the nearest whole per cent?

2 The tennis team called Seekers won 21 of their 30 matches. If $\frac{2}{3}$ of the wins were won by more than 10 games, what fraction of all matches were won by more than 10 games?

Solutions

1 Fraction of marks obtained

$= \frac{47}{48}$

(changing to a percentage)

$= \frac{47}{48} \times \frac{100}{1}\%$

$= \frac{47}{\cancel{48}_{12}} \times \frac{\cancel{100}^{25}}{1}$

$= \frac{1175}{12}\%$

$= 97\frac{11}{12}\%$

$\div 98\%$

Mary scored approximately 98% in her Mathematics test.

2 Fraction of matches which were won

$= \frac{21}{30} \overset{\div 3}{\underset{\div 3}{}} = \frac{7}{10}$

Fraction of matches won by more than 10 games

$= \frac{2}{3}$ of $\frac{7}{10}$

$= \frac{\cancel{2}^{1}}{3} \times \frac{7}{\cancel{10}_{5}}$

$= \frac{7}{15}$

$\therefore \frac{7}{15}$ of all matches were won by more than 10 games.

Exercise 6:14

1 If Naomi shared 350 g of birdseed equally among her seven budgerigars, how much would she put in the cage containing two budgerigars?

2 If eighteen football players are each to be given $\frac{1}{4}$ of an orange at half-time, how many oranges would have to be purchased for this purpose? How many oranges would have to be purchased if each player receives $\frac{3}{4}$ of an orange?

3 On Tuesday night 312 students attended the square dance. This is exactly $\frac{1}{3}$ of my school.

- **a** How many students belong to my school?
- **b** How many students did not attend the dance?
- **c** If each 'square' consists of 8 students, how many squares could be formed for dancing?



4 Our school committee has 60 members. Two-thirds of the members must be present before any decisions can be made. Yesterday only $\frac{3}{5}$ of the members were present.

- **a** How many members attended the meeting yesterday?
- **b** How many members are needed before decisions can be made?
- **c** How many more members were needed yesterday for a decision to be made?

5 Rhonda had $\frac{3}{10}$ of her money invested in shares, $\frac{6}{10}$ was in a credit union account and the rest she kept in her safe at home. What fraction of her money was kept in the safe? What percentage of her money is that?

6 It takes me $\frac{3}{4}$ of an hour to write one page of a letter. How long would it take me to write a letter seven pages long?

7 For 20 hours of one day Tom stayed awake in order to build a canoe. For $\frac{3}{4}$ of this time he was actually at work on the canoe.

a For how many hours did he work on the canoe?
b For how many hours did he sleep during this day?



8 Charlie has 11 guinea pigs and one rabbit. What fraction of his pets are guinea pigs? What percentage is this?

9 During the season, I. Inthenet was successful with exactly $\frac{3}{5}$ of his attempts at kicking goals.

a What was his percentage success?
b What fraction of his attempts were unsuccessful?

10 In a terrible battle, the Roman soldiers lost three of every eight men in their army. If the army had 10 000 soldiers before the battle, how many soldiers were lost?

11 76% of my class has brown hair. What percentage of my class does not have brown hair? If my class has less than 50 students, how many people are in my class?

12 Alan and Rhonda have five children: 4 girls and 1 boy.

a What percentage of the children are girls?
b What percentage of the children are boys?
c What fraction of the family are male?
d What percentage of the family are female?
(Give your answer to the nearest whole per cent.)



13 There are 1000 students in our school.
Males make up 0·52 of the school.

a How many students are male?
b What percentage of students are male?
c What fraction of students are female?

14 Interest of $150 was charged on a loan of $750 for one year.

a What fraction of the loan is the interest?
b What percentage of the loan is the interest?

15 Ravi enrolled in a speed-reading course. He was given a two-minute speed test at the beginning and end of the course. In the first test he read $2\frac{3}{4}$ pages of the book used. In the second test he read $5\frac{1}{2}$ pages of the same book.

a How much of the book did he read altogether?
b What was his average speed in the first test, in pages per minute?
c What was his average speed in the second test, in pages per minute?

16 When Marita was 14 years old she had a fine stamp collection. After a further 16 years she had increased her collection by 300%.

a If she had 2500 stamps when she was 14, how many stamps did she have when she was 30?

b How many times as big was her collection when she was 30 than when she was 14?

17 In Africa, 3 out of every 10 people attend church. If 2000 people were chosen at random, how many from this group would you expect to be church attenders?

18 What percentage of marks did I have if I scored $\frac{18}{20}$ in my spelling test?

19 Zac spent 20% of his money to buy a car for $8600. How much did he have before buying the car?

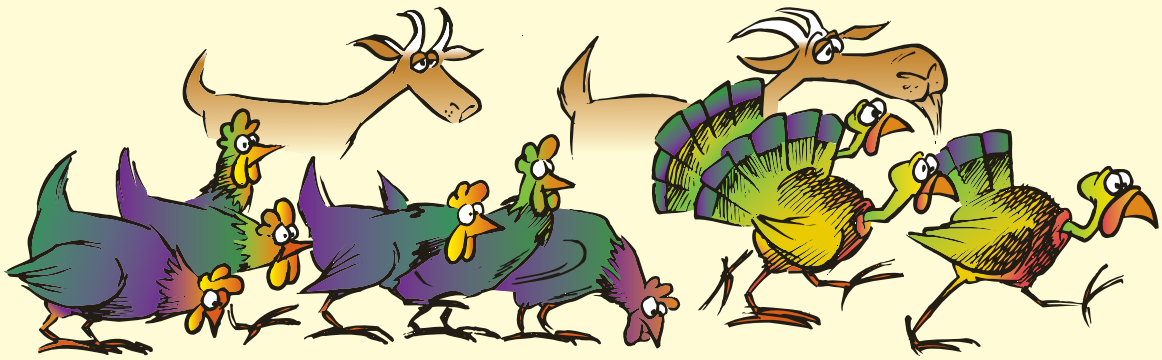
20 Miriam's petrol tank was half full but she used one-third of a tank of petrol in travelling to the City and back. What fraction of a tank of petrol remained?

21 Ten full drums of oil were stored in a warehouse. Three-eighths of one drum was lost. If the value of the oil that was lost was $90, what is the value of the remaining oil?

22 Miriam's petrol tank was half full when she started out, but she used one-quarter of this petrol in travelling to Tamworth and back. What fraction of a tank did she use, and what fraction of a tank remained after the trip?

23 Taxation takes $\frac{1}{3}$ of Joel's salary. He banks $\frac{1}{10}$ of what is left. If he banks $40 every week, how much does he earn each week before tax is removed?

24 It takes $\frac{3}{8}$ of a bottle of milk to fill a large glass. How many of these glasses can be filled with 40 bottles of milk?



25 James left the farm gate open and $\frac{3}{4}$ of the turkeys, $\frac{3}{4}$ of the chickens and $\frac{1}{2}$ of the goats got out. What fraction of the turkeys and chickens got out altogether?

26 Tom practised his violin for exactly the same length of time each day. After five days he had practised for $8\frac{3}{4}$ hours altogether. How much did he practise each day?

27 The Hudsons bought 3 pizzas to share between eight people. If each person was given the same amount, how much did each receive?

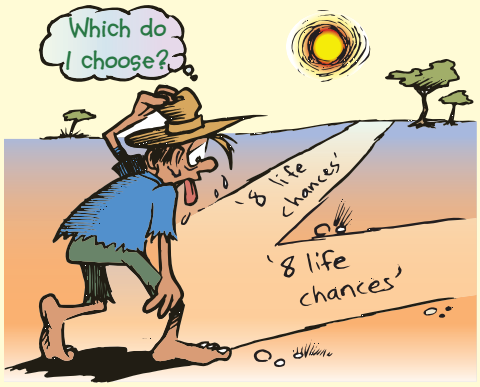
28 Mrs Daley had $\frac{1}{3}$ of a pie. She gave each of her two sons half of this. What fraction of the pie was each given?

6:15 | What Chance of Survival?

Lost in the outback, Mac the prospector decides that he must follow the tracks he finds because some of the tracks are sure to lead to water. When he comes to a point where there is more than one path to choose from, he is just as likely to choose any of them.

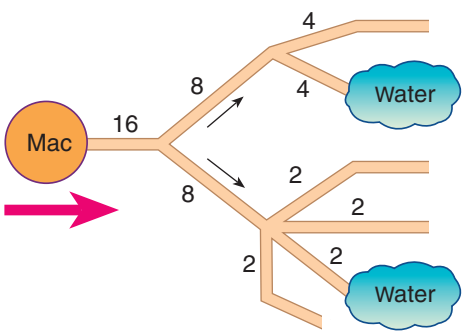
In each of the cases below Mac will begin with a number of 'life chances' and we will work out how many of these 'chances' lead to water. Once we have found what fraction of his chances lead to water, we can calculate his percentage chance of survival.

Note: Mac never retraces his steps.



worked example

Starting with 16 life chances, work out Mac's percentage chance of survival for the following tracks. At each fork in the track, the life chances must be shared equally among the new tracks.



Solution

From the diagram, you can see that 6 out of 16 life chances reach water,

$\therefore$ Mac's chance of survival

$$= \frac{\cancel{6}^{3}}{\cancel{16}_{4_2}} \times \frac{\cancel{100}^{25}}{1}\,\%$$

$$= \frac{75}{2}\,\%$$

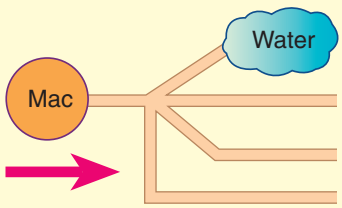
$$= 37\tfrac{1}{2}\,\%$$

Mac has a $37\frac{1}{2}\%$ chance of survival.

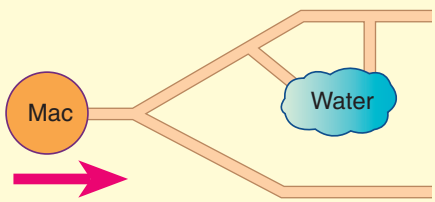
Exercise 6:15

Find Mac's percentage chance of survival for each of the following cases.

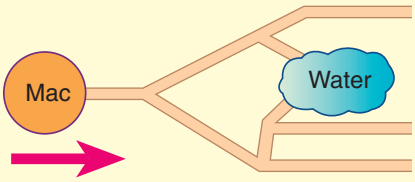
1 Start with 4 life chances.



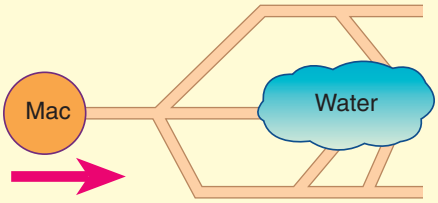
2 Start with 8 life chances.



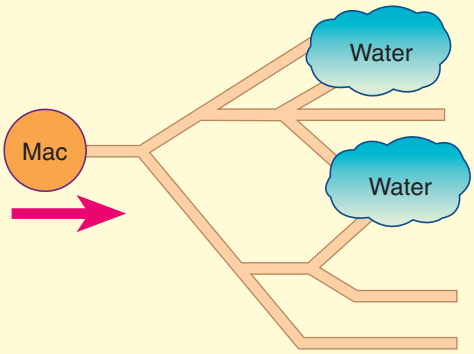
3 Start with 8 life chances.



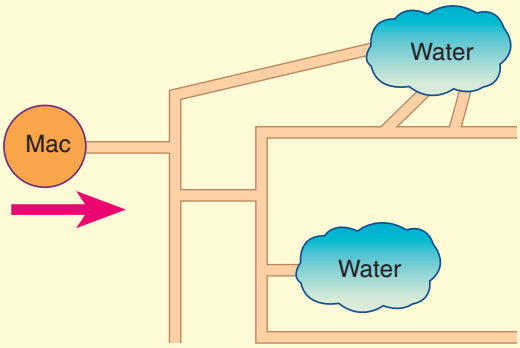
4 Start with 12 life chances.



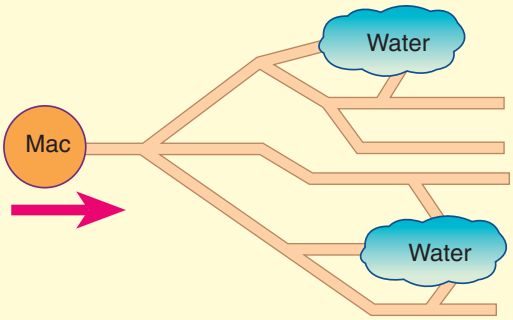
5 Start with 24 life chances.



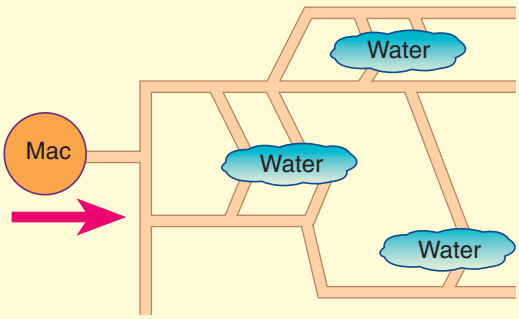
6 Start with 32 life chances.



7 Start with 24 life chances.



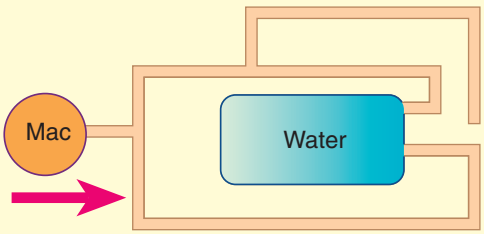
8 Start with 96 life chances.



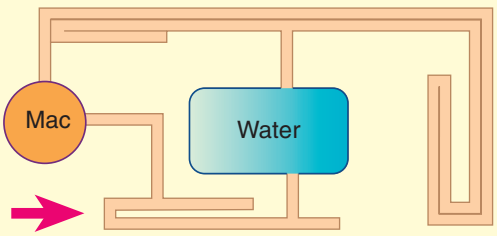
Note: If we start with 100 life chances, the answer is a percentage.

In the following questions find the chance (as a percentage) that Mac will find water without having to turn back.

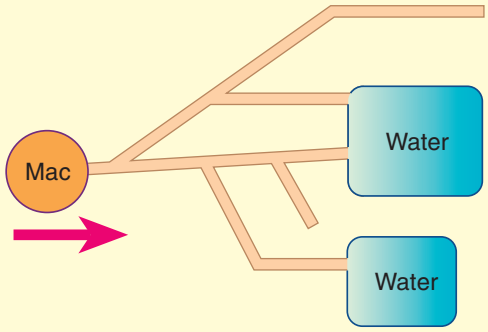
9 Start with 100 chances.



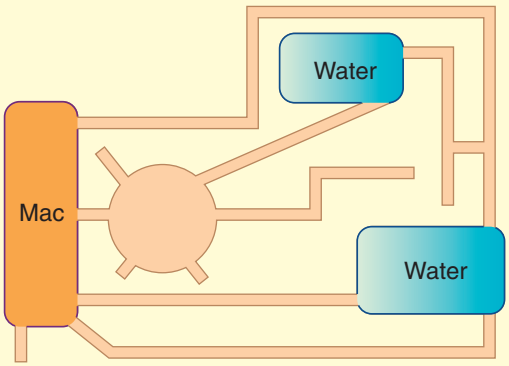
10 Start with 100 chances.



11 Start with 100 chances.



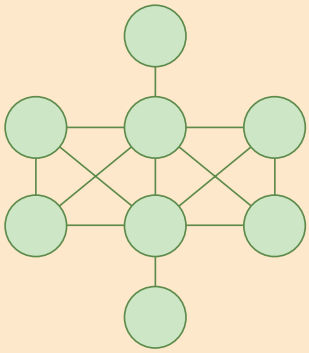
12 Start with 100 chances.



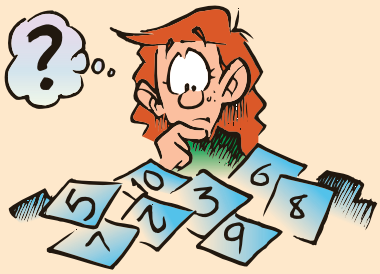
fun spot
6:15

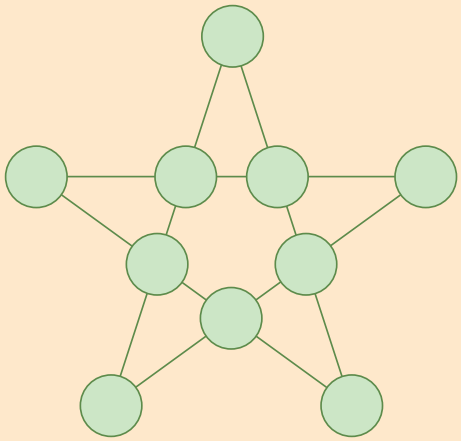
Fun Spot 6:15 | Where should the numbers go?

1 Arrange the numbers 1, 2, 3, 4, 5, 6, 7, 8 inside these circles so that no two consecutive numbers are in connected circles. (*Remember:* 'Consecutive' means 'one after the other', eg 2 and 3 or 6 and 7.)



2 Place 10 of the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 in the circles on this star so that each line of 4 numbers adds to give 24. (*Note:* Two numbers cannot be used; but which ones?)





6:16 | Probability

In the last section we saw one example of the many cases where probability is involved in making decisions. We often use probability: 'Where is the most likely place to find my wallet?' 'How can I best avoid an accident?' 'Which is the most likely answer?' 'Is it possible that a pedestrian will step out from behind that car?'

We describe *the chance of something happening* using either words or numbers.

Words: **impossible** | **unlikely** | **even chance (fifty-fifty)** | **likely** | **certain**

Numbers: **0% or 0** | **between 0 and $\frac{1}{2}$** | **50%, $\frac{1}{2}$ or 0·5** | **between $\frac{1}{2}$ and 1** | **100% or 1**

worked examples

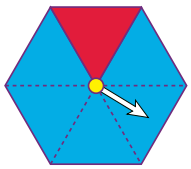
1 List all possible outcomes if a dice is tossed.



The possible outcomes are:

1, 2, 3, 4, 5 and 6.

2 What is the probability that this spinner will stop on *red*?



Probability $= \frac{1}{6}$

(One section of the 6 is red.)

$= \frac{1}{6} \times \frac{100}{1}\%$

$= 16\frac{4}{6}\%$ or $16\frac{2}{3}\%$

Exercise 6:16

1 Choose a label for each part below.

impossible | unlikely | even chance (fifty-fifty) | very likely | certain

a You will travel in a train next week.

b Your next teacher will be over 50 years old.

c You will get over 95% in your next Maths test.

d You will see a living Tyranosaurus Rex today.

e If you throw a dice you will get a number less than 7.

f If you toss a coin you will get a *tail*.

If all outcomes are equally likely:

Probability $= \dfrac{\text{number of successful outcomes}}{\text{total number of outcomes}}$

2 Arrange the parts **a** to **f** in Question 1 in order, from least likely to most likely.

3

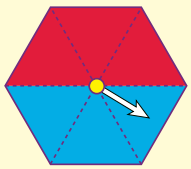
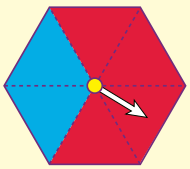
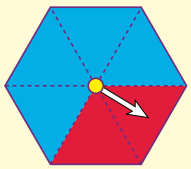
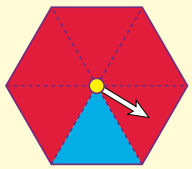


Write as a fraction, the probability that *if I throw a dice it will be*:

a a 5 **b** a zero **c** an even number

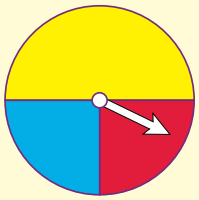
d an odd number **e** a number less than 7

4 What is the probability, as a fraction, that the spinner will stop on red?

a  **b**  **c**  **d** 

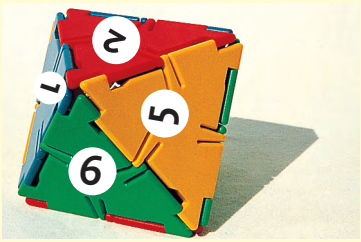
5 Write each of the probabilities in Question 4 as a percentage.

6 Use the diagrams in Question 4 to give the probability, as a fraction and as a percentage, that the spinner will stop on blue.

7 

a What is the probability, written as a fraction, that this spinner will stop on:
i yellow? **ii** blue? **iii** red?

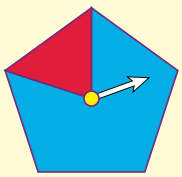
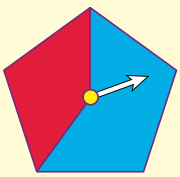
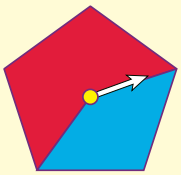
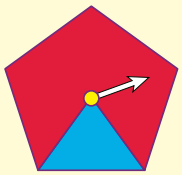
b What answer do you get when you add all three possible probabilities together?

8 

A regular octahedron is marked 1 to 8 on each of its faces. If this is rolled, what is the probability that the number showing will be:

a a 2? **b** even? **c** odd? **d** a 3, 4 or 5?
e a single-digit number? **f** a multiple of 3?
g a multiple of 2? **h** a number less than 1?
i between 1 and 8? **j** between 1 and 7?

9 What is the probability that the spinner will stop on red?

a  **b**  **c**  **d** 

see 3:04

10 A dice is thrown. What is the probability that the result will be:



a a prime number? **b** a composite number?
c neither prime nor composite? **d** a Fibonacci number?
e a square number? **f** a triangular number?

Activity CW
6:16 Heads and tails
Probability animation — spinner simulation

Interactive CW Animation

Mathematical terms 6

fraction (or common fraction)

- Part of a whole.
  eg $\frac{1}{4}$ of a pie



- A number between 0 and 1 is formed by dividing a number by a bigger number.
  eg $\frac{7}{8}$ or $7 \div 8$

numerator

- Top number of a fraction.
  eg 7 in $\frac{7}{8}$

denominator

- Bottom number of a fraction.
  eg 8 in $\frac{7}{8}$

improper fraction

- A fraction which has a numerator that is bigger than the denominator.
  eg $\frac{9}{8}$

rational number

- A number that can be written as a common or improper fraction.

mixed number

- A number that has a whole number part and a fraction part.
  eg $1\frac{2}{3}$

equivalent fractions

- Fractions of equal size.
  eg $\frac{1}{2} = \frac{5}{10} = \frac{7}{14} = \ldots$

common denominator

- A denominator that allows two or more fractions to be written with the same denominator.
  eg Some common denominators for $\frac{3}{4}$ and $\frac{5}{6}$ are 12, 24, 36, ...

lowest common denominator (LCD)

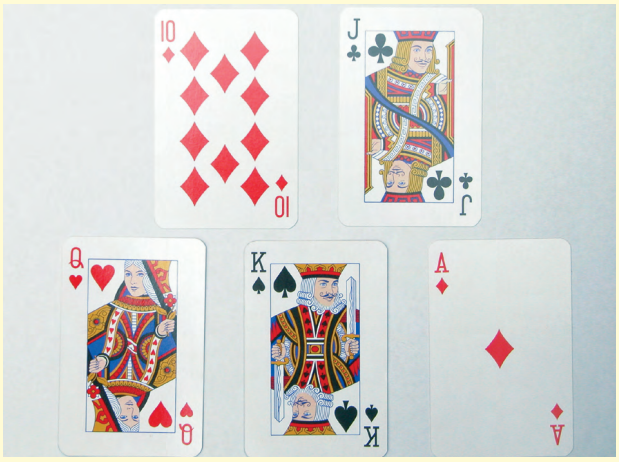
- The smallest of the common denominators.

reciprocal

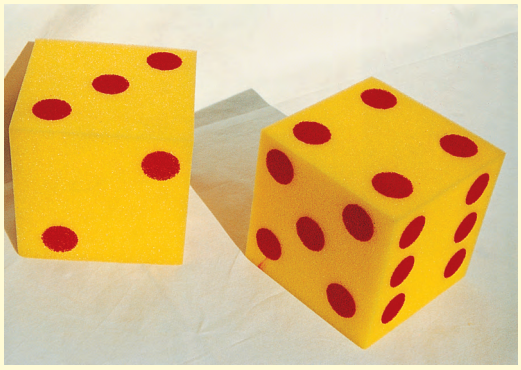
- The number formed by turning a fraction upside down.
  eg The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.
  The reciprocal of 4 is $\frac{1}{4}$.
- Reciprocals multiply to give 1.

percentage (or percentage fraction)

- It means 'out of 100'. A percentage fraction has a denominator of 100.
- The symbol % is used for percentages.
  eg $15\% = \frac{15}{100}$



- The court cards are the *Jack*, *Queen* and *King*. What is the probability of picking a court card at random from these cards?



- In how many ways can a *seven* be made using two dice?

diagnostic test

6

Diagnostic Test 6: | Fractions, Percentages and Probability

- Each section of the test has similar items that test a certain type of question.
- Failure in more than one item will identify an area of weakness.
- Each weakness should be treated by going back to the section listed.

				Section
1	a $\frac{3}{10} + \frac{5}{8}$	b $\frac{3}{4} + \frac{2}{5}$	c $\frac{1}{3} + \frac{1}{10}$	6:04
2	a $\frac{3}{4} - \frac{2}{5}$	b $\frac{9}{10} - \frac{1}{4}$	c $\frac{1}{4} - \frac{1}{5}$	6:04
3	a $3\frac{2}{5} + 1\frac{3}{4}$	b $6\frac{1}{5} + 4\frac{1}{2}$	c $1\frac{7}{8} + 2\frac{1}{4}$	6:05
4	a $5 - 2\frac{3}{5}$	b $6\frac{3}{4} - 2\frac{1}{10}$	c $13\frac{1}{5} - 1\frac{1}{2}$	6:05
5	a $\frac{7}{8} \times \frac{9}{10}$	b $\frac{3}{5} \times \frac{2}{3}$	c $\frac{9}{10} \times \frac{15}{16}$	6:06
6	a $4 \times \frac{3}{5}$	b $1\frac{7}{8} \times 3$	c $3\frac{3}{4} \times 1\frac{1}{3}$	6:06
7	a $\frac{9}{10} \div \frac{1}{2}$	b $\frac{3}{4} \div \frac{5}{8}$	c $\frac{3}{4} \div \frac{2}{5}$	6:07
8	a $\frac{3}{5}$ of 2 km	b $\frac{7}{10}$ of 3 L	c $\frac{2}{3}$ of 2 hours	6:07
9	a What fraction of 2 m is 40 cm? b What fraction of 2 hours is 12 minutes? c What fraction of 5 kilometres is 300 metres?			6:07
10	Change to percentages. a $\frac{5}{8}$	b $\frac{3}{4}$	c $\frac{4}{5}$	6:10
11	Change to percentages. a 0·25	b 0·7	c 0·625	6:10
12	Write each as a fraction and as a decimal. (Extension) a $3\frac{1}{4}\%$	b 17%	c 18·6%	6:11
13	a 45% of 600 mL	b 7% of \$15 200		6:12
14	a What percentage is 200 g of 1 kg? b Express \$750 as a percentage of \$4000.			6:13
15	Ten bus tickets were misplaced. One is mine. Three of these have now been found. What is the probability that: a my ticket is one of those found? b my ticket is not one of those found?			6:16
16	A dice is thrown. What is the probability that the result will be: a an even number?	b greater than 1?		6:16

Chapter 6 | Revision Assignment

assignment 6A

1 **a** $0{\cdot}6 + 0{\cdot}7$ **b** $2{\cdot}15 + 3{\cdot}6$
c $7{\cdot}8 + 1{\cdot}95$ **d** $12{\cdot}16 + 3{\cdot}178$

2 **a** $0{\cdot}9 - 0{\cdot}3$ **b** $1{\cdot}6 - 0{\cdot}8$
c $12{\cdot}1 - 6{\cdot}72$ **d** $5{\cdot}68 - 1{\cdot}7$

3 **a** $0{\cdot}6 \times 0{\cdot}5$ **b** $(0{\cdot}2)^2$
c $11{\cdot}5 \times 2$ **d** $3{\cdot}1 \times 0{\cdot}4$

4 **a** $12{\cdot}6 \div 0{\cdot}2$ **b** $0{\cdot}3 \div 0{\cdot}1$
c $2{\cdot}5 \div 0{\cdot}02$ **d** $0{\cdot}7 \div 2{\cdot}5$

5 **a** $1{\cdot}1 \times 5$ **b** $0{\cdot}11 \times 5$
c $0{\cdot}011 \times 5$ **d** $1{\cdot}1 \times 0{\cdot}5$

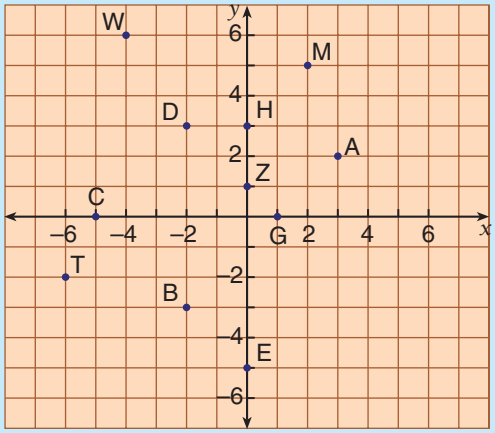
6 **a** $6{\cdot}5 \div 5$ **b** $0{\cdot}65 \div 5$
c $6{\cdot}5 \div 0{\cdot}5$ **d** $0{\cdot}65 \div 0{\cdot}5$

7 Are the following true or false?
a $16 > 12$ **b** $398 \div 400$
c $0{\cdot}3 > 0{\cdot}033$ **d** $5{\cdot}1 + 4 = 4 + 5{\cdot}1$

8 If $x = 3$, find the value of y for each of the following rules.
a $y = 5 \times x$ **b** $y = 7 - x$
c $y = 2 \times x + 3$ **d** $y = 24 \div x$

9 Write down the coordinates of the points on the diagram at right:
a A **b** B **c** C **d** D **e** E

10 Which point is at:
a $(0, 3)$? **b** $(-6, -2)$ **c** $(2, 5)$?
d $(-4, 6)$? **e** $(0, 1)$?



11 Which of the points given are on the same line as:
a H and Z? **b** C and B?
c A and B? **d** D and Z?

12 Write each numeral in simplest form.
a $(7 \times 1000) + (3 \times 100) + (2 \times 10)$
b $(6 \times 1000) + (4 \times 10) + (8 \times 1)$
c $(6 \times 10\,000) + (7 \times 1000) + (4 \times 100)$
d $(3 \times 1\,000\,000) + (5 \times 10\,000)$

assignment 6B

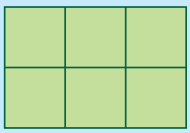
id 6

Chapter 6 | Working Mathematically

1 Use ID Card 7 on page xix to identify:

a 5	b 6	c 12	d 17
e 18	f 19	g 20	h 21
i 22	j 23		

2 How many squares can you find in this rectangle? (Be careful!)

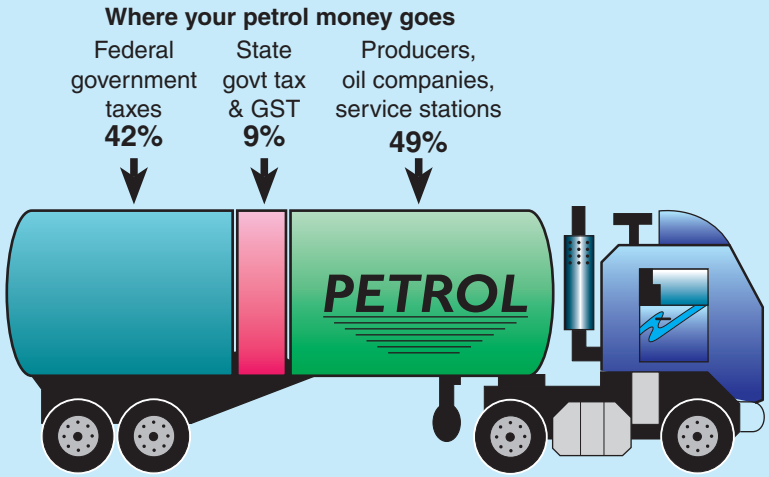


3 When staying at a motel, a person notices that the menu has a choice of 3 entrees, 4 main meals and 2 desserts. How many different three-course meals are possible?

4 A train 1 km long is travelling at 60 km/h. The train enters a tunnel 1 km long. How long is it before the train emerges completely from the tunnel?

5 A man has a business in which he makes plastic figures representing the numbers 0 to 9. He receives an order from a school for a set of room numbers from 1 to 50. How many plastic figures of each digit are needed for this order?

6 A plumber, carpenter and electrician all work for a large building company. Their names are Bagnell, Jones and Farrant but not necessarily in that order. It is known that the plumber who was an only child earns the least, while Bagnell, who married Jones' sister, earns more than the electrician. Give the names of the plumber, the carpenter and the electrician in that order.



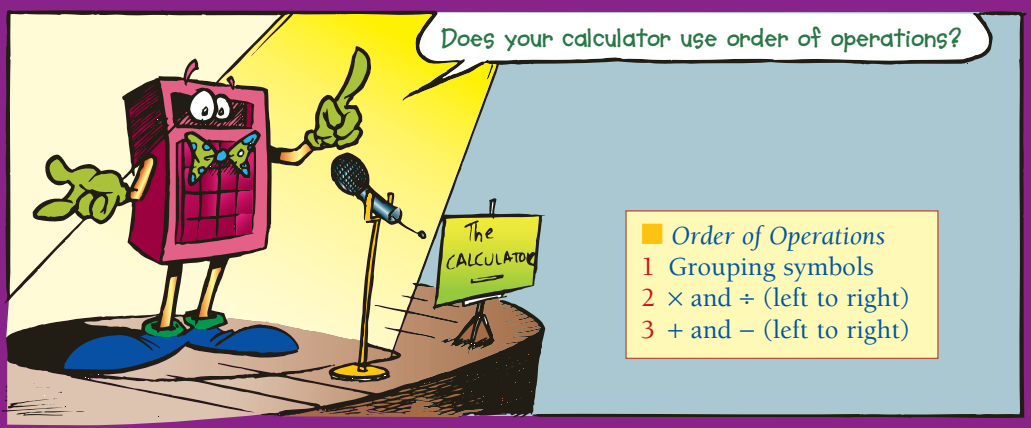
7 The above diagram is an eye-catching way of showing how petrol revenue is divided.
- a Is the diagram a graph? Give reasons for your answer.
- b What percentage of petrol revenue goes to governments?
- c If petrol costs 90 cents a litre, how much of this goes to the federal government in taxes?
- d If the percentages were left off the diagram, would it be an accurate way to represent the information?

Drag and Drop Interactive
1 Percentages to fractions and decimals
2 Probability of spinners
3 Fractions

Review Questions

7

Calculators



Chapter Contents

7:01 Using a calculator
7:02 Making estimates
7:03 2nd F and C
7:04 Speed and accuracy
7:05 Multiples, factors and patterns
7:06 Problem solving with the calculator
Fun Spot: Make words with your calculator
Challenge: The Australian Grand Prix
Mathematical Terms, Diagnostic Test, Working Mathematically

Learning Outcomes

- Analyse a mathematical or real-life situation, solving problems using technology where appropriate.
- Recognise the properties of special groups of whole numbers and apply a range of strategies to aid computation.

Areas of Interaction

Approaches to Learning and Homo Faber

7:01 | Using a Calculator

Calculators usually have 'order of operations' built in. Check the operation of your calculator by pushing the keys shown and comparing your answer with the one given.

Question	*Calculator sentence*	*Answer*
1 8000 + 400 + 30 + 2	8000 [+] 400 [+] 30 [+] 2 [=]	8432
2 789·6 – 89·75 – 564·2	789·6 [–] 89·75 [–] 564·2 [=]	135·65
3 15·6 × 147 ÷ 1·4	15·6 [×] 147 [÷] 1·4 [=]	1638
4 1782·96 × (6·8 + 7·6)	1782·96 [×] [(] 6·8 [+] 7·6 [)] [=] OR 6·8 [+] 7·6 [=] [×] 1782·96 [=]	25 674·624
5 162 ÷ (16 + 2)	162 [÷] [(] 16 [+] 2 [)] [=] OR 16 [+] 2 [=] [STO] 162 [÷] [RCL] [=] Here the [STO] key puts what is on the screen into the memory and [RCL] brings it back when needed. (STO = store, RCL = recall)	9
6 6·8 + 14 ÷ 0·5 – 4	6·8 [+] 14 [÷] 0·5 [–] 4 [=] if it has 'order of operations', or if not; 14 [÷] 0·5 [=] [STO] 6·8 [+] [RCL] [–] 4 [=]	30·8
7 Add the answers to: 9 × 1·5, 6 + 3·8, 2·1 ÷ 3.	9 [×] 1·5 [=] [STO] 6 [+] 3·8 [=] [M+] 2·1 [÷] 3 [=] [M+] [RCL] Here [M+] adds what is on the screen to what is already in the memory.	24



Important Notice
Not all calculators are the same. Your calculator may not have all of the keys mentioned in this book, or it may expect you to use a key in a different way. Check your instruction booklet to be sure.

Exercise 7:01

1 First do these in your head, then check your answers using a calculator.

a $7000 + 300 + 90 + 1$	**b** $20\,000 + 700 + 10 + 6$	**c** $90\,000 + 5000 + 60 + 4$
d $15 - 5 - 3 - 2$	**e** $800 - 200 - 200 - 200$	**f** $90 - 10 - 20 - 30 - 30$
g $60 + 10 + 20 + 30 + 40$	**h** $7 + 3 + 8 + 2 + 9 + 1$	**i** $800 + 200 + 600 + 400$
j $5 \times 20 \div 5$	**k** $14 \div 7 \times 7$	**l** $25 \times 4 \div 10$
m $600 \div (3 + 3)$	**n** $35 - (10 + 5)$	**o** $10 - (20 - 10)$
p $10 + 3 \times 10$	**q** $20 - 4 \times 5$	**r** $9 - 14 \div 2$
s $9 + 12 \div 3 - 10$	**t** $6 - 10 \div 2 + 4$	**u** $10 + 4 \times 5 - 20$

2 Use a calculator to find the answers.

a $72\,104 + 38\,697$	**b** $6{\cdot}18 + 9{\cdot}72 + 6{\cdot}15$	**c** $34\,947\,688 + 6\,904\,203$
d $309\,178 - 297\,309$	**e** $1547 - 13{\cdot}814$	**f** $15\,050\,000 - 47\,999$
g 5097×704	**h** $87\,415 \times 73{\cdot}8$	**i** $8{\cdot}318 \times 97{\cdot}8$
j $626\,745 \div 635$	**k** $792{\cdot}022 \div 8{\cdot}14$	**l** $20\,044{\cdot}8 \div 58$
m $3007 + 51{\cdot}8 \times 95$	**n** $475 - 14\,050 \div 562$	**o** $1\,247\,000 - 946 \times 384$
p $89{\cdot}14 \times 3 - 205{\cdot}65$	**q** $7044 \times 81 - 70\,562$	**r** $223\,584 \div 32 + 3013$
s $8{\cdot}9 \times 3{\cdot}8 + 1{\cdot}1 \times 3{\cdot}8$	**t** $104 \times 7{\cdot}3 - 4 \times 7{\cdot}3$	**u** $9{\cdot}8 \times 7 - 9{\cdot}8 \times 6$

3 **a** Tyndale school bought 4350 exercise books at 89 cents per book. How much did these books cost altogether?

b Five stamp albums contain Alan's stamp collection. There are 3814, 1426, 899, 1796 and 99 stamps in the albums. How many stamps are there altogether?

c Mr Rich plans to start a rabbit farm in Bourke. His intention is to build the number of rabbits in the farm from 480 000 in 2003 to 645 800 in 2004. What increase is he planning for in that year?

d If $15 000 million worth of goods were to be exported from NSW over 12 months, what value of goods would need to be exported in the first month to be on target?

e Inside a beehive, 7560 bees are beating their wings 185 times every second to keep the hive cool. How many beats would occur altogether in one minute?

f Light travels at about 17·9 million kilometres per minute. How many million kilometres would light travel in one year? (Use 1 year = 365 days.) The distance that light travels in one year is called a *light year*.

g Cyclone Blowhard travelled across the countryside at 1·08 km per minute for 128 minutes, and at 0·84 km per minute for a further 117 minutes. How far did the cyclone travel altogether in that 245 minutes?

7:02 | Making Estimates

prep quiz

7:02

Round off each number as instructed.

1 387 (to the nearest hundred)
2 94·18 (to the nearest ten)
3 8·9104 (to the nearest whole)
4 3437 (to the nearest thousand)

Round off correct to one decimal place.

5 0·7615
6 0·349
7 0·8888
8 0·274

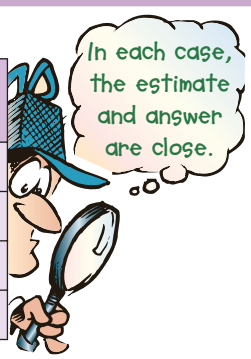
Round off each number as instructed.

9 3·8 million (to the nearest million)
10 0·05338 (to the nearest hundredth)

When making an estimate, round off each number to the place holding the first non-zero digit and then perform the operation.
eg 8174 × 3·814 ≈ 8000 × 4 or 32 000

If our estimate is close to the calculator answer, we would accept that answer. If not, we would do the question again.

worked examples



	Question	*Rounding off*	*Estimate*	*Calculator display*
1	432·8 ÷ 16·95	400 ÷ 20	20	25·533923
2	41·85 × 19·3	40 × 20	800	807·705
3	684·15 + 34·9	700 + 30	730	719·05
4	649 − 3·42	600 − 3	597	645·58

5 John gets an answer of 403·6669 on his calculator when he does the multiplication 63·37 × 63·37.
His estimate, however, is 3600. What should he do next?
As his estimate of 3600 is very different from his answer, he should carefully enter the question into the calculator again. If the answer is still unchanged, he should check the estimate.

63·37 [×] 63·37 [=] gives [4015·7569]

The last answer is close enough to the estimate, but John should do it once again in order to be sure of his answer.

■ Estimate
$60 \times 60 = 3600$

6 Find an estimate for $\sqrt[3]{743}$, correct to the nearest whole number.
We can use trial and error to estimate the cube root of 743.
Try 20. $20^3 = 8000$. This is too big.
Try 10. $10^3 = 1000$. This is better.
Try 8. $8^3 = 512$. This is too small.
Try 9. $9^3 = 729$. This is close.
A good estimate for $\sqrt[3]{743}$ is 9.

Exercise 7:02

1 Use the method of rounding off shown in the previous examples to find estimates for:

a $58{\cdot}165 + 342{\cdot}844$	**b** $8416 + 93 + 497$	**c** $56\,814 + 93\,147$
d $710\,943 - 241\,385$	**e** $879 - 94 - 12{\cdot}6$	**f** $345{\cdot}85 - 90{\cdot}312$
g $8416 \times 3{\cdot}086$	**h** $19{\cdot}467 \times 0{\cdot}4996$	**i** $873\,400 \times 214{\cdot}6$
j $563{\cdot}81 \div 18{\cdot}3$	**k** $9{\cdot}3814 \div 2{\cdot}746$	**l** $8\,374\,600 \div 472$

2 Copy and complete the table below, doing questions again where your estimate and calculator display are not very similar.

	Question	*Rounding off*	*Estimate*	*Calculator display*
a	$46{\cdot}5 \times 21{\cdot}7 \times 42{\cdot}6$	$50 \times 20 \times 40$	40 000	42 985·53
b	$93{\cdot}75 \div 27{\cdot}8 + 8{\cdot}67$			
c	$87\,814 - 10\,456 - 7141$			
d	$\$496.72 \times 27 + \$93.40$			
e	$\$6{\cdot}7$ million $\times 12 \times 39$			
f	$856{\cdot}84 - 73{\cdot}8 \times 4{\cdot}96$			
g	$9486 + 3447 \times 8369$			

3 In each case select the most probable answer from those provided, and then work out the answer using your calculator.

	Question	*Possible answers*	*Choice*	*Answer*
a	$\$476.35 + \$37.63$	\$513.98, \$852.65, \$480.98		
b	$\$9000 - \$193.46$	\$7069.54, \$8806.54, \$8906.54		
c	$\$78.95 \times 4{\cdot}12$	\$225.27, \$3252.74, \$325.27		
d	$\$92.50 \div 7{\cdot}4$	\$12.50, \$125.00, \$1.25		
e	$\$99\,412 \div 980$	\$101 440.82, \$101.44, \$1014.41		



4 Estimate the answer to each question and then use your calculator to find the solution. By comparing the estimate and calculator solution, decide whether to accept the solution as your answer. Then round off your answer to one decimal place.

a $435{\cdot}3 \times 27{\cdot}14$	**b** $8{\cdot}3 \times 6{\cdot}9 \times 18{\cdot}3$	**c** $675{\cdot}8 \times 24 \times 74$
d $5346{\cdot}82 + 3741{\cdot}06$	**e** $9{\cdot}4186 + 23{\cdot}5894$	**f** $500{\cdot}6 + 93{\cdot}4 + 73{\cdot}25$
g $7000 - 183{\cdot}65 \times 9{\cdot}48$	**h** $17\,350 - 96\,342 \div 37$	**i** $38{\cdot}814 - 6{\cdot}618 \times 5{\cdot}14$
j $8914 \div 4{\cdot}6 \div 3{\cdot}8$	**k** $214{\cdot}6 \div 87{\cdot}5 + 93$	**l** $7864{\cdot}5 \div 365 + 852$

5 Find an estimate to the nearest whole number for:

a $\sqrt{1020}$	**b** $\sqrt{8000}$	**c** $\sqrt{24\,336}$
d $\sqrt[3]{2200}$	**e** $\sqrt[3]{5832}$	**f** $\sqrt[3]{50\,653}$

7:03 | 2nd F and C

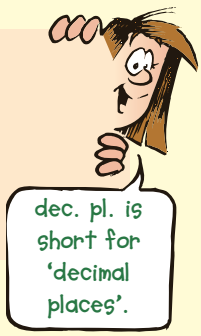
prep quiz 7:03

Round off each number as instructed.

1 4·795 (correct to one decimal place)
2 498·125 (correct to 2 dec. pl.)
3 45 874·3857 (correct to 3 dec. pl.)
4 0·004165 (correct to 4 dec. pl.)

Do these using your calculator, giving answers correct to 2 decimal places.

5 8·73 × 8·95
6 $3.95 × 4·7
7 $97 ÷ 13
8 37·325 + 19·18
9 $506.53 ÷ 103
10 1000 − 34·878



Check the keys on your calculator by entering each calculator sentence and comparing your answer with the one given.

Key or question	*Explanation or calculator sentence*	*Answer*
C (or CI or CE)	These keys can be used to clear the last number only. Keys pressed beforehand are remembered.	■ Note that the use of C may vary for different calculators. Check by pushing 5 × 6 C 2 =. You should get either 10 or 2.
C C together	These keys should wipe out everything in the calculator except what is stored in the memory.	
What happens when two different operation keys are pressed one after the other?	The second key cancels the first, eg 12 × ÷ 2 = gives 6 (ie 12 ÷ 2). 11 − + 10 = gives 21 (ie 11 + 10).	■ Before using your calculator, estimate the size of the answer. eg $\sqrt{93{\cdot}65}$ is between 9 and 10, as $9^2 = 81$ and $10^2 = 100$.
SHIFT 2nd F or ☐	On some keys there is a second function (usually written above the key) which is used by pushing the 2nd function key before the function key.	
$\sqrt[3]{125}$	$\sqrt[3]{\ }$ 2nd F √ 125 =	5
$\sqrt[4]{81}$	$\sqrt[x]{\ }$ 4 2nd F y^x 81 =	3
Subtract 21 from the memory.	M− 21 2nd F M+	
7!	(This means 7 × 6 × 5 × 4 × 3 × 2 × 1.) 7 2nd F % gives 5040. $n!$	

Exercise 7:03

1 Often a special discount price is given when a large number of items is bought. Find the cost to the nearest cent for one item if:

a 1000 nails cost $38
b $45 will buy 400 screws
c $200 gives you 144 cans
d 600 books cost $350
e 3000 pamphlets cost $67
f 40 000 sheets of paper cost $860
g 600 calculators cost $16 300
h 450 tennis balls cost $860
i $1800 buys 700 glue sticks
j 800 rubber bands cost $3.20

2 Explain why if one rubber band costs $0.00 to the nearest cent, 800 rubber bands can cost $3.20.

3 Assuming that **A** [C] clears only the number on the calculator display,
B [C] [C] clears everything except the memory, and
C a second operation key pushed cancels the first,
write down, without using your calculator, the resulting answers when these keys are pushed.

a 6 [×] 8 [C] 9 [=]
b 12 [−] 5 [C] [C] 3 [=]
c 4 [×] [−] 3 [=]
d 30 [−] [÷] 5 [C] 10 [=]
e 5 [×] [C] [C] 4 [×] 7 [=]
f 100 [−] [+] 4 [+] [−] 3 [=]
g 2 [×] [÷] 2 [×] [+] 8 [=]
h 10 [÷] [×] 10 [+] [−] 1 [=]

4 Estimate the two counting numbers between which the answer lies, then use your calculator to find the answer correct to 2 decimal places.

a $\sqrt{5{\cdot}186}$
b $\sqrt{27{\cdot}155}$
c $\sqrt{94{\cdot}125}$
d $\sqrt{63{\cdot}888}$
e $\sqrt[3]{1{\cdot}891}$
f $\sqrt[3]{10{\cdot}473}$
g $\sqrt[3]{30{\cdot}042}$
h $\sqrt[3]{0{\cdot}834}$

5 Use the second function key when necessary to find the answer to these questions.

a $\sqrt[3]{1000}$
b $\sqrt[3]{1}$
c $\sqrt[3]{4913}$
d $\sqrt[3]{46\,656}$
e $\sqrt[3]{1331}$
f $\sqrt[3]{373\,248}$
g $\sqrt[4]{16}$
h $\sqrt[4]{1}$
i $\sqrt[4]{625}$
j $\sqrt[4]{6561}$
k $\sqrt[4]{38\,416}$
l $\sqrt[4]{2\,313\,441}$



6 Noting that $8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$, find the value of:

a 3!
b 8!
c 5!
d 12!
e $9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$
f $11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$
g $13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

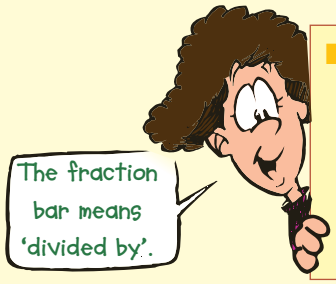
7:04 | Speed and Accuracy

Exercise 7:04

Use your watch to time how quickly you can do each of the following sets of questions.
Add 10 seconds for each mistake.

1

a $3{\cdot}4 + 29{\cdot}725$ **b** $17\,050 - 9347$
c $4428 \div 36$ **d** $25{\cdot}3 \times 541$
e Which is the best buy: 12 kg of potatoes for \$3 or 7 kg for \$1.70?
f How many 37 cent stamps can you buy for \$6.35?
g What is the cost of one litre of petrol, if 28 litres cost \$24.78?
h Change $\frac{7}{16}$ to a decimal.
i Find the value of $\frac{1272}{3} + 6{\cdot}142$.
j Find $\frac{1}{12}$ of 12·3 metres.



■ $\frac{25}{4} = 25 \div 4$
$= 6{\cdot}25$
$\frac{3}{8} = 3 \div 8$
$= 0{\cdot}375$
$\frac{2}{7} = 2 \div 7$
$= 0.\dot{2}85714\dot{}$

■ Use your calculator.
$\frac{1}{8}$ of 36·9 metres:
1 ÷ 8 × 36·9 =
Answer = 4·6125 m

2

a $8{\cdot}6 + 999{\cdot}56$ **b** $90\,761 - 4172$
c $22\,442 \div 49$ **d** $98 \times 52{\cdot}4$
e Which is the best buy: 14 kg of potatoes for \$4 or 9 kg for \$2.61?
f How many 42 cent stamps can you buy for \$8.65?
g What is the cost of one litre of petrol, if 26 litres cost \$24.05?
h Change $\frac{7}{8}$ to a decimal.
i Find the value of $\frac{3471}{15} + 81{\cdot}67$.
j Find $\frac{1}{15}$ of 37·2 metres.

3

a $14{\cdot}98 + 7{\cdot}65$ **b** $19\,674 - 8475$
c $20\,412 \div 28$ **d** $81{\cdot}6 \times 493$
e Which is the best buy: 500 g of tuna for \$3 or 300 g for \$2.05?
f How many 39 cent stamps can you buy for \$9.20?
g What is the cost of one litre of petrol, if 46 litres cost \$44.85?
h Change $\frac{11}{16}$ to a decimal.
i Find the value of $\frac{4369}{17} + 6{\cdot}142$.
j Find $\frac{1}{14}$ of 32·2 metres.

4

a $374{\cdot}1 + 88{\cdot}8$ **b** $17\,340 - 8096$
c $15\,456 \div 92$ **d** $741 \times 8{\cdot}66$
e Which is the best buy: 450 g of tuna for \$2.85 or 250 g for \$1.60?
f How many 89 cent rulers can you buy for \$8.85?
g What is the cost of one litre of petrol, if 16 litres cost \$15.12?
h Change $\frac{5}{8}$ to a decimal.
i Find the value of $\frac{7470}{15} + 17{\cdot}86$.
j Find $\frac{1}{18}$ of 97·2 metres.

5

a $419 + 34{\cdot}361$ **b** $91\,310 - 694$
c $12\,168 \div 156$ **d** $4{\cdot}886 \times 9{\cdot}1$
e Which is the best buy: 375 g of tuna for \$2.37 or 300 g for \$1.92?
f How many 78 cent rulers can you buy for \$6.56?
g What is the cost of one litre of petrol, if 72 litres cost \$64.44?
h Change $\frac{9}{16}$ to a decimal.
i Find the value of $\frac{8412}{12} + 32{\cdot}45$.
j Find $\frac{1}{11}$ of 73·7 metres.

7:05 | Multiples, Factors and Patterns

- **A *multiple* of a counting number is obtained when you multiply it by another counting number.**
- **A *factor* of a counting number divides it exactly.**
  eg $15 \times 7 = 105$. Here 15 and 7 are factors of 105, and 105 is a multiple of both 15 and 7.

Exercise 7:05

1 Use your calculator to list the first seven multiples of:

a 12 **b** 37 **c** 56 **d** 128 **e** 433

2 Find which of the prime numbers 2, 3, 5, 7, 11, 13, 17, 19, 23, 29 and 31 are factors of:

a 24 242 **b** 49 335 **c** 589 589 **d** 785 726 **e** 4 365 515

3 Find the four numbers in each row that have a product equal to the answer given.

							Answer
a	9	13	17	19	23	29	84 303
b	9	13	17	19	23	29	51 129
c	9	13	17	19	23	29	215 441
d	7	11	23	31	37	41	244 237
e	7	11	23	31	37	41	290 191
f	7	11	23	31	37	41	517 297

							Answer
g	9	18	41	43	53	59	840 951
h	9	18	41	43	53	59	1 681 902
i	9	18	41	43	53	59	2 420 298
j	7	49	43	59	61	64	1 083 299
k	7	49	43	59	61	64	1 612 352
l	7	49	43	59	61	64	1 339 072

4 Use the rule given to write the next four numbers in the pattern.

	Explanation of rule	*Pattern*
a	Multiply the last term by 11 and subtract 77.	8, 11, . . ., . . ., . . ., . . .
b	Multiply the last term by 27 and subtract 368.	15, 37, . . ., . . ., . . ., . . .
c	Multiply the last term by 0·6 and add 625.	3125, 2500, . . ., . . ., . . ., . . .
d	Multiply the last term by 7·5 and subtract 64.	15, 56, . . ., . . ., . . ., . . .
e	Divide the last term by 8 and add 32 768.	32 768, . . ., . . ., . . ., . . .
f	Divide the last term by 16 and add 2048.	4096, . . ., . . ., . . ., . . .

5 The square of a number is found by multiplying that number by itself. Find the square of:

a 37 **b** 125 **c** 536 **d** 89 **e** 15·64 **f** 0·999

6 By 'guess, check and refine' find the number that has a square of:

a 1764 **b** 5329 **c** 3364 **d** 15 129 **e** 125 316 **f** 112 896

7 By 'guess, check and refine' (trial and error), find the number that when used three times in a product gives the given number. (For example, $\square \times \square \times \square = 2197$, $\square = ?$ After trying a few numbers we see that $\square = 13$.)

a 125 **b** 343 **c** 1728 **d** 4096 **e** 140 608 **f** 551 368

8 When 11 864 is divided by 37 on a calculator, the screen shows 320·64865. This is an approximation, as the answer is a repeating decimal. To write this answer as a whole number plus remainder, we subtract from the screen the 'whole number part' (ie 320) and multiply what is left by the divisor (ie 37). The answer is the remainder.

eg 320·64865 [−] 320 [=] [×] 37 [=] The remainder is 24

$\therefore$ 11 864 ÷ 37 = 320 r 24 or $320\frac{24}{37}$.

Use this method to find the answers to these as mixed numbers.

a 41 867 ÷ 46 **b** 38 947 ÷ 93 **c** 11 014 ÷ 33

d 403 204 ÷ 107 **e** 934 608 ÷ 847 **f** 500 427 ÷ 761

7:06 | Problem Solving with the Calculator

Exercise 7:06

1 If 23 cm 4 mm = 23·4 cm, find the distance in centimetres of:

a 23 cm 4 mm × 14 **b** 23 cm 4 mm ÷ 13 **c** 23 cm 4 mm × 8·5

2 If 9 m 52 cm = 9·52 m, find the distance in metres of:

a 9 m 52 cm ÷ 7 **b** 9 m 52 cm × 58 **c** 9 m 52 cm ÷ 17

3 If 81 m 765 mm = 81·765 m, find the distance in metres of:

a 81 m 765 mm × 18 **b** 81 m 765 mm ÷ 45 **c** 81 m 765 mm ÷ 23

4 The dimensions of buildings are usually given in millimetres. Find the perimeter of a building that has side lengths of 12 064 mm, 28 565 mm, 13 405 mm and 29 405 mm. Give your answer in metres.

5 A power pole is 13 m 67 cm high. A wire from a house is joined to the pole at a point 2 m 83 cm from the top. How far is this point from the ground? Luke can climb 1 m 35 cm up the pole every minute. How long would it take him to reach the top? (Give your answer correct to 2 decimal places of a minute.)

6 Muzi lives in South Africa and was trying to find out what the population of his province KawaZulu-Natal was in 2001. He discovered that the population of South Africa was approximately 44 820 000 and that for the remaining provinces, the population of Eastern Cape was 6 437 000, Free State was 2 707 000, Gauteng was 8 837 000, Limpopo was 5 274 000, Mpumalanga was 3 123 000, Northern Cape was 823 000, North West Province was 3 669 000 and Western Cape was 4 524 000. From this information he found the population of KwaZulu-Natal correct to the nearest 100 000. What was this figure?

7 In an attempt to improve the standard of Mathematics at her school, Diane ordered 17 maths books which cost $22.95 per book. She was told that if she ordered 10 or more books, she would receive a discount of 10%. (This means that she need only pay $\frac{9}{10}$ or 0·9 of the bill.) She also had to pay $7 for postage and handling. Since she wished to pay in advance, how much money (cheque) should she send? Answer to the nearest 5 cents.

8 These days weighing machines have been produced which are so accurate that they are used for counting the number of items being weighed. If the weight of 1000 items is known, the machine, by weighing the items, can tell you how many are being weighed. To find the number of items, the machine divides the 'WEIGHT kg' by the 'UNIT-WEIGHT' and multiplies the result by 1000. Use you calculator to do the job of the machine to complete the table below. Answer to the nearest whole number.



	WEIGHT kg of items	*UNIT WEIGHT per 1000 items*	*QUANTITY (no. of items)*
a	0·162	0·27	
b	0·254	0·27	
c	0·444	0·27	
d	1·816	1·94	
e	0·877	1·94	
f	2·042	3·46	
g	0·472	3·46	

■ If 1000 items weigh 0·27 kg

the weight of 1 item $= \frac{0{\cdot}27}{1000}$ kg.

These items weigh 0·162 kg.

$\therefore$ Quantity $= 0{\cdot}162 \div \frac{0{\cdot}27}{1000}$

$= 0{\cdot}162 \times \frac{1000}{0{\cdot}27}$

$= \frac{0{\cdot}162}{0{\cdot}27} \times 1000$

9 If 45 pencil sharpeners weighed 0·012 kg, what would be the weight of:

a 1000 pencil sharpeners? **b** 800 pencil sharpeners? **c** 973 pencil sharpeners?

10 If one Malaysian ringgit (MYR) is worth 43.72 Singapore cents, find the amount of Singaporean money (to the nearest cent) that could be exchanged for Malaysian money to the value of:

a MYR 6000 **b** MYR 14 678 **c** MYR 987.46

11 If one American dollar is worth 131·78 Japanese yen or 1·886 Australian dollars, give to the nearest cent the value in American money of:

a 143 yen **b** 963 yen **c** 4400 yen

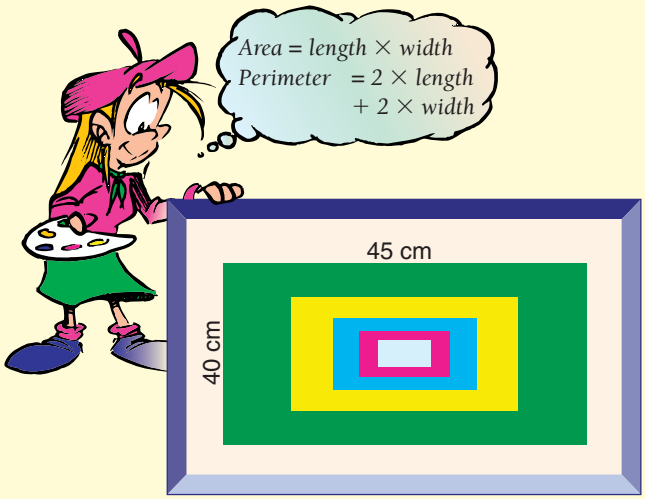
d 8142 yen **e** 320 400 yen **f** 2 496 000 yen

g $500 Australian **h** $25 000 Australian **i** $496 800 Australian

j $1 000 000 Australian **k** $10 760 427 Australian **l** $247 674.80 Australian

12 Miss Easton invested $37 680 for 6 years. Interest payments were the same each year, with all interest payments and the money invested being paid at the end of the six years. How much money did she receive after six years if the interest each year was $10 173.60?

13 Julie decided to paint a pattern of rectangles as her major work in Art for the IB Diploma. She decided to paint one large rectangle 45 cm long by 40 cm wide and to paint each new rectangle within the last one and 0·625 times as long and wide. She made up the table below. Complete the table for her, giving all answers correct to one decimal place. (*Remember:* The length of rectangle 3 will be 0·625 times the length of rectangle 2. Area = length × width, Perimeter = 2 × length + 2 × width.)



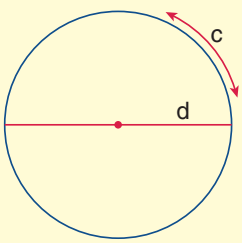
Rectangle	*1*	*2*	*3*	*4*	*5*
Length	45 cm	28·1 cm			
Width	40 cm				
Area	1800 cm^2				
Perimeter	170 cm				

14 Mr Rich worked out that his fortune of $10 million could be used to buy 5130 computer systems or 48 780 guitars or 402 foreign cars or 7168 Persian rugs or 5 025 126 jars of peanut butter or 16 666 667 newspapers. This was in April 1987. Find the approximate cost at this time of:

a a computer system **b** a guitar **c** a foreign car
d a Persian rug **e** a jar of peanut butter **f** a newspaper

15 Charlie Black wanted to find how many times the circumference of a circle (ie its perimeter) was bigger than the diameter, so he measured several circles and for each divided the circumference by the diameter. Find this answer for each circle correct to five decimal places.

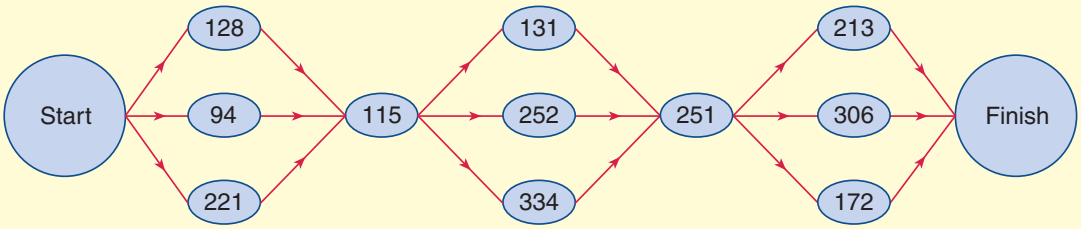
a diameter = 14·2 cm
circumference = 44·6 cm
b diameter = 8·55 cm
circumference = 26·86 cm
c diameter = 1·85 m
circumference = 5·81 m



16 When the Panama Canal was built, it took 42 000 men 10 years to dig out 270 million cubic metres of earth. A machine used in the Musckingum mine in Ohio in 1984 could remove 3 million cubic metres of earth in one month.

- **a** How many months would it take the machine to remove the same amount of earth as was removed when the Panama Canal was built?
- **b** By first finding the amount of earth removed in one year, then dividing by 365, find the amount of earth the machine can remove in one day. (Answer to the nearest 100 cubic metres.)
- **c** Use your answers to **b** and the fact that the machine, called 'Big Muskie', works for 22·5 hours each day to find how much earth can be removed in one minute. (Answer to the nearest cubic metre.)

17 Use your calculator to list the five numbers along the path that have a sum of 1000.



18 For each sum or difference, find two numbers that have the given product. The first one has been done for you. (*Hint:* Guess two numbers with the given sum.)

	Sum	*Product*	*Numbers*
a	53	612	17 and 36
b	100	2331	
c	150	4536	
d	178	6696	

	Difference	*Product*	*Numbers*
e	0	7744	
f	1	3192	
g	2	8280	
h	37	19 680	



- Use a calculator to find the total value of these stamps.

fun spot

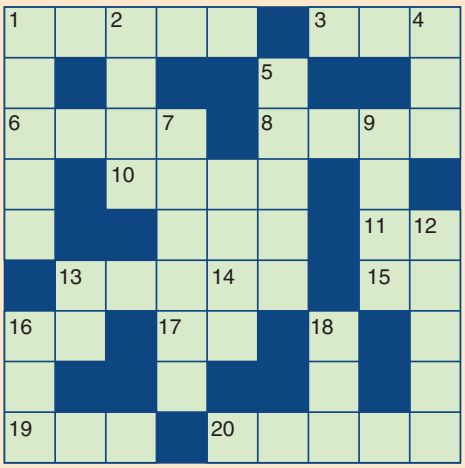
7:06

Fun Spot 7:06 | Make words with your calculator

Use the calculations and the clues to complete the crossword. You read the answers to the calculations by turning your calculator around and looking at the numbers upside down. (Ignore any decimal points in the answers.)

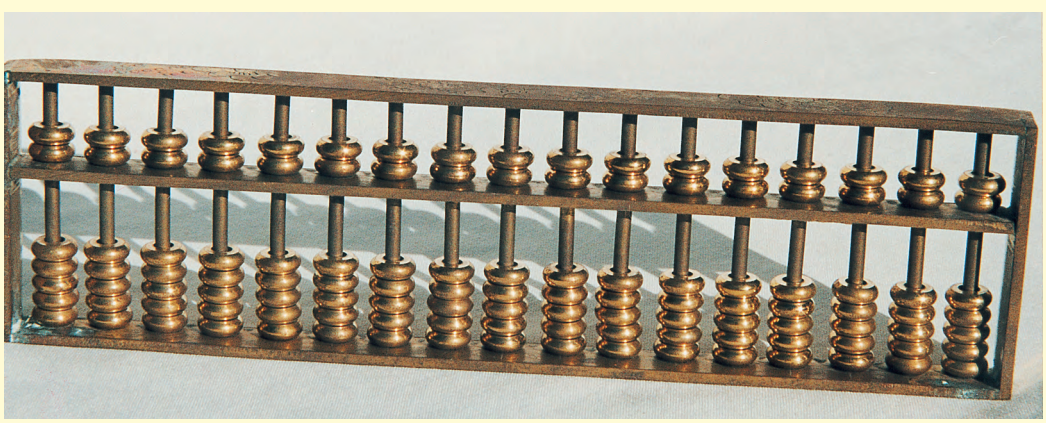
Across

1 '. . . .', st'at'u? (30·1626 ÷ 39)
3 Not hers. ($26^2 - 162$).
6 Beginnings of matchsticks. ($\frac{1}{12}$ of 70 884)
8 Rhymes with 's'. (4000 ÷ 0·625 – 863)
10 A real 'soul' mate. (203 × 15)
11 '. . . no!' (The number of 45 cent stamps for $18.)
13 Unpopular 'male'? (8657·7 ÷ 0·15)
15 Don't come! ($1 - \frac{111}{1110}$)
16 Not she or it. ($3^2 + 5^2$)
17 Same as 14 Down. ($\frac{189}{270}$)
19 It needs a horse. (66 816 – 65 897)
20 Removes phlegm. (35 842 + 17 876)



Down

1 'Alive' with the Sound of Music. (240 × 241 – 126)
2 Borrow some from a centipede. (37 × 47 × 4 – 1019)
4 Save our Souls. (303 × 5 ÷ 3)
5 Opposite of curse. (193 823 ÷ 3·5)
7 Nut cases. ($\frac{1}{8}$ of 923 752 × 5)
9 Hit hard. (925 less than 10 000)
12 Garden tubes. (17 121·28 × 3·125)
13 It won't sting. (6·1644 × 6·1644 correct to the nearest whole number.)
14 Not the opposite of high. (0·2401 ÷ 0·7 ÷ 0·7 ÷ 0·7)
16 To take more than your share. (933·832 ÷ 1·033)
18 What you get when you flip a leg. (56·3 × 98·3 – 4795·29)



- The abacus was the first hand-held calculator.

Challenge 7:06 | The Monaco Grand Prix



Length of track = 3·78 km
Longest straight = 0·9 km

Grid positions + lap times

(1) A. Senna (170·344 km/h)
(2) N. Mansell 1 : 20·537
(3) K. Rosberg 1 : 21·887
(4) A. Prost 1 : 21·889
(5) M. Alboretto 1 : 22·337
(6) M. Surer 1 : 22·561
(7) G. Berger 1 : 22·592
(8) P. Tambay 1 : 22·683
(9) N. Piquet 1 : 22·718
(10) E. deAngelis 1 : 23·007
(11) T. Boutsen 1 : 23·196
(12) D. Warwick 1 : 23·426
(13) E. Cheever 1 : 23·597
(14) R. Patrese 1 : 23·758

Questions

1 Convert each of the lap times from minutes and seconds to seconds, eg 1 : 29·031 = 60 + 29·031 = 89·031 s.

2 Ayrton Senna averaged 170·344 km/h for his fastest lap of 3·78 km. Using $T = D \div S$, find his lap time in hours and convert to minutes and seconds.

3 Calculate the average speed of each of the next 13 drivers using $S = D \div T$. Remember, the distance is in km and the time will be in seconds, so the speed will be km/s. Convert this to km/h by multiplying by 60×60.

4 The length of the race was 82 laps. How fast would a car need to travel to complete the race in 2 hours?

Mathematical terms 7

estimate

- To calculate roughly.
- A good guess or the result of calculating roughly.

factor

- A factor of a counting number divides it exactly.
  eg The factors of 15 are 1, 15, 3, 5.

multiple

- A multiple of a counting number is found by multiplying it by another counting number.
  eg The multiples of 5 are 5, 10, 15, 20, . . .

order of operations

- 1 Grouping symbols 2 × and ÷ 3 + and −.
- These are built into most modern calculators.

keys on a calculator

- C This clears the last entry. Press it twice to clear all but the memory.
- M+ Adds what is on the screen to the memory.
- M− Subtracts what is on the screen from the memory.
- RCL Recalls to the screen what is in the memory.
- STO or Min This replaces what is in the memory with the number on the screen. It **stores** the number displayed.
- 2nd F or SHIFT Pushing this before pushing a key gives you the 2nd function for the key, usually written above it.

Diagnostic Test 7: | Calculators

- Each section of the test has similar items that test a certain type of example.
- Failure in more than one item will identify an area of weakness.
- Each weakness should be treated by going back to the section listed.

Use a calculator to find:			Section
1 a $7000 + 600 + 5$	b $6{\cdot}8 + 14 \div 0{\cdot}5$	c $15{\cdot}6 \times 147 \div 1{\cdot}4$	7:01
2 a $178 \times (6{\cdot}8 + 7{\cdot}6)$	b $162 \div (16 + 2)$	c $19{\cdot}5 - (1{\cdot}8 + 2{\cdot}54)$	7:01
3 Estimate as a whole number:			7:02
a $\sqrt{150}$	b $\sqrt{96}$	c $\sqrt{5086}$	
d $\sqrt[3]{72}$	e $\sqrt[3]{743}$	f $\sqrt[3]{5086}$	
4 Use a calculator to find:			7:03
a $\sqrt[3]{512}$	b $\sqrt[3]{1331}$	c $\sqrt[3]{2744}$	
d $\sqrt[4]{256}$	e $\sqrt[4]{6561}$	f $\sqrt[4]{28\,561}$	
5 Write down, without using your calculator, the resulting answer if these keys are pushed.			7:03
a 7 × 5 C 9 =	b 30 − ÷ 5 =		
c 5 × 4 C C 5 × 2 =			

7A

Chapter 7 | Revision Assignment

1 Simplify:

a
$$\begin{array}{r} 121 \\ 78 \\ \hline 4362 \\ \hline \end{array}$$

b
$$\begin{array}{r} 4296 \\ -\ 709 \\ \hline \\ \hline \end{array}$$

c
$$\begin{array}{r} 107 \\ \times\ \ 9 \\ \hline \\ \hline \end{array}$$

d $4\overline{)2060}$

2 **a** $7.65 + $3.80 + $19.05
b $100 − $63.90
c $2.75 × 22
d $16.80 ÷ 8

3 Simplify:
a $3 + 2 \times 2$ **b** $16 \div 2 - 2$
c $3 \times 2 + 4 \times 5$ **d** $12 - 6 \times 2 \div 3$

4 Write the basic numeral for:
a 4^2 **b** 10^3
c $\sqrt{100}$ **d** 8×10^4

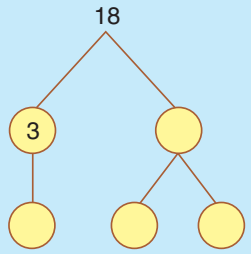
5 Write as a power:
a $4 \times 4 \times 4$ **b** 10×10
c $3 \times 3 \times 3 \times 3$ **d** $7 \times 7 \times 7$

6 List all the factors of:
a 15 **b** 24
c 7 **d** 100

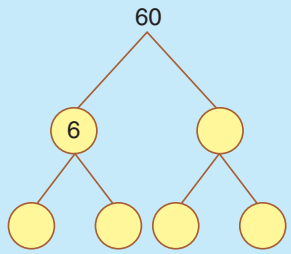
7 **a** Write down the first three multiples of 10.
b Write down the prime numbers between 10 and 20.
c What is the highest common factor of 30 and 40?
d What is the lowest common multiple of 6 and 8?

8 Complete these factor trees.

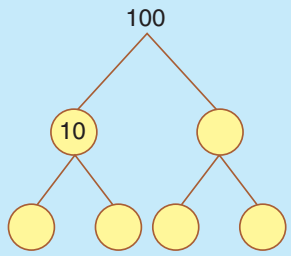
a



b



c



9 Simplify:
a $\frac{5}{10}$ **b** $\frac{2}{8}$
c $\frac{3}{10} + \frac{6}{10}$ **d** $\frac{3}{4} - \frac{2}{4}$

10 Which of the fractions is the larger?
a $\frac{3}{4}$ or $\frac{1}{2}$ **b** $\frac{3}{4}$ or $\frac{1}{10}$
c $\frac{9}{10}$ or $\frac{7}{8}$ **d** $\frac{3}{5}$ or $\frac{2}{10}$

Chapter 7 | Working Mathematically

1 Fred inherited a large sum of money. He gave $5000 to charity, $1000 to each of his three brothers and $1500 each to his school and church. He then paid $5328 for his house loan. After this he had $3572 left. How much did he inherit?

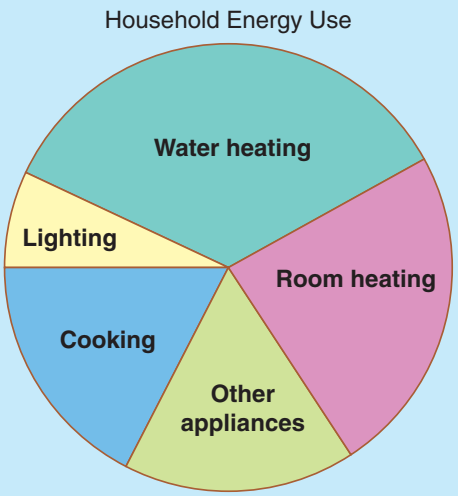
2 The petrol tank of my car holds 55 litres. I drive into a petrol station and fill the tank. It costs me $34.40. If the petrol costs 96·9 cents a litre, how many litres did I buy? How much was in the tank before it was filled? (Give answers correct to 1 decimal place.)

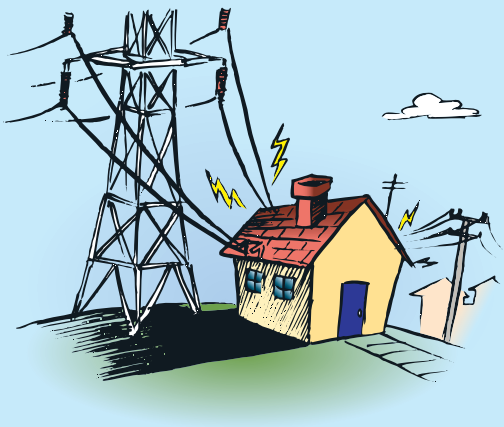
3 In a bag there are six black cards and four red cards. If cards are taken unseen and one at a time, what is the smallest number of cards you must pick out to be sure of getting a pair of cards the same colour? What is the smallest number of cards you must pick to be sure of getting a pair of red cards and a pair of black cards?

4 If 25 people entered a knockout tennis competition, how many byes should be given in the first round so that there are no byes in the following rounds?

5 Is it possible to have a year without a Friday the 13th?

6





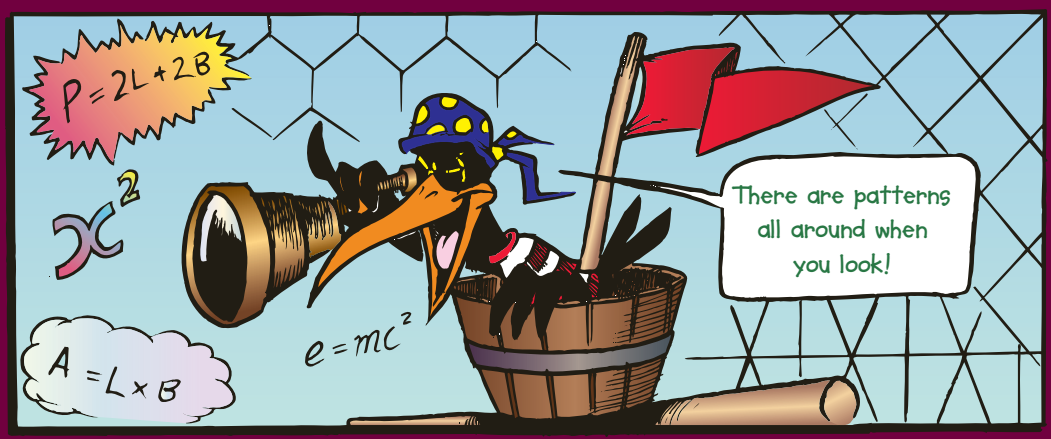
a What is the largest user of household energy?
b What is the smallest user of household energy?
c Measure the water heating angle with your protractor. What fraction of 360° is this?
d What percentage of household energy is used on water heating?
e If the annual power bill is $960, how much is spent on water heating?

1 Calculating percentages

Drag and Drop CW Interactive

8

Patterns and Algebra



Chapter Contents

8:01 Number patterns
8:02 Describing number patterns
Fun Spot: Mysteries mixing matches
8:03 Pronumerals and rules
Reading Mathematics: Quality of life
8:04 Graphs of patterns
Mathematical Terms, Diagnostic Test, Revision Assignment, Working Mathematically

Learning Outcomes

Students will:

- Use letters to represent numbers and translate between words and algebraic symbols.
- Create, record, analyse and generalise number patterns using words and algebraic symbols in a variety of ways.

Areas of Interaction

Approaches to Learning, Homo Faber

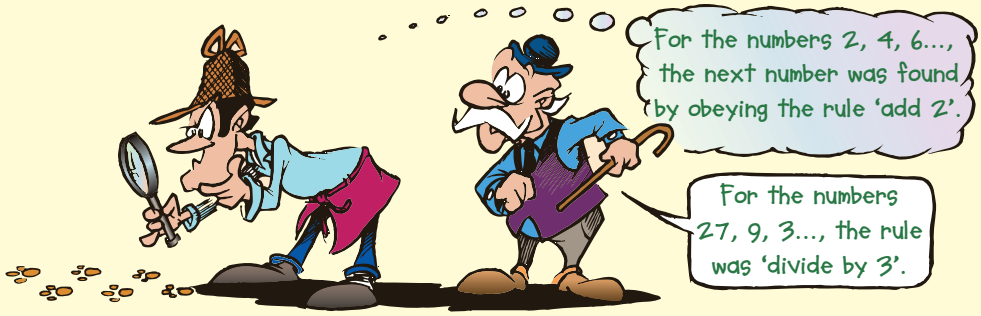
8:01 | Number Patterns

prep quiz 8:01

What is the next number in the following patterns of numbers?

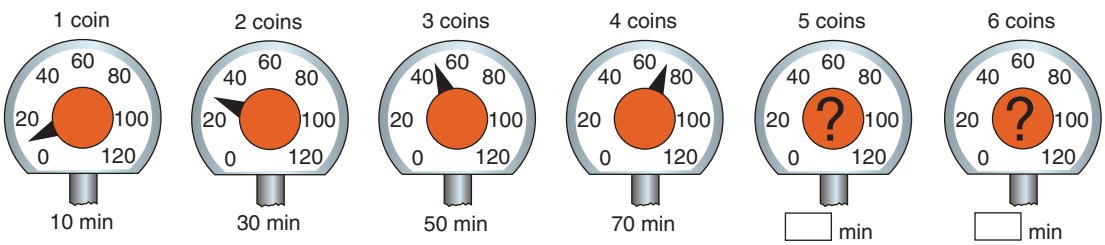
1 2, 4, 6, . . .
2 7, 9, 11, . . .
3 1, 4, 7, . . .
4 4, 8, 12, . . .
5 5, 10, 15, . . .
6 20, 18, 16, . . .
7 2, 4, 8, 16, . . .
8 27, 9, 3, . . .
9 0, $\frac{1}{2}$, 1, $1\frac{1}{2}$, . . .
10 1·0, 1·3, 1·6, . . .

In Prep Quiz 8:01, the next number in the line could be found because each set of numbers was following a pattern or rule.

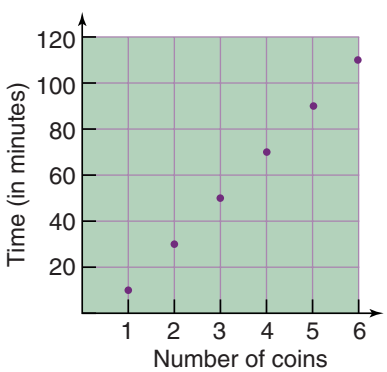


worked example 1 (discussion)

- A parking meter takes only $1 coins. The pictures below show what happens as Tran puts in his coins.
- Is there a pattern? What is the rule used? What parking time would you expect for:
  a 5 coins? **b** 6 coins?



- Let's draw a graph.



Solution

There is a pattern.

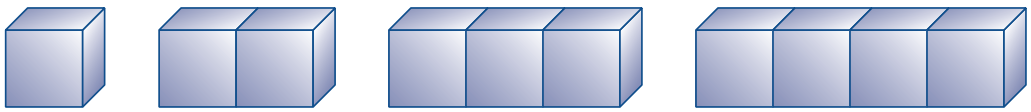
You start at 10 and then follow the rule, 'add 20'.

a For 5 coins you could park for 90 minutes.

b For 6 coins you could park for 110 minutes.

worked example 2 (discussion)

- To make display counters for our school display, we joined boxes (cubes) and painted all the square faces we could see.
- If one box is used, 5 squares can be seen as one is on the ground. For two boxes there are 2 squares on top and 6 around the sides.
- How many squares would need to be painted if the number of boxes joined was:
  a 3? **b** 4? **c** 5?



1 box
5 squares

2 boxes
8 squares

3 boxes
..... squares

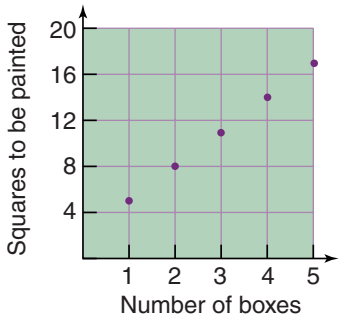
4 boxes
..... squares

As we add extra boxes, do the numbers of squares to be painted form a pattern?
If we know any number in the pattern, how do we find the next?

- First we should make a table.

Number of boxes (n)	1	2	3	4	5
Squares to be painted (s)	5	8	11	14	...

From the table we can see that for every extra box, three more sides need to be painted.



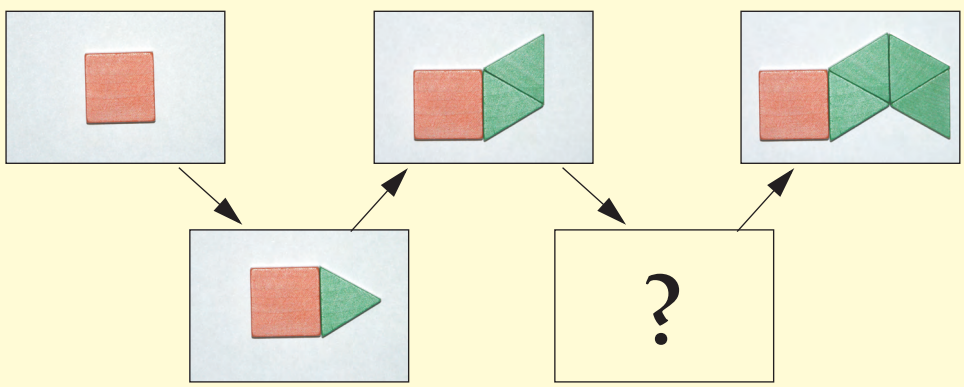
Solution

a For 3 boxes we paint 11 squares.
b For 4 boxes we paint 14 squares.
c For 5 boxes we paint 17 squares.

The squares form a pattern.

To get the next term of the pattern, 'add 3'.

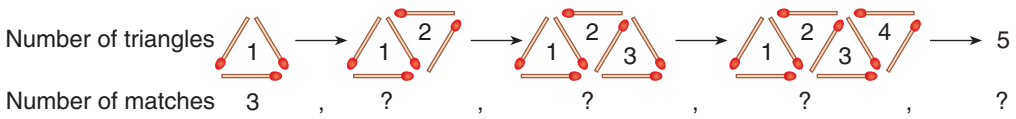
Tables and graphs can be used to show patterns.



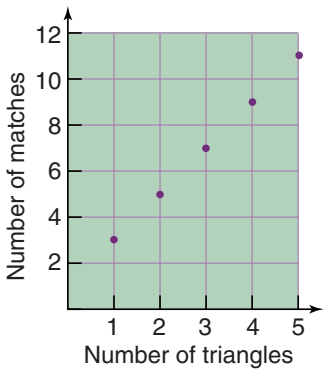
- What is the missing part of this pattern?

worked example 3 (discussion)

- Complete the number pattern for the number of matches needed to make these triangles.



- What is the rule used to get from one number in the pattern to the next?



Solution

3, 5, 7, 9, 11, . . .

The rule is 'add 2'.



If we can recognise and use patterns, we will better understand the world around us.

Exercise 8:01

1 Using the first number and the rule given, write down the next *two* numbers in each pattern.

a 5; add 4 **b** 7; add 11 **c** 12; add 9
d 26; subtract 9 **e** 37; subtract 13 **f** 45; subtract 15
g 3; multiply by 3 **h** 9; multiply by 4 **i** 7; multiply by 3
j 100; divide by 2 **k** 500; divide by 5 **l** 81; divide by 3
m 3; multiply by 2 and add 1 **n** 5; multiply by 3 and add 4
o 2; multiply by 4 and subtract 3 **p** 81; add 1 then divide by 2
q 125; divide by 5 and add 10 **r** 3; multiply the number by itself

2 For each number pattern, write down the next number and the rule you used to get that number.

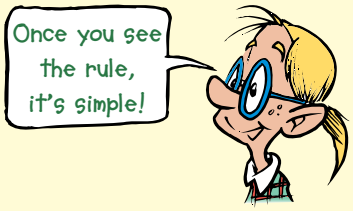
a 3, 7, 11, . . . **b** 5, 11, 17, . . . **c** 1, 7, 13, . . .
d 20, 15, 10, . . . **e** 37, 30, 23, . . . **f** 17, 15, 13, . . .
g 2, 4, 8, . . . **h** 1, 3, 9, . . . **i** 1, 10, 100, . . .
j 80, 40, 20, . . . **k** 81, 27, 9, . . . **l** 64, 16, 4, . . .

3 Write down the next three numbers in each pattern.

a 10, 25, 40, . . . , . . . , . . . **b** 16, 27, 38, . . . , . . . , . . .
c 36, 59, 82, . . . , . . . , . . . **d** 76, 62, 48, . . . , . . . , . . .
e 117, 99, 81, . . . , . . . , . . . **f** 151, 123, 95, . . . , . . . , . . .
g 8, 16, 32, . . . , . . . , . . . **h** 33, 66, 132, . . . , . . . , . . .
i 10, 30, 90, . . . , . . . , . . . **j** 1, 3, 6, 10, . . . , . . . , . . .
k 4, 9, 16, 25, . . . , . . . , . . . **l** 50, 40, 31, 23, . . . , . . . , . . .

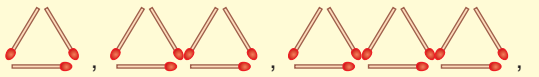
4 Write down the missing number from each pattern.

a	4, 9, . . . , 19, 24	**b**	3, 10, . . . , 24, 31
c	11, 23, . . . , 47, 59	**d**	24, 31, . . . , 45, 52
e	29, 22, . . . , 8, 1	**f**	43, 32, . . . , 10
g	2, . . . , 18, 54, 162	**h**	6, . . . , 24, 48, 96
i	4, 9, 19, . . . , 54, 79	**j**	5, 8, . . . , 17, 23
k	100, . . . , 25, $12\frac{1}{2}$	**l**	1000, 200, . . . , 8, 1·6

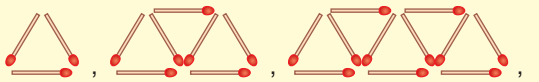


5 Matches were used to construct these patterns. Write down the number pattern that the matches represent. Also write down the number you think would come next in the pattern.

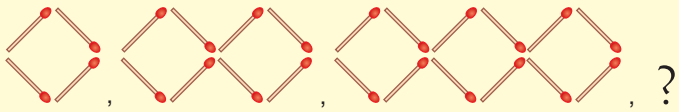
a



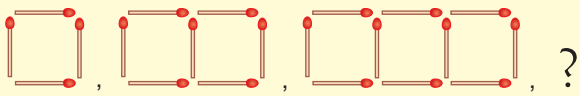
b



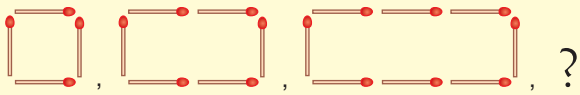
c



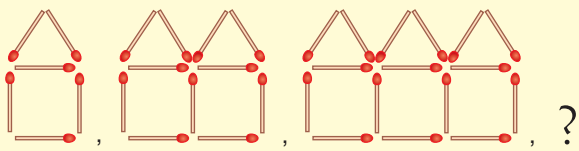
d



e



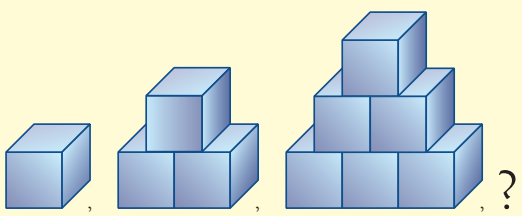
f



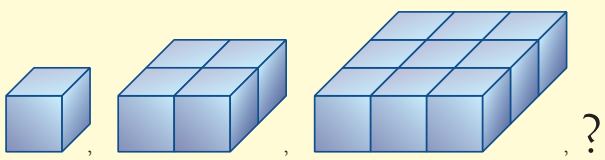


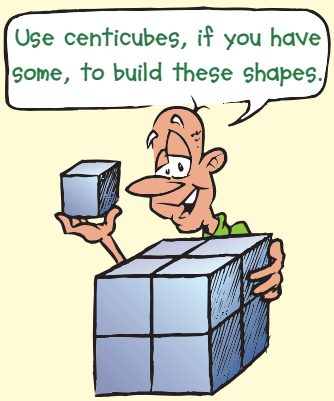
6 Find the number pattern represented by the number of blocks in each set of figures. Also find the next number in each pattern.

a

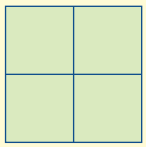


b

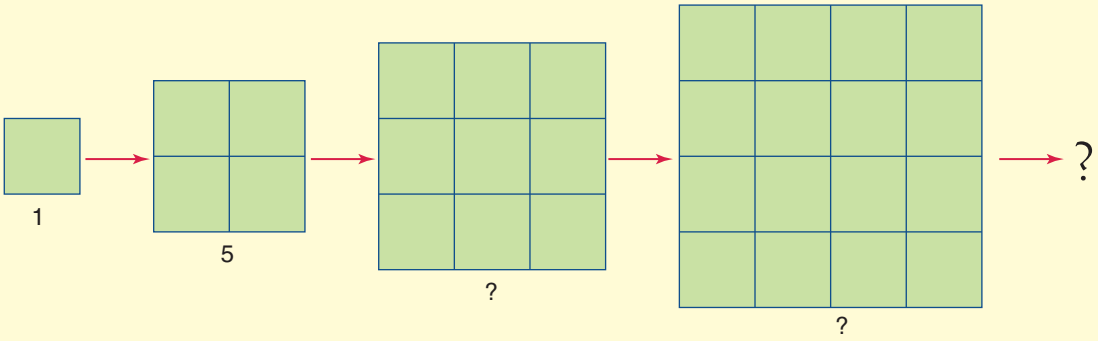




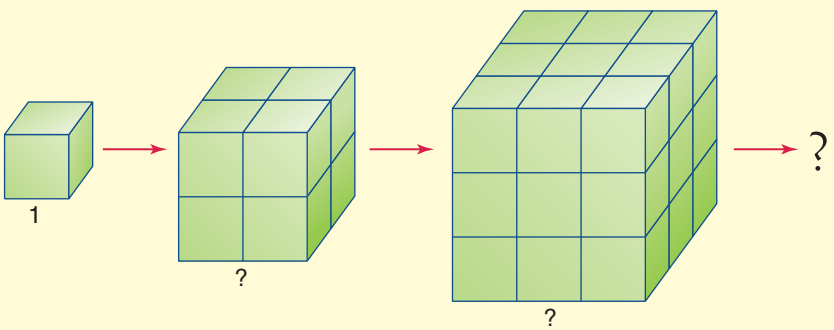
7 Can you see that there are 5 squares in this figure? There are 4 small squares with sides 1 unit long and 1 bigger square with a side length of 2 units.

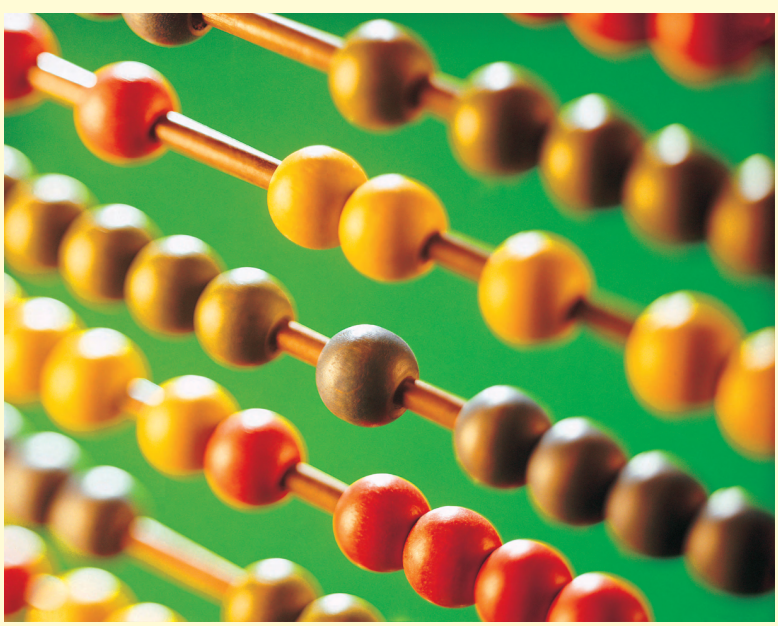


a Find the total number of squares in the third and fourth figures below. What would be the next number in the pattern?



b Find the total number of different cubes, of any size, in the second and third figure. How many cubes would be present if there were four cubes along each edge?

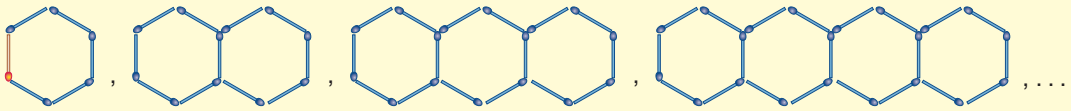




- Each term of a pattern is linked to the next by a rule or pattern.

8:02 | Describing Number Patterns

Discussion



Matchsticks have been used to make this pattern of hexagons.
This table shows the number of matchsticks needed for each part of the pattern.

Number of hexagons (h)	1	2	3	4	5	100
Number of matchsticks (N)	6	11	16	21	...	...

■ What is the rule for this number pattern?

1 Let's describe this number pattern **starting from 1 match**.

Number of hexagons (h)	1	2	3
Number of matchsticks (N)	$1 + 5 = 6$	$1 + 5 + 5 = 11$	$1 + 5 + 5 + 5 = 16$
or	$1 + (5 \times 1) = 6$	$1 + (5 \times 2) = 11$	$1 + (5 \times 3) = 16$
or	$1 + (1 \times 5) = 6$	$1 + (2 \times 5) = 11$	$1 + (3 \times 5) = 16$

We could describe the pattern as: The number of matchsticks is 'one plus, 5 times the number of hexagons', or $N = 1 + 5 \times h$.

2 Let's describe this number pattern **starting from 1 hexagon** (6 matches).

Number of hexagons (h)	1	2	3
Number of matchsticks (N)	6	$6 + 5 = 11$	$6 + 5 + 5 = 16$
or	6	$6 + (5 \times 1) = 11$	$6 + (5 \times 2) = 16$

We could describe the pattern as: The number of matchsticks is '6 plus, 5 times one less than the number of hexagons', or $N = 6 + 5 \times (h - 1)$.

Now for 5 hexagons	Method **1** would lead to	$1 + (5 \times 5) = 26$
	Method **2** would lead to	$6 + (5 \times 4) = 26$
Now for 100 hexagons	Method **1** would lead to	$1 + (5 \times 100) = 501$
	Method **2** would lead to	$6 + (5 \times 99) = 501$

- **If we can find a rule, we can work out any term in the pattern.**
- **In the rule, the letters used are called *variables*, because they can stand for any chosen value. They can *vary*.**

Exercise 8:02

Matchsticks or counters may be useful when doing this exercise.

1



Figure 1 Figure 2 Figure 3 Figure 4



a Complete this table of values using the pictures above.

Figure	1	2	3	4
Number of coins				

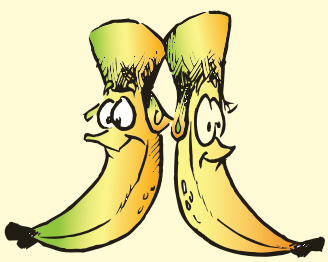
b What rule can you see that connects the number of the figure and the number of coins?

c How many coins would be in:

i the 5th figure of this pattern? **ii** the 6th? **iii** the 10th?

2 This table links the number of bananas to their cost.

Number of bananas	1	2	3	4
Cost (cents)	14	28	42	56



a What is the simple rule that links the number of bananas to their cost?

b What would be the cost of 9 bananas?

3 **a** Complete the table of values for this pattern made by joining square tiles.

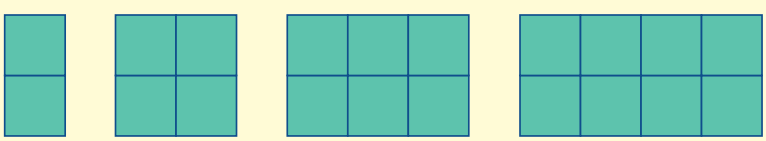


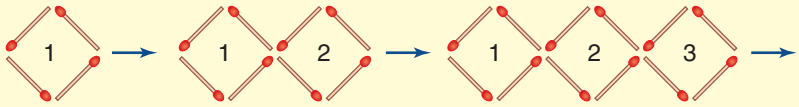
Figure 1 Figure 2 Figure 3 Figure 4

Figure number	1	2	3	4
Number of tiles				

b What rule links the number of square tiles to the number of the figure?

c How many tiles would be in the 15th figure?

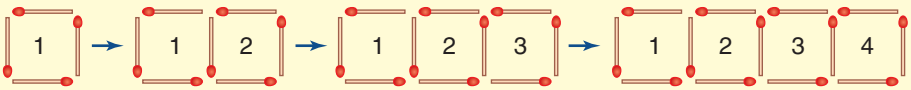
4 Matches have been used to make the following figures.



a Find a rule that links the number of matches used to the number of squares in each figure.
b Complete this table.

Number of squares	1	2	3	4	5	6	7	8
Number of matchsticks								

5



a How many matches are being added to get the next figure?
b Find a rule that links the number of matches to the number of squares in each figure.
c Complete a table of values showing the number of matches for figures with up to 8 squares.

6 Each 'bottom' number in the following tables is obtained from the 'top' number by following a rule.

Example

'Top' number	3	4	5	6
'Bottom' number	10	13	16	19

Rule:

'Bottom' = 3 times 'top' plus 1
or $B = 3 \times T + 1$

Find the rule for each table of values below.

a

Top	1	2	3	4
Bottom	5	10	15	20

b

Top	1	2	3	4
Bottom	7	8	9	10

c

Top	2	4	6	8
Bottom	5	7	9	11

d

Top	1	3	5	7
Bottom	2	6	10	14

e

Top	1	2	3	4
Bottom	3	5	7	9

f

Top	1	2	3	4
Bottom	2	5	8	11

g

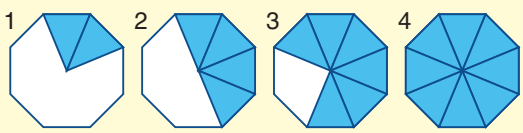
Top	2	3	4	5
Bottom	0	1	2	3

h

Top	5	6	7	8
Bottom	11	13	15	17

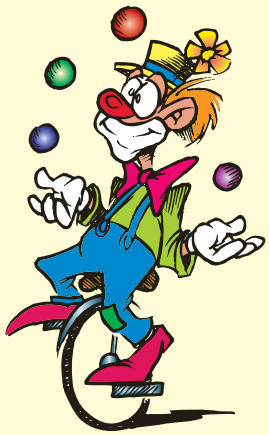
7 **a** Complete the table of values for this pattern made by colouring an octagon.

Figure number	1	2	3	4
Fraction coloured				



b Find a rule that links *Fraction coloured* (F) to the *Figure number* (n).

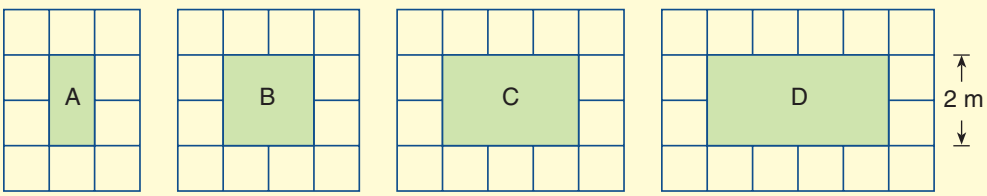
8 Sue-Lin was at the circus. She noticed that during the parade three unicycles (cycles having only one wheel) were always present while the number of bicycles changed often. She made the table below, showing the total number of wheels present as the number of bicycles changed.



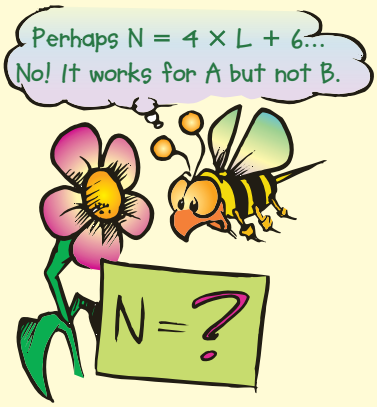
Number of bicycles (B)	1	2	3	4	5	6
Number of wheels (W)	5	7				

a Copy and complete the table above.
b Find a rule connecting the total number of wheels and the number of bicycles.
c Could the total of the wheels ever be 36? Why or why not?

9



These pictures show gardens which are 2 m wide.
Each garden is surrounded by a path made of one-metre squares of concrete. Below is a table showing the number of squares of concrete needed for each garden length.

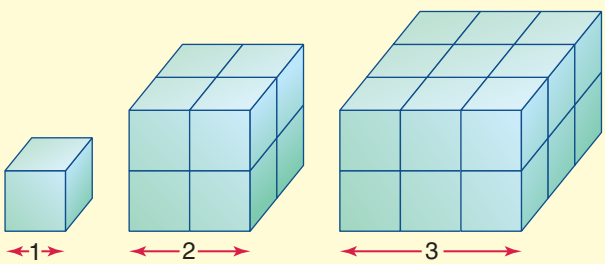


Length of garden (L)	1	2	3	4
Number of squares (N)	10	12	14	16

You can see that for each extra metre of garden, we must 'add two squares'. Find a rule that connects the number of squares with the garden length.

a How many concrete squares would be needed for a garden of length:
i 5 metres? **ii** 30 metres?
b Does the rule $N = 2 \times (L + 1) + 6$ also hold?

10 Each solid is made of small cubes.
a What rule links the number of small cubes (C) to its length (L)?
b How many small cubes would be in a solid of length 5 units?



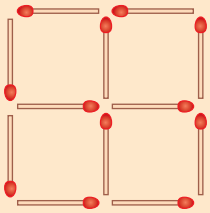


Challenge worksheet 8:02 Describing number patterns

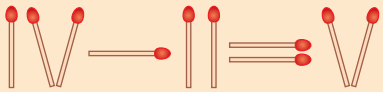
Fun Spot 8:02 | Mysteries mixing matches

Try to solve each of these tricky match problems.

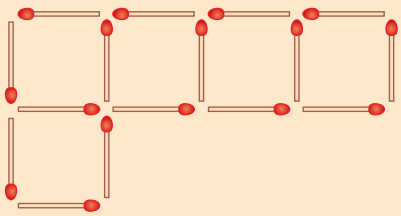
1 Arrange 12 matches to look like this. Then move 3 matches to make only 3 squares.



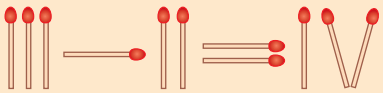
2 The equation in Roman numerals is not correct. Move just one match to make it a correct statement.



3 Here are 5 equal squares made from 16 matches. Move 3 matches to make only 4 squares of equal area.



4 Move just one match to make this equation correct. (There are two different solutions.)





• Patterns can take many forms.

8:03 | Pronumerals and Rules

prep quiz 8:03

Write down the next *two* numbers in each pattern.

1 5, 11, 17, . . . , . . .
2 30, 24, 18, . . . , . . .
3 3, 6, 12, . . . , . . .
4 80, 40, 20, . . . , . . .
5 4, 9, 16, . . . , . . .
6 64, 16, 4, . . . , . . .

What is the missing number in each table of values?

7

Top	1	2	3	4
Bottom	3	6		12

8

Top	2	4	6	8
Bottom	5		9	11

9

Top	1	3	5	7
Bottom	5	13		29

10 What is the rule for the table of values in question **7**?

Foundation Worksheet 8:03

Patterns and rules

1 Complete these patterns:
a 5, 10, 15, . . ., . . .
2 Complete these tables:

a

Top	1	2	3	4
Bottom	2	4	6	

3 Write the rule for each table in question 2.
a Bottom = . . . × Top

In the last exercise you found many rules for different number patterns, and you were asked to write out these rules. Now we will make the writing of the rules much easier by using *symbols* to represent the numbers in the pattern. These symbols are called **pronumerals** and the most often used pronumerals are letters of the alphabet.

A *pronumeral* takes the place of a *numeral*.

worked example 1

If apples cost 25 cents each, then to find the cost of a number of apples we obey the rule:

cost of apples = number of apples times 25 cents.

Now if we let C represent the cost of apples in cents and n represent the number of apples, then the rule could be simply written like this:

$C = n \times 25$

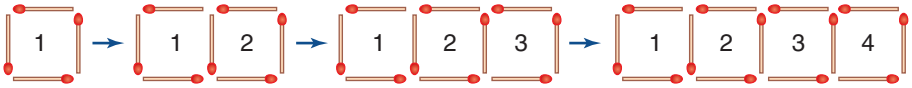
and a table of values would look like this:

n	1	2	3	4	5
C	25	50	75	100	125

$C = n \times 25$ could also be written as $C = 25 \times n$.



worked example 2



For this pattern, the rule linking the number of matches to the number of squares is:

number of matches = 1 plus [number of squares times 3]

If we let the number of matches be represented by m, and the number of squares be represented by s, then the rule would be simply written as:

$m = 1 + (s \times 3)$

and the table of values would be:

s	1	2	3	4	5
m	4	7	10	13	16

Now we can use the rule $m = 1 + (s \times 3)$ to find the number of matches, m, for any number of squares, s. For example, for 40 squares, s becomes 40 in the rule, and so:

$m = 1 + (40 \times 3)$
$= 121$

So m would have the value of 121 if s had the value of 40.

The use of pronumerals to represent numbers in solving problems is called *algebra*.

Exercise 8:03

1 Complete the tables of values using the rules shown at the top of each table.

a $y = x + 3$

x	0	1	2	3
y				

b $y = 3 \times a$

a	1	2	3	4
y				

c $q = p + 7$

p	1	3	5	7
q				

d $k = l \times 5$

l	2	4	6	8
k				

e $m = n - 5$

n	8	9	10	11
m				

f $s = 10 - t$

t	1	2	3	4
s				

g $y = x \div 2$

x	8	10	12	14
y				

h $a = 12 \div b$

b	1	2	3	4
a				

i $m = n \div 5$

n	5	15	25	35
m				

j $t = 2 \times u + 1$

u	1	2	3	4
t				

k $b = a \times 3 - 2$

a	2	3	4	5
b				

l $y = (x + 2) \times 4$

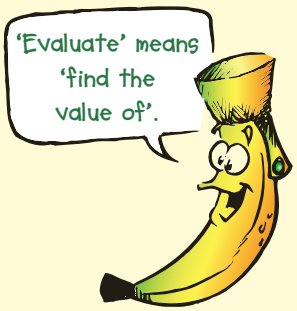
x	0	2	4	6
y				

2 Using the rules below, find the value of y if x is given the value 4.

a $y = x + 7$ **b** $y = x - 3$ **c** $y = 10 + x$ **d** $y = 12 - x$
e $y = 6 \times x$ **f** $y = x \times 10$ **g** $y = x \times x$ **h** $y = x \div 4$
i $y = 24 \div x$ **j** $y = 2 \times x + 3$ **k** $y = 5 \times x - 15$ **l** $y = 20 - 3 \times x$

3 Using the rules given, evaluate n if m has the value 7.

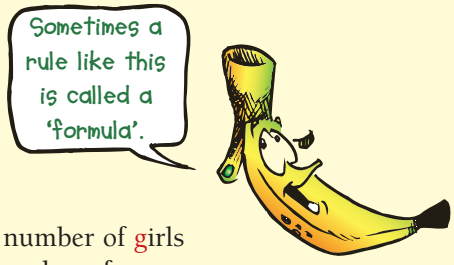
a $n = m + 12$ **b** $n = 30 + m$
c $n = 18 - m$ **d** $n = 11 \times m$
e $n = m \times 13$ **f** $n = m \times m$
g $n = 5 \times m + 3$ **h** $n = m \times 3 - 4$
i $n = 16 - m \times 2$ **j** $n = (m + 1) \times 5$
k $n = (m - 1) \times (m + 2)$ **l** $n = (m - 5) \times (m - 5)$



4 Rewrite these rules using the pronumerals indicated in colour,

eg Cost of bananas = number of bananas times 18 cents
Answer: $C = n \times 18$

a Distance travelled = Speed times Time
b Area of a rectangle = Length times Breadth
c Perimeter of a square = four times its side length
d Total number of students = number of boys plus the number of girls
e Pieces of fruit in a bag = number of apples plus the number of pears
f Mother's age = Son's age plus 23
g Cost per item = Total cost divided by the number of items
h Average of two numbers = first number a plus the second number b; then divide by 2



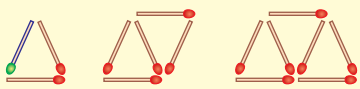
5 For the patterns below, t stands for the number of triangles in each pattern, and m stands for the number of matches. Describe the pattern in words. Then describe the pattern in symbols using t and m. Use the rules to complete the tables of values.

a



t	1	2	3	8	12
m					

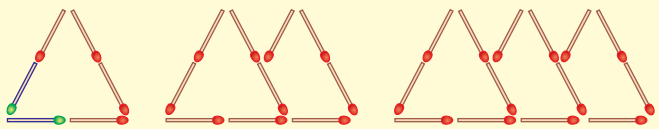
b



t	1	2	3	10	15
m					

(*Hint:* Two more matches are needed to form each new triangle.)

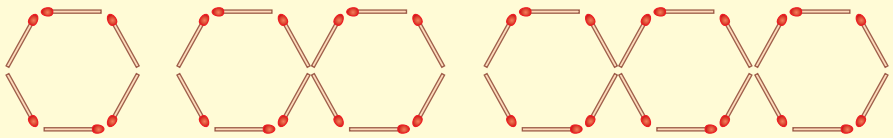
c



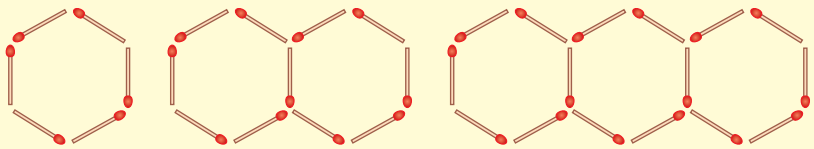
t	1	2	3	9	20
m					

6 Describe each pattern in words. Also use the symbols h (the number of hexagons) and m (the number of matches) to describe the pattern. In each case find how many matches would be needed to form 20 of the hexagons.

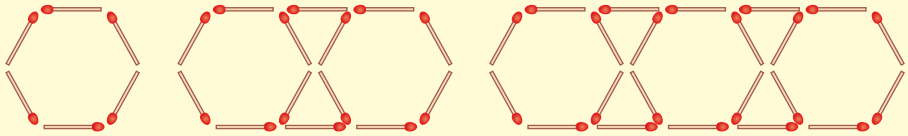
a



b

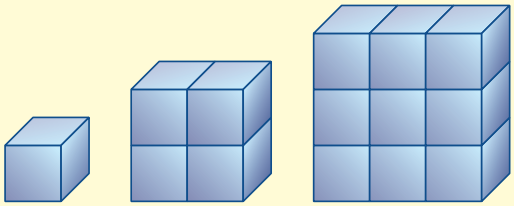


c



7 For each pattern of blocks, complete the table and describe the pattern using the pronumerals indicated.

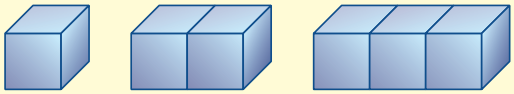
a



Number of blocks in height (s)	1	2	3	4
Total number of blocks (b)				

$b = ?$

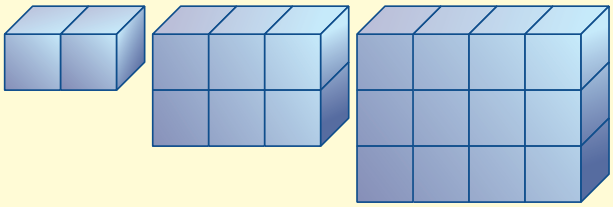
b

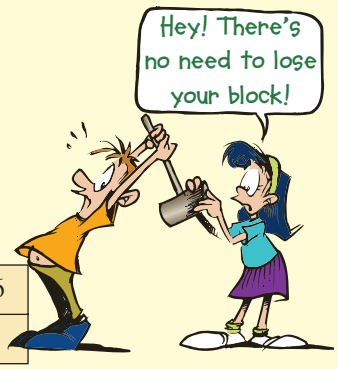


Number of blocks (b)	1	2	3	4
Number of faces showing (f)				

$f = ?$

c





Number of blocks along the bottom (b)	2	3	4	5	6
Total number of blocks (n)					

$n = ?$

8 For the table of values shown here, the rule using the pronumerals given would be:
$b = 3 \times t$

t	1	2	3	4
b	3	6	9	12

Find the rules that would give the following tables of values, using the pronumerals given.

a

t	1	2	3	4
b	4	8	12	16

b

t	1	2	3	4
b	3	4	5	6

c

t	1	2	3	4
b	0	1	2	3

d

a	1	2	3	4
b	1	4	9	16

e

a	1	2	3	4
b	9	8	7	6

f

a	1	2	3	4
b	5	4	3	2

g

x	1	2	3	4
y	2	5	8	11

h

x	1	2	3	4
y	10	20	30	40

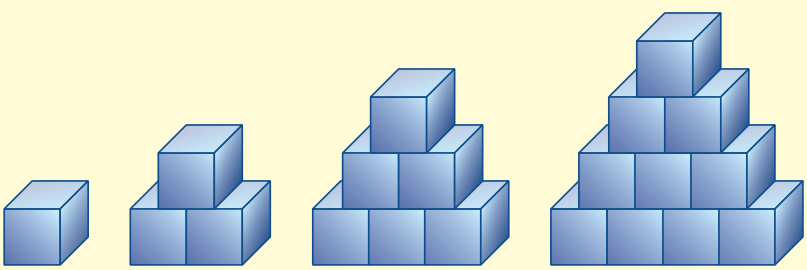
i

x	1	2	3	4
y	9	19	29	39

j

p	2	4	6	8
q	1	2	3	4

9



It is easy to see how this pattern is growing, but it is not so easy to find the rule that links the total number of blocks to the number of blocks along the bottom. Try to find the rule.

Let N stand for the total number of blocks and b stand for the number of blocks in the bottom row.

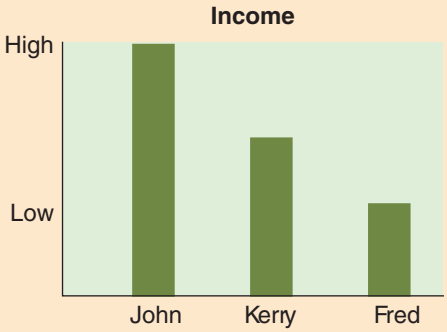
$$N = \frac{\ldots \times \ldots}{2}$$

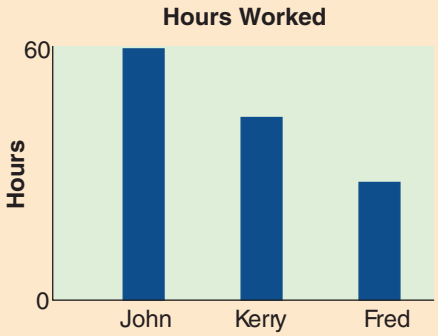


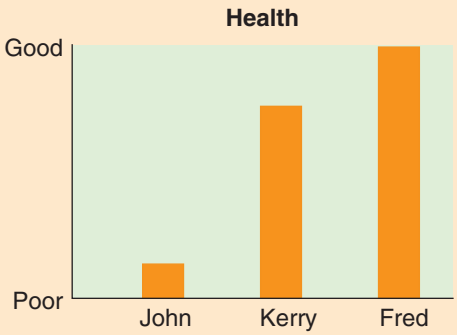
b	1	2	3	4	5	6
N	1	3	6	10	15	21

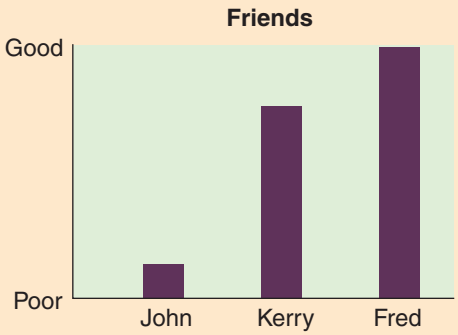
Reading Mathematics 8:03 | Quality of life

Study the four graphs below carefully.



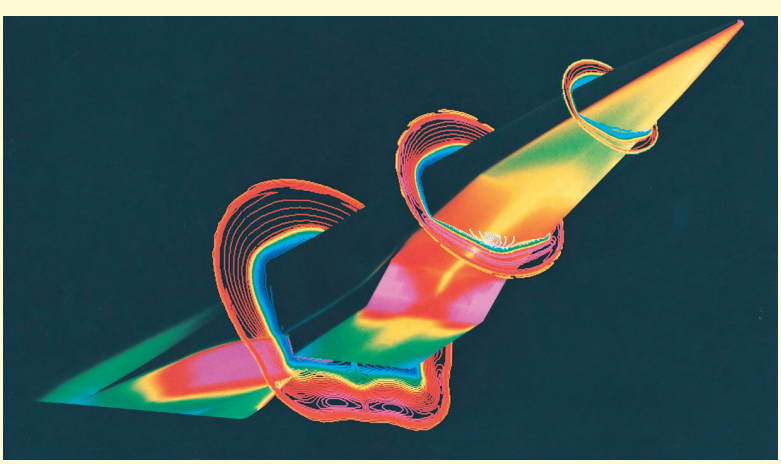






a Write as much as you can about John's life.
b Write as much as you can about Kerry's life.
c Write as much as you can about Fred's life.
d Which lifestyle would you prefer? Why?

Appendix D D:01 Finite differences (Finding the rule)

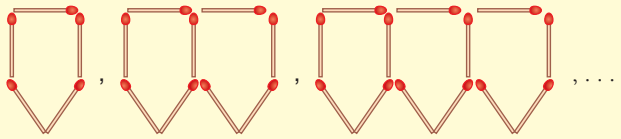


- Use mathematical terms to describe this picture.

8:04 | Graphs of Patterns

Step 1

- Use the pattern to make a table of values.



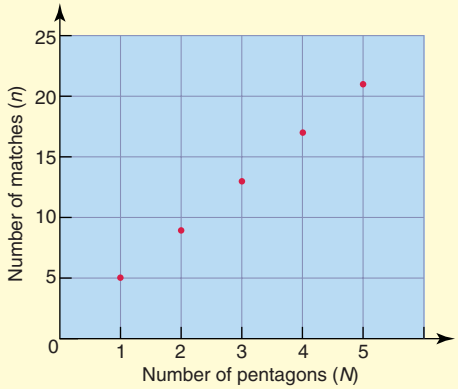
Number of pentagons	1	2	3	4	5
Number of matches	5	9	13	17	21

Step 2

- Look for a pattern and complete the table.

Step 3

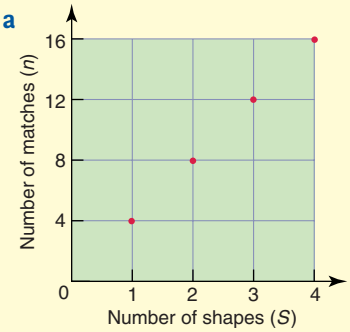
- Make up a graph putting 'Number of pentagons' along the bottom and 'Number of matches' up the side.
- Choose numbers 0 to 5 across the bottom and 0 to 25 up the side to suit the numbers in the table. Plot the points from the table.



Exercise 8:04

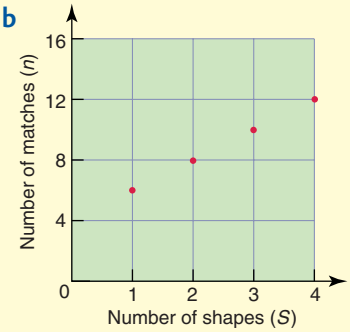
1 In each case, use the graph to fill in the table. Describe the dot pattern.

a



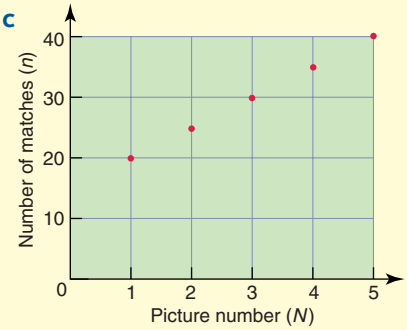
S	1	2	3	4
n				

b



S	1	2	3	4
n				

c



N				
n				

2 In each case, use the table to graph the pattern.

a

S	1	2	3	4
n	16	12	8	4

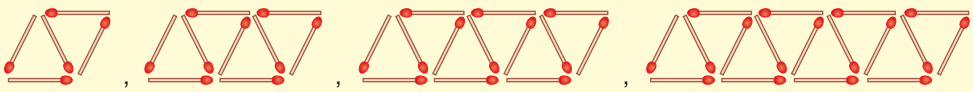
b

S	1	2	3	4
n	12	10	8	6

c

S	1	2	3	4
n	10	12	14	16

3 Make a table and a graph to describe the pattern below.



Mathematical terms 8

algebra
- The use of pronumerals in solving problems.

evaluate
- To find the value of . . .

number pattern
- A set of numbers connected by a rule or relationship.

pronumeral
- A symbol (usually a letter) that takes the place of a numeral.

value
- The number represented by a pronumeral.
  eg For the rule $y = x + 7$, find the value of y if x is given the value, 4.

Diagnostic Test 8: | Patterns and Algebra

- Each section of the test has similar items that test a certain type of example.
- Failure in more than one item will identify an area of weakness.
- Each weakness should be treated by going back to the section listed.

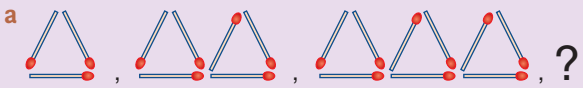
Section

1 Write down the next number in each number pattern. — 8:01
a 2, 6, 10, . . . **b** 15, 12, 9, . . . **c** 2, 6, 18, . . . **d** 40, 20, 10, . . .

2 Write down the rule you used to get each answer in question 1. — 8:01

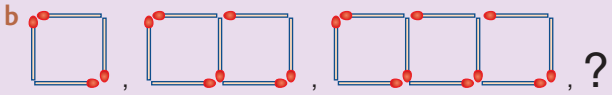
3 Complete the table of values for each matchstick pattern. — 8:02

a



Number of triangles	1	2	3	4
Number of matches				

b



Number of squares	1	2	3	4
Number of matches				

4 Complete each table using the rule given. — 8:03
a $y = x + 4$ **b** $b = 4 \times t$ **c** $n = 2 \times m + 3$

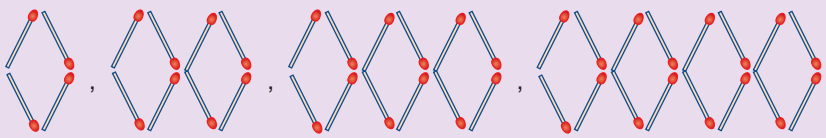
x	1	2	3	4
y				

t	0	2	4	6
b				

m	0	1	2	3
n				

5 If $x = 3$, find the value of y in each case. — 8:03
a $y = 5 \times x$ **b** $y = 10 - x$ **c** $y = x \times x$ **d** $y = 24 \div x$

6 Make a table and a graph to describe this pattern. — 8:04 and 8:02



assignment 8A

Chapter 8 | Revision Assignment

1 Complete:

a 70 cm = ... mm

b 8500 m = ... km

c 960 mm = ... cm

2 Simplify:

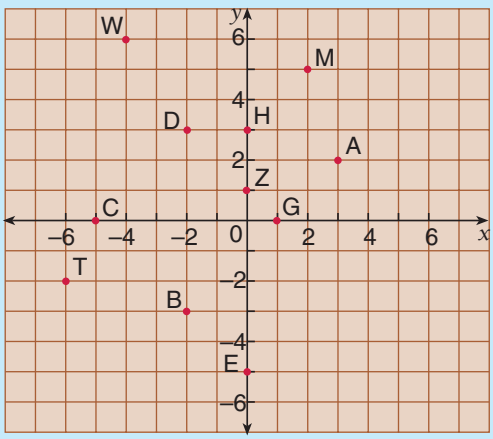
a 0·6 + 0·12 **b** 12 − 2·5

c 6 × 0·4 **d** 12·36 ÷ 6

3 Write down the coordinates of the points:

a A **b** B **c** C

d D **e** E



4 Which point above is at:

a (0, 3)? **b** (−6, −2)?

c (2, 5)? **d** (−4, 6)?

e (0, 1)?

5 Simplify:

a $\frac{7}{5} - \frac{3}{5}$ **b** $\frac{2}{10} + \frac{1}{10}$

c $5 \times \frac{2}{3}$ **d** $\frac{8}{10} \div 2$

6 **a** What fraction of 1 m is 40 cm?

b Change $\frac{3}{10}$ to a percentage.

c Change 0·8 to a percentage.

d Write 56% as a decimal.

7 Write the following times as 24-hour times.

a 3:15 am **b** 3:15 pm

c 11:30 pm **d** 10:18 am

8 Add these times together, giving the answer in hours and minutes.

a 3 h 20 min + 1 h 40 min

b 2 h 27 min + 3 h 5 min

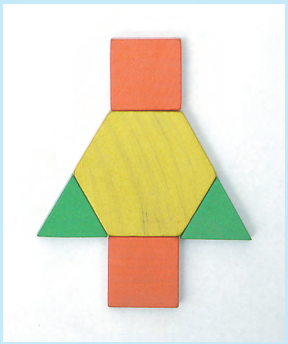
c 1 h 48 min + 1 h 42 min

d 2 h 28 min + 2 h 38 min

9 Write as decimals:

a $\frac{72}{100}$ **b** $\frac{105}{1000}$

c $\frac{7}{100}$ **d** $\frac{45}{1000}$



- If the length of the square is ***b*** cm, what is the perimeter of this shape?
- If the perimeter of the shape is 36 cm, what is the value of ***b***?

Drag and Drop Interactive 1 Number patterns

Review Questions

Chapter 8 | Working Mathematically

assignment 8B

id 8

1 Use ID Card 2 on page xiv to identify (1) to (24). Learn the symbols you do not know.

2 A boy looks at a truck and notices that it has 12 pallets on it. Each pallet has 100 cases of drink on it and each case holds 24 cans. How many cans of drink are on the truck?

3 In a Test cricket match each team has two turns (called *innings*) to score runs. If the West Indies scored 310 runs and 337 runs in their two innings, and Australia scored 287 runs in their first innings, how many runs would Australia have to score in their second innings to win?

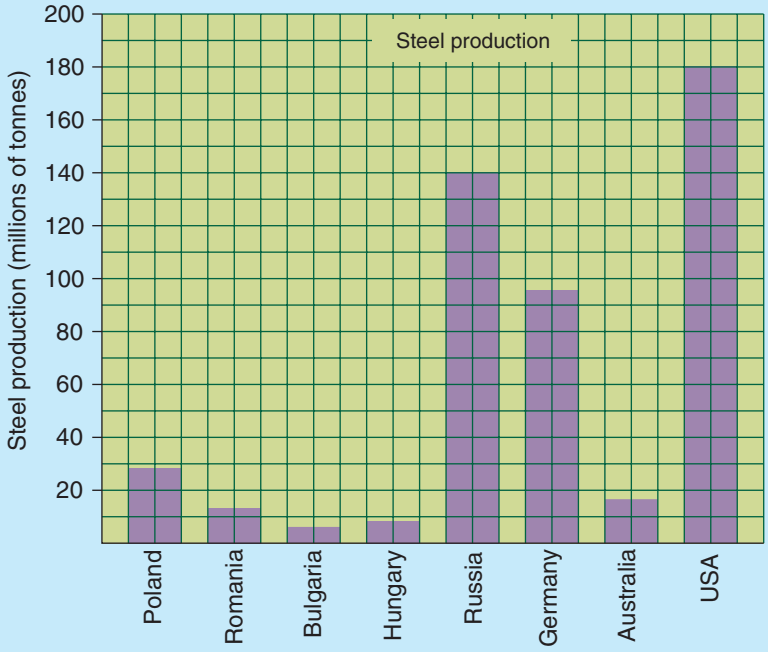


4 At a restaurant I bought 3 orange juices and a glass of wine. The drinks cost $6.30. Later I bought 4 orange juices and it cost $6. How much was the glass of wine?

5 A house is to be built from 30 000 bricks. For each thousand bricks that are laid, 1 tonne of sand and 4 bags of cement are used. How much will it cost to buy the bricks and lay them if the bricks cost $380 a thousand, the bricklayer charges $300 a thousand to lay them, sand costs $12 a tonne and a bag of cement is $7?

6 Soula and her brother Kosta have $2 and $1 pocket money respectively. After going to the shops, Soula has three times as much money left as Kosta. If they spent $2 altogether, how much money does each of them have left?

7 a Who is the largest producer of steel?
b Who is the smallest producer of steel?
c Which country has the larger production, Poland or Australia?
d Japan's production is 117 million tonnes. If it were marked on this graph, how many countries would have a higher production?
e How much steel does Germany produce?



9

Further Algebra



Chapter Contents

9:01 Algebraic abbreviations
9:02 Making sense of algebra
9:03 Substitution
9:04 Simplifying algebraic expressions
Fun Spot: Let's improve our algebra skills
9:05 Grouping symbols
9:06 Index notation
9:07 Graphing tables of values
9:08 Graphs involving directed numbers
9:09 Simplifying expressions with directed numbers
Reading Mathematics: What's in food?
9:10 Algebraic sentences (solving equations)
9:11 Solving problems using algebra
9:12 Investigation of real problems
Fun Spot: Solving pyramids
Mathematical Terms, Diagnostic Test, Revision Assignment, Working Mathematically

Learning Outcomes

Students will:

- Use letters to represent numbers and translate between words and algebraic symbols.
- Create, record analyses and generalise number patterns using words and algebraic symbols in a variety of ways.
- Use the algebraic symbol system to simplify, expand and factorise simple algebraic expressions.
- Use algebraic techniques to solve linear equations and simple inequalities.
- Graph and interpret linear relationships on the number plane.

Areas of Interaction

Approaches to Learning, Homo Faber

9:01 | Algebraic Abbreviations

prep quiz 9:01

If x is replaced by the numeral 4, what is the value of the following expressions?

1 $x+5$ **2** $x-4$ **3** $3\times x$ **4** $x\times x$ **5** $x\div 2$

6 $5\times(x+3)$

Find the value of C, if $C=5\times n+2$ and n is replaced by the numeral:

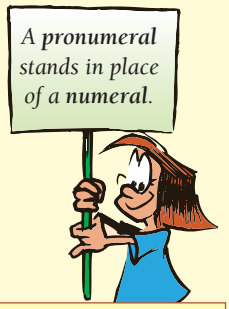
7 3 **8** 1 **9** 0 **10** 13

Algebra provides a short way of writing ideas in mathematics. Since pronumerals take the place of numerals, they behave the same way when we perform operations with them.

When writing algebraic expressions, we try to shorten or simplify the expression as much as possible. Here are some important abbreviations.



- **$3\times a$ is written as $3a$, and $a\times b$ is written as ab**
- **$x\div 5$ is written as $\frac{x}{5}$, and $a\div b$ is written as $\frac{a}{b}$**



- $3+3+3+3$ is the same as 4 lots of 3, or 4 times 3, so $y+y+y+y$ is 4 lots of y, or $4y$.
- $2\times m=2m$, $y\times 6=6\times y=6y$, $2a\div 3=\frac{2a}{3}$, $5\times a\times b=5ab$.

When a numeral is multiplied by a pronumeral, the numeral should be written first.

Important notice! $n\times 5$ is always written as $5n$, not as $n5$.



$m^2=m\times m$, in the same way that $7^2=7\times 7$.
m^2 is read 'm squared'.

worked examples

1 Rewrite these expressions without multiplication or division signs.

a $5\times k$ **b** $a\div 7$ **c** $a\times b\times a$ **d** $3\times x\times y$

e $(a+3)\div 2$ **f** $5\times b\div 8$ **g** $6\times(c+7)$ **h** $(c\times d)\div(e\times f)$

2 Rewrite these expressions using multiplication and division signs where they belong.

a $3a+b$ **b** $5c-\frac{d}{2}$ **c** $3(x+2)$ **d** $\frac{a+7}{3}$

3 Simplify:

a $3\times 2m$ **b** $5a\times 3$ **c** $4\times 2a\times b$ **d** $6\times 3c\times 10$

Solutions

1 **a** $5\times k$
$=5k$

b $a\div 7$
$=\frac{a}{7}$

c $a\times b\times a$
$=a^2b$

d $3\times x\times y$
$=3xy$

continued ➜➜➜

e $(a+3) \div 2$
$= \frac{a+3}{2}$

f $5 \times b \div 8$
$= \frac{5b}{8}$

g $6 \times (c+7)$
$= 6(c+7)$

h $(c \times d) \div (e \times f)$
$= \frac{cd}{ef}$

2 **a** $3a + b$
$= 3 \times a + b$

b $5c - \frac{d}{2}$
$= 5 \times c - d \div 2$

c $3(x+2)$
$= 3 \times (x+2)$

d $\frac{a+7}{3}$
$= (a+7) \div 3$

Here we must have grouping symbols, because the whole numerator is divided by 3.



3 **a** $3 \times 2m$
$= 3 \times 2 \times m$
$= 6m$

b $5a \times 3$
$= 3 \times 5a$
$= 3 \times 5 \times a$
$= 15a$

c $4 \times 2a \times b$
$= 4 \times 2 \times a \times b$
$= 8ab$

d $6 \times 3c \times 10$
$= 6 \times 10 \times 3 \times c$
$= 180c$

Exercise 9:01

1 **a** How can 'three lots of 5' be written in a shorter way?
b How can 'three lots of n' be written in a shorter way?
c How can $3 \times n$ be written in a shorter way?
d Why can't 3×5 be written in the same way?
e What is another way of writing $d \times t$?
f Rewrite $5ab$, showing the multiplication signs.
g Write $q \div 3$ in another way.
h What does $\frac{m}{2}$ mean?
i What is the difference between $34y$ and $3 \times 4 \times y$?



2 Write these without multiplication signs.

a $7 \times a$	**b** $10 \times x$	**c** $2 \times p$	**d** $8 \times q$
e $9 \times b$	**f** $6 \times m$	**g** $3 \times r$	**h** $12 \times c$
i $m \times 9$	**j** $n \times 5$	**k** $y \times 4$	**l** $t \times 12$
m $1 \times a$	**n** $1 \times x$		
o $n \times 1$	**p** $b \times 1$		
q $1 \times y$	**r** $1 \times m$		
s $5 \times (a+2)$	**t** $6 \times (x+7)$		
u $2 \times (3x-2)$	**v** $5 \times (m-n)$		
w $5 + 2 \times q$	**x** $1 \times n - 10$	**y** $4 \times a + 5 \times b$	**z** $q \times 7 - p \times 6$

If you multiply something by 1, it does not change, ie $1 \times n = n \times 1 = n$
$\therefore 1 \times m = 1m = m$

3 Write these divisions as fractions.

a $x \div 2$	**b** $m \div 4$	**c** $b \div 3$	**d** $n \div 5$
e $a \div 1$	**f** $y \div 10$	**g** $t \div 8$	**h** $k \div 100$
i $2b \div 3$	**j** $3x \div 5$	**k** $5m \div 6$	**l** $3a \div 4$
m $a \div b$	**n** $6 \div n$	**o** $2a \div x$	**p** $10 \div a$
q $(a+1) \div 4$	**r** $(8-x) \div 5$	**s** $7a \div 8b$	**t** $4x \div 3y$

4 Rewrite each of these expressions in its simplest form.

a $5 \times a \times b$	**b** $4 \times m \times n$	**c** $a \times b \times c$	**d** $3 \times a \times a$
e $x \times 3 \times y$	**f** $a \times b \times 10$	**g** $6 \times r \times s$	**h** $m \times 1 \times n$
i $3 \times 8 \times a$	**j** $6 \times 5 \times b$	**k** $4 \times 7 \times x$	**l** $9 \times 9 \times p$
m $5 \times 3a$	**n** $4 \times 2b$	**o** $3 \times 10y$	**p** $2 \times 6m$
q $6 \times m \times 5$	**r** $p \times 3 \times 2$	**s** $4a \times 7$	**t** $8b \times 5$
u $7 \times 3a$	**v** $10 \times 8x$	**w** $10 \times 6b$	**x** $3 \times 3y$
y $4 \times a + 5 \times b$	**z** $q \times 7 - p \times 6$		

5 Rewrite these in a shortened form.

a $2 \times p \times (q - 1)$	**b** $5 \times 3 \times (a - 3)$	**c** $a \times b \times (c - 7)$
d $4 \times (3 \times a + 1)$	**e** $6 \times (1 + 2 \times x)$	**f** $3(5 \times y - 2 \times z)$
g $(x + 2) \times (x + 3)$	**h** $(a - 3) \times (a + 3)$	**i** $(4p + 1) \times (3q - 1)$

6 Show the meaning of these expressions by including multiplication signs.

a $5x$	**b** $7a$	**c** $12y$	**d** $14p$
e $2p + 7$	**f** $9x - 5$	**g** $11 - 7m$	**h** $1 + 10a$
i $7x + 3y$	**j** $10a - 9b$	**k** $7a + 3p$	**l** $11m - 9n$
m $ab + 3$	**n** $k - 3l$	**o** abc	**p** $8xy$
q $6(a + 3)$	**r** $4a(6 - a)$	**s** $3(2x + 1)$	**t** $(a + 7)(a - 7)$
u x^2	**v** $a^2 + c^2$	**w** $2y^2$	**x** $m^2 - 3$

7 Rewrite these expressions, showing all multiplication and division signs. Remember to use grouping symbols if they are needed.

a $\frac{a}{3}$	**b** $\frac{5}{k}$	**c** $\frac{m}{7}$	**d** $\frac{n}{a}$
e $\frac{2x}{5}$	**f** $\frac{3y}{2}$	**g** $\frac{5}{8m}$	**h** $\frac{6}{5p}$
i $\frac{5a}{6b}$	**j** $\frac{11m}{10n}$	**k** $\frac{a+7}{3}$	**l** $\frac{m-1}{2}$
m $\frac{6+a}{a-5}$	**n** $\frac{9+x}{7-x}$	**o** $\frac{3(x+1)}{5}$	**p** $\frac{a+6}{3a}$



- If each packet holds n cards, how many cards are there altogether?

9:02 | Making Sense of Algebra

The pronumerals in algebra stand for numbers that we need to find, or they stand for variables that we use to explain rules.

In the discussion below, we use two ways of packaging counters. We don't know how many are in each type of package so we will let x represent the number in one type of package and y represents the number in the other type of package.



x counters

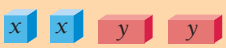
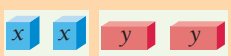
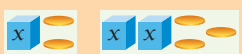
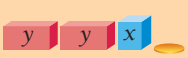


y counters

4 counters

x might be 12 and y might be 20. We have not yet opened the boxes. (Many of each box have been delivered, along with a number of unpacked counters.)

These models will help you understand the algebra used.

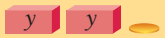
Model	*Number*	*Model*	*Number*
	$x + 3$		$x + x + y + y$ $= 2x + 2y$
	$x + x + x$ $= 3x$		$(x + x) + (y + y)$ $= 2x + 2y$
	$x + x + 1$ $= 2x + 1$		$(x + y) + (x + y)$ $= 2(x + y)$ $= 2x + 2y$
	$x + y$		$(x + 2) + (2x + 3)$ $= 3x + 5$
	$(x + 1) + (x + 1)$ $= 2x + 2$		$(x + 2x) + (2 + 3)$ $= 3x + 5$
	$2 + x + x + x$ $= 2 + 3x$		$3x + 5$
	$y + y + x + 1$ $= 2y + x + 1$		
	$(x + y) \div 2$ $= \frac{x + y}{2}$ $= \frac{x}{2} + \frac{y}{2}$		



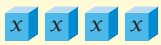
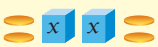
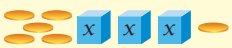
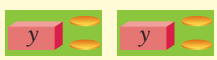
From the examples above it can be seen that:

- $x + x + y + y = (x + x) + (y + y) = x + y + x + y$
  $= 2x + 2y$
- $(x + 2) + (2x + 3) = (x + 2x) + (2 + 3)$
  $= 3x + 5$

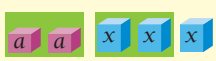
Exercise 9:02

Foundation Worksheet 9:02
Making sense of algebra
1 a 
2 a 

1 Use pronumerals to write the number of counters in each.

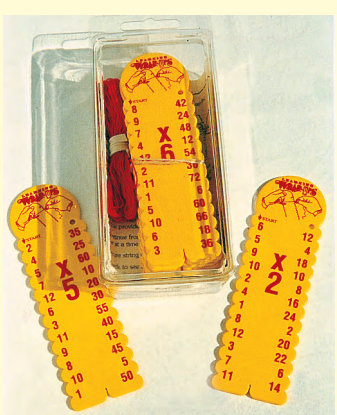
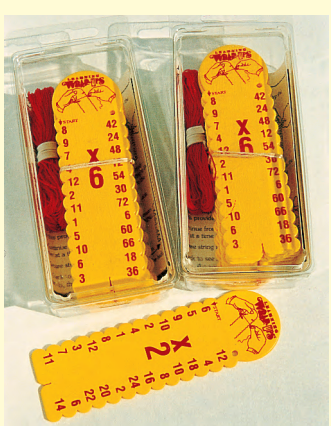
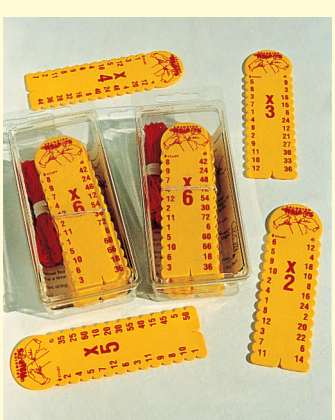
a 
b 
c 
d 
e 
f 
g 
h 
i 
j 
k 
l 
m 
n 
o 
p 
q 
r 

2 Use pronumerals to write the number of counters as simply as you can.

a 
b 
c 
d 
e 
f 
g 
h 
i 
j 
k 

3 Write the number of counters in simplest form.

a 
b 
c 
d 
e 
f 

- If there are *n* wrap-ups in each packet, how many wrap-ups are in each picture?

9:03 | Substitution

prep quiz 9:03

Rewrite without multiplication or division signs:

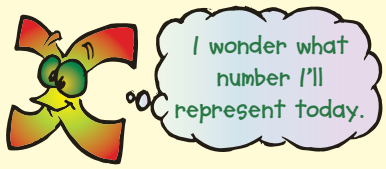
1 $4 \times y$ **2** $m \times 2$ **3** $6 \times a \times b$ **4** $9 \times n + 2$

5 $9 \times (n + 2)$ **6** $p \div 7$ **7** $a \times b \div c$

Show the meaning of the following by using multiplication signs.

8 $3pq$ **9** y^2 **10** $6a + 4b$

As we saw in Chapter 4, an algebraic expression can have different values, depending upon the values we **substitute** for each pronumeral. Look at the following examples.



If we have 3 boxes of counters (x counters in each) we can work out the number of counters if we are told that $x = 12$.



$x + x + x = 3x = 3 \times x = 3 \times 12 = 36$

$\therefore$ There would be 36 counters if $x = 12$.

worked examples

1 If x is replaced by the numeral 3, find the value of these expressions.

a $6x$ **b** $2x + 5$ **c** $5x^2$ **d** $x(4x - 7)$

2 If $a = 2$ and $b = 5$, evaluate:

a ab **b** $3a + 7b$ **c** $\frac{10a}{b}$ **d** $(b + a)(b - a)$

'Evaluate' *means* 'find the value of'.

Solutions

1 If we substitute 3 for x, each expression becomes:

a $6x \rightarrow 6 \times 3$
$= 18$

b $2x + 5 \rightarrow 2 \times 3 + 5$
$= 11$

c $5x^2 \rightarrow 5 \times 3^2$
$= 5 \times 9$
$= 45$

d $x(4x - 7) \rightarrow 3 \times (4 \times 3 - 7)$
$= 3 \times 5$
$= 15$

Note: The multiplication signs must be written in each expression once the values have been substituted for the pronumerals.

2 If $a = 2$ and $b = 5$ then:

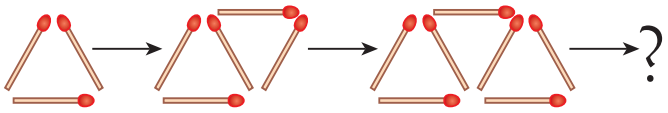
a $ab = 2 \times 5$
$= 10$

b $3a + 7b = 3 \times 2 + 7 \times 5$
$= 6 + 35$
$= 41$

c $\frac{10a}{b} = \frac{10 \times 2}{5}$
$= 20 \div 5$
$= 4$

d $(b + a)(b - a) = (5 + 2)(5 - 2)$
$= 7 \times 3$
$= 21$

3 The expression $2n + 1$ gives the number of matches needed to make n triangles in the pattern.
How many matches are needed to make 25 triangles?



4 Complete each table using the formula given.

a $b = 3t$

t	1	2	3	4
b				

b $m = 5 + 4n$

n	2	4	6	8
m				

c $y = 10 - 2x$

x	0	1	2	3
y				

Solutions

3 In the expression $2n + 1$, n represents the number of triangles.
To find the number of matches in 25 triangles, substitute 25 for n.
ie $2n + 1 = 2 \times 25 + 1$
$= 51$
$\therefore$ Number of matches needed = 51.

4 For a table of values we simply substitute the top numbers one at a time into the rule to find the value of the bottom pronumeral.
Check to see that you can get the given answer.

a $b = 3t$

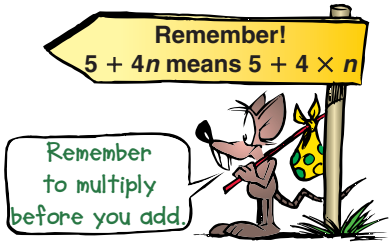
t	1	2	3	4
b	3	6	9	12

b $m = 5 + 4n$

n	2	4	6	8
m	13	21	29	37

c $y = 10 - 2x$

x	0	1	2	3
y	10	8	6	4



Exercise 9:03

1 Find the value of each expression if n is replaced by 4.

a $2n$ **b** $7n$ **c** $3n + 1$
d n^2 **e** $n^2 + 3$ **f** $n^2 - n$
g $5 - n$ **h** $7 + 2n$ **i** $3(n + 1)$
j $\frac{n}{2}$ **k** $\frac{6n}{3}$ **l** $\frac{20}{n}$

Foundation Worksheet 9:03
Substitution
1 Find the value of:
a $3x$ if $x = 12$
b $2y$ if $y = 20$
c $a + 3$ if $a = 8$
2 Further types.

2 If $p = 2$ and $q = 5$, find the value of:

a pq	**b** $p + q$	**c** $2p + 3$	**d** $3q - 2$
e $2p + q$	**f** $p + 3q$	**g** $3p + 2q$	**h** $4p - q$
i $3pq$	**j** $\frac{10p}{q}$	**k** $\frac{5 + q}{p}$	**l** $\frac{5p}{2q}$

3 If each pronumeral is replaced by the value shown in this table, evaluate each expression.

a	b	c	d	e	f	g	h	i	j
0	1	2	3	4	5	6	7	8	9

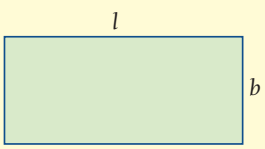


a ef	**b** bj	**c** di	**j** d^2f
d $3a + g$	**e** $4h + 2c$	**f** $9d - 2f$	
g beh	**h** ace	**i** dij	
k $c^2 + d^2$	**l** $5h^2$	**m** $2c + 3d - 5b$	**n** $ac + bd + fh$
o $3(b + g)$	**p** $f(h + 3)$	**q** $j(2b - c)$	**r** $(b + d)(f - c)$
s $\frac{c + e}{g}$	**t** $\frac{i}{b + d}$	**u** $\frac{h + j}{2e}$	**v** $\frac{di}{eg}$

4 Evaluate each expression, using the values given.

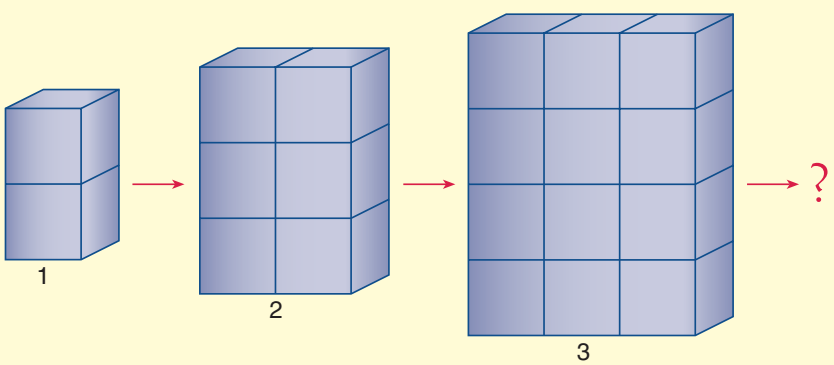
a $5a$, given that $a = \ldots$	**i** 3	**ii** 0	**iii** 11	**iv** $\frac{1}{2}$
b $2x + 7$, given that $x = \ldots$	**i** 5	**ii** 1	**iii** 15	**iv** 0·5
c $20 - 3n$, given that $n = \ldots$	**i** 0	**ii** 6	**iii** 1·5	**iv** $\frac{1}{3}$
d $5(2y + 1)$, given that $y = \ldots$	**i** 3	**ii** 10	**iii** 0	**iv** 2·5

5 The perimeter P of a rectangle is given by the formula $P = 2l + 2b$,
where l is the length and b is the breadth of the rectangle.
Find the value of P if:



a $l = 5, b = 3$	**b** $l = 7, b = 6$
c $l = 21, b = 15$	**d** $l = 1{\cdot}6, b = 1{\cdot}2$

6



The total number of cubes in each figure is given by the expression $n(n + 1)$, where n is the number of cubes along the bottom of each stack. How many cubes would there be if:

a $n = 8$?	**b** $n = 12$?	**c** $n = 20$?	**d** $n = 100$?

7 If apples cost 15 cents each and pears cost 20 cents each, then the cost of buying n apples and m pears would be given by the expression $15n + 20m$.
What is the total cost of buying:

a 10 apples and 5 pears? **b** 5 apples and 10 pears? **c** 25 of each?

8 Complete each table, using the formula given.

a $b = 4t$

t	2	3	4	5
b				

b $y = 3x$

x	0	1	2	3
y				

c $n = 7m$

m	1	3	5	7
n				

d $b = 3t + 1$

t	0	1	2	3
b				

e $y = 2x + 3$

x	2	4	6	8
y				

f $q = 5p + 2$

p	1	4	5	10
q				

g $b = 5a - 2$

a	1	2	3	4
b				

h $w = 20 - 2t$

t	0	2	4	6
w				

i $g = 3h - 7$

h	3	4	5	6
g				

j $y = \frac{x}{2}$

x	2	4	6	8
y				

k $b = \frac{a}{5}$

a	0	5	10	20
b				

l $n = m^2$

m	0	1	2	3
n				

9 Write each as an equation. Let the first number be x, and the second number y.

a You add 5 to the first number to get the second number.
b If you subtract 23 from the first number, you get the second number.
c Doubling the first number gives the second number.
d Dividing the first number by 7 produces the second number.

10 Match each table of values with the correct rule or formula.

a

x	1	2	3	4
y	6	12	18	24

b

x	0	2	4	6
y	0	6	12	18

c

x	3	4	5	6
y	10	11	12	13

d

x	5	7	9	11
y	1	3	5	7

e

x	1	2	3	4
y	3	5	7	9

f

x	0	2	4	6
y	2	8	14	20

g

x	5	6	7	8
y	6	8	10	12

h

x	9	12	15	18
y	3	4	5	6

A $y = x + 7$
B $y = 2x + 1$
C $y = 6x$
D $y = 2x - 4$
E $y = \frac{x}{3}$
F $y = 3x + 2$
G $y = 3x$
H $y = x - 4$

Write these equations in everyday language as shown in question 9.

9:04 | Simplifying Algebraic Expressions

prep quiz 9:04

Rewrite each expression without multiplication signs.

1 $3 \times a$ **2** $m \times n$ **3** $6 \times p \times 2 \times q$ **4** $3 \times (5 + a)$ **5** $2 \times a + 5 \times b$

If $x = 3$ and $y = 5$, evaluate each expression.

6 $4x$ **7** xy **8** $2y + x$ **9** $2x - y$ **10** $x(y + 3)$

An algebraic expression is made up of one or more **terms** joined together by plus or minus signs.

Expression	$3x + 5$	$5x - 2t + 7$	$a^2 - 2a + 4x - 5$
Number of terms	2	3	4



Like terms have identical pronumeral parts, eg $2x$ and $3x$, $5ax$ and $7ax$, $10n$ and $\frac{n}{2}$.

If the pronumeral parts are different in any way they are *not* like terms. In that case they are called **unlike terms**, eg $5a$ and $2b$, $2xy$ and $3xz$, $7m^2$ and $7m$.

Discussion

1 Let n represent the number of marbles in a packet. If Jane buys 2 packets and Alan buys 5 packets then:

The number of marbles Jane has $= 2 \times n$, *or* $2n$.

The number of marbles Alan has $= 5 \times n$, *or* $5n$.

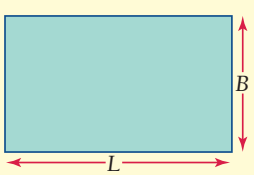
Now the number of marbles altogether would be given by the expression $2n + 5n$.

Also, we know that the number of marbles bought altogether was $7n$, as seven packets were bought,

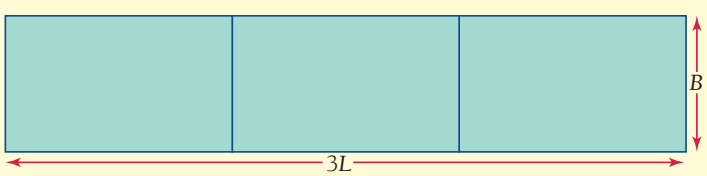
$\therefore\ 2n + 5n = 7n$

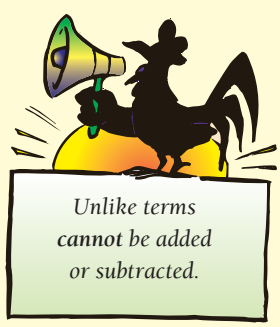
(ie 2 lots of n marbles plus 5 lots of n marbles must equal 7 lots of n marbles.)

2



- The area of a rectangle is equal to its length times its breadth.
  $\therefore$ Area $= L \times B$ or LB.
- If we joined 3 such rectangles the total area would be $LB + LB + LB$.
  But the area can also be seen to be $3LB$.
  $\therefore\ LB + LB + LB = 3LB$





Only like terms can be added or subtracted.

worked examples

1 $7a + 5a = 12a$

2 $20pq - 11pq = 9pq$

3 $8x + x = 9x$

4 $5m + 7m - 10m = 2m$

5 $5x + 2y + 7y = 5x + 9y$

6 $7p + 2q + 3p + q$
$= 10p + 3q$

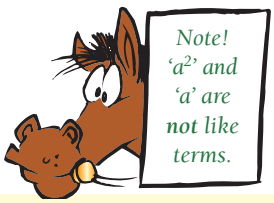
7 $6a + 7b - 2a + 5b$

$= \boxed{6a} \boxed{-2a} \boxed{+7b} \boxed{+5b}$

$= 4a + 12b$

Important notice! The plus or minus sign always belongs to the term after it!

8 $7a^2 - 4a + 2a^2$
$= 9a^2 - 4a$



Exercise 9:04

Foundation Worksheet 9:04
Simplifying algebraic expressions
Simplify:
1 a 
2 a $2x + 3x$
3 a $5a + 3 + 4$

1 How many terms are there in each expression?

a $7a + 2a$	**b** $5x$	**c** $7p + 9q$
d $7xy$	**e** $9 - m + n$	**f** 2
g $ab + 3a - 4b$	**h** $2x - 5y - x + y$	**i** $9 - 2x + x^2$
j $9x - 4y + 7z - 8w + t$	**k** $6a + 4b - 2c + 11d - e$	
l $7 + x - x^2 + y - y^2$	**m** $25a + 9bc - 8def$	

2 Write down the *like terms* from each group.

a $2x,\ 3y,\ 4x$	**b** $a,\ 3b,\ 7b$
c $4m,\ 7n,\ 8m,\ 6p$	**d** $2a,\ 2b,\ 3a,\ 3c$
e $ab,\ a^2,\ 3a,\ 2a^2$	**f** $xy,\ yz,\ 3yx$
g $9x,\ 7y,\ x,\ 5b,\ 3x$	**h** $2mn,\ 7m,\ 8n,\ 6mn,\ 9$
i $a,\ 2a,\ a^2,\ 3ab,\ 7a$	**j** $ab,\ abc,\ a,\ ba,\ bc,\ ac$
k $a^2x,\ 2ax,\ 3ax^2,\ 4a^2x$	**l** $9m,\ 7mn,\ 8m^2,\ n^2,\ 5m^2$

Notice: $ab = ba$ since $a \times b = b \times a$

3 Simplify each expression.

a $5a + 2a$	**b** $3x + 4x$	**c** $3m + 10m$
d $8y + 2y$	**e** $6n + n$	**f** $b + 7b$
g $a + a$	**h** $7x + x$	**i** $9a - 2a$
j $8m - 5m$	**k** $10k - 2k$	**l** $19b - 12b$
m $8q - q$	**n** $11x - x$	**o** $a - a$
p $15m - 14m$	**q** $5x + 2x + 3x$	**r** $7a + 2a + 4a$
s $9m + 4m + m$	**t** $6x + x + 5x$	**u** $8m + 2m - 9m$
v $10y + 4y - 6y$	**w** $5p - 4p + 2p$	**x** $8a - a + 2a$
y $4ab + 7ab - 2ab$	**z** $9xy + 4yx$	

Note: $1x = x$

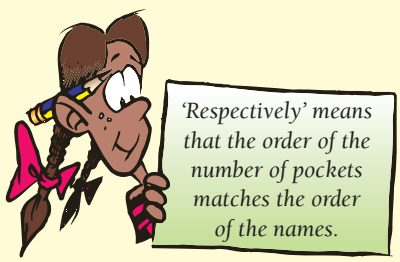
4 Simplify each expression by adding or subtracting like terms only.

	A	B	C
a	$7a + 2a + 3b$	$9x + 2x + 7y$	$5m + 2n + 7m$
b	$8p + 2p - 7q$	$10k + 5l - 2k$	$8x - 7x - 2y$
c	$9a^2 + 2a^2 + 5a$	$4x^2 + 5x + 3x^2$	$9m + 3m^2 - 2m^2$
d	$5ab + 2ab - 7a$	$11x^2 + 7xy - 2xy$	$5ab + 3bc - 3ba$
e	$7 + 2a - a$	$6x + 9 - 5x$	$10a^2 - 5a^2 - 7$
f	$2a + 3b + 5a + 2b$	$6x + 2y + 4x + y$	$9m + 4m + 7n + 5n$
g	$6p + 2q - 4p + 4q$	$8k + 10l - 7l + 2k$	$5x^2 + 7x - 2x^2 - 3x$
h	$9m^2 + 7m^2 + 5m - 3m$	$6ab + 3bc - 4ab + bc$	$10pq + 8p^2 - 7p^2 - 8pq$
i	$7a^2 + 4a - 2a^2 + 7$	$10k + 5l - 2m + 3l$	$5x + 7y + 3z - 5y$
j	$10p - 3q + p - 7p$	$15a + 3a + 6b + 2a$	$t + 6r + 7t - 5t$
k	$3x^2 + 2x - 2x^2 + 3x + x^2$	$10p + 7q - 2q + 3p - q$	$3a + 7a + 2b + a - b$

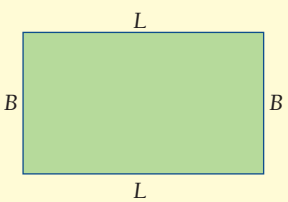
5 **a** If there are n chocolates in each box, and Julie buys 4 boxes while Michelle buys 7 boxes, how many chocolates are bought altogether?

b There are b bananas in a kilogram. Jeff buys 5 kg but they are so good he goes back to buy 3 kg more. How many bananas did Jeff buy altogether?

c If a packet of biscuits contains x biscuits, and Bill, Kylie and Allyson buy 3, 4 and 6 packets respectively, how many biscuits are bought altogether?

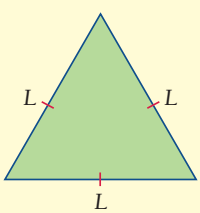


6 **a** The perimeter of this rectangle could be written as $L + B + L + B$.
How could this be written in a shorter form?

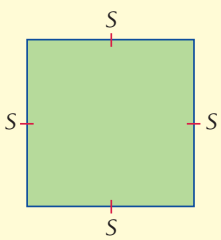


b Write down the simplest expression for the perimeters of these figures:

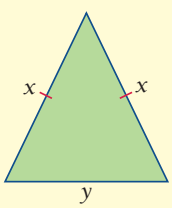
i



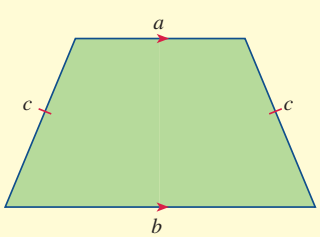
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iii

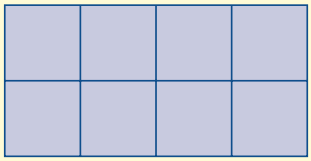


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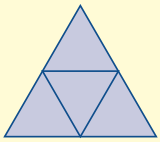


7 If the area of each square tile is s cm^2 and the area of each triangular tile is t cm^2, find the total area of the following patterns.

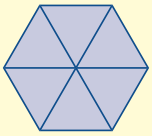
a



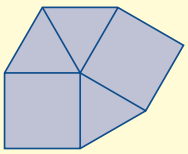
b



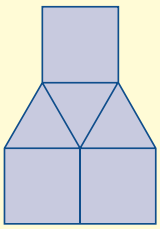
c



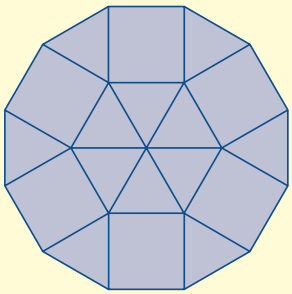
d



e



f



g If a pattern contained 10 squares and 12 triangles, what would be its area?
h If a pattern contained n squares and m triangles, what would be its area?

fun spot 9:04

Fun Spot 9:04 | Let's improve our algebra skills

Mr Breakwell, the principal, visited my Year 6 class and gave them a series of instructions to see how many students could end up with the right answer.

'Begin with x,' he said. 'Multiply it by 8. Add 4 to your answer. Subtract 8 from your last answer. Divide the result by 4 and find the value of this latest answer if x is equal to 7.' Stuart, who was good at maths, did it this way.

x	$\times 8$	$8x$	$+4$	$8x + 4$	-8	$8x - 4$	$\div 4$	$2x - 1$	$x = 7$	$2(7) - 1 = 13$

Activity

See if you can do as well as Stuart.

1. Begin with x, multiply by 6, add 2, subtract 6, divide by 2, let x equal 8.
2. Begin with y, multiply by 10, add 5, multiply by 2, subtract 20, divide by 10, let y equal 5.
3. Begin with m, multiply by 3, subtract 12, multiply by 2, add 6, divide by 3, let m equal 2.
4. Begin with $3d$, add $5d$, subtract 7, multiply by 3, subtract 24, let d be 4.
5. Begin with $10k$, subtract k, multiply by $2k$, add 18, divide by 2, let k equal -3.



9:05 | Grouping Symbols

prep quiz 9:05

Evaluate:

1	two lots of $(6 + 7)$	2	two lots of 6 + two lots of 7
3	$3 \times (6 + 10)$	4	$3 \times 6 + 3 \times 10$
5	$10 \times (7 + 3)$	6	$10 \times 7 + 10 \times 3$
7	3 lots of $(100 - 1)$	8	3 lots of 100 – 3 lots of 1
9	$5 \times (100 - 3)$	10	$5 \times 100 - 5 \times 3$

An expression like $3(2 + 5)$ means $3 \times (2 + 5)$ or 3 lots of $(2 + 5)$,

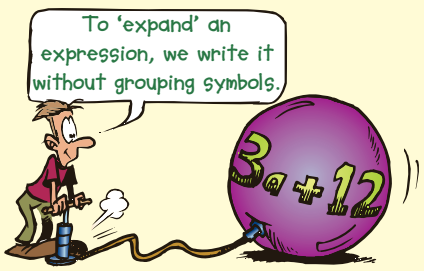
so $3(2 + 5) = (2 + 5) + (2 + 5) + (2 + 5)$
$= 2 + 5 + 2 + 5 + 2 + 5$
$= 2 + 2 + 2 + 5 + 5 + 5$
$= 3 \times 2 + 3 \times 5$
$\therefore 3(2 + 5) = 3 \times 2 + 3 \times 5$

Also: $6(5 - 2) = 6 \times 5 - 6 \times 2$
and $(10 + 7)8 = 10 \times 8 + 7 \times 8$

This process is the same if pronumerals are involved.

$3(a + 4) = (a + 4) + (a + 4) + (a + 4)$
$= a + 4 + a + 4 + a + 4$
$= a + a + a + 4 + 4 + 4$
$= 3 \times a + 3 \times 4$
$= 3a + 12$
$\therefore 3(a + 4) = 3a + 12$

Notice
The most common grouping symbols are:
- parentheses ( )
- brackets [ ]



To 'remove' the grouping symbols, we multiply each term inside the grouping symbols by the term

$7(2a + 3) = 7 \times 2a + 7 \times 3$
$= 14a + 21$

worked examples

1 $5(x + 2) = 5 \times x + 5 \times 2$
$= 5x + 10$

2 $10(2a - 3) = 10 \times 2a - 10 \times 3$
$= 20a - 30$

3 $m(m + 2) = m \times m + m \times 2$
$= m^2 + 2m$

Expand and simplify:

4 $5(3x + 8) + 10$
$= 15x + 40 + 10$
$= 15x + 50$

5 $2(8m - 3) + 4m$
$= 16m - 6 + 4m$
$= 20m - 6$

- $a(b + c) = ab + ac$
- $a(b - c) = ab - ac$
- $(a + b)c = c(a + b)$

Exercise 9:05

Foundation Worksheet 9:05

Grouping symbols

1 Remove grouping symbols.
  a $2(x + 3)$

2 Expand:
  a $3(5m - 1)$

1 Remove grouping symbols.

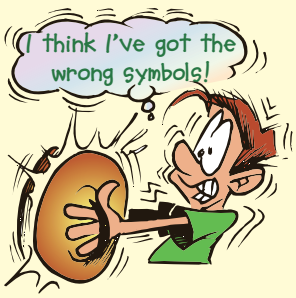
a $2(a + 3)$	**b** $3(x + 5)$	**c** $5(y + 3)$
d $3(x - 1)$	**e** $7(n - 7)$	**f** $4(a - 4)$
g $7(2 + x)$	**h** $6(1 + y)$	**i** $9(5 - y)$
j $(5 + x)2$	**k** $(x + 9)3$	**l** $(a - 2)3$

2 Expand:

a $2(3a + 2)$	**b** $3(2x + 1)$	**c** $5(4y + 3)$	**d** $10(3p + 7)$
e $6(2y - 1)$	**f** $5(3q - 4)$	**g** $4(2k - 7)$	**h** $9(4n - 3)$
i $5(4 + 2x)$	**j** $7(3 + 4x)$	**k** $5(1 + 3p)$	**l** $8(3 - 2y)$
m $(2m + 3)4$	**n** $(3y + 6)2$	**o** $(2 + 4x)5$	**p** $(7 + 3q)10$
q $(5n - 3)6$	**r** $(8m - 1)7$	**s** $(2 - 3x)7$	**t** $(5 - 9y)11$

3 Remove the grouping symbols.

a $x(x + 2)$	**b** $y(y + 5)$	**c** $m(m + 7)$
d $q(q - 1)$	**e** $m(m - 10)$	**f** $n(n - 4)$
g $a(3 + a)$	**h** $k(4 + k)$	**i** $p(6 - p)$
j $x(2x + 3)$	**k** $y(3y + 4)$	**l** $n(5n - 1)$
m $2p(p + 7)$	**n** $3q(q - 4)$	**o** $(m + 3)2m$
p $5a(2a + 3)$	**q** $2m(4m - 1)$	**r** $6n(5 - 3n)$



4 Expand:

a $5(x + y)$	**b** $4(a + b)$	**c** $7(p - q)$	**d** $10(m - n)$
e $6(2a + b)$	**f** $7(3x + 2y)$	**g** $9(4p - 2q)$	**h** $5(2k - 7l)$
i $x(x + y)$	**j** $p(p - q)$	**k** $2y(y + z)$	**l** $5g(3g - 2h)$

5 Expand and simplify:

a $2(a + 4) + 3$	**b** $4(m + 6) + 5$	**c** $8(x + 1) + 6$
d $3(a + 6) + 2a$	**e** $5(7 + y) + 3y$	**f** $10(m + 5) + 3m$
g $6(2n + 3) + 2$	**h** $4(2n + 1) + 7$	**i** $3(4a + 2) + 11$
j $3(m + 7) - 1$	**k** $8(a + 6) - 48$	**l** $3(m + 3) - 8$
m $2(5a + 1) - 4a$	**n** $3(2m + 2) - 6a$	**o** $5(10m + 3) - 20m$
p $4(2a - 1) + 3a$	**q** $5(2x - 4) + x$	**r** $7(5y - 2) + 10y$



9:05 Expanding grouping symbols

Activity CW

- Each packet holds n cards. How many cards altogether?
- $3(2n + 2) = \ldots$





9:06 | Index Notation

prep quiz 9:06

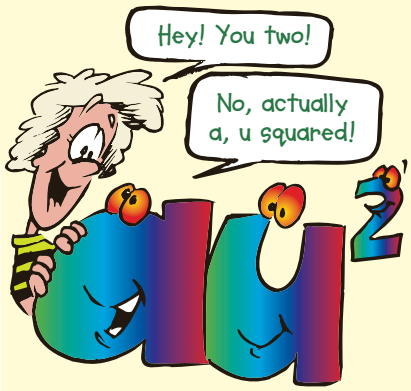
Simplify these expressions.

1 $5a + 7a$ 2 $10p - 6p$ 3 $11x + x$ 4 $7m - m$

5 $9p + 7q - 2q$ 6 $5ab + 6ba$ 7 $9a + 5 + 2a + 3$

8 $7p + 8q - 2p$ 9 $5x + 7y - 2x + 3y$ 10 $10t^2 + 7t - 4t - 2t^2$

- $a \times a$ is best written as a^2.
- When a pronumeral is multiplied by itself a number of times we can simplify the expression using **index notation**.



examples

1 $a \times a \times a \times a = a^4$

2 $p \times p \times p \times p \times p \times p \times p = p^7$ (seven *p*s multiplied together)

3 $m \times m \times m \times n \times n \times n \times n \times n = m^3n^5$

4 $5 \times t \times t \times 7 \times y = 35t^2y$

■ Note that the index belongs to the *t* only.

■ The plural of *index* is *indices* (pronounced 'in-diss-ease').

Exercise 9:06

1 Simplify the following, using index notation.

a $a \times a \times a$ b $p \times p \times p \times p \times p$ c $t \times t \times t \times t$

d $y \times y$ e $k \times k \times k \times k \times k$ f $m \times m \times m$

g $n \times n \times n \times n \times n \times n$ h $b \times b \times b \times b$ i $a \times a \times b \times b \times b$

j $x \times x \times x \times y \times y \times y$ k $a \times a \times a \times a \times t$ l $m \times n \times n \times n$

m $p \times q \times p \times p \times q$ n $a \times a \times b \times a \times b \times b$ o $x \times y \times x \times y \times x$

2 Rewrite in expanded form (eg $t^4 = t \times t \times t \times t$).

a a^5 b q^4 c t^7 d x^3 e m^2

f n^5 g p^1 h a^6 i m^2n^3 j a^4b^3

k xy^5 l k^4l m a^3y n h^2n^5 o tw^3

p x^4y^4 q $n^2p^3q^4$ r ab^5c^2 s $x^3y^2z^4$ t m^3np^4

3 Simplify, using index notation.

a $5 \times a \times 6 \times a$ b $3 \times y \times 4 \times y \times y$ c $9 \times m \times m \times 2 \times m$

d $t \times t \times 7 \times t$ e $7 \times y \times y \times 4 \times y \times y$ f $8 \times n \times 3 \times n \times 2$

g $5 \times m \times m \times m \times 9$ h $6 \times a \times 3 \times a \times 4 \times a$ i $2 \times x \times 3 \times y \times 4 \times x$

j $10 \times a \times 3 \times b \times 5 \times a \times b$ k $k \times l \times 3 \times l \times 5 \times k$ l $t \times 5 \times t \times w \times 3 \times t$

Appendix H H:01 Operations with indices (extension)

9:07 | Graphing Tables of Values

prep quiz 9:07

If $n = 8$, evaluate: **1** $n + 7$ **2** $2n$ **3** $4n - 1$ **4** $\frac{n}{2}$ **5** n^2

If $x = 3$, find the value of y, if: **6** $y = x + 5$ **7** $y = 4x$ **8** $y = 2x + 3$ **9** $y = 10 - 2x$ **10** $y = \frac{5 + x}{2}$

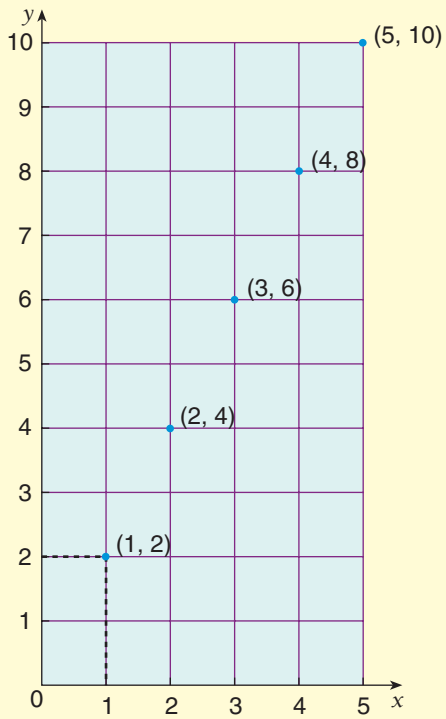
We have seen how a number pattern can be displayed in a table of values. These can also be displayed on a graph. For example, from the table of values,

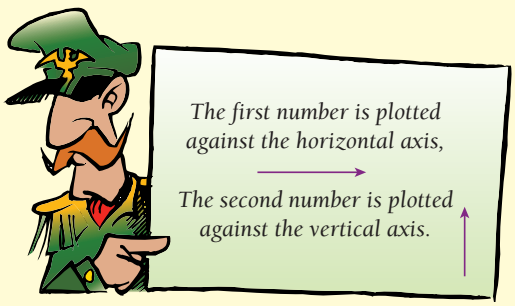
$y = 2x$

x	1	2	3	4	5
y	2	4	6	8	10

We get the 'ordered pairs': (1, 2) (2, 4) (3, 6) (4, 8) (5, 10)

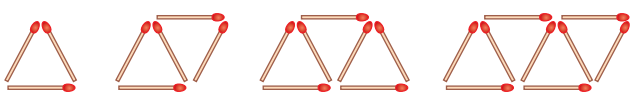
If these pairs of numbers are plotted on the number plane, we get a graph like the one shown.





worked examples

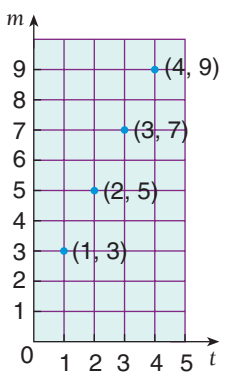
1



This pattern of triangles formed from matches gives the following table.

Number of triangles (t)	1	2	3	4
Number of matches (m)	3	5	7	9

If these ordered pairs of numbers are plotted, we get the graph shown.



continued →→→

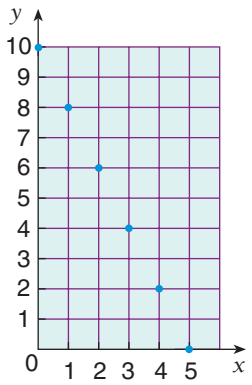
2 This is how we graph the points given by a particular pattern rule.
Rule: $y = 10 - 2x$

Step 1 Complete a table of values:

x	0	1	2	3	4	5
y	10	8	6	4	2	0

Step 2 Identify the ordered pairs from the table:
(0, 10) (1, 8) (2, 6) (3, 4) (4, 2) and (5, 0).

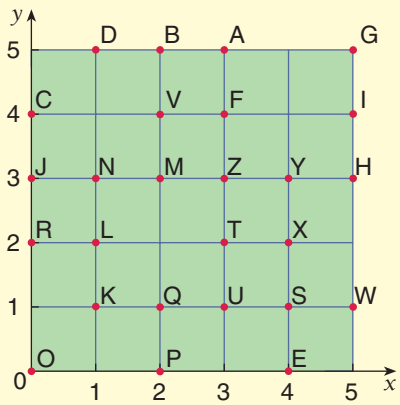
Step 3 Plot the points on the graph. Be sure to label each axis correctly!



Exercise 9:07

1 Write down the letter of the point that has as its ordered pair:

a (1, 2)	**b** (3, 1)	**c** (5, 3)
d (2, 4)	**e** (3, 3)	**f** (1, 5)
g (0, 2)	**h** (4, 0)	**i** (0, 3)
j (4, 2)	**k** (1, 3)	**l** (3, 5)
m (2, 0)	**n** (0, 4)	**o** (5, 1)



2 Write down the ordered pair for each point.

a M	**b** F	**c** G
d B	**e** Y	**f** E
g O	**h** I	**i** K
j Q	**k** T	**l** S

3 Graph on separate diagrams the set of ordered pairs given by each table of values.
(Don't forget to label each axis with the pronumerals given.)

a

t	0	1	2	3	4
b	1	2	3	4	5

b

t	0	1	2	3	4
b	3	4	5	6	7

c

t	0	1	2	3	4
b	0	2	4	6	8

d

x	1	2	3	4	5
y	5	4	3	2	1

e

x	1	2	3	4	5
y	10	8	6	4	2

f

x	1	2	3	4	5
y	1	3	5	7	9

4 Complete each table of values, using the given rule, and then plot the ordered pairs on a number plane. What do you notice about each set of points?

a $b = t + 1$

t	0	1	2	3
b				

b $y = 2x$

x	0	1	2	3
y				

c $m = 5 - n$

n	1	2	3	4
m				

d $y = 2x - 1$

x	1	2	3	4
y				

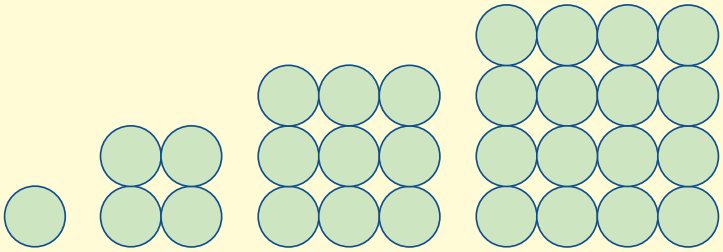
e $p = \frac{1}{2}q$

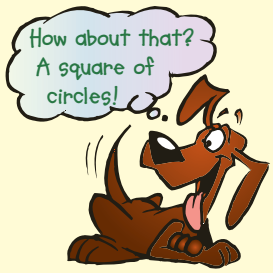
q	0	2	4	6
p				

f $b = 2(a - 1)$

a	1	2	3	4
b				

5



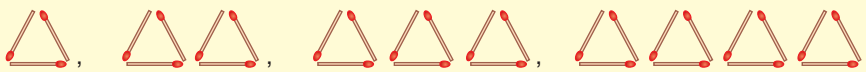


a Complete the table of values for the pattern of circles above.

Number of circles on each edge (C)	1	2	3	4
Total number of circles (T)				

b Graph this set of ordered pairs on a number plane.

6



a Each triangle is constructed out of matches. Complete the table of values.

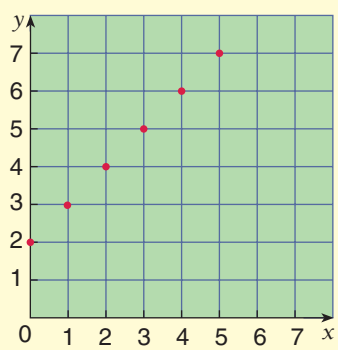
Number of triangles (t)	1	2	3	4	5
Number of matches (m)					

b What is the rule for this number pattern?

c Graph this set of ordered pairs on a number plane.

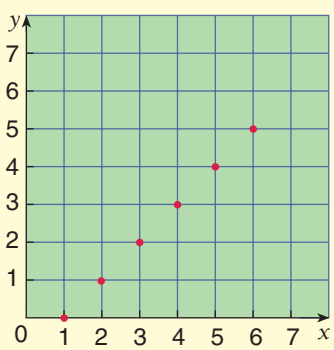
7 Complete a table of values from each of the graphs given. Also find the pattern rule for each table.

a



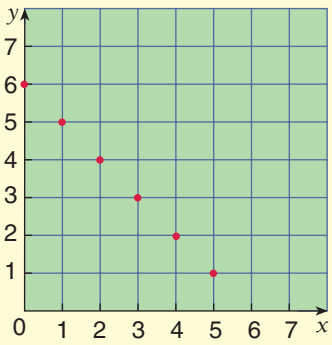
x	0	1	2	3	4	5
y						

b



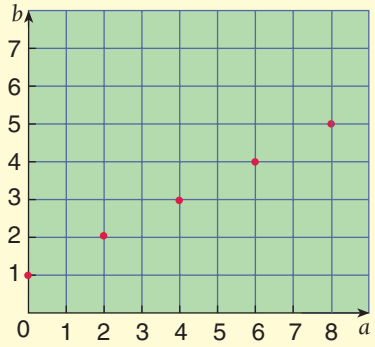
x	1	2	3	4	5	6
y						

c



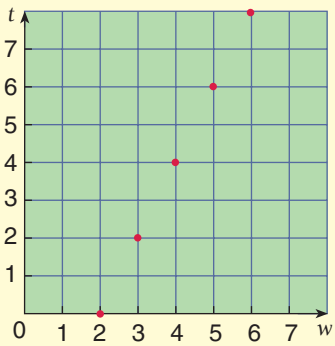
x	0	1	2	3	4	5
y						

d



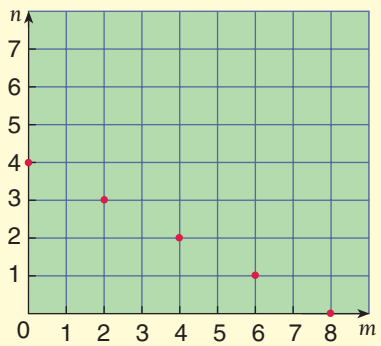
a	0	2	4	6	8
b					

e



w	2	3	4	5	6
t					

f



m	0	2	4	6	8
n					

9:08 | Graphs Involving Directed Numbers

prep quiz

9:08

Answer the following.

1 $7 - 11$ **2** $-5 + 7$ **3** $-2 - 3$

4 $9 - (-3)$ **5** $(-5) \times 4$ **6** $(-4) \times (-3)$ **7** $12 \div (-2)$

8 $(-24) \div (-6)$ **9** $\dfrac{4 - 10}{-2}$ **10** $\dfrac{-5 - (-11)}{6}$

- When substituting directed numbers into algebraic expressions, it is often useful to substitute parentheses as well.
  eg When $x = -7$, x^2 becomes $(-7)^2$, which is -7×-7.
- $-7^2 \neq (-7)^2$ because for -7^2 only the 7 is squared.

worked examples

1 If $x = -2$, $y = -3$ and $z = 5$, find the value of:

a $x + y$ **b** xy **c** $z - x$ **d** $\dfrac{y+z}{x}$

2 Complete each table, using the rule given, and graph the result.

a $y = 2x$

x	-2	-1	0	1
y				

b $b = 5 + a$

a	-4	-2	0	2
b				

Solutions

1 a $x + y \to -2 + (-3)$
$= -5$

b $xy \to -2 \times -3$
$= 6$

c $z - x \to 5 - (-2)$
$= 7$

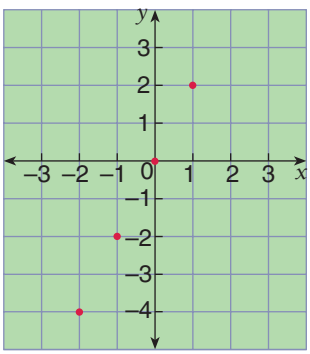
d $\dfrac{y+z}{x} \to \dfrac{-3+5}{-2}$
$= -1$

2 a When $x = -2$, $y = 2 \times -2$
$= -4$
When $x = -1$, $y = 2 \times -1$
$= -2$
When $x = 0$, $y = 2 \times 0$
$= 0$
When $x = 1$, $y = 2 \times 1$
$= 2$

$\therefore$ The table is:

x	-2	-1	0	1
y	-4	-2	0	2

The points to be plotted are: $(-2, 4)$, $(-1, -2)$, $(0, 0)$ and $(1, 2)$.

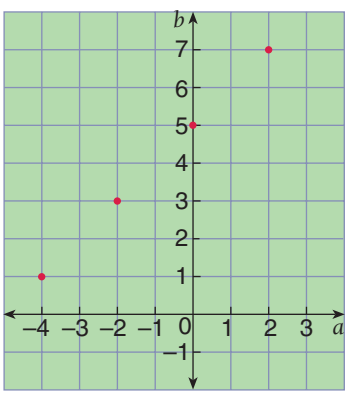


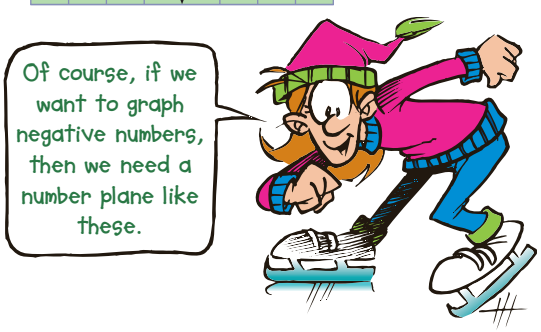
b When $a = -4$, $b = 5 + -4$
$= 1$
When $a = -2$, $b = 5 + -2$
$= 3$
When $a = 0$, $b = 5 + 0$
$= 5$
When $a = 2$, $b = 5 + 2$
$= 7$

$\therefore$ The table is:

a	-4	-2	0	2
b	1	3	5	7

The points to be plotted are: $(-4, 1)$, $(-2, 3)$, $(0, 5)$ and $(2, 7)$.





Exercise 9:08

Foundation Worksheet 9:08

Directed numbers

1 Evaluate, if $x = 7$ and $y = 3$.
a $2x$ **b** $x + 3$

2 If $x = -2$ and $y = 3$, find:
a $x + 1$ **b** $x + y$

1 Find the value of each expression if $n = -3$.

a $2n$ **b** $7n$ **c** $5 + n$
d $n - 1$ **e** $10 - n$ **f** $-2n$
g $n + n$ **h** $n \times n$ **i** $2n + 1$
j $10 + 3n$ **k** $5n + 15$ **l** $\frac{6}{n}$

2 If $x = -3$, $y = -4$, $z = 2$, evaluate:

a $x + y$ **b** $y + z$ **c** $z + x$ **d** $x + y + z$
e $x - y$ **f** $y - z$ **g** $x - z$ **h** $x - y - z$
i xy **j** yz **k** $2xz$ **l** $x^2 + z^2$
m $y \div z$ **n** $\frac{z - y}{x}$ **o** $3x + 2z$ **p** xyz

3 Evaluate each expression using the values given.

a	$3a$	if a is:	**i** -2	**ii** -4	**iii** 5	**iv** 0
b	$2a - 1$	if a is:	**i** 0	**ii** -1	**iii** -3	**iv** -10
c	$10 - 2a$	if a is:	**i** 3	**ii** 6	**iii** -1	**iv** -3
d	$2(3a - 4)$	if a is:	**i** 1	**ii** 0	**iii** -2	**iv** -10

4 Graph these tables of values on separate number planes.

a $y = x + 2$

x	-2	-1	0	1
y	0	1	2	3

b $y = 5 - x$

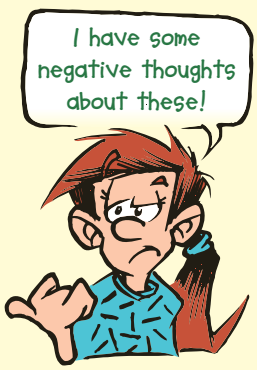
x	4	5	6	7
y	1	0	-1	-2

c $y = 3x$

x	-2	-1	0	1
y	-6	-3	0	3

d $y = 2x - 3$

x	0	1	2	3
y	-3	-1	1	3



5 Complete these tables of values and then graph them on separate number planes.

a $y = 1 + x$

x	-2	-1	0	1
y				

b $b = 2a$

a	-2	-1	0	1
b				

c $q = 3 - p$

p	2	3	4	5
q				

d $t = s - 5$

s	1	2	3	4
t				

e $y = 2x + 1$

x	-2	-1	0	1
y				

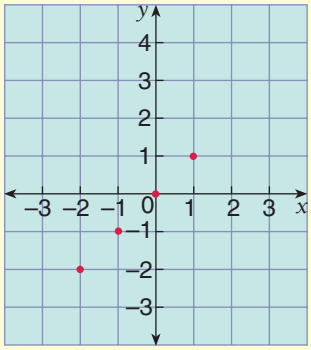
f $n = 3 - 2m$

m	0	1	2	3
n				

6 Complete the table of values with each graph and match it with the correct pattern rule.

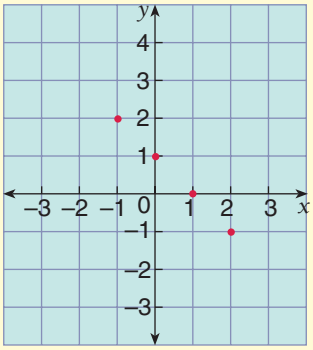
A $y = 2x - 1$ **B** $y = x$ **C** $x + y = 1$

a



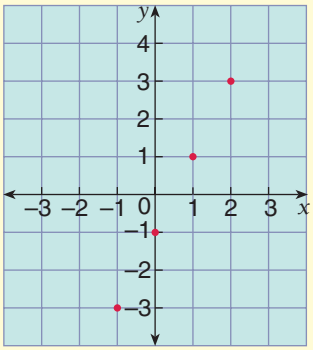
x	-2	-1	0	1
y				

b



x	-1	0	1	2
y				

c



x	-1	0	1	2
y				

7 Sofia plays a quiz game in which a right answer scores 10 points but a wrong answer scores −5 points. Therefore if r = number of right answers, and w = number of wrong answers, her score S would be given by the rule:
$S = 10r - 5w$

a What would Sofia's score be if she got:

i 3 right and 1 wrong?

ii 4 right and 8 wrong?

iii 2 right and 5 wrong?

iv 5 right and 14 wrong?

b If Sofia answered 20 questions and her score was −10, how many questions did she answer correctly?



9:08 Make your own 3 × 3 magic squares Activity

9:09 | Simplifying Expressions with Directed Numbers

prep quiz 9:09

Give the simplest answer.

1 $-6-4$	**2** $-8+12$	**3** $2-5$	**4** -3×5	**5** $-12\div4$
6 $\frac{6a}{2}$	**7** $3m+8m$	**8** $5\times3a$	**9** $3x+2y+x$	**10** $2(a+3)-6$

see 9:04 Diagnostic Test 8

If you know your work on directed numbers and algebra, you should have no trouble using these rules together.

Only like terms can be added or subtracted.



Note: $-1x=-x$

worked examples

1 $-4a-3a=-7a$

2 $5ab-8ab=-3ab$

3 $-8x+x=-7x$

4 $6m-2n+m-n$

$=$ | $6m$ | $-2n$ | $+m$ | $-n$ |

$=7m-3n$

5 $-3\times a=-3a$

6 $-3\times-4x=12x$

7 $10x\div-2=\frac{10x}{-2}$

$=-5x$

8 $2(a+4)-10$

$=2a+8-10$

$=2a-2$

Exercise 9:09

Foundation Worksheet 9:09

Simplifying expressions with directed numbers

Simplify:

1 a $-3-2$ **b** $2-6$

2 a -4×5 **b** $-2\times3x$

3 a $-2x-x$ **b** $2y-7y$

1 Simplify the following expressions.

a $3a-6a$	**b** $4m-5m$	**c** $2x-7x$
d $-3x+2x$	**e** $-2n+n$	**f** $-7p-2p$
g $5a^2-9a^2$	**h** $6ab-8ab$	**i** $-3x^2-7x^2$
j $6a+3b-9a$	**k** $5y-3x-10y$	**l** $2x-7x+3y$
m $8p+7q-9q$	**n** $6a^2-5a-9a^2$	**o** $7t+8-8t$

2 Simplify:

a $-4\times x$	**b** $-10\times y$	**m** $-24a\div8$
c $-1\times a$	**d** $b\times(-1)$	**p** $3(a-5)+10$
e $-4\times2m$	**f** $6\times-3a$	**s** $4(3-x)+9x$
g $-8\times-3a$	**h** $-6m\times4$	
i $-7\times8n$	**j** $-a\times-3$	
k $6x\div-2$	**l** $-10m\div-5$	
n $2(m+3)-8$	**o** $5(m-1)+9$	
q $4(2x+1)-8x$	**r** $3(10-2x)+3x$	

- **plus × minus = minus**
  minus × plus = minus
- **minus × minus = plus**

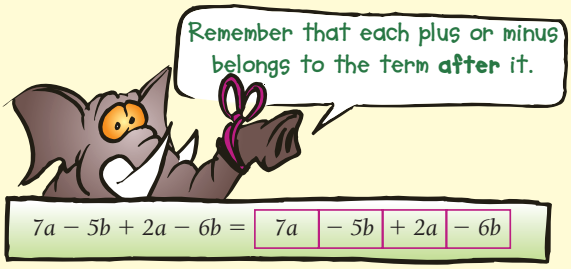
3 Simplify by collecting like terms:

a $7a - 5b + 2a - 6b$ **b** $11x - 2y - 5x + 7y$ **c** $3m + 2g - 5g - 4m$

d $6a - 7 - 9a + 10$ **e** $7p - 2q - 6p + 3q$ **f** $3x + 7 - 12 - 5x$

g $2ab + 3bc - 5ab + bc$ **h** $6t^2 + 3t - 5t^2 - 8t$ **i** $9y - 6z - 9y + 5z$

j $2k - 3k^2 - 4k + k^2$ **k** $10t + 5w + t - 7w$ **l** $7a - 3b - 8a - 5b$



4 Simplify each expression. Remember, like terms must have the same pronumeral part.

a $6a + 7A - 2a + 7a^2 - 10A - 5 + 6a^2 - 2A^2 + 7 - 2A + 6a^2 - 2a + 4A^2$

b $3ab - 2bc + 5ac - 6ab + 2ac + bc - 7ca + 9cb + 3ba - 2bc + 6ac - 9cb$

Reading Mathematics 9:09 | What's in food?

reading mathematics 9:09

Do you ever wonder what is in what you are eating?

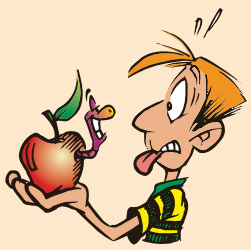
Food	*% protein*	*% fat*	*% carbohydrate*	*kJ/100 g*
Chicken, fried	32·0	24·0	2	1386
Bananas	1·1	Trace	19·2	322
Lamb, roasted	50.0	30.0	0	1176
Butter	0·4	85·1	Trace	3318
Carrots	0·7	Trace	5·4	96
Eggs, poached	12·4	11·7	Trace	669
Mars Bar	5·0	20·0	70·0	1920
Milk	3·4	3·7	4·8	276
Peas	5·0	Trace	7·7	205

1 What percentage of these foods is protein?
a lamb **b** carrots **c** Mars Bar

2 What percentage of these foods is fat?
a fried chicken **b** poached eggs **c** milk

3 How many kilojoules per 100 g are there in:
a bananas? **b** butter? **c** peas?

4 Which foods contain more fat than Mars Bars?



9:10 | Algebraic Sentences (Solving Equations)

prep quiz 9:10

Make the number sentences true by finding the missing number.

1 $\square + 3 = 7$ **2** $\triangle - 5 = 2$ **3** $\star \times 5 = 20$ **4** $12 - \square = 3$ **5** $24 \div \triangle = 6$

State whether the following are true or false.

6 If $6 + \square = 15$, then $\square = 11$.

7 If $5 \times \triangle = 45$, then $\triangle = 9$.

8 If $\theta - 6 = 10$, then $\theta = 4$.

9 If $2 \times \square + 1 = 11$ then $\square = 5$.

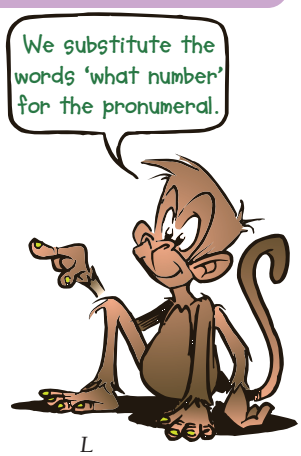
10 If $\frac{\triangle + 1}{3} = 4$, then $\triangle = 11$.

- To '**solve**' an algebraic sentence or equation, we must find the numerical value of the pronumeral that makes the sentence a true mathematical statement. Often we can just 'see' the answer to the equation, especially if we say in words what the sentence means.
- To **check** an answer, we substitute the answer into the original equation to see if it gives a number sentence that is true.

worked examples

1 $x + 5 = 9$
We have to find the value of x, so we say: 'What number plus 5 is equal to 9?'
Answer: $x = 4$

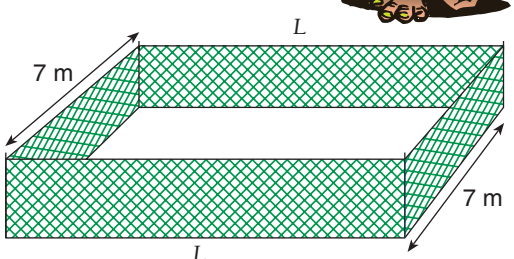
2 $12 - a = 5$
For this equation we would say: '12 minus what number is equal to 5?'
Answer: $a = 7$



3 $6m = 42$
'6 times what number is equal to 42?'
Answer: $m = 7$

4 $\frac{x}{3} = 21$
'What number divided by 3 is equal to 21?'
Answer: $x = 63$

5 Brian and Kathy wanted to build an enclosure. They wanted it to be rectangular in shape, 7 metres wide, and to use all 36 metres of fencing material they had bought.



They knew that:
perimeter = 2 × length + 2 × width
ie $P = 2L + 2W$
The perimeter had to be 36 metres and the width 7 metres,
$\therefore \quad 36 = 2L + 14$
To find the length, the value of L had to be found.
Question: 'What number plus 14 is equal to 36?' Answer: 22
$\therefore \quad 2L = 22$
Question: '2 times what number is equal to 22?' Answer: 11
$\therefore \quad L = 11$
The enclosure had to be 11 metres long.

Exercise 9:10

Foundation Worksheet 9:10

Algebraic sentences

Find the number to make the equation true.

1 a $\square + 3 = 10$ **b** $4 \times \square = 20$

2 a $x - 2 = 7$ **b** $a \div 2 = 9$

1 There are x counters in each blue box and y counters in each red box. In each question the x and y can stand for a different number. In each question find the value of x or y.

a x = 11 counters

b $x\ x\ x\ x\ x$ = 50 counters

c y = 24 counters

d $y\ y\ y$ = 33 counters

e $x\ x$ = 100 counters

f x = 20 counters

g y = 15 counters

h $y\ y$ = 21 counters

i $y\ y$ = 32 counters

j $x\ x\ x$ = 27 counters

2 Write in words what each equation means and then solve it.

a $x + 5 = 7$ **b** $a + 9 = 21$ **c** $10 - p = 7$ **d** $m - 5 = 2$

e $4n = 20$ **f** $7m = 56$ **g** $q \div 2 = 7$ **h** $\frac{n}{4} = 4$

3 Solve these equations.

	A	B	C	D
a	$a + 3 = 6$	$x + 9 = 10$	$m + 3 = 11$	$k + 7 = 15$
b	$n + 4 = 12$	$q + 10 = 17$	$p + 8 = 13$	$a + 11 = 22$
c	$6 + x = 9$	$7 + b = 14$	$10 + q = 23$	$14 + m = 29$
d	$12 + l = 30$	$15 + x = 19$	$21 + a = 40$	$25 + n = 48$
e	$10 - a = 9$	$12 - x = 8$	$9 - q = 0$	$8 - m = 8$
f	$15 - p = 9$	$19 - m = 8$	$21 - x = 12$	$30 - a = 18$
g	$x - 9 = 1$	$p - 7 = 3$	$q - 10 = 4$	$t - 7 = 10$
h	$y - 6 = 6$	$x - 5 = 0$	$k - 12 = 13$	$l - 21 = 19$
i	$6x = 12$	$3p = 18$	$9m = 27$	$5m = 25$
j	$20 = 4y$	$24 = 6m$	$15 = 5a$	$16 = 2k$
k	$8m = 0$	$9n = 9$	$100 = 25x$	$144 = 12n$
l	$x \div 2 = 3$	$p \div 5 = 2$	$a \div 3 = 4$	$n \div 8 = 2$
m	$10 \div a = 5$	$12 \div x = 4$	$24 \div m = 4$	$20 \div m = 5$
n	$\frac{x}{3} = 6$	$\frac{a}{2} = 6$	$\frac{m}{5} = 3$	$\frac{n}{7} = 4$
o	$\frac{16}{x} = 2$	$\frac{12}{a} = 1$	$\frac{32}{m} = 4$	$\frac{48}{p} = 4$

4 Check the given solution for each equation by substituting it back into the equation. State whether it is correct or incorrect.

a $x + 12 = 26$
$x = 14$

b $14 + a = 29$
$a = 16$

c $15 - m = 9$
$m = 4$

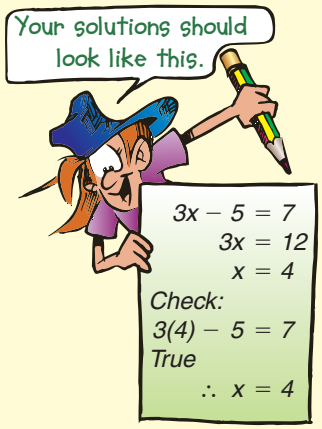
d $n - 19 = 22$
$n = 3$

e $7a = 63$
$a = 9$

f $13x = 65$
$x = 6$

g $p \div 7 = 3$
$p = 21$

h $m \div 3 = 6$
$m = 2$



5 These equations involve two steps. (See worked example 5.)

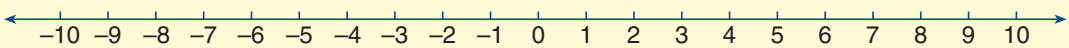
a $2x + 1 = 7$
b $2a + 7 = 11$
c $3a + 4 = 13$
d $4y + 3 = 15$
e $2x - 3 = 5$
f $3m - 1 = 14$
g $5m - 3 = 12$
h $9y - 2 = 7$
i $7 + 2x = 17$
j $10 + 5a = 20$
k $9 - 2a = 3$
l $12 - 5p = 2$
m $16 = 2x + 6$
n $17 = 4x + 5$
o $5 = 5p - 10$
p $2 = 6m - 10$
q $19 = 10 + 3p$
r $26 = 12 + 7q$
s $0 = 20 - 4m$
t $2 = 24 - 2n$

see Chapter 4

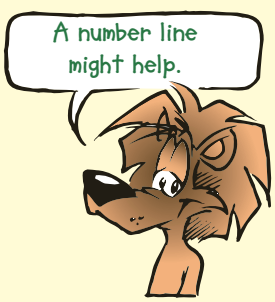
6 Solve these equations involving decimal numerals.

a $x + 2 = 3{\cdot}2$
b $a + 5 = 6{\cdot}1$
c $m + 4{\cdot}2 = 5{\cdot}2$
d $a + 1{\cdot}7 = 4{\cdot}8$
e $2{\cdot}3 + b = 3{\cdot}7$
f $6{\cdot}9 + x = 10{\cdot}0$
g $x - 2{\cdot}3 = 5{\cdot}0$
h $a - 1{\cdot}7 = 0{\cdot}6$
i $9{\cdot}2 - p = 6{\cdot}8$
j $4m = 4{\cdot}8$
k $5m = 12{\cdot}5$
l $7a = 1{\cdot}33$
m $0{\cdot}9x = 1{\cdot}8$
n $0{\cdot}4a = 1{\cdot}6$
o $1{\cdot}2p = 0{\cdot}36$
p $q \div 0{\cdot}5 = 6$
q $1{\cdot}2 \div x = 0{\cdot}4$
r $24 \div y = 2{\cdot}4$
s $2x + 1{\cdot}2 = 1{\cdot}8$
t $5y + 2{\cdot}4 = 7{\cdot}9$
u $3m - 2{\cdot}1 = 3{\cdot}0$

7 To solve these equations you need to remember how to operate with integers.



a $3 - x = -1$
b $5 - m = -5$
c $4 - a = -2$
d $a - 5 = -1$
e $x - 3 = -6$
f $p - 2 = -5$
g $x + 3 = 1$
h $a + 1 = -4$
i $m + 7 = 1$
j $a + (-2) = 1$
k $x + (-3) = -1$
l $n + (-9) = -10$
m $6m = -12$
n $3x = -18$
o $-5m = -15$
p $a \div 4 = -6$
q $x \div (-2) = 3$
r $20 \div n = -4$
s $x - (-2) = 5$
t $a - (-1) = -3$
u $-6 - x = 1$



8 An enclosure must be rectangular in shape and 2 metres wide. It must use all 10 metres of fencing material available. Use the formula $P = 2L + 2W$ to find how long the enclosure must be. (See example 5.)

9 Faye bought 5 plums and an apple, for 86 cents.

a Write down an equation to show this information if the cost of a plum is P cents and the cost of an apple is A cents.

b Use your equation to find the cost of a plum if the apple cost 26 cents.

9:11 | Solving Problems Using Algebra

prep quiz 9:11

What mathematical symbol (+, −, ×, ÷) is correct for these words?

1 sum **2** product **3** minus **4** difference **5** total

6 times **7** quotient

Solve these equations.

8 $x + 12 = 23$ **9** $a - 7 = 10$ **10** $6m = 42$

The main use of algebra, and particularly equations, is to help us solve problems.

The use of algebra is a problem-solving strategy.



■ *Take Note!*
Many of the problems in this section are very easy. You may be able to see the answer immediately. But you must practise writing out solutions correctly in order to be able to solve harder equations and problems later!

We are going to *translate* or rewrite problems written in words into algebraic sentences, which we call **equations**. It is therefore important to know key words and their meanings, like those in the Prep Quiz.

We will let a pronumeral represent what we are trying to find.

Look at the following examples and note the way the solutions are set out.

worked examples

Example 1

The sum of a certain number and 7 is 12. What is the number?

Let the number be n.	Step 1 Introduce pronumeral	
$n + 7 = 12$	Step 2 Write down equation	
$n = 5$	Step 3 Solve equation	
∴ The number is 5.	Step 4 Answer the problem	

Example 2

The product of a number and 9 is 72. What is the number?

	Step 1 Introduce pronumeral	Let the number be x.
	Step 2 Write down equation	$9x = 72$
	Step 3 Solve equation	$x = 8$
	Step 4 Answer the problem	∴ The number is 8.

Example 3

After buying a drink I have 35 cents left from $1. How much was the drink?

Let the cost of the drink be c cents.

$c + 35 = 100$ [or $100 - c = 35$]

$c = 65$

∴ The cost of the drink was 65 cents.

■ ∴ means 'therefore'.



continued →→→

Example 4

If an integer is multiplied by 7, and 3 is added to the product, the result is 38. What is the integer?

Let the integer be x.

So $7x + 3 = 38$

$7x = 35$

$x = 5$

$\therefore$ The integer is 5.

■ This is a two-step equation.



In the next two examples you have to collect like terms before solving the equation.

Example 5

One number is 5 more than a second number. Their sum is 21. What are the numbers?

Let the first number be n.

The second number must be $n - 5$.

So $n + n - 5 = 21$

ie $2n - 5 = 21$

$2n = 26$

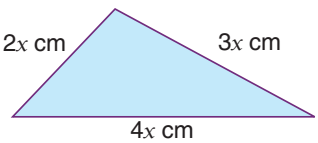
$n = 13$

$n - 5 = 8$

$\therefore$ The numbers are 8 and 13.

■ Collect like terms.

Example 6



The perimeter of this triangle is 36 cm. Find the length of each side.

In this question the pronumerals are given.

$2x + 3x + 4x = 36$

$9x = 36$

$x = 4$

Since the sides are $2x$, $3x$ and $4x$, the side lengths are 8 cm, 12 cm and 16 cm.

Exercise 9:11

Foundation Worksheet 9:11

Solving problems using algebra

1 Solve each equation.

a $x + 9 = 10$ **b** $5x = 30$

2 Find the value of x.

a 

Perimeter = 18

1 Solve these problems by first writing an equation.

a The sum of a certain number and 4 is 13. What is the number?

b The sum of 9 and a certain number is 17. What is the number?

c A certain number minus 7 is equal to 9. What is the number?

d The product of 8 and a number is 56. What is the number?

e A certain number is bigger than 7 and the difference from 7 is 3. What is the number?

f The total of a number, 6 and 14 is 35. What is the number?

g If a number is divided by 7, the result is 12. What is the number?

h The product of a number and 9 is 117. What is the number?

i If 96 is divided by a number, the result is 16. What is the number?

j If a number is subtracted from 73, the result is 31. What is the number?

2 Write an equation and then solve each problem.

- **a** An exercise book and pen cost me $2.20. If the pen cost 75 cents, how much did the book cost? (*Hint:* Let the cost of the book be b cents.)
- **b** It takes me 30 minutes to get dressed and have breakfast each morning. If it takes me 17 minutes to have breakfast, how long do I need to get dressed? (*Hint:* Let the time to get dressed be t minutes.)
- **c** A set of books cost me $63. If each book cost $5.25, how many were in the set? (*Hint:* Let the number of books be n.)
- **d** Tom, Dick and Harry's ages add to give 52. If Tom's age is 18 and Dick's is 15, how old is Harry?
- **e** 156 pieces of candy were divided among a group of children so that each got 12 pieces of candy. How many children were there?

3 To solve each of these problems you need a two-step equation.

- **a** If a number is multiplied by 3, and 5 is added to that product, the answer is 17. What is the number?
- **b** If 10 is subtracted from the product of 2 and a certain number, the result is 4. What is the number?
- **c** A number is divided by 5 and then 3 is added to the result to give 7. What is the number?
- **d** The product of a number and 8 is subtracted from 100. The result is 28. What is the number?

4 For each of these problems, collect like terms before solving the equation.

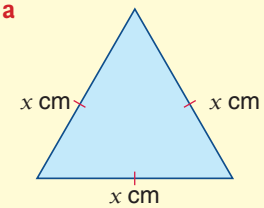
- **a** One number is 7 more than another. Their sum is 13. What are the numbers?
- **b** The sum of two numbers is 20. Their difference is 6. What are the numbers?
- **c** The difference of two numbers is 19 and their sum is 29. Find the smaller number.
- **d** The sum of two consecutive integers is 17. What are these integers?
- **e** The sum of three consecutive integers is 18. What is the smallest integer?
- **f** Bill is 5 years older than Meg, and their ages add to give 37. How old is Bill? (*Hint:* Let Meg's age be x.)
- **g** Kylie has twice as many dolls as Michelle. Together they have 36. How many dolls does Michelle have?



Note: If one integer is x, then the next consecutive integers would be $x + 1$, $x + 2$, etc.

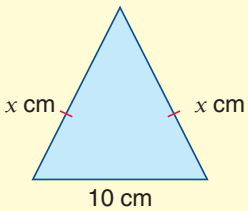
5 The perimeter of each figure is given. Find the value of x for each problem.

a



$P = 27$ cm

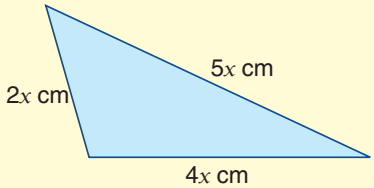
b



10 cm

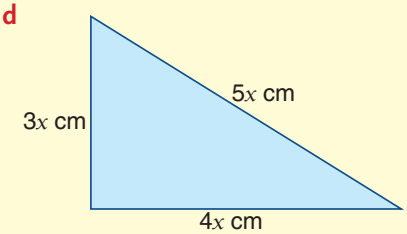
$P = 38$ cm

c



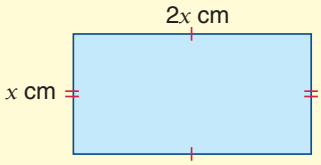
$P = 110$ cm

d



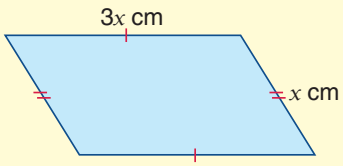
$P = 48$ cm

e



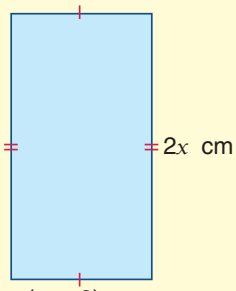
$P = 60$ cm

f



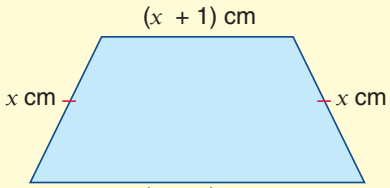
$P = 96$ cm

g



$P = 52$ cm

h



$P = 42$ cm

6 These problems may involve negative integers and/or decimal fractions. (Remember to write the equations first.)

a The sum of a number and 3 is -4. Find the number.

b The product of a number and 1·3 is 1·17. What is the number?

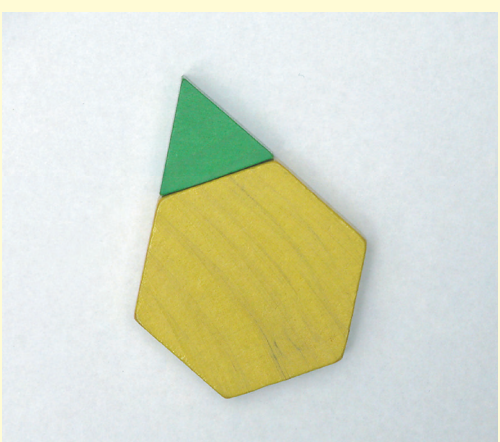
c If 7 is subtracted from the product of 3 and a number, the result is -10. What is the number?

d If the sum of a number and 5 is divided by 4, the result is 3. What is the number?

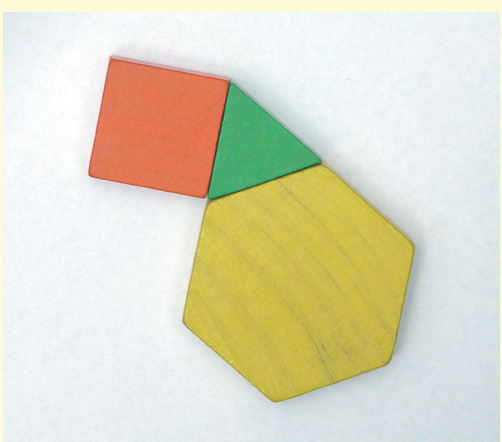
e If twice a number is subtracted from 10, the result is 12. What is the number?

f Three drinks and five ice-creams cost a total of \$4.35. If a drink costs 25 cents more than an ice-cream, find the cost of an ice-cream.

g The sum of three consecutive even numbers is 36. What are the numbers? (*Hint:* If the first even number is n, what is the next even number?)



- The total area of the top surface is 14 unit^2. What is the area of the top of the triangle?



- The perimeter of this shape is 108 units. What is the length of one side of the square?

9:12 | Investigation of Real Problems

A variety of real-life problem-solving tasks, investigations and discussions are considered here. You will find that algebra can be a valuable tool.

Exercise 9:12

1 A car-hire firm uses the following formula to work out the rental per day on a van they have for hire.

$$\text{Rental per day} = \left(25 + \frac{\text{kilometres travelled}}{2}\right)\$$

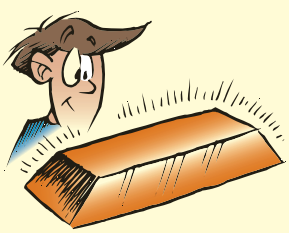


Lisa wishes to hire the van for one day. Why does she not know exactly how much it will cost? If she hires the van and travels 120 km, what will be the rental for the day? If Yasar hired the van and paid a total of $111, how many kilometres did he travel? (Set this up as an equation to solve.) What would it cost for you to hire the van and travel to your favourite spot?

2 Asif wishes to make a picture frame from a single strip of wood 300 cm in length. He wants the frame to be 12 cm longer than the width. Write an equation to represent this situation and use the equation to solve the problem.

3 A 400 g ingot of silver is to be made into a 40 g brooch and some 15 g rings. Use r to represent the number of rings and complete the equation.

$$\ldots = 40 + \ldots \times r$$



Solve this equation to find the value of r, the number of rings made.

4 An industrious student decided to investigate the relationship between shoe size and length of foot among her classmates. The table shows the results she obtained in the investigation.

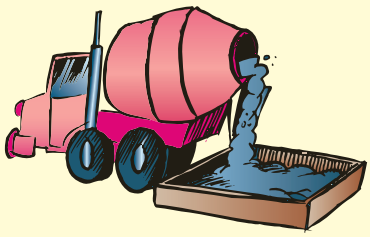


Foot length	*Size*
7	1
8	4
9	7
10	10

Can you find a rule connecting size (s) and foot length (L)? Is this rule useful for people of all ages? Predict the foot length of someone with a size 13 shoe. Investigate for yourself among your classmates to see if a rule can be found?

5 A concrete contractor uses wooden boards to 'box' in his concrete while it sets. In making a path, he records the amount of concrete he uses and the number of boards he lays down along the path. The results appear below.

Concrete (m^3)	*No. of boards*
2	7
3	12
4	17
5	22



He wishes to find a rule connecting the amount of concrete with the number of boards used. Can you find a rule? For a large project, he estimates he will need about 30 m^3 of concrete. How many boards will he require?

6 A company executive found a formula that gives a good approximation to the relationship between sales and profit for her company. The formula is

Profit = (2 × sales − 300) thousand dollars.

Over a five-year period, the sales figures have been 180, 210, 300, 170, 140. Find the profit in each year using the formula. What can you say about the profit in the last year? If the company wishes to make a profit of $50 000, how many sales would need to be made? How many sales need to be made so that the company breaks even?



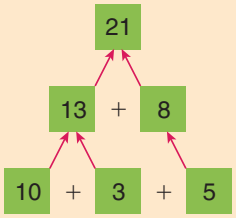
7 A student decides to study the relationship between the area of a circle and the radius. He is only able to measure roughly, and comes up with the following results.

Radius (cm)	*Area (cm^2)*
2	12
3	27
4	48
5	75

Can you find a rule to fit these results? (*Hint:* Try comparing the radius with one-third of the area.) Find out the actual formula connecting radius and area. Is the rule you have found a good approximation? If the radius of the circle on a football oval is about 400 cm, use your result to find an approximate area for the circle.

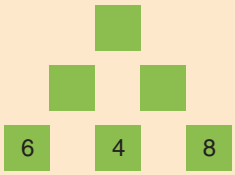
Fun Spot 9:12 | Solving pyramids

In these pyramids, the numbers in two squares are added to give the number in the one above. By doing this, we can fill in any blank squares.

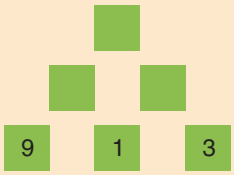


Complete these pyramids.

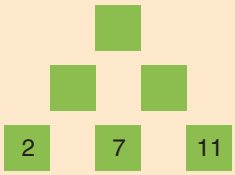
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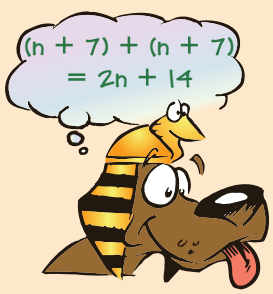
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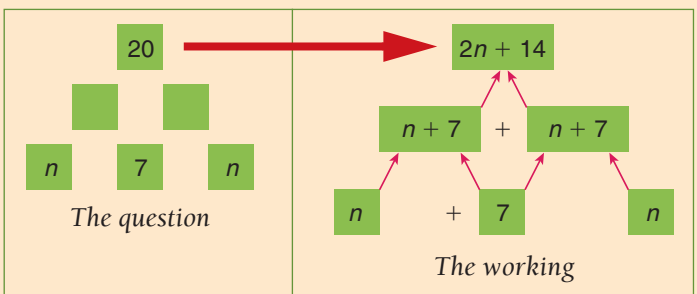


3



If one of the bottom numbers is missing, we can call it n. If enough information is given we may be able to find the value of n.

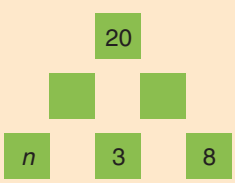




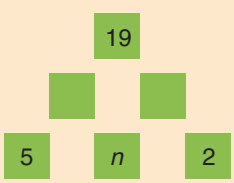
We can see that $2n + 14 = 20$
So $2n = 6$
$\therefore n = 3$

Complete these pyramids and find the value of n.

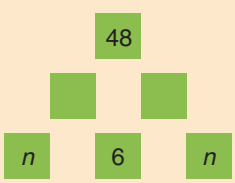
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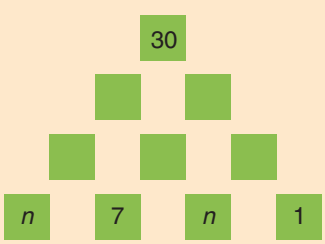
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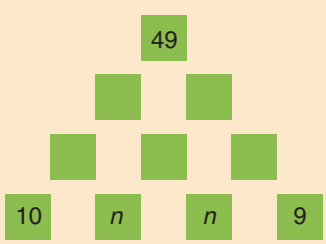
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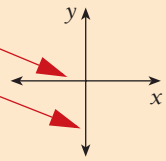
8



Mathematical terms 9

algebra (algebraic)
- The use of pronumerals in solving problems.

axis (axes)
- Horizontal axis
- Vertical axis



consecutive
- Following one after the other.
  eg 3, 4, 5, 6 are consecutive numbers.

cubed
- Used three times as a factor.
  eg $a^3 = aaa$

directed numbers
- Numbers that use direction (+ and –).

equation
- A number sentence.
  eg $x + 5 = 12$

evaluate
- To find the value of ...

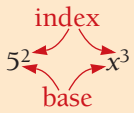
expand
- To remove grouping symbols.
  eg $3(2x - 7) = 6x - 21$

expression
- Algebraic expressions are made up of one or more terms joined by + or – signs.
  eg $2x - 7$, $5x + 2y + 1$, $a^2 - 2a + 1$

formula
- A number sentence (or equation) written using pronumerals and numerals.
  eg $C = 50M + 6$, $A = L \times B$

grouping symbols
- These group numbers or terms.
  eg parentheses ( ), brackets [ ], braces { }, fraction bar $\frac{x+1}{2}$, square root sign $\sqrt{4x+1}$

index (indices)
- The raised symbol when a number is written as a power.



integer
- Whole number.
  eg ..., –3, –2, –1, 0, 1, 2, 3, ...

like terms
- These have the same pronumeral parts.
  eg $3xy$ and $7xy$, $8x^2$ and $-x^2$
- Unlike terms have different pronumeral parts.
  eg x^2 and x, $5xy$ and $2y$

opposite operations
- + is the opposite of –
- × is the opposite of ÷

power
- Repeated as a factor.
  eg 3 to the power of $5 = 3^5$
  $= 3 \times 3 \times 3 \times 3 \times 3$

pronumeral
- A symbol (usually a letter) that takes the place of a numeral.
  eg x or θ

rule
- An instruction or formula that explains how the numbers in a pattern are related.

simplify
- To give the shortest or simplest answer.

squared
- Multiplied by itself.
  eg $5^2 = 5 \times 5$
- Used twice as a factor.

substitute (substitution)
- 'To put in place of.'
- Usually a numeral replaces a pronumeral.
  eg Substitute $x = 7$ in $2x + 3$.

value
- The number represented by a pronumeral.
  eg For the rule $y = x + 7$, find the value of y if x is given the value 4.

Diagnostic Test 9: | Further Algebra

- Each section of the test has similar items that test a certain type of example.
- Failure in more than one item will identify an area of weakness.
- Each weakness should be treated by going back to the section listed.

diagnostic test 9

Section

1 Write each expression without the multiplication signs. 9:01

a $5 \times y$ **b** $a \times b$ **c** $12 \times m \times n$ **d** $2 \times s + 5 \times t$

2 Write each expression in a shorter form. 9:01

a $a \times a$ **b** $6 \times p \times p$ **c** $2 \times m \times 3 \times m$ **d** $t \times w \times t$

3 Write these divisions in fraction form. 9:01

a $y \div 5$ **b** $10 \div a$ **c** $(a + 2) \div 3$ **d** $x \div (3 + y)$

4 There are x counters in the blue box and y counters in the red box. Use pronumerals to write the number of counters. 9:02

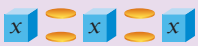
a 

b 

c 

d 

e 

f 

5 Find the value of each expression if $n = 5$. 9:03

a $4n$ **b** n^2 **c** $2n + 1$ **d** $\frac{n + 7}{3}$

6 If $a = 3$ and $b = 6$, evaluate: 9:03

a ab **b** $2a + b$ **c** $a^2 + 2b$ **d** $\frac{b}{a}$

7 Complete each table of values. 9:03

a $b = 3t$

t	1	2	3	4
b				

b $y = 2x + 1$

x	0	2	4	6
y				

c $m = n^2$

n	0	1	2	3
m				

8 How many terms are there in each expression? 9:04

a $2x + 1$ **b** ab **c** $3a + 2b + 3c$ **c** $6 - 2x^2$

9 Simplify each expression. 9:04

a $2a + 5a$ **b** $7y - y$ **c** $2ab + 3ba$ **d** $6m - 5m$

10 Simplify: 9:04

a $2a + 3 + a$ **b** $6x + 2y - 4x$

c $2m + 3n + 4m + 5n$ **d** $3p^2 + 5p + 2p^2 - 3p$

11 Expand (remove grouping symbols). 9:05

a $2(x+4)$ **b** $5(1-a)$ **c** $(2y+5)3$ **d** $x(2x+y)$

12 Expand and simplify. 9:05

a $4(x+3)+5$ **b** $3(2a+1)+8a$ **c** $5(2m-4)-8m$

13 Rewrite, using index notation. 9:06

a $a \times a \times a \times a$ **b** $y \times y \times y \times y \times y$

c $3 \times a \times 4 \times a \times a$ **d** $x \times y \times x \times y \times x$

14 Graph each table of values on a number lattice. Label each axis. 9:07

a

x	0	1	2	3
y	1	2	3	4

b

t	0	1	2	3
b	5	4	3	2

c

n	1	2	3	4
m	1	3	5	7

15 Evaluate the following if $a = -2$. 9:08

a $5a$ **b** a^2 **c** $2a+5$ **d** $6-a$

16 Complete each table of values. 9:08

a $y = x + 2$

x	−2	−1	0	1
y				

b $b = 2t + 1$

t	−2	−1	0	1
b				

c $n = 2 - m$

m	−2	0	2	4
n				

17 Graph each table of values in question **16** on a number plane. 9:08

18 Simplify: 9:09

a $4a - 6a$ **b** $-5x + 6x$ **c** $-3x^2 - 4x^2$ **d** $5a + 5 - 2a - 8$

19 Simplify: 9:09

a $-4 \times 2a$ **b** $6 \times -3a$ **c** $-m \times -3$ **d** $-1 \times a$

20 Solve these equations. 9:10

a $a + 5 = 11$ **b** $6 + x = 15$ **c** $y - 7 = 8$ **d** $10 - m = 4$

21 Solve: 9:10

a $3x = 24$ **b** $5m = 100$ **c** $p \div 2 = 8$ **d** $\frac{n}{6} = 6$

22 Solve these two-step equations. 9:10

a $2a + 1 = 7$ **b** $5m + 3 = 18$ **c** $3x - 1 = 8$ **d** $12 - 2m = 6$

23 Write down an equation to represent each problem. Let the unknown number be n, then solve the problem. 9:11

a The sum of a number and 7 is 19. What is the number?

b The product of 3 and a certain number is 36. Find the number.

c An unknown number minus 13 is equal to 23. What is the number?

d If 72 is divided by a certain number, the result is 12. Find the number.

Chapter 9 | Revision Assignment

assignment 9A

1 Simplify:

a $22 - 4 \times 3$ **b** $2 + 4 \times 2 \times 3$ **c** $\frac{24 + 6}{4}$ **d** $\frac{6 + 4}{6 - 4}$

2 What is the value of the 5 in each?

a 537 **b** 15 384 **c** 2·54 **d** 20·05

3 Write each expanded notation in its simplest form.

a $(7 \times 1000) + (8 \times 100) + (2 \times 10) + (9 \times 1)$

b $(9 \times 10) + (7 \times 1) + (3 \times \frac{1}{10}) + (8 \times \frac{1}{100})$

c $(4 \times 10^5) + (3 \times 10^4) + (0 \times 10^3) + (7 \times 10^2) + (5 \times 10) + (1 \times 1)$

4 Write each in expanded notation.

a 817 **b** 2143 **c** 6·35 **d** 60 081

5 Find the value of the □ in each.

a $\frac{1}{2} = \frac{\square}{8}$ **b** $\frac{3}{4} = \frac{\square}{100}$ **c** $\frac{50}{100} = \frac{\square}{2}$ **d** $\frac{4}{16} = \frac{\square}{4}$

6 Simplify:

a 5^2 **b** 6^2 **c** 7^2 **d** 8^2

7 Find:

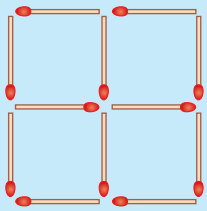
a $5 \times \$1.50$ **b** half of \$3.70 **c** $\$10.50 \div 2$ **d** $\$10 - \$1.75$

Chapter 9 | Working Mathematically

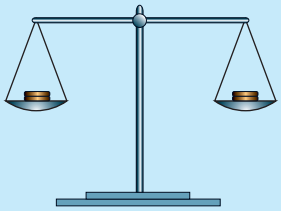
1 Use ID Card 5 on page xvi to identify:

a 1 b 2 c 3 d 4
e 8 f 9 g 10 h 11

2 Five squares are made from 12 matches as shown in the diagram. By removing two of the matches, form only two squares.



3 A girl has five coins, one of which is known to be counterfeit. The counterfeit coin is known to be either lighter or heavier than the real coins. What is the smallest number of weighings needed on the balance to be sure of identifying the counterfeit coin?

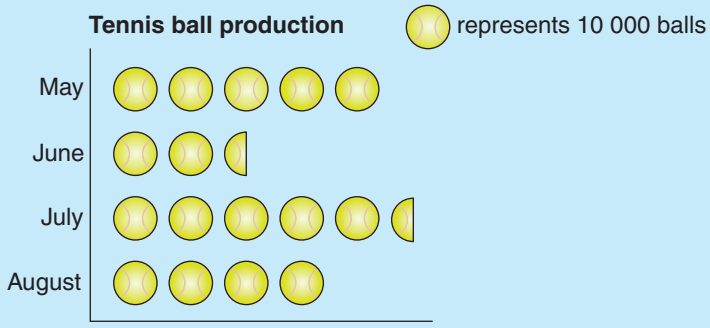


4 A carpark charges $1.80 per hour for parking. How much will it cost to park a car that is left at 6:45 pm and collected at 9:10 pm? (Note that part of an hour costs the same as 1 hour.)

5 Thirty-five children are to go on an excursion. They each bring in $6 to cover the cost of the bus. At the last moment, an extra fifteen children decide to come on the excursion. How much must they pay and how much must be returned to each of the children who have already paid?

6 A mother organises a party to which 30 children are coming. If each child has to have 2 drinks of lemonade and each glass holds 125 mL, find how many 1250 mL bottles she needs to buy.

7





a In which month were the most balls produced?
b In which month were the fewest balls produced?
c What is the title of this picture graph?
d How many balls were produced in May?
e How many balls were produced in July?
f How many more balls were produced in August than in June?

Drag and Drop CW Interactive
1 Algebraic abbreviations
2 Making sense of algebra
3 Substitution
4 Simplifying algebraic expressions

Review CW Questions

10 Angles



Chapter Contents

10:01 Measuring turning
10:02 Naming angles
10:03 Measuring the size of an angle
Investigation: Triangulation
Fun Spot: Making set squares
10:04 Types of angles
Reading Mathematics: Smoking and health facts!
10:05 Discovering more about angles
Reading Mathematics: Need some help?
10:06 Angles and parallel lines
Challenge: Getting the angle on things: Triangulation
10:07 Identifying parallel lines
Mathematical Terms, Diagnostic Test, Revision Assignment, Working Mathematically

Learning Outcomes

Students will identify and name angles formed by the intersection of straight lines, including those related to transversals on sets of parallel lines, and make use of the relationships between them.

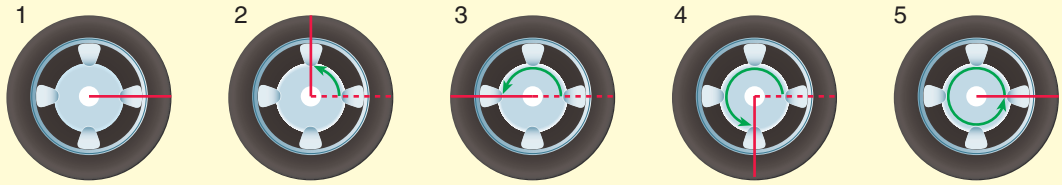
Areas of Interaction

Approaches to Learning, Homo Faber, Community

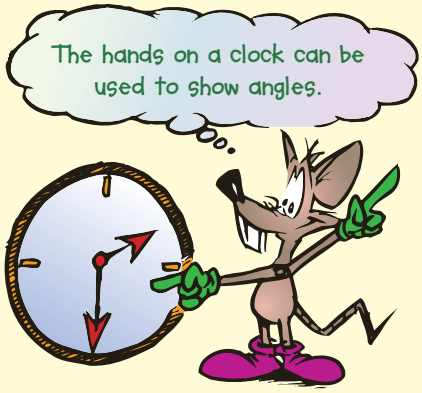
Angles are used almost everywhere, from rowing a boat to building a bridge. They are used in manufacturing, surveying, navigation and engineering.

10:01 | Measuring Turning

Angles are used to measure how far something has turned. This is shown with the wheel below.

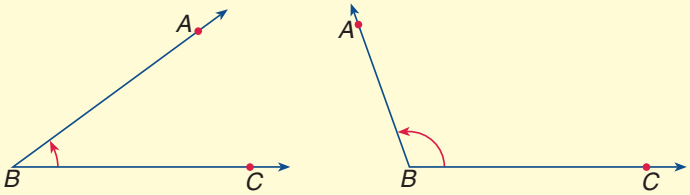


- The wheel can spin about its centre. The sequence of diagrams shows how the wheel makes one revolution.
- Diagram 1 shows the starting position, with the red line pointing horizontally to the right.
- As the wheel is spun, the red line moves in an anticlockwise direction from its starting position, which is shown by the dotted line. As the wheel spins, the angle increases in size.
- The last diagram shows the wheel returning to its original starting position after one revolution.



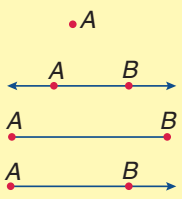
Angles can also be found in two- and three-dimensional shapes such as triangles, quadrilaterals and prisms.

If we look at an angle, we see that it is made of **two arms** or **rays**, which meet at a point called the **vertex**.



In the angles above, AB and BC are the arms of the angle while B is the vertex.

Capital letters are used to name points, lines, intervals and rays.



The point A

The line AB

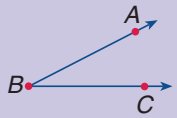
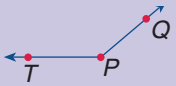
The interval AB

The ray AB



10:02 | Naming Angles

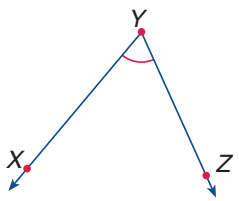
An angle can be named in several ways. Two are shown here.

Angle	*Method 1*	*Method 2*
	$\angle ABC$ $\angle CBA$ or $A\hat{B}C$	$\angle B$ or $\hat{B}$
	$\angle TPQ$ $\angle QPT$ or $Q\hat{P}T$	$\angle P$ or $\hat{P}$

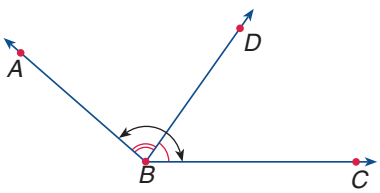
- In method 1, we go from the letter on one arm to the vertex and then to the letter on the other arm. The **vertex** is in the middle.
- Method 2 can only be used when there is no chance of confusion.

worked examples

1 Name the angle pictured below.

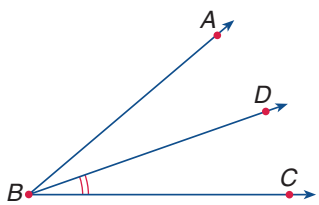


2 The figure below has three angles marked, all with vertex B. Name them.

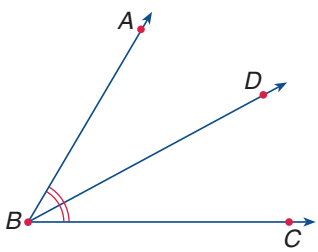


3 Name the angle marked with the '))' symbol in each of the following.

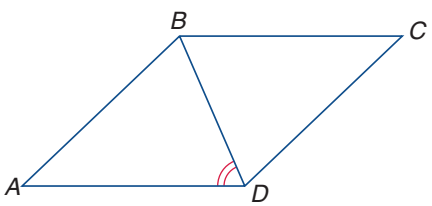
a



b

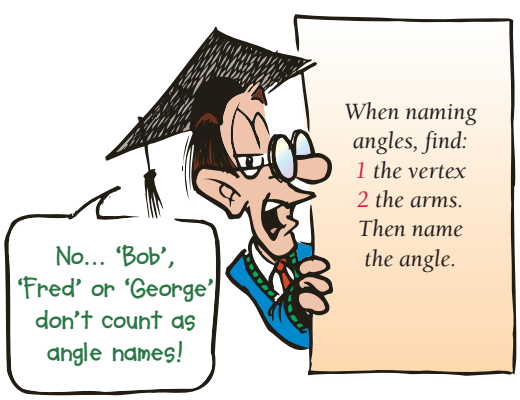


c

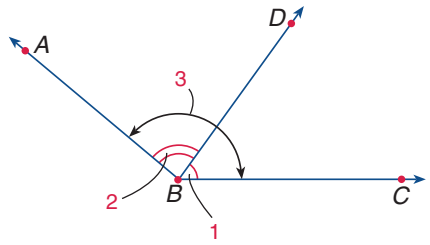


Solutions

1 Since Y is the vertex, we could write:
$\angle XYZ$ or $\angle ZYX$ or $X\hat{Y}Z$ (method 1)
$\angle Y$ *or* $\hat{Y}$ (method 2)



2



1 is $\angle DBC$ or $\angle CBD$.
2 is $\angle ABD$ or $\angle DBA$.
3 is $\angle ABC$ or $\angle CBA$.
(Method 2 could not be used.)

3 **a** B is the vertex. BD and BC are the arms. Hence the name of the angle is $\angle DBC$ (or $\angle CBD$).

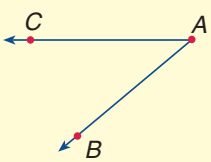
b B is the vertex. AB and BC are the arms. The angle is named $\angle ABC$ (or $\angle CBA$).

c D is the vertex. AD and BD are the arms. The angle is named $\angle ADB$ (or $\angle BDA$).

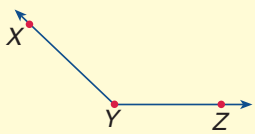
Exercise 10:02

1 Name the vertex of the following angles.

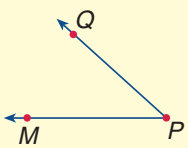
a



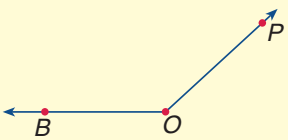
b



c



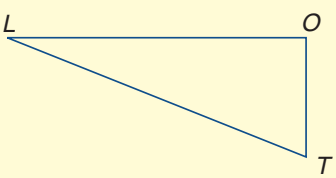
d



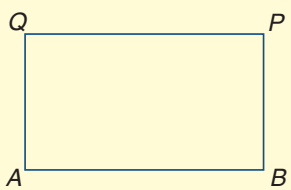
2 Name each of the angles in question 1.

3 Use methods 1 and 2 to name the angles in the following shapes.

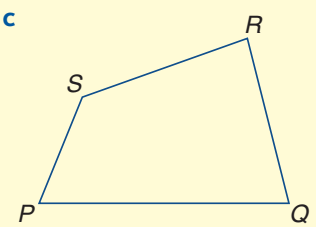
a



b

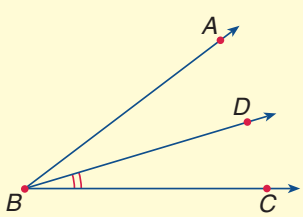


c

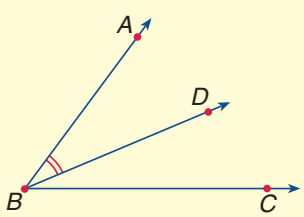


4 Name the angle marked with the '))' symbol in each of the following.

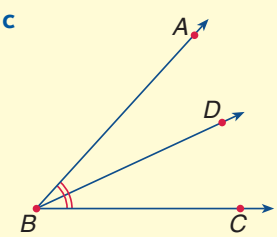
a



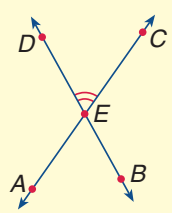
b



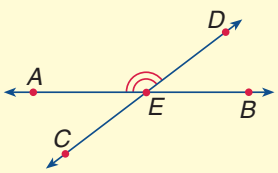
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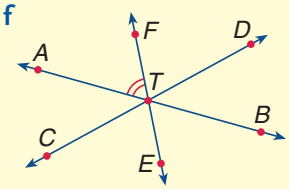
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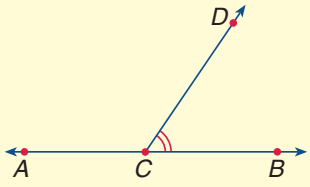
e



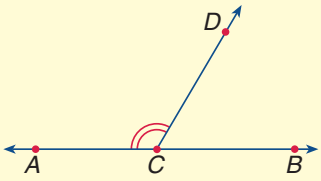
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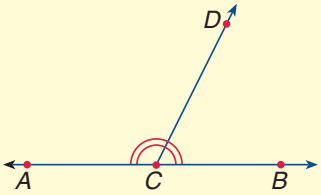
g



h

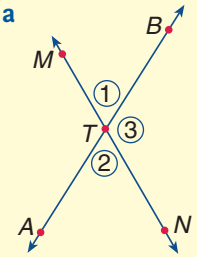


i

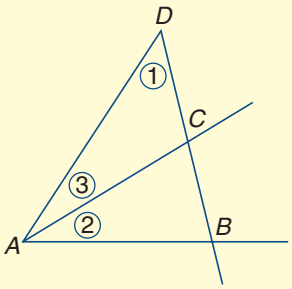


5 Name the angles ①, ② and ③ in each of the following.

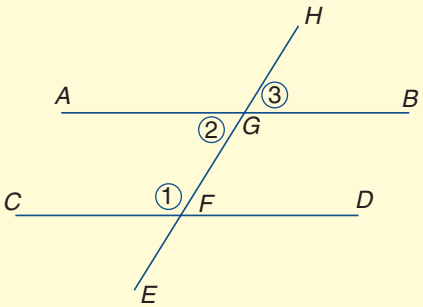
a



b

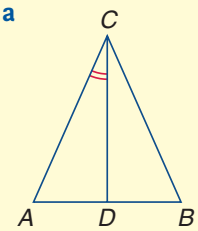


c

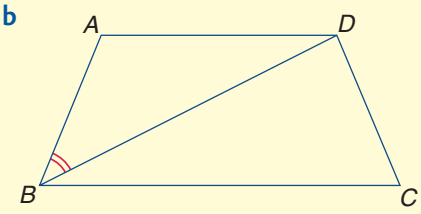


6 Name the angle marked with the '))' in each of the following.

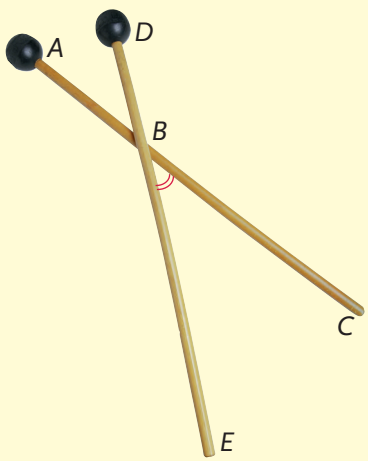
a



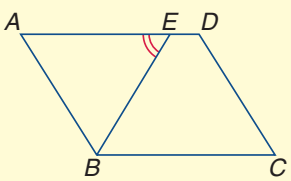
b



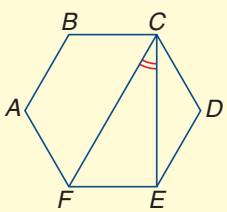
c



d

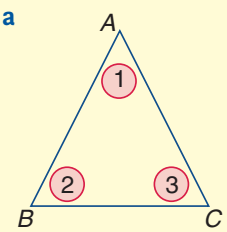


e

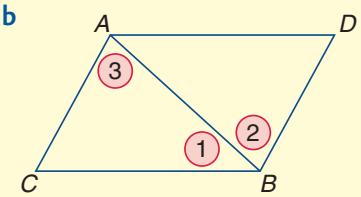


7 Which of the numbers ①, ②, or ③ is used to mark $\angle ABC$?

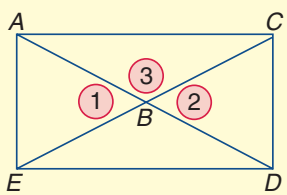
a



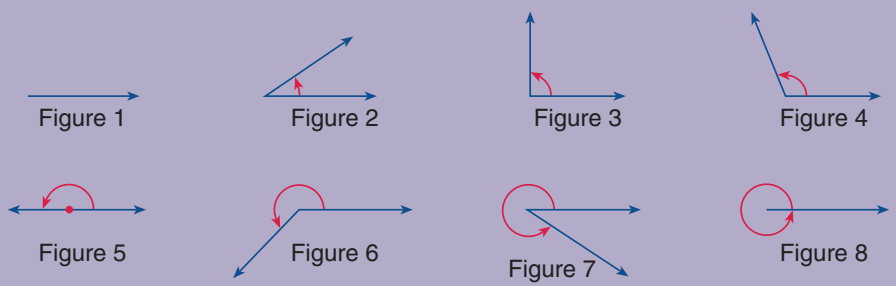
b



c

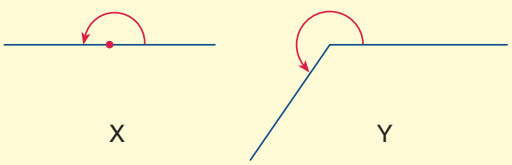


In the diagrams below, the angles increase in size from Figure 1 (no rotation) to Figure 8 (one revolution). Use these figures to answer the following questions.

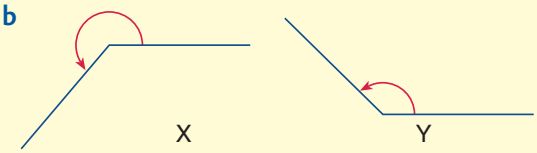


8 Which angle is the larger in each of the following pairs?

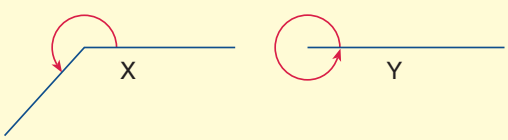
a



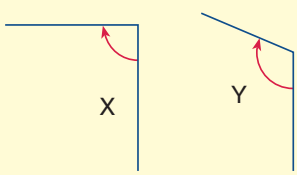
b



c

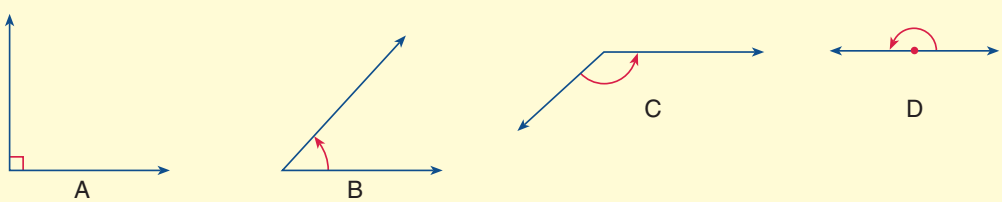


d

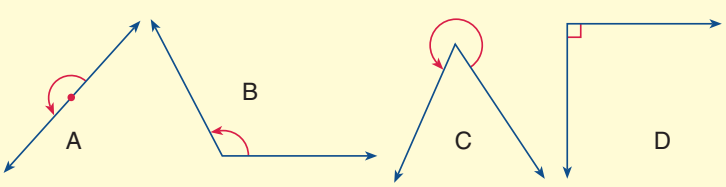


9 Arrange the angles in each of the following in order of increasing size.

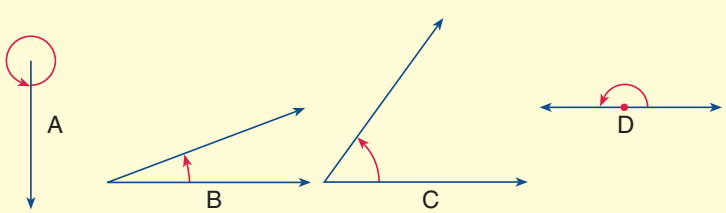
a

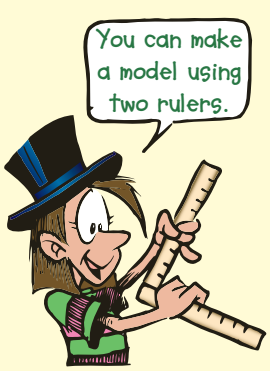


b



c



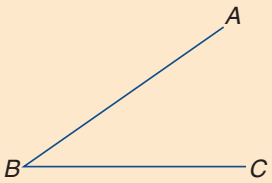


10:03 | Measuring the Size of an Angle

prep quiz 10:03

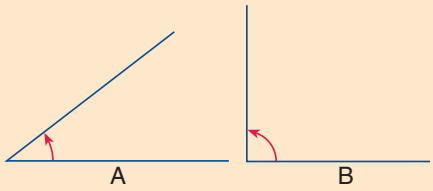
For the angle shown:

1 name the vertex
2 name the arms
3 name the angle

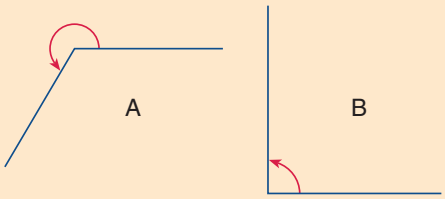


Which of the following angles is the larger?

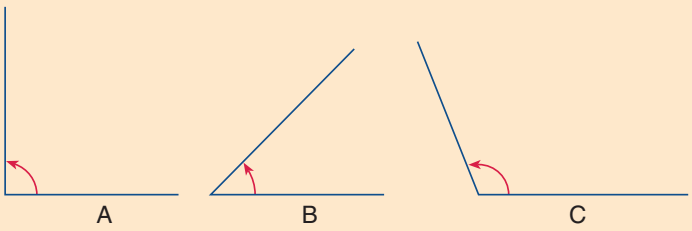
4



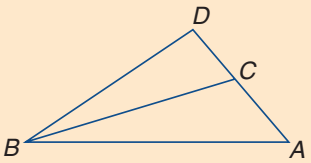
5



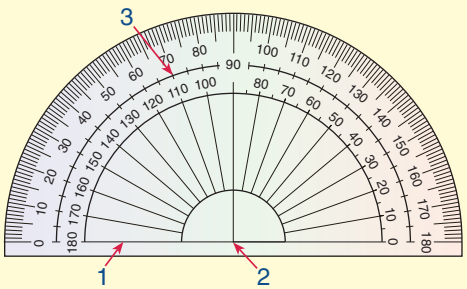
6 Arrange in order of size, smallest to largest.



7 Which angle is the larger, $\angle ABC$ or $\angle ABD$?
8 Which is larger, $\angle D$ or $\angle A$?
9 Which is larger, $D\hat{C}B$ or $B\hat{C}A$
10 Which is larger, $\angle BDC$ or $\angle BCA$?



Earlier in this chapter we learned that angles are used to measure turning. The instrument used to measure angles is called a **protractor**, and the unit of measurement is the **degree**. The degree symbol is '°'.



As you can see, the protractor has:

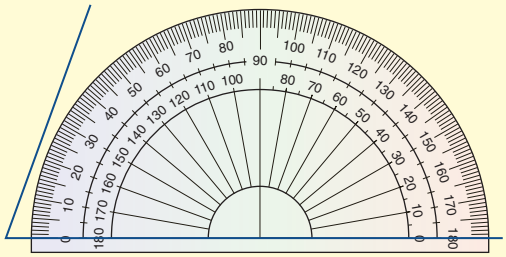
1 a base line
2 a centre point where the 90° line meets the base line
3 two scales (an inside and and outside) each ranging from 0° to 180°.



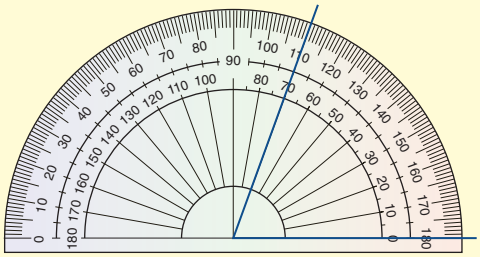
Study this carefully and you will know all the angles!

To measure an angle, follow these steps.

Step 1 Set the protractor down with the base on the lower arm of the angle.



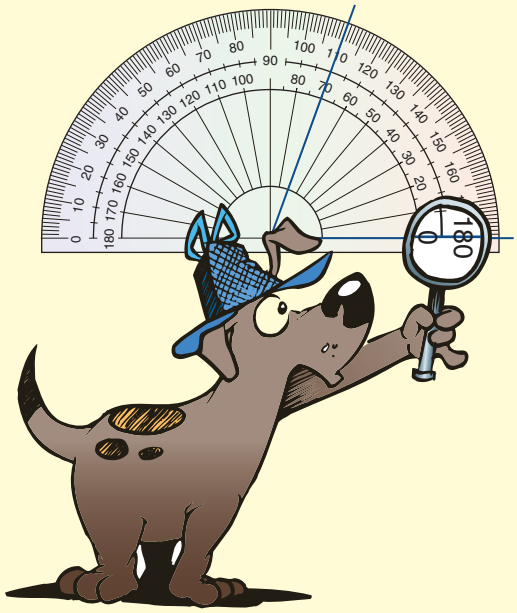
Step 2 Slide the protractor along until the centre of the protractor meets the vertex of the angle.



Step 3 Find 0° on the lower arm and use that set of figures to measure the angle.

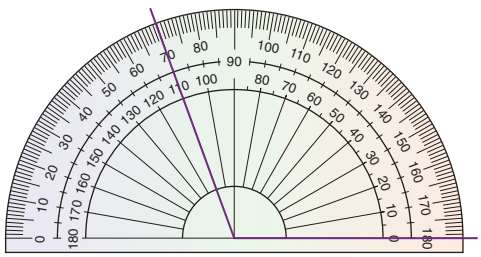


- Measure the angle.



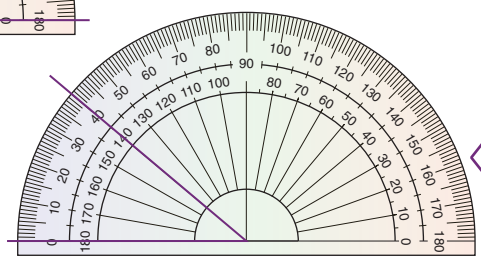
worked examples

1



This angle measures 110°

2

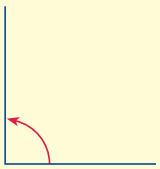


This angle measures 40°

Exercise 10:03 (Practical)

1 Without using your protractor, estimate the size of each angle and match each angle with a measurement on the right.

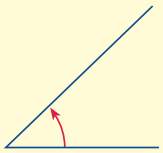
a



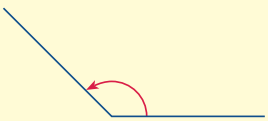
b



c



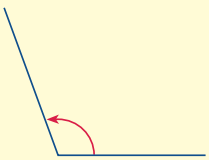
d



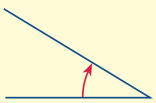
e



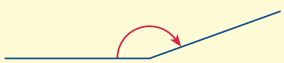
f



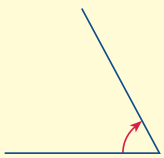
g



h



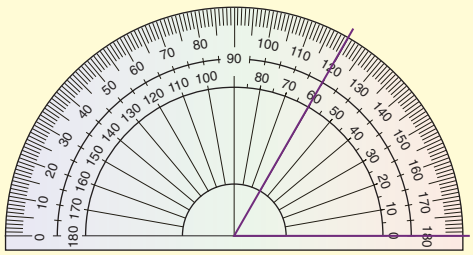
i



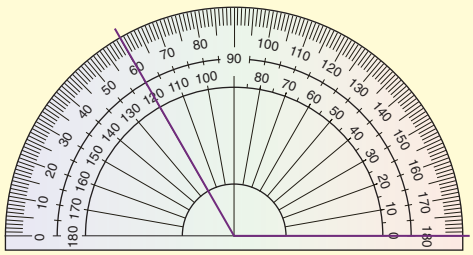
Size
10°
30°
45°
60°
90°
110°
135°
160°
180°

2 Write down the size of the angle in each of the following.

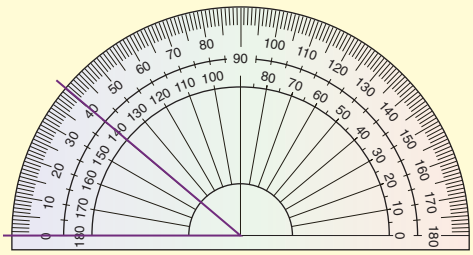
a



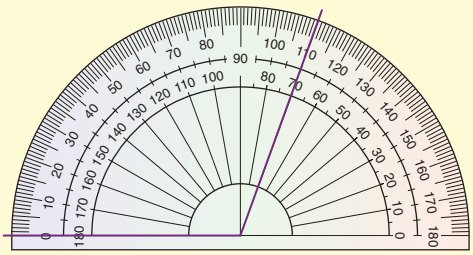
b



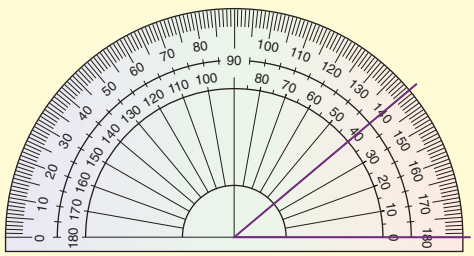
c



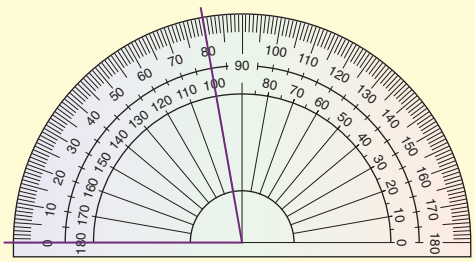
d



e

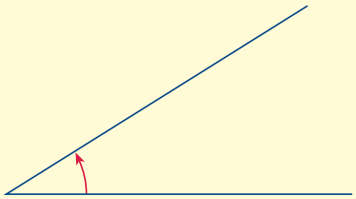


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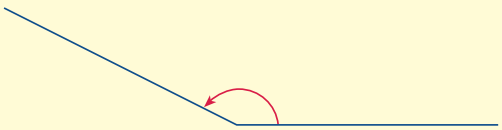


3 Estimate the size of each angle. Then use a protractor to measure its size, giving the answer to the nearest 10°.

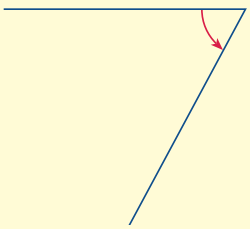
a



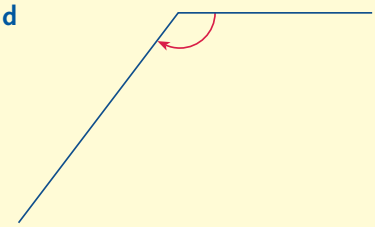
b



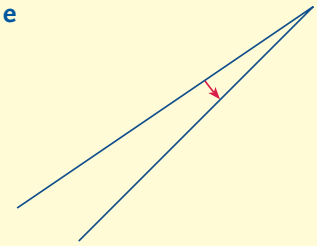
c



d



e



4 Use your protractor to draw angles of size:

a 40° **b** 80° **c** 120° **d** 160° **e** 90°

5 Using a ruler only, draw angles as close as you can to:

a 90° **b** 45° **c** 30° **d** 60° **e** 135°

Check each of your efforts above by measuring with a protractor.

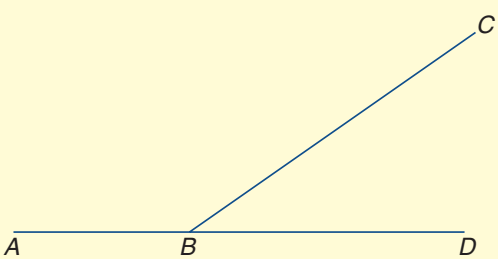
6 Measure the sizes of the angles in question **3** to the nearest degree.

7 Use a protractor to draw angles measuring:

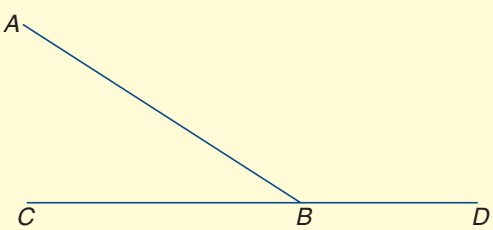
a 35° **b** 75° **c** 87° **d** 104° **e** 138°

8 Measure the size of $\angle ABC$ in each of the following.

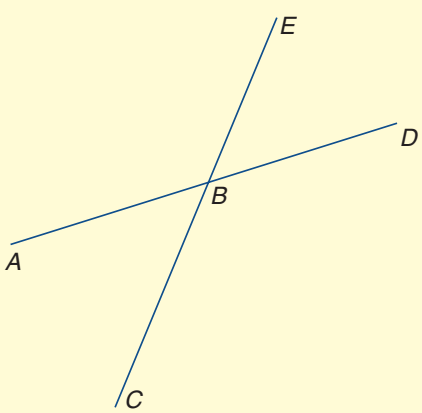
a



b



c

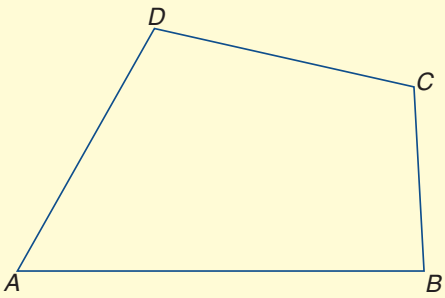




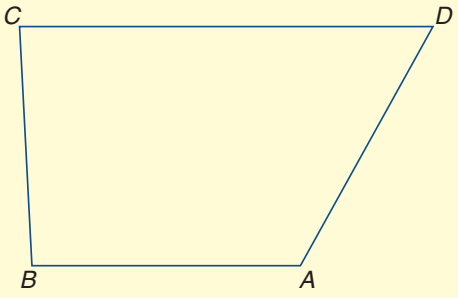
- What angle is made by the legs of this athlete?

9 Measure the size of $\angle A$ and $\angle C$ in each of the following.

a

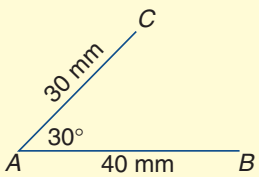


b

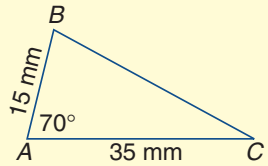


10 Make accurate full-sized drawings of the following using a ruler, compasses and protractor. (You may not need all the instruments for each drawing.)

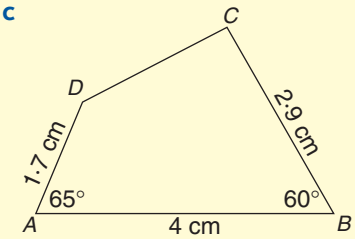
a



b

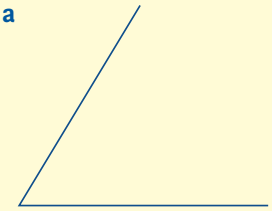


c

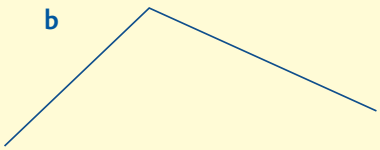


11 Use a protractor to bisect the following angles.

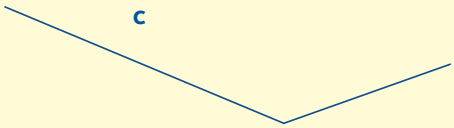
a



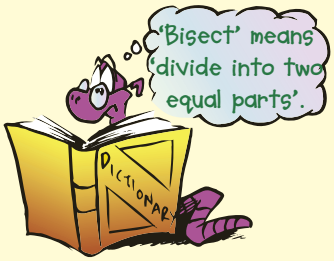
b



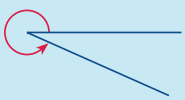
c



12 Use a ruler to draw three different angles. Estimate where the bisector (line which bisects the angle) is and rule it in. Check your estimate with a protractor.



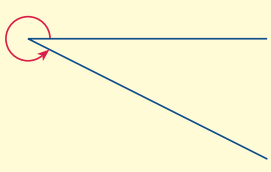
To measure angles larger than 180°, measure the smaller angle and subtract this from 360°.



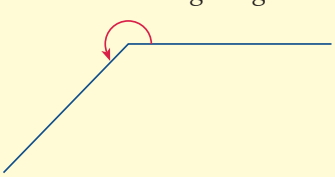
The smaller angle measures 23°.
The larger angles measures 360° – 23°.
∴ The larger angle measures 337°.

13 Use the method above to find the sizes of these large angles.

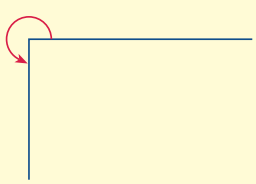
a



b



c

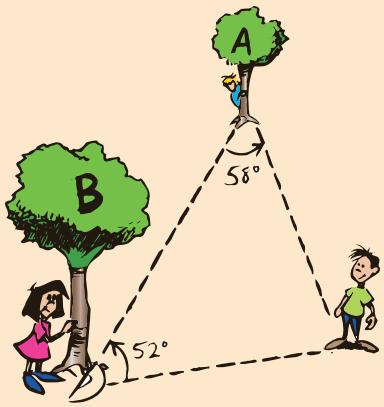


Investigation 10:03 | Triangulation

Please use the Assessment Grid on the following page to help you understand what is required for this Investigation.

Triangulation is a way of fixing a position so that it can be found later by someone else. It is also useful when we want to make a scale drawing.

Bronwyn and Barney wanted to plant a time capsule in the school grounds. They would leave directions in the school records so that the capsule could be found in 20 years.



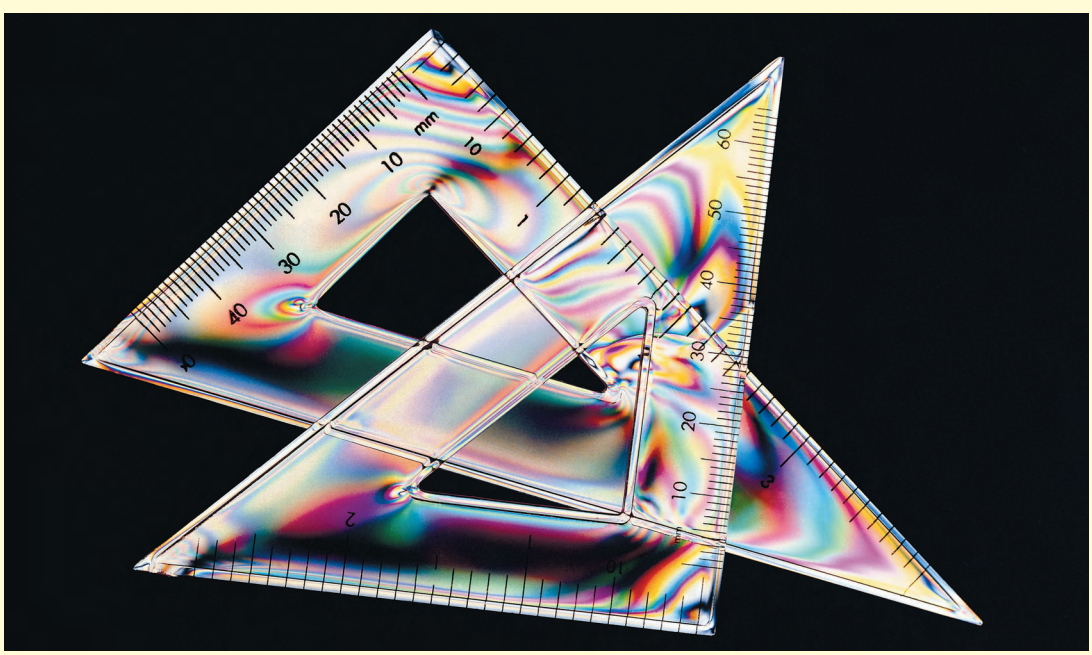
Step 1 They selected two strong young trees and joined them with string.

Step 2 A friend stood at the spot where the time capsule was to be buried.

Step 3 Bronwyn and Barney used a blackboard protractor to measure the angles between the string and the lines from each tree to their friend.

Step 4 They recorded the results: *From the string joining A and B, 58° at A, 52° at B.*

- In a group of three, choose a spot where you would place a time capsule. Record the directions to the spot using the steps listed above.
- Swap information with another group and locate the spot they have chosen.
- Investigate the use of a **theodolite** in surveying.



- Discuss the angles on these set squares.
- What are their sizes?

Assessment Grid for Investigation 10:03 | Triangulation

The following is a sample assessment grid for this investigation. You should carefully read the criteria *before* beginning the investigation so that you know what is required.

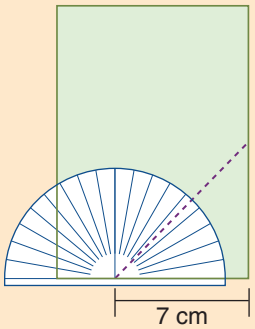
Assessment Criteria (B, C, D)				Achieved ✓
Criterion B Application & Reasoning	a	No systematic approach has been used.	1	
			2	
	b	A systematic and organised approach has been used. An attempt has been made to explain how triangulation works.	3	
			4	
	c	A systematic and organised approach has been used. An explanation of how triangulation works is given.	5	
			6	
	d	A systematic approach has been used successfully and a description of a theodolite is given.	7	
			8	
	e	A systematic approach has been used successfully and the use of a theodolite has been investigated.	9	
			10	
Criterion C Communication	a	No working out is given and directions are hard to follow.	1	
			2	
	b	Directions can be followed and a description of the survey technique is given.	3	
			4	
	c	Full directions are given including diagrams. These are easy to follow.	5	
			6	
Criterion D Reflection & Evaluation	a	Some attempt has been made to explain how the problem was set out, and to check how well the problem was solved.	1	
			2	
	b	The processes are explained and the reliability of the directions has been tested with some success.	3	
			4	
	c	The processes are well explained and the reliability of the directions has been tested successfully.	5	
			6	
	d	Full and reasoned justifications of why triangulation works are given, including possible uses in real life.	7	
			8	

fun spot

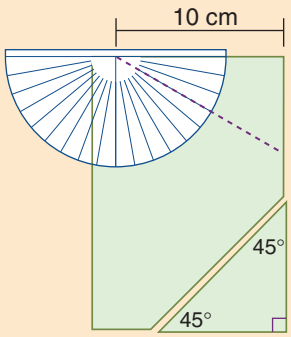
10:03

Fun Spot 10:03 | Making set squares

- Start with cardboard that has square corners.
- **a** Mark a point 7 cm from one of the square corners.
  b Measure an angle of 45° at this point as shown.
  c Continue the arm of this angle until it reaches the side of the cardboard.
  d Cut along the line you have drawn. You have made a 45° set square.

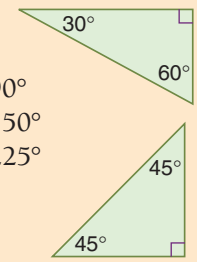


- **a** Mark a point 10 cm from one of the other square corners.
  b Measure an angle of 30° at this point as shown.
  c Continue the arm of this angle until it reaches the side of the cardboard.
  d Cut along the line you have drawn. You have made a 60°/30° set square.



- Use the set squares you have made to draw angles of:

a 30°	**b** 60°	**c** 90°
d 45°	**e** 120°	**f** 150°
g 135°	**h** 210°	**i** 225°

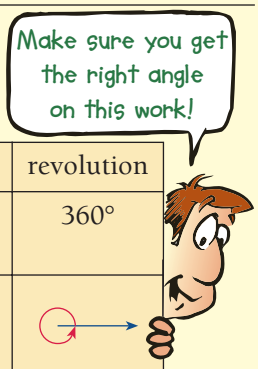


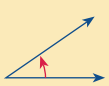
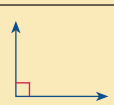
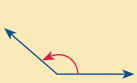
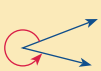


- Stick your set squares onto the inside back cover of your work book using sticky tape. You may have use for them later.

10:04 | Types of Angles

Angles can be classified into different types depending on their size. The different types of angles are given in the table below.



Type	acute	right	obtuse	straight	reflex	revolution
Size	less than 90°	90°	between 90° and 180°	180°	between 180° and 360°	360°
Diagram						

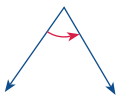
Note:
- A revolution is one complete turn.
- A straight angle is a half turn.
- A right angle is a quarter turn.
- The arms of a straight angle form a straight line.
- The special symbol for a right angle is ⌞.
- The right angle, straight angle and angle of revolution each have one size; 90°, 180° or 360°.



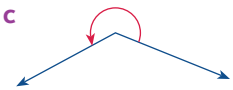
• Why are triangles used in buildings?

worked examples

1 Classify the following angles.

a 

b 

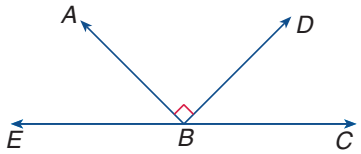
c 

d 

2 From the diagram, classify the following angles.

a $\angle ABC$ **b** $\angle EBC$

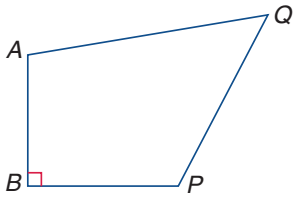
c $\angle ABD$ **d** $\angle DBC$



3 In this diagram, what type of angle is:

a $\angle ABP$? **b** $\angle BAQ$?

c $\angle BPQ$?



Solutions

1 **a** The angle is smaller than a right angle. Hence it is acute.

b A right angle.

c This angle is bigger than a straight angle. Hence it is a reflex angle.

d This angle is bigger than a right angle but smaller than a straight angle. Hence it is an obtuse angle.

2 **a** $\angle ABC$ is obtuse.

b $\angle EBC$ is a straight angle.

c $\angle ABD$ is a right angle.

d $\angle DBC$ is acute.

3 **a** A right angle.

b An obtuse angle.

c An obtuse angle.

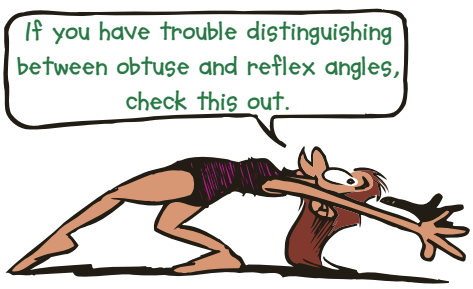


Figure 1

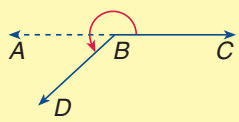
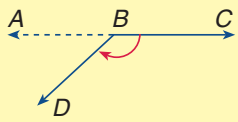


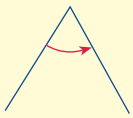
Figure 2

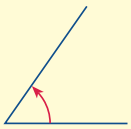


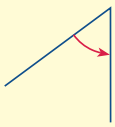
In both figures $\angle ABC$ is a straight angle.
- In Figure 1, $\angle DBC$ is bigger than a straight angle. Hence $\angle DBC$ is reflex.
- In Figure 2, $\angle DBC$ is smaller than a straight angle. Hence $\angle DBC$ is obtuse.

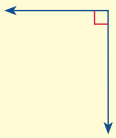
Exercise 10:04

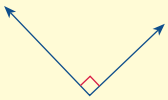
1 Classify the following angles.

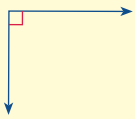
a 

b 

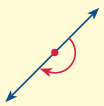
c 

d 

e 

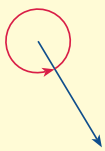
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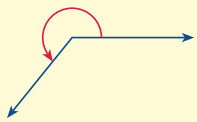
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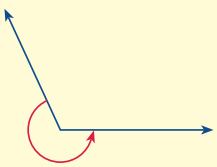
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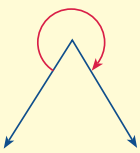
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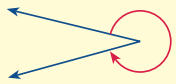
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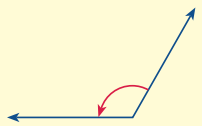
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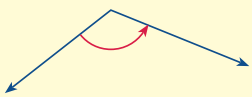
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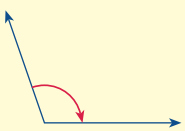
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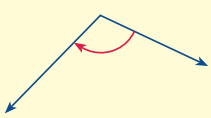
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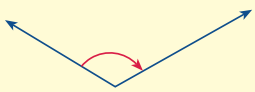
o 

p 

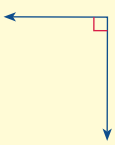
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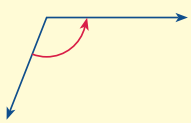
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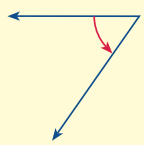
s 

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v 

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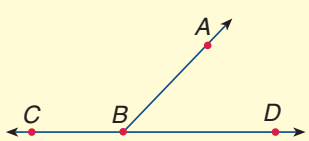
x 

2 Classify angles that have the following sizes.

a 90°	**b** 180°	**c** 360°	**d** 100°	**e** 89°	**f** 98°
g 210°	**h** 270°	**i** 27°	**j** 135°	**k** 300°	**l** 170°
m 30°	**n** 60°	**o** 95°	**p** 360°	**q** 90°	**r** 179°

3 In this diagram, what type of angle is:

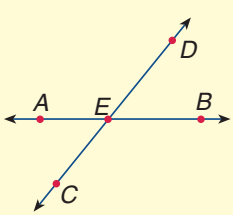
a $\angle ABD$? **b** $\angle ABC$? **c** $\angle CBD$?



4 In the diagram, what type of angle is:

a $\angle AEB$? **b** $\angle DEC$? **c** $\angle AEC$?

d $\angle BED$? **e** $\angle AED$? **f** $\angle BEC$?

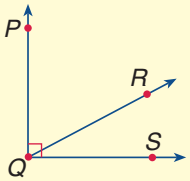


5 In the diagram, what type of angle is:

a $\angle PQS$?

b $\angle PQR$?

c $\angle RQS$?

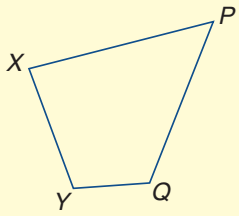


6 In the diagram, what type of angle is:

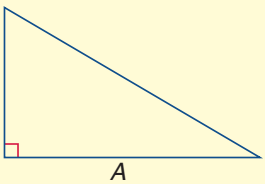
a $\angle XYQ$?

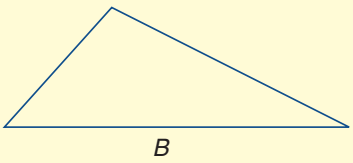
b $\angle PXY$?

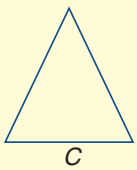
c $\angle XPQ$?



7 Study each of the triangles below and then answer the questions.





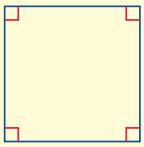


a Which triangle has 3 acute angles?

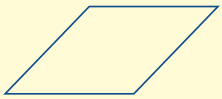
b Which triangle has an obtuse angle?

8 Classify the angles in each of the following quadrilaterals.

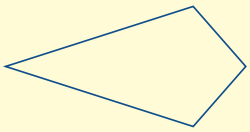
a

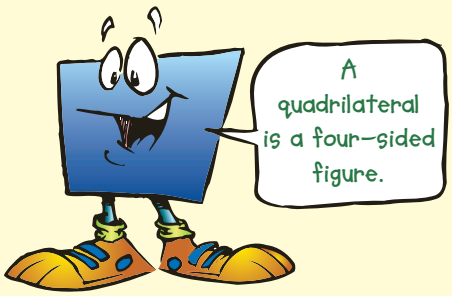


b



c





Reading Mathematics 10:04 | Smoking and health facts!

reading mathematics 10:04

- Each year, at least 154 900 US residents die needlessly of diseases caused by their smoking.
- This is more than three times the number of road deaths.
- Cigarette smoke contains more than 1000 different substances, many of which cause serious damage to health.

What are the risks?

- Smoking is a major risk factor for myocardial infarction, sudden cardiac death and atherosclerotic peripheral vascular disease, which can result in gangrene and loss of limbs. Up to 4 out of 10 smokers will die due to their smoking.
- Heavy smokers are the group of smokers most at risk of premature death.
- 40% of heavy (more than 25/day) smokers will die before the age of 65, compared with 15% of non-smokers.

- Habitual 20-a-day smokers shorten their lives by 5 years, on average. Forty-a-day smokers shorten their lives by 8 years, on average. This means that the average habitual smoker's life is shortened by 5·5 minutes for each cigarette smoked — not much less than the time it takes to smoke it.
- Nine out of every 10 lung cancer victims have been smokers.
- A smoker is more than twice as likely as a non-smoker to die of a heart attack. But once the smoker quits, the risk of heart attack is reduced to no more than the risk for a non-smoker.
- Men below the age of 45 who smoke 15 to 24 cigarettes per day are 9 times more likely to die from coronary heart disease than men of the same age who don't smoke. For those smoking more than 25 cigarettes a day, the risk is about 14 times greater.
- Smokers are 20 times more likely than non-smokers to die from crippling respiratory diseases such as emphysema and chronic bronchitis.



Exercise

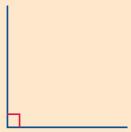


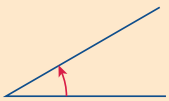
1 How many US residents die of diseases caused by their smoking?
2 How do deaths caused by smoking compare with road deaths?
3 What fraction of smokers would probably have died due to their smoking?
4 What percentage of people would have died before the age of 65 if they were:
   a heavy smokers?
   b non-smokers?
5 By how much does each cigarette shorten the life of the average habitual smoker?
6 What fraction of lung cancer victims have been smokers?
7 How much more likely is a smoker to have a heart attack than a non-smoker?
8 Men smoking more than 25 cigarettes a day and who are below the age of 45 have a greater risk of heart disease. How much greater is this risk?

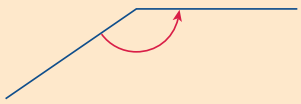
10:05 | Discovering more about Angles

prep quiz 10:05

What type of angle is each of the following?

1 

2 

3 

4 

What is the size in degrees of:

5 a right angle? **6** a straight angle?

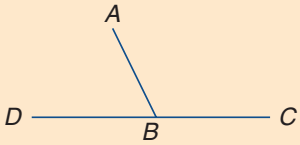
7 an angle of revolution?

Copy the diagram on the right.

8 Mark $\angle ABD$ with a '$*$'.

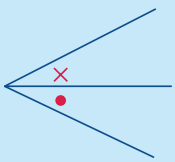
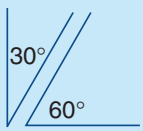
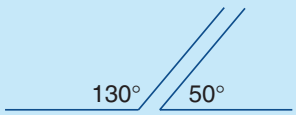
9 Mark $\angle CBA$ with a '$\bullet$'.

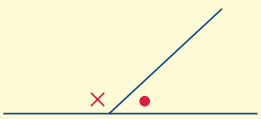
10 What is the size of $\angle DBC$?



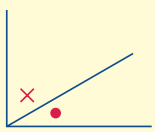


Describing angles

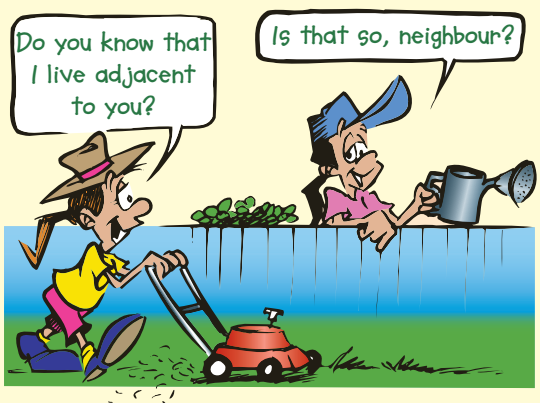
Adjacent angles	*Complementary angles*	*Supplementary angles*
• Next to each other.  They have the same vertex and are alongside each other.	• Add up to 90°.  30° is the complement of 60°.	• Add up to 180°.  130° is the supplement of 50°.



These angles are adjacent *and* supplementary.



These angles are adjacent *and* complementary.

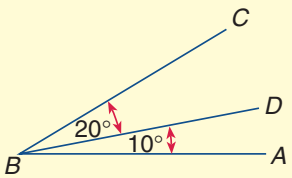


Adjacent angles

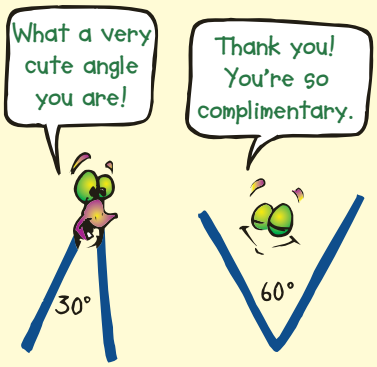
- **have a common vertex**
- **have a common arm**
- **and lie on opposite sides of that common arm.**

Exercise 10:05

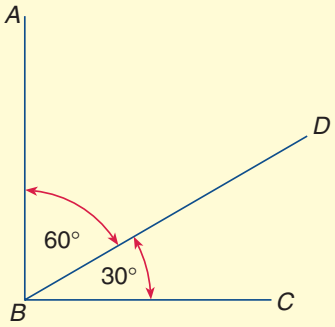
1



a Name a pair of adjacent angles.
b Find the size of $\angle ABC$.

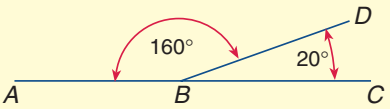


2



a Name a pair of adjacent angles.
b Name a pair of complementary angles.
c Find the size $\angle ABC$.

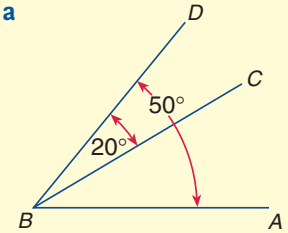
3



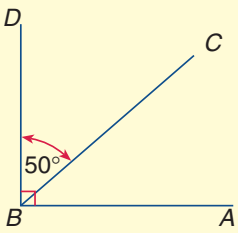
a Name a pair of adjacent angles.
b Name a pair of supplementary angles.
c Is $\angle ABC$ a straight angle?

4 Find the size of $\angle ABC$ in each of the following.

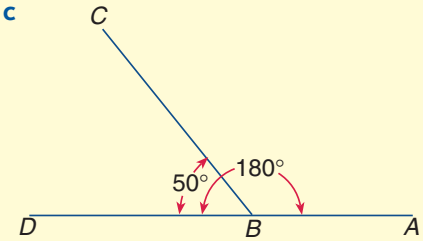
a



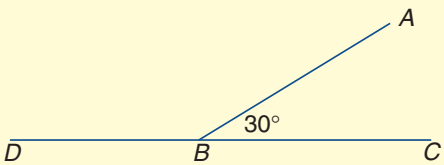
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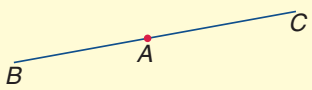
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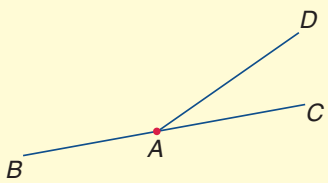


5
a Draw a 30° angle as shown.
b Extend the arm CB with a ruler to any point D.
c Is $\angle DBC$ a straight angle? What is its size?
d Measure $\angle DBA$. What is its size?
e Is $\angle ABC + \angle DBA = 180°$ true?
f Are $\angle DBA$ and $\angle ABC$ supplementary? Are they adjacent?

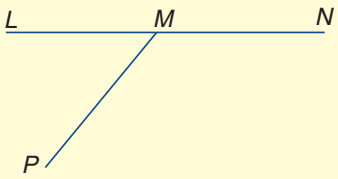


6
a Draw a straight line BC and mark a point A between B and C, as shown in the diagram.
b What is the size of $\angle BAC$?
c Select any point D and join it to A.
d Measure $\angle BAD$ and $\angle DAC$.
e Is $\angle BAD + \angle DAC = 180°$ true? Should it be?
f Name a pair of adjacent angles that are supplementary.



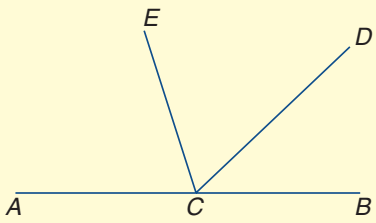


7



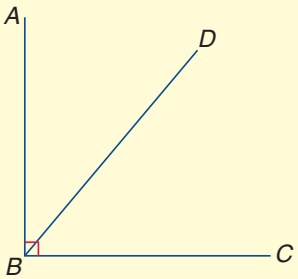
a Use a ruler to check if LN is a straight line.
b What is the size of $\angle LMN$?
c Measure $\angle LMP$ and $\angle PMN$.
d Are they supplementary? Are they adjacent?
e Is $\angle LMP + \angle PMN = \angle LMN$ true?
f Is $\angle LMP + \angle PMN = 180°$ true?

8



Draw a diagram like the one shown.
a Measure $\angle ACE$, $\angle ECD$ and $\angle DCB$.
b Is $\angle ACB$ a straight angle? What is its size?
c Is $\angle ACE + \angle ECD + \angle DCB = 180°$ true? Should it be?

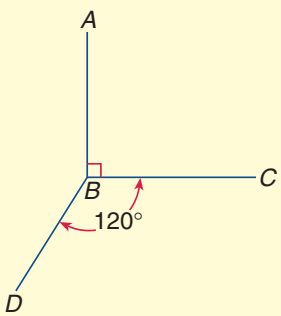
9 **a** Construct a right angle, $\angle ABC$, with vertex at B, and choose any point D between the arms of the angle and join it to B (see diagram).
b Measure $\angle ABD$ and $\angle DBC$.
c Is $\angle ABD + \angle DBC = 90°$ true? Should it be?



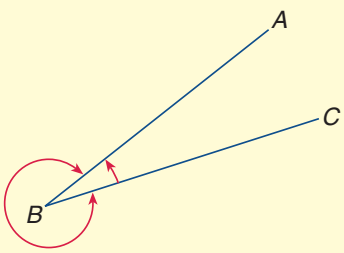
10 Construct the figure shown in the diagram. Can you predict the size of $\angle ABD$ from what you know about an **angle of revolution**? Check your prediction by measurement.



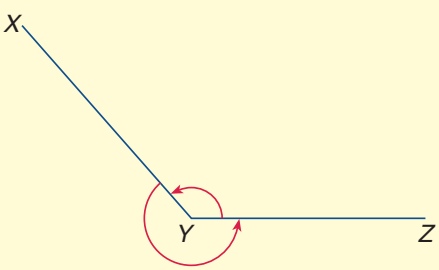
The angles at a point add up to 360°.



11 The diagram shows an acute angle ABC and a reflex angle ABC.
a Measure the size of the acute angle ABC.
b Using what you know about an angle of revolution and the answer to part **a**, calculate the size of the reflex angle ABC.

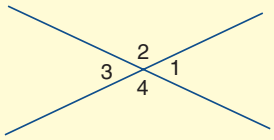


12



The diagram shows an obtuse angle XYZ and a reflex angle XYZ.
a Measure the size of the obtuse angle XYZ.
b Using your answer to part **a**, calculate the size of the reflex angle XYZ.

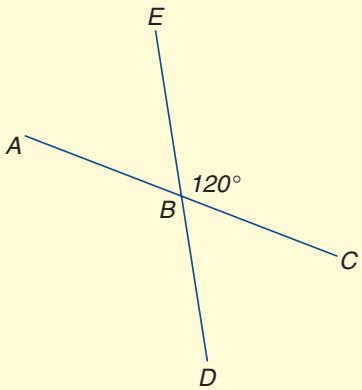
13 When two straight lines cross, two pairs of **vertically opposite angles** are formed (the pairs are 1 and 3, and 2 and 4).



a Which angles look to be equal?
b Check your answer to **a** by measurement.
c Check your answer to **a** by tracing the lines, cutting out the angles and matching them.
d What can you say about vertically opposite angles?

Vertically opposite angles are equal.

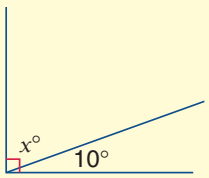
14



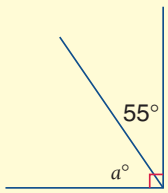
AC and *ED* are straight lines.
a What is the size of $\angle ABE$? Why?
b What is the size of $\angle DBC$? Why?
c Is it true that $\angle ABE = \angle DBC$?
d Are vertically opposite angles equal?

15 Without measuring, work out the value of the pronumeral in each of the following.

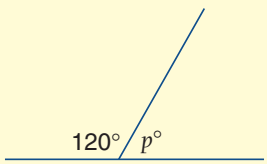
a



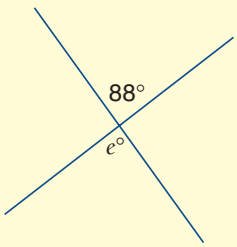
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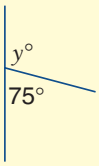
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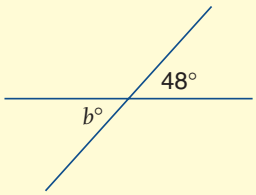
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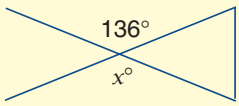
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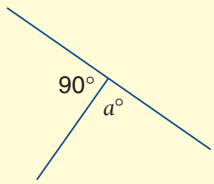
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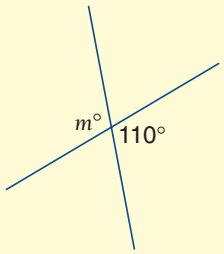
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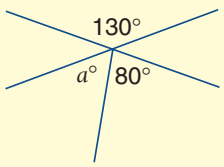
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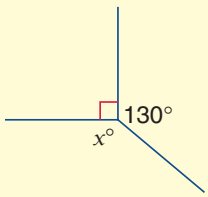
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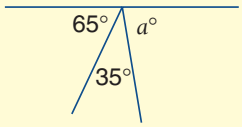
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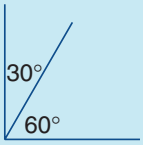


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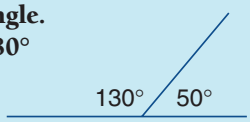


From the last exercise we discovered that:

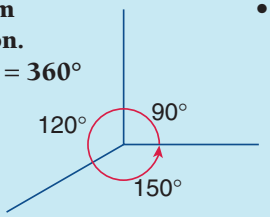
- ***Adjacent complementary* angles form a right angle.**
  eg 30° + 60° = 90°



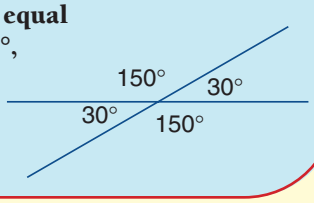
- ***Adjacent supplementary* angles form a straight angle.**
  eg 130° + 50° = 180°



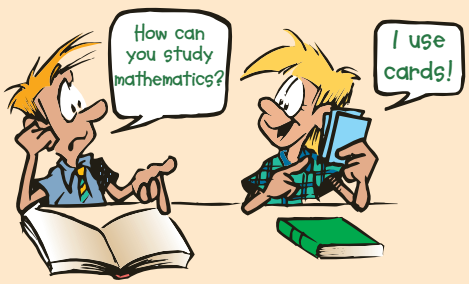
- ***Angles at a point* form an angle of revolution.**
  eg 120° + 90° + 150° = 360°



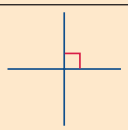
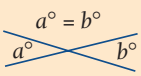
- ***Vertically opposite angles*, formed when straight lines intersect, are equal**
  eg 150° = 150°,
  30° = 30°



Reading Mathematics 10:05 | Need some help?



- On one side of the card write the things you need to learn.
- On the back of the card write the answers,
  eg **1** interval *AB*
  2 perpendicular lines
  3 collinear points
  4 1, 12, 2, 6, 3 and 4
  5 vertically opposite angles
  6 $6 \times 6 \times 6 = 216$
  7 ( ), × and ÷, + and –
  8 49
- Carry the card with you and test yourself over and over again until you know every fact.

1 	**2**  . . . . . lines
3  . . . . . points	**4** The factors of 12 are: . . . . .
5  . . . . . angles	**6** 6^3
7 What's the order of operations?	**8** 7×7

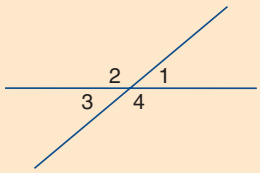
10:05A Supplementary and complementary angles
10:05B Flower power

10:06 | Angles and Parallel Lines

prep quiz 10:06

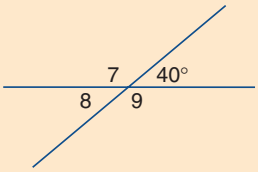
In the diagram

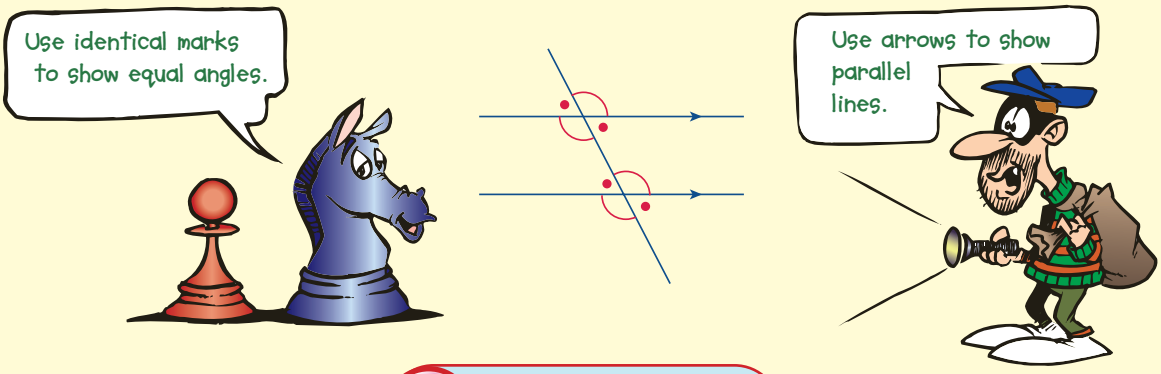
1 Which angle is equal to angle 1?
2 Which angle is equal to angle 2?
3 What can you say about the sizes of angles 2 and 3?
4 What can you say about the sizes of angles 3 and 4?
5 What can you say about the sizes of angles 1 and 4?
6 $\angle 1 + \angle 2 + \angle 3 + \angle 4 = \ldots\ldots\ldots$



In the diagram, find the size of angles **7**, **8** and **9**.

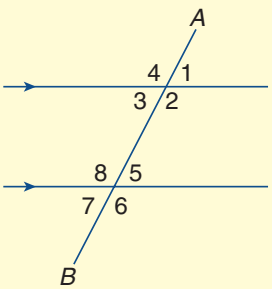
10 Are vertically opposite angles equal?

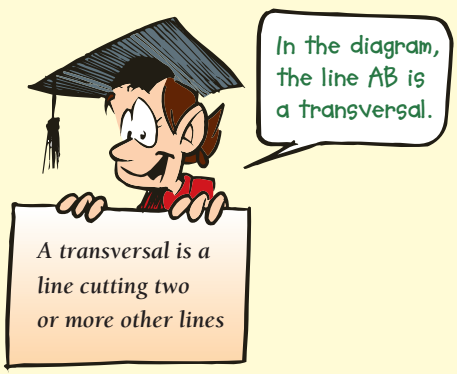




Angles marked in the same way are equal.

If two parallel lines are cut by another line (called a **transversal**), eight angles are formed, as shown in the diagram.



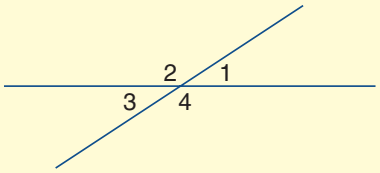


In the next exercise we will learn about some of the relationships between these angles.

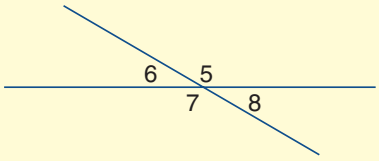
Exercise 10:06

1 In each diagram, which angles are equal?

a

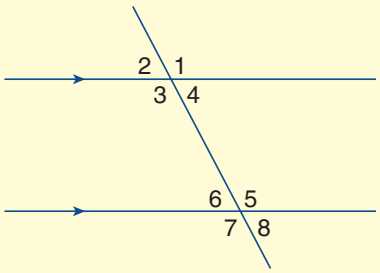


b

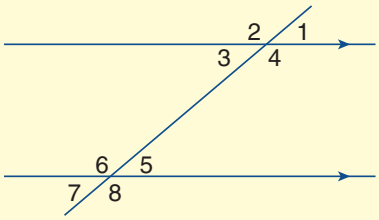


2 In each diagram, measure the angles 1 to 8. Which angles are equal?

a



b



3 From the answers to question **2**, what did you find out about the sizes of the following pairs of angles?

a 1 and 5 **b** 3 and 7 **c** 2 and 6 **d** 4 and 8

The pairs of angles in question **3** are called **corresponding angles** because they are in corresponding positions relative to the transversal and a parallel line. For instance:

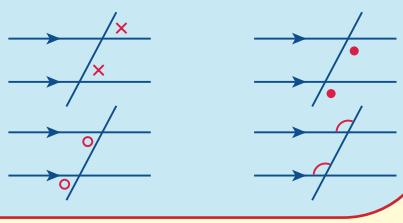
- 1 and 5 are both above a parallel line and to the right of the transversal.
- 2 and 6 are both above a parallel line and to the left of the transversal.

4 How would you describe the positions of angles

a 3 and 7? **b** 4 and 8?

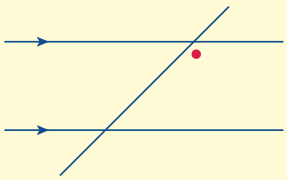


When two lines are parallel, four pairs of equal *corresponding angles* are formed by a transversal.

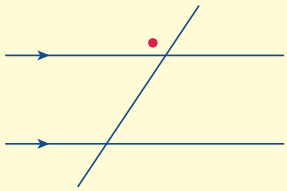


5 Sketch the diagrams below and mark in the angle that is **corresponding** to the angle marked with the dot.

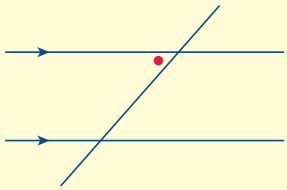
a



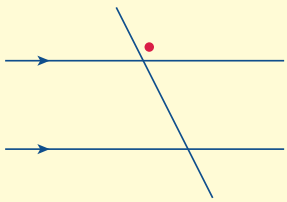
b



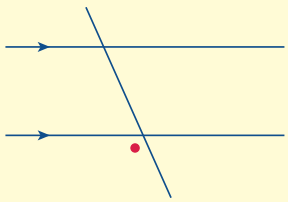
c



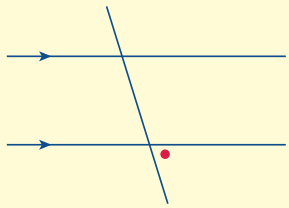
d



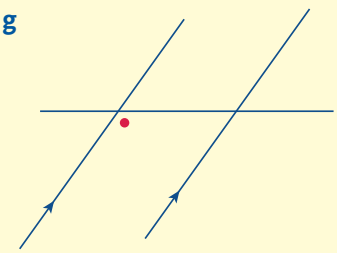
e



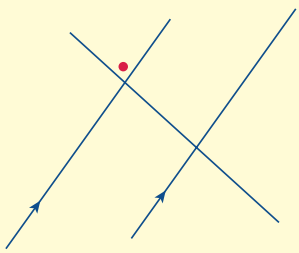
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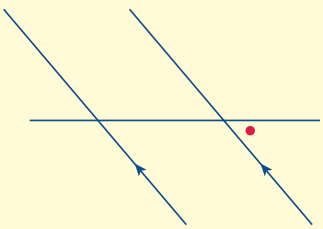
g



h

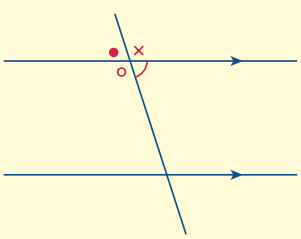


i

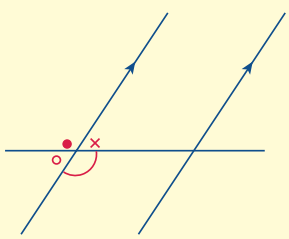


6 Sketch each diagram and mark in the four pairs of corresponding angles. Use the symbols **x**, ●, ○ and).

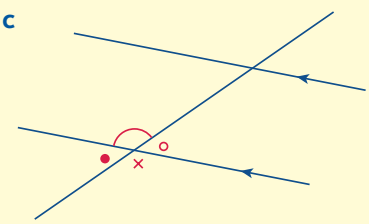
a



b



c

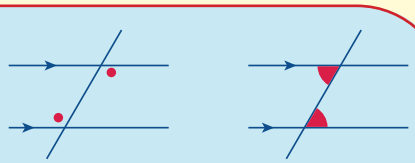


7 Using the results of question **2**, what did you find out about the sizes of the following pairs of angles?

a 3 and 5 **b** 4 and 6

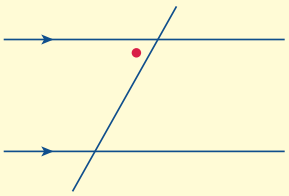
The pairs of angles used in this question are called **alternate angles**.

When two lines are parallel, two pairs of equal *alternate angles* are formed by a transversal.

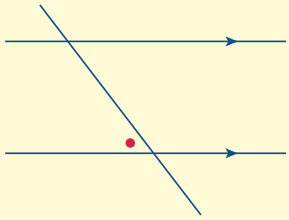


8 Sketch these diagrams and mark in the angle that is **alternate** to the one given in the diagram.

a

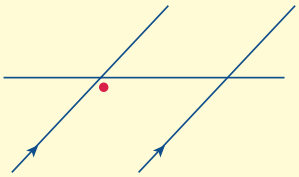


b

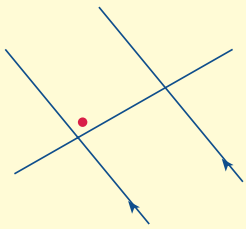




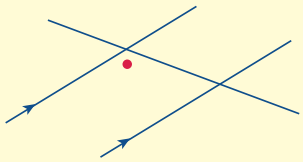
c



d

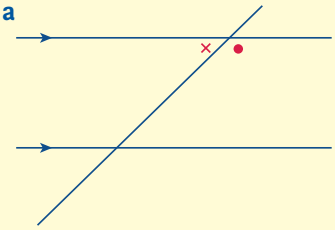


e

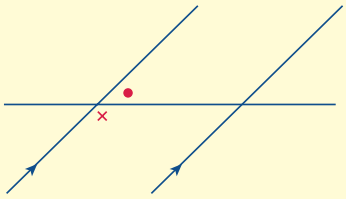


9 Sketch the following diagrams and mark in both pairs of alternate angles. Use the symbols **x** and ●.

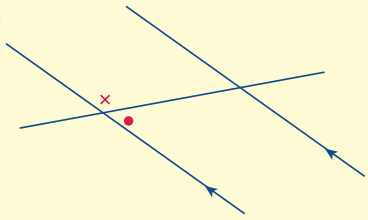
a



b



c

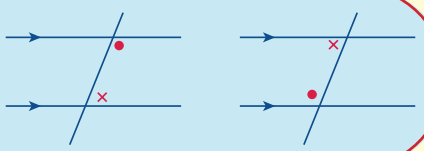


10 Using the results of question **2**, what did you find out about the sizes of the following pairs of angles?

a 4 and 5 **b** 3 and 6

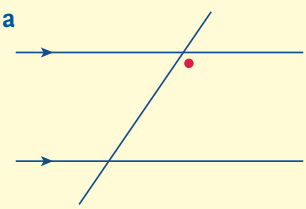
The pairs of angles used in this question are called **co-interior angles**. Co-interior angles lie between the parallel lines and are on the same side of the transversal.

When two lines are parallel, the two pairs of *co-interior angles* formed by a transversal are supplementary.

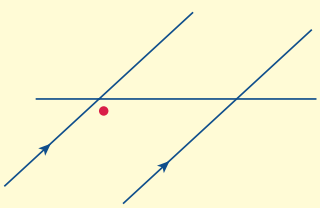


11 Sketch the diagrams and mark in the angle that is **co-interior** to the one given in the diagram.

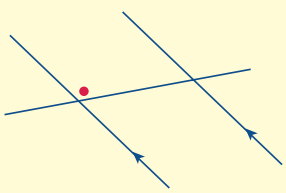
a



b

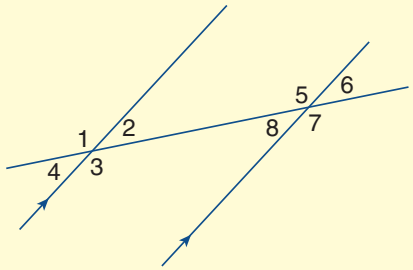


c



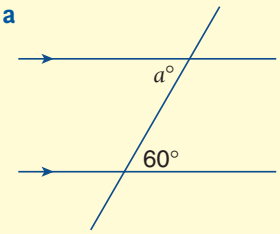
12 In the diagram:

a Which angle is co-interior to angle 2?
b Which angle is co-interior to angle 8?
c Which angle is alternate to angle 3?
d Which angle is alternate to angle 2?
e Which angle is corresponding to angle 1?
f Which angle is corresponding to angle 7?

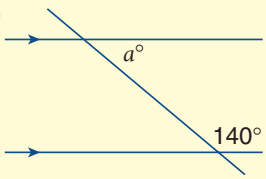


13 Find the value of the pronumeral in each of the following by first identifying corresponding, alternate or co-interior angles.

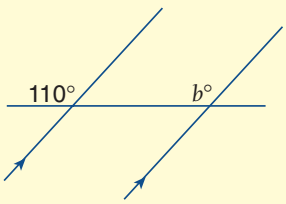
a



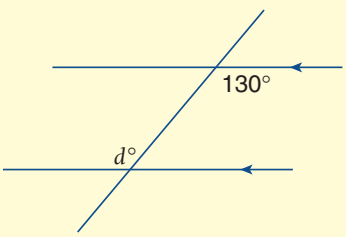
b



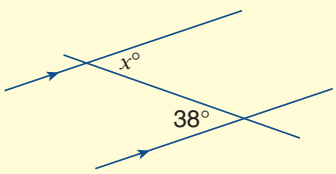
c



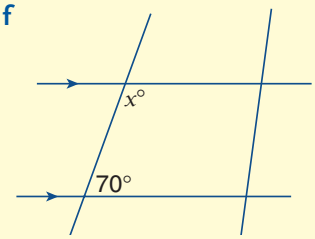
d



e

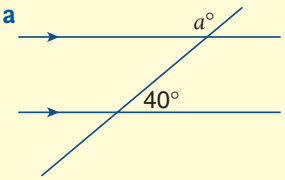


f

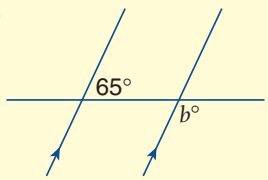


14 Use the properties of vertically opposite angles, adjacent angles making straight angles, and corresponding, alternate and co-interior angles to find the value of each pronumeral.

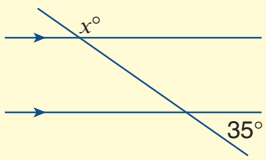
a



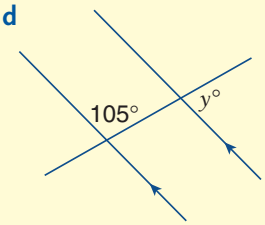
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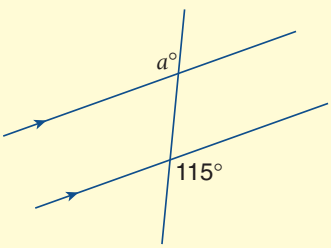
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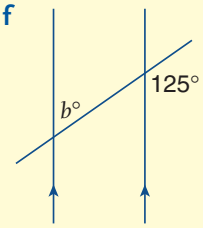
d



e



f

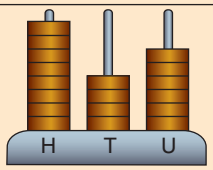
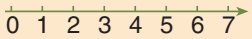
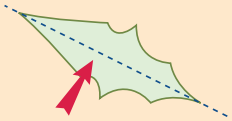
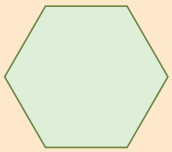
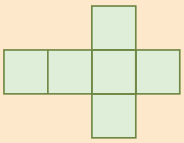
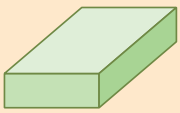


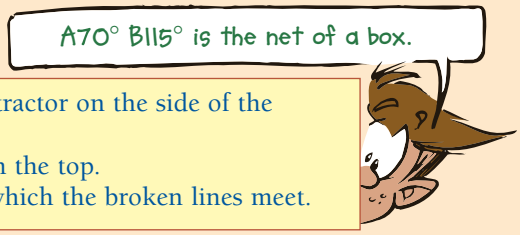
id

Refer to ID card 6 on page xviii.
Identify figures 1 to 20. Learn the terms you do not know.

10:06

Challenge 10:06 | Getting the angle on things: Triangulation

2, 4, 6, . . . are . . . . . numbers	1, 3, 5, . . . are . . . . . numbers	814 has three	 . . . . .
 is a number	 line of		 . . . . .
	 . . . . . of a box	 . . . . . prism	 . . . . .

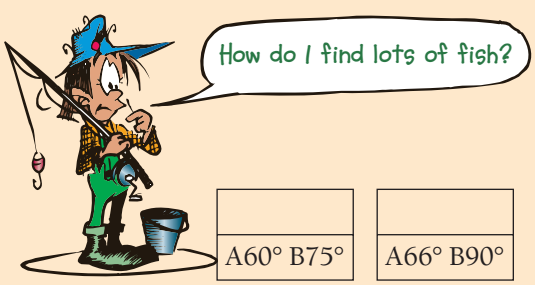


- To do the questions below, place your protractor on the side of the grid as shown above.
- Measure the angles at *A* and *B* around from the top.
- Write the missing word for the square in which the broken lines meet.

1 Find the missing word for each reference.

a A55° B85°	**b** A10° B25°	**c** A70° B115°	**d** A40° B60°
e A60° B75°	**f** A52° B70°	**g** A70° B87°	**h** A25° B70°
i A66° B90°	**j** A80° B105°	**k** A50° B140°	**l** A85° B102°

2 Use the first letter of each missing word to decode the answer to the question below.



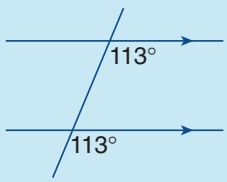
	A60° B75°	A66° B90°	A85° B102°	A10° B25°	A70° B115°	A52° B70°
A55° B85°	A50° B140°	A70° B87°	A40° B60°	A40° B60°	A25° B70°	A55° B85°

10:07 | Identifying Parallel Lines

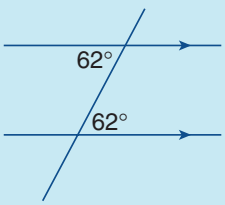
- To determine whether a pair of lines are parallel we could extend them in both directions to see if they meet. However, if they do not meet we could not be sure that they are parallel as we may not have extended the lines far enough.
- In real life, there are other problems caused by accuracy of drawing and measurement, but in diagrams, knowing the size of angles around a transversal will tell us whether two lines are parallel.

Two lines are parallel if:

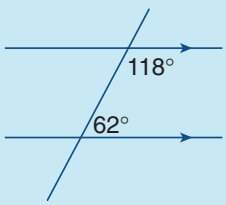
- **corresponding angles are equal**



- **alternate angles are equal**



- **co-interior angles are supplementary**

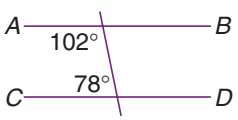


In each case above the lines must be parallel.

Exercise 10:07

worked example

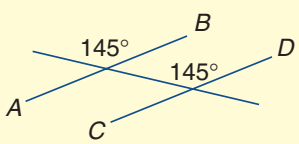
Is *AB* parallel to *CD*? Give reasons.



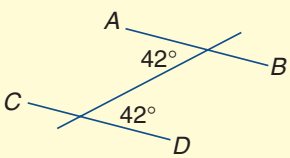
Yes. *AB* || *CD* because co-interior angles are supplementary.

1 In each case, is *AB* parallel to *CD*? Give reasons.

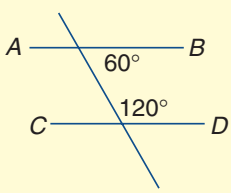
a



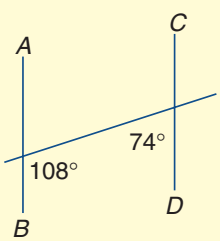
b



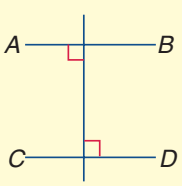
c



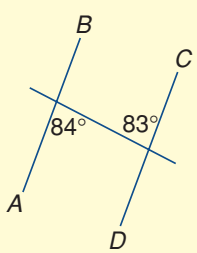
d



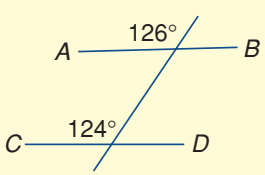
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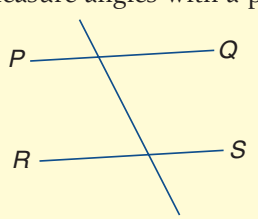
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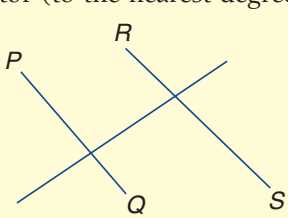
|| means 'is parallel to'
⊥ means 'is perpendicular to'

2 Measure angles with a protractor (to the nearest degree) to see if *PQ* is parallel to *RS*.

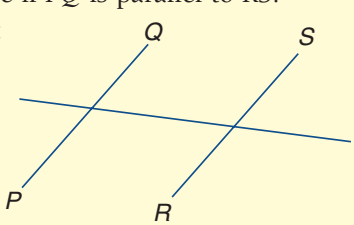
a



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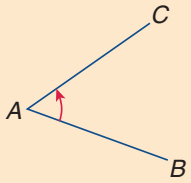


c



Mathematical terms 10

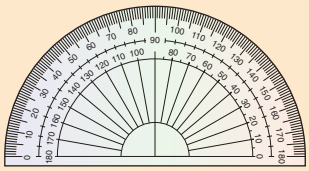
angle
- The amount of turning between two rays (half-lines) that start from the same point.



arm
- A ray of an angle.

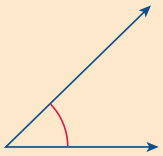
vertex
- The point where the two arms of an angle meet.

protractor
- An instrument used for measuring the size of an angle.

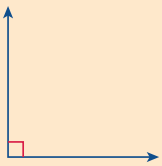


degree
- The unit used to measure angles.
- The symbol for degree is '°'.
  eg 60°

acute angle
- An angle that is less than 90°.



right angle
- An angle that measures 90°.



obtuse angle
- An angle that measures between 90° and 180°.



straight angle
- An angle that measures 180°.



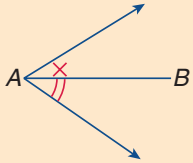
reflex angle
- An angle that measures between 180° and 360°.



revolution
- An angle that measures 360°.
- One complete turn or rotation.



adjacent angles
- Angles that have the same vertex and a common arm. They lie on opposite sides of the common arm.



complementary angles
- Two angles that add up to 90°.

complement
- The angle that needs to be added to a given angle to give 90°.
  eg The complement of 60° is 30°.

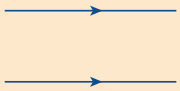
supplementary angles
- Two angles that add up to 180°.

supplement
- The angle that needs to be added to a given angle to give 180°.
  eg The supplement of 120° is 60°.

vertically opposite angles
- These are equal angles, formed when two straight lines intersect.

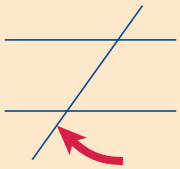


parallel lines
- Straight lines, in the same plane, that do not meet.



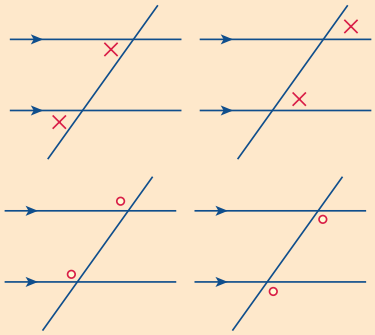
transversal

- A line that crosses two or more other lines.



corresponding angles (and parallel lines)

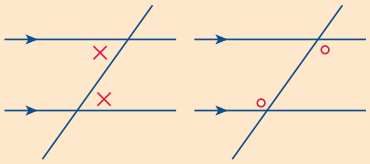
- These are in corresponding or matching positions relative to the transversal and a parallel line.



- They are equal if the lines are parallel.
- There are 4 pairs in the diagrams above.

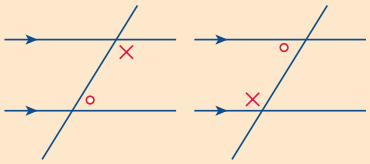
alternate angles (and parallel lines)

- They lie 'inside' the parallel lines and on opposite sides of the transversal.
- They are equal if the lines are parallel.



co-interior angles (and parallel lines)

- They lie 'inside' the parallel lines and on the same side of the transversal.
- They are supplementary.





- What is the total value of these stamps?

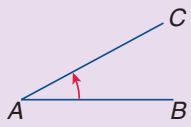
Diagnostic Test 10: | Angles

- Each section of the test has similar items that test a certain type of example.
- Failure in more than one item will identify an area of weakness.
- Each weakness should be treated by going back to the section listed.

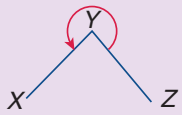
Section

1 Name the following angles and their angle type. 10:02 10:04

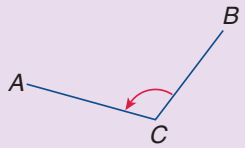
a



b

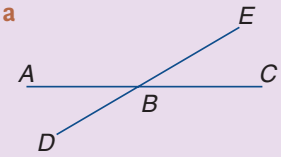


c

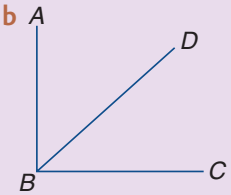


2 Copy each figure and mark $\angle ABC$ with a '))'. 10:02

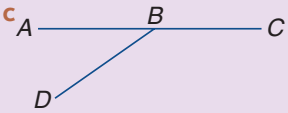
a



b

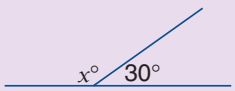


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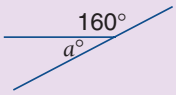


3 Find the value of the pronumeral in each of the following. 10:05

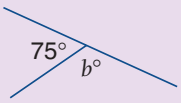
a



b

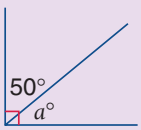


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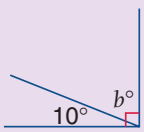


4 Find the value of the pronumeral in each of the following. 10:05

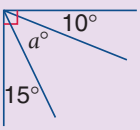
a



b

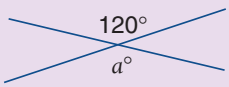


c

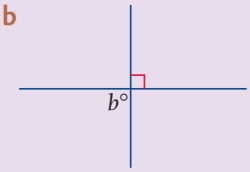


5 Find the value of the pronumeral in each of the following. 10:06

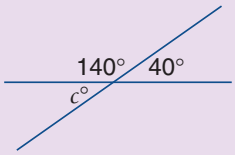
a



b

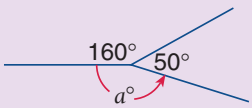


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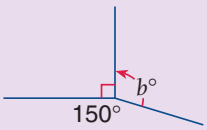


6 Find the value of the pronumeral in each of the following. 10:06

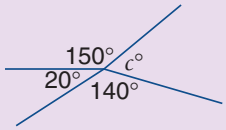
a



b

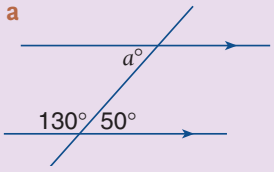


c

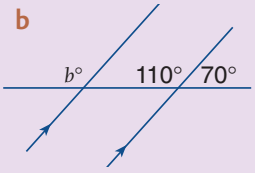


7 Find the value of the pronumeral in each of the following. 10:06

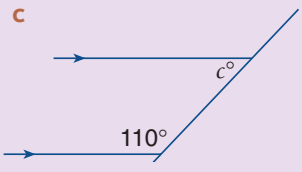
a



b



c



assignment 10A

Chapter 10| Revision Assignment

1 Convert to decimals:

a $\frac{7}{10}$ **b** $\frac{42}{100}$ **c** $\frac{175}{1000}$

d $\frac{5}{100}$ **e** $\frac{27}{1000}$

2 Complete each table using the rule given.

a $y = x + 3$

x	1	2	3	4
y				

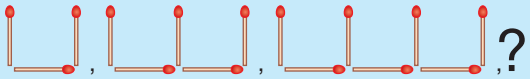
b $b = 8 \times t$

t	0	2	4	6
b				

c $n = 5 \times m + 1$

m	0	1	2	3
n				

3 Complete the table of values for this matchstick pattern.



Number of cups	1	2	3	4
Number of matches				

4 Write each expression without multiplication signs.

a $6 \times m$ **b** $3 \times m \times n$

c $4 \times y \times 3$ **d** $2 \times a + 3 \times b$

5 Find the value of each expression if $a = 3$, and $b = 6$.

a ab **b** $(a + b)^2$

c $a^2 + b^2$ **d** $\frac{b + 6}{a}$

6 Evaluate the following if $a = -2$.

a $3a$ **b** $a + 2$ **c** a^2

d $5 - a$ **e** $3a + 4$

7 Find the time difference between the following times:

a 1 pm and 4 pm

b 9 pm and 3 am

c 10 am and 2 pm

d 2 am and 7 am

8 Simplify:

a $-2x - 3x$ **b** $x + 8x - y$

c $2x - 10x$ **d** $-1 + 3x - 4$

9 Write in index form:

a $2 \times 2 \times 2$ **b** $3 \times 3 \times 3 \times 3$

c $a \times a \times b \times b \times b$ **d** $2 \times a \times a \times b \times a$

10 How many sides has a:

a pentagon? **b** hexagon?

c octagon? **d** trapezium?

11 **a** Which fraction is the larger, $\frac{2}{3}$ or $\frac{3}{4}$?

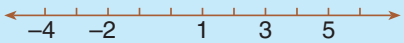
b How many fifths in $3\frac{3}{5}$?

c Write $\frac{7}{2}$ as a mixed numeral.

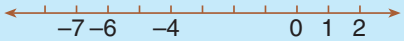
d Which is larger, $\frac{5}{3}$ or $\frac{5}{4}$?

12 Place a number under each mark to complete the number lines below.

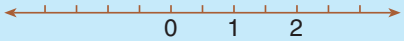
a



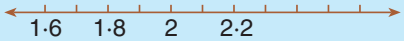
b



c



d



Drag and Drop Interactive

1 Types of angles
2 Finding the size of angles A
3 Finding the size of angles B
4 Finding the size of angles C

Review Questions

Chapter 10 | Working Mathematically

1 A saleswoman drove from town A to town B and back again. At the start of her journey her car's odometer read 21 857 km, while at the end of her journey it read 22 173 km. How far was it from town A to town B?

2 Jan works 4 hours a day from Monday to Friday and 6 hours on Saturday. She is paid $7.50 an hour from Monday to Friday and $10.75 an hour on Saturday. What are her weekly earnings?

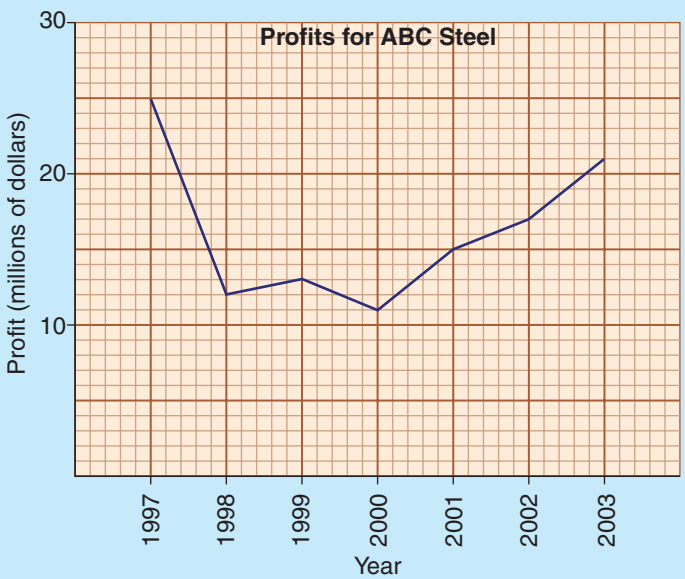
3 I buy three bags of chips and four pieces of fish at the local fish shop. From the price list I can see that chips cost 80c a bag but no price is given for a piece of fish. If I give $10 and receive $1.20 change, how much was a piece of fish?

4 Tyres for a car cost $58 each. To balance a tyre costs $3.50 and tubes for the tyres cost $10.75 each. Find how much it will cost to put four tyres on the car if three of them require tubes and the two front tyres have to be balanced.



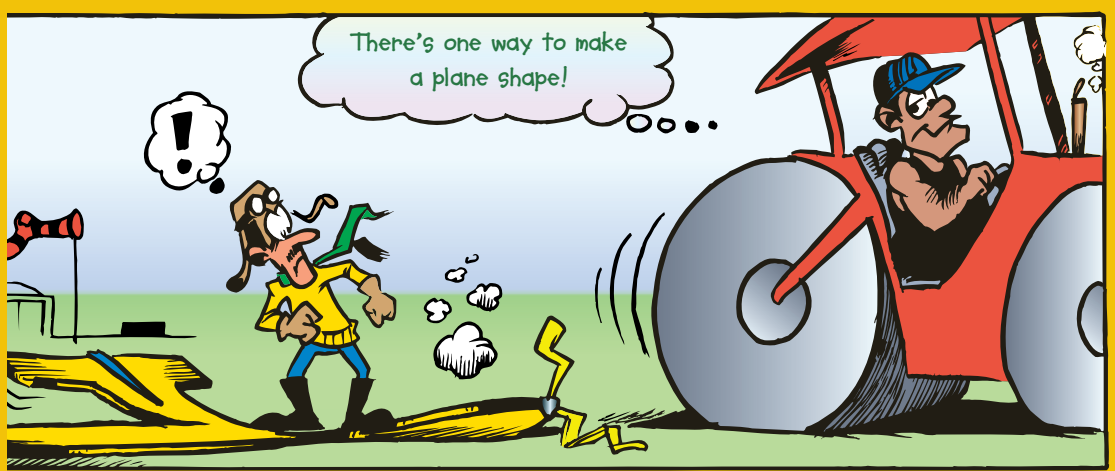
5 In a 1500 m run Fred came in first. Noel came last. Bob was ahead of Steve, and Alan was just behind Steve. If there were five people in the race, who came in second?

6



a What was the profit in:
i 1997? ii 2001?
b When was the largest profit made?
c When was the smallest profit made?
d By how much did the profit fall from 1997 to 1998?
e In which years was the profit greater than $15 million?

11 Shapes



Chapter Contents

11:01 Plane shapes
11:02 Types of triangles and their properties
11:03 Properties of quadrilaterals
ID Card
11:04 Finding the size of an angle
11:05 Angle sum of a polygon
11:06 Symmetry
11:07 Solids
11:08 Nets of solids
11:09 Drawing pictures of solids
11:10 Building solids from diagrams
11:11 Looking at solids from different views
Reading Mathematics: The Platonic solids
Mathematical Terms, Diagnostic Test, Revision Assignment, Working Mathematically

Learning Outcomes

Students will:

- Describe and sketch three-dimensional solids including polyhedra and classify them in terms of their properties.
- Classify, construct and determine the properties of triangles and quadrilaterals.

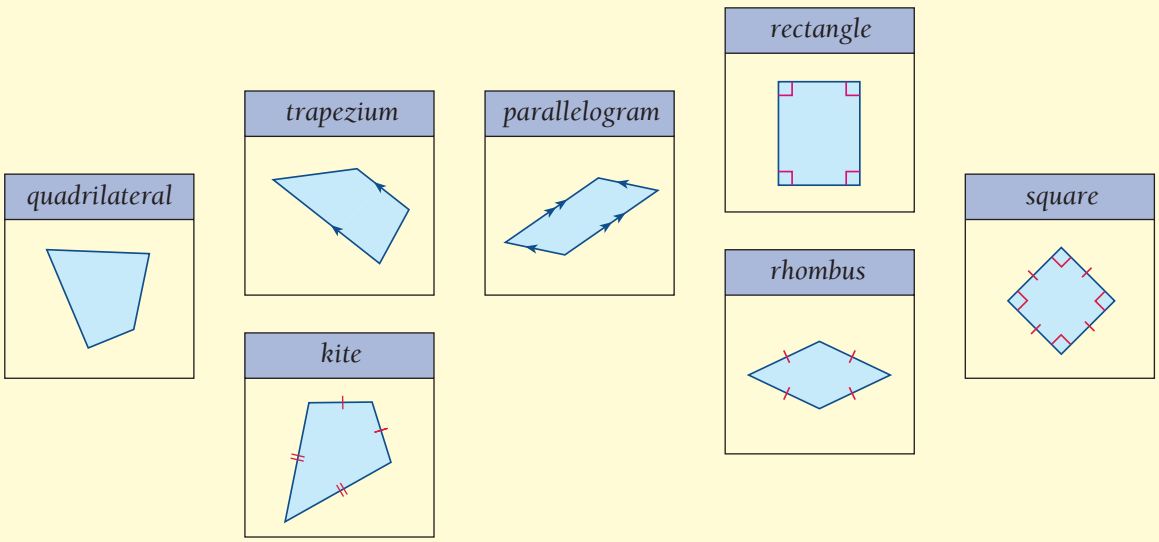
Areas of Interaction

Approaches to Learning, Homo Faber, Environment

11:01 | Plane Shapes

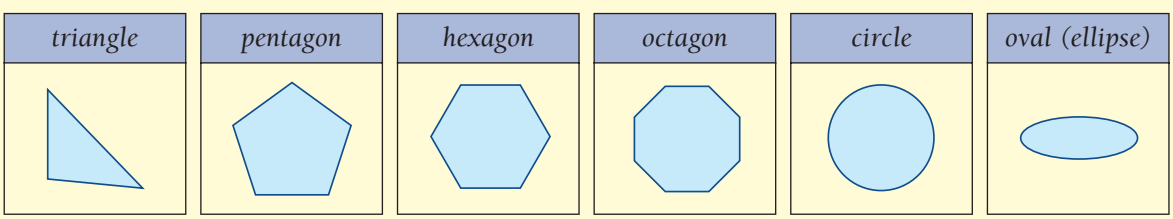
Plane shapes are shapes that lie in one plane, or flat surface. They have area but not volume. We say that they are **two-dimensional** (2D).

Below are shown some of the common plane shapes. Taken together, they make up the family of **quadrilaterals**, or four-sided shapes.



Each member of this family is a special kind of each of the shapes on its left, eg a rhombus is a special parallelogram, trapezium, kite and quadrilateral. A parallelogram is a special trapezium and quadrilateral (but not a kite).

Other plane shapes include:



Shapes that have only straight sides are called **polygons**.

Exercise 11:01

The following questions refer to the shapes shown above.

1 Why aren't the figures below triangles?

a 

b 

c 

2 A plane shape with four straight sides is called a quadrilateral. Name six special kinds of quadrilaterals.



3 A parallelogram has two pairs of parallel sides.

a What does *parallel* mean?

b Name all of the special kinds of quadrilateral that are also parallelograms.

c Which quadrilateral has only one pair of parallel sides?

d Give a real-life example of a trapezium.

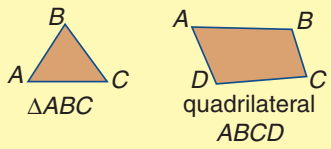
4 Name two plane shapes that do not have straight sides.

5 What is another name for an ellipse?

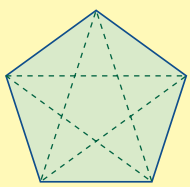
6 Copy and complete the table below.

Shape	*Number of sides*	*Number of angles*	*Number of diagonals*
triangle			
quadrilateral			
pentagon			
hexagon			
octagon			

■ We use the letters at the vertices to name shapes.



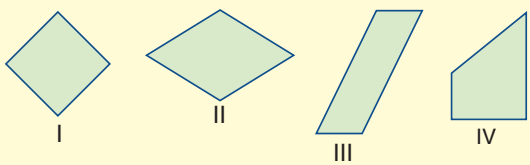
■ *Diagonals* are lines drawn from one corner of a polygon, across the polygon, to another corner.

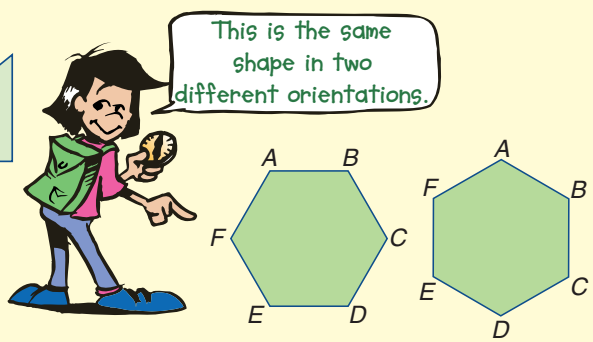


This shape has 5 diagonals.

7 Some people think of a rhombus as a squashed square. What other name is often given to the rhombus?

8 Sometimes when a shape is drawn in a different position or *orientation*, it can look like a different shape. Figures I to IV are four plane shapes drawn in unusual orientations.



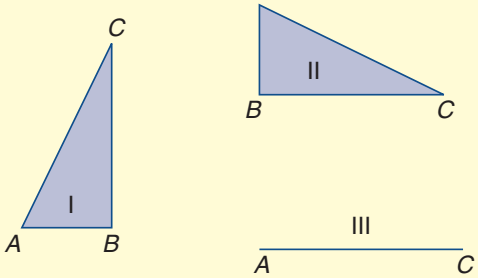


a Which figure shows a square?

b Which figure shows a rhombus?

c Which figure shows a trapezium?

9 Figures I and II below show a triangle in two different orientations. Figure III shows part of the triangle in a new orientation. Copy and complete the sketch.

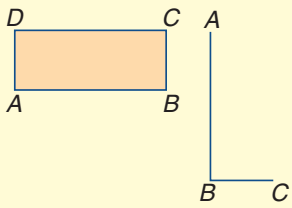


■ **Did you know?**
An electrocardiogram (ECG) measures the electrical activity of the heart, in relation to three points; the left and right shoulders and the navel, forming an equilateral triangle.

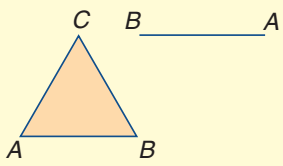


10 Each of the following diagrams shows a plane shape with part of the same shape drawn alongside it. Complete the sketch of the shape in its new orientation.

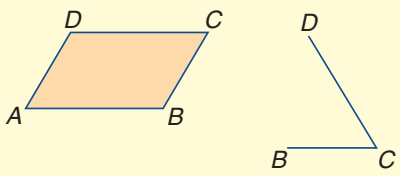
a



b

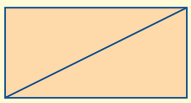


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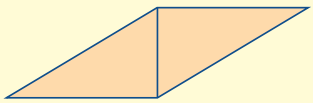


11 Name the quadrilateral that has been formed by joining the two identical triangles in each of the following.

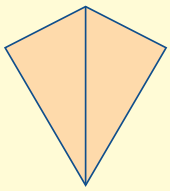
a



b

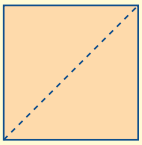


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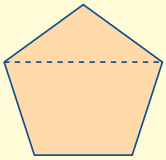


12 If each of the plane shapes shown were cut into pieces along the dotted line, what shapes would be formed?

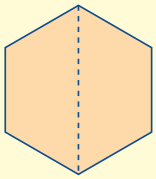
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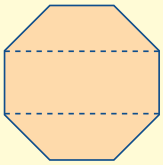
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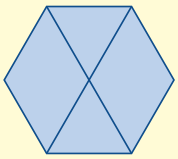


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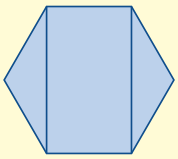


13 In each of the following a regular hexagon has been made from different-shaped pieces. Name the pieces.

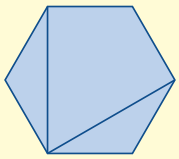
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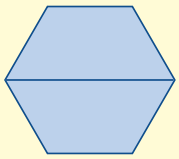
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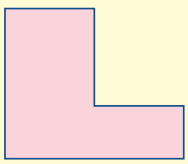


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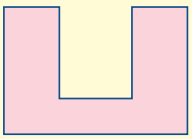
14 In later work it will be important to be able to divide a complicated shape into simpler shapes.

a



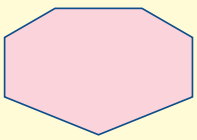
Divide this shape into 2 rectangles. Can this be done in more than one way?

b



Divide this shape into 3 rectangles. Can this be done in more than one way?

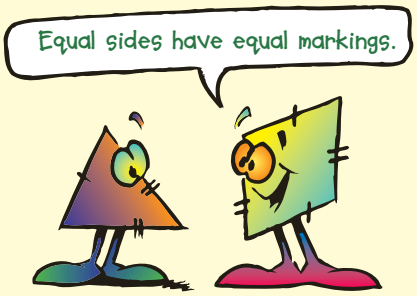
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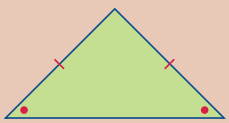
Divide this shape into a triangle, rectangle and trapezium.

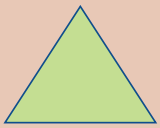
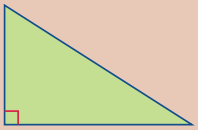
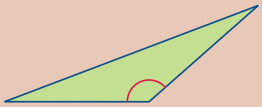
11:02 | Types of Triangles and Their Properties

Triangles have three sides and three angles. They can be sorted into different types according to the lengths of their sides or the sizes of their angles. The different types are shown below.



These are 6 terms used to put triangles into categories.

Scalene	*Isosceles*	*Equilateral*
		
• No sides equal. • No angles equal.	• Two sides equal. • Angles opposite equal sides are equal.	• All sides equal. • Three 60° angles.

Acute-angled	*Right-angled*	*Obtuse-angled*
		
• All angles are acute.	• One angle is a right angle.	• One angle is obtuse.

Triangles can be of more than one type. For example, a triangle can be both isosceles and acute-angled. We call this an **acute-angled isosceles triangle**.

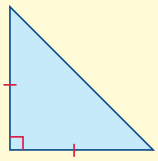
- For construction of triangles see section 14:04.



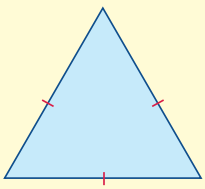
Exercise 11:02

1 Classify each triangle according to the lengths of its sides and the sizes of its angles.

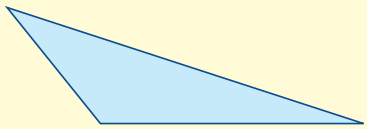
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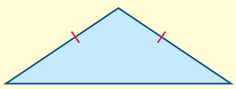
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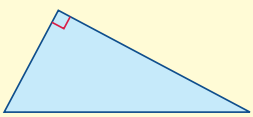
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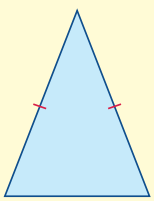
d



e



f



2 Use a ruler to draw an isosceles triangle that:

- **a** is acute-angled and has two sides 5 cm long
- **b** is right-angled and has two sides 5 cm long
- **c** is obtuse-angled and has two sides 5 cm long

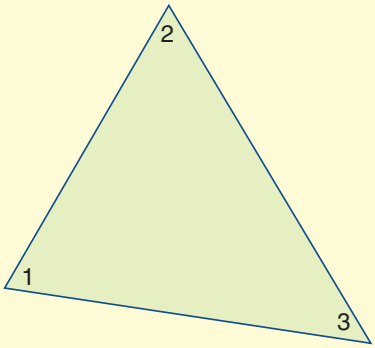
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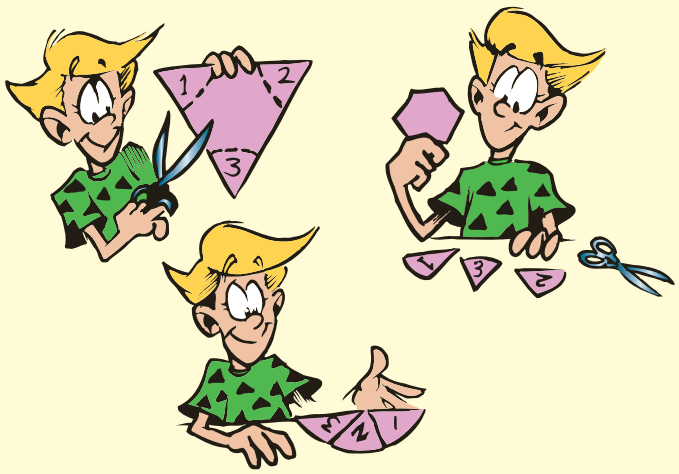
- **a** Draw a scalene triangle. Are any angles equal in size?
- **b** Draw an isosceles triangle. Are any angles equal in size?
- **c** Draw an equilateral triangle. Are any angles equal in size?

4 For each of the triangles in question 3, measure the angles and find their sum.

5

- **a** Draw any triangle. Mark the angles 1, 2 and 3.
- **b** Cut off the corners of the triangle and arrange them so that they form a straight angle.
- **c** What does this suggest about the angles of a triangle?





6 Draw an isosceles triangle and measure its angles. Are any of the angles equal? If so, how could you describe their position in the triangle?

7 Draw a triangle with two 50° angles.

- **a** What is the size of the third angle?
- **b** Are any of the sides equal in length? If so, can you describe their position in the triangle?

8

- **a** Draw an equilateral triangle. What are the sizes of the angles?
- **b** Draw a triangle with two 60° angles. What is the size of the third angle? Is the triangle equilateral?

Our discoveries

1. **The angles of a triangle add to 180°.**
2. **A scalene triangle has no equal angles.**
3. **An isosceles triangle has two equal angles opposite its equal sides.**
4. **An equilateral triangle has three 60° angles.**
5. **The largest angle of a triangle is opposite the largest side and the smallest angle is opposite the smallest side.**

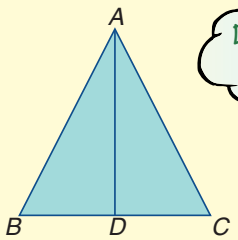
9 Use your compasses to draw a circle. Take any two points on the circle and the centre of the circle and join them to form a triangle. What type of triangle is it?

see 10:02

10 The diagram shows an isosceles triangle *ABC*. *AD* is an axis of symmetry of the triangle.

a Write down the lengths that are equal in the diagram.

b Write down the angles that are equal in the diagram.





11 **a** A triangle has three sides of length 5 cm. What type of triangle is it? What are the sizes of the angles?

b A triangle has all its angles 60° in size. What can you say about the length of its sides?

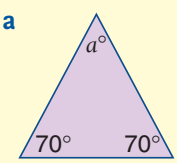
c A triangle has sides of length 3 cm, 4 cm and 5 cm. What type of triangle is it?

d A triangle has angles of 40°, 60° and 80°. Can it have any sides equal in length?

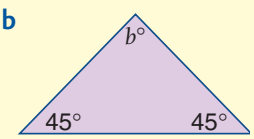
e Draw a triangle that has a 90° angle and two sides of length 2 cm. What are the sizes of the other angles?

12 Find the value of the pronumeral in each of the following.

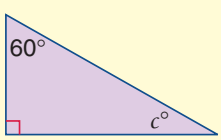
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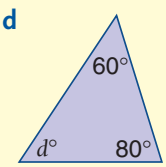
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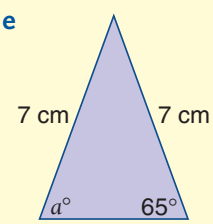
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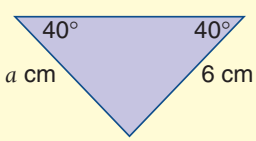
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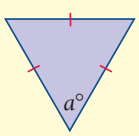
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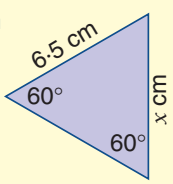
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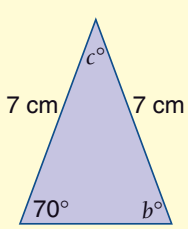
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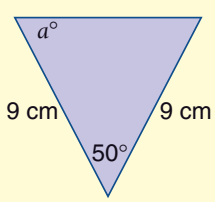
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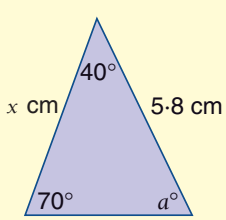
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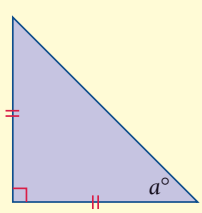
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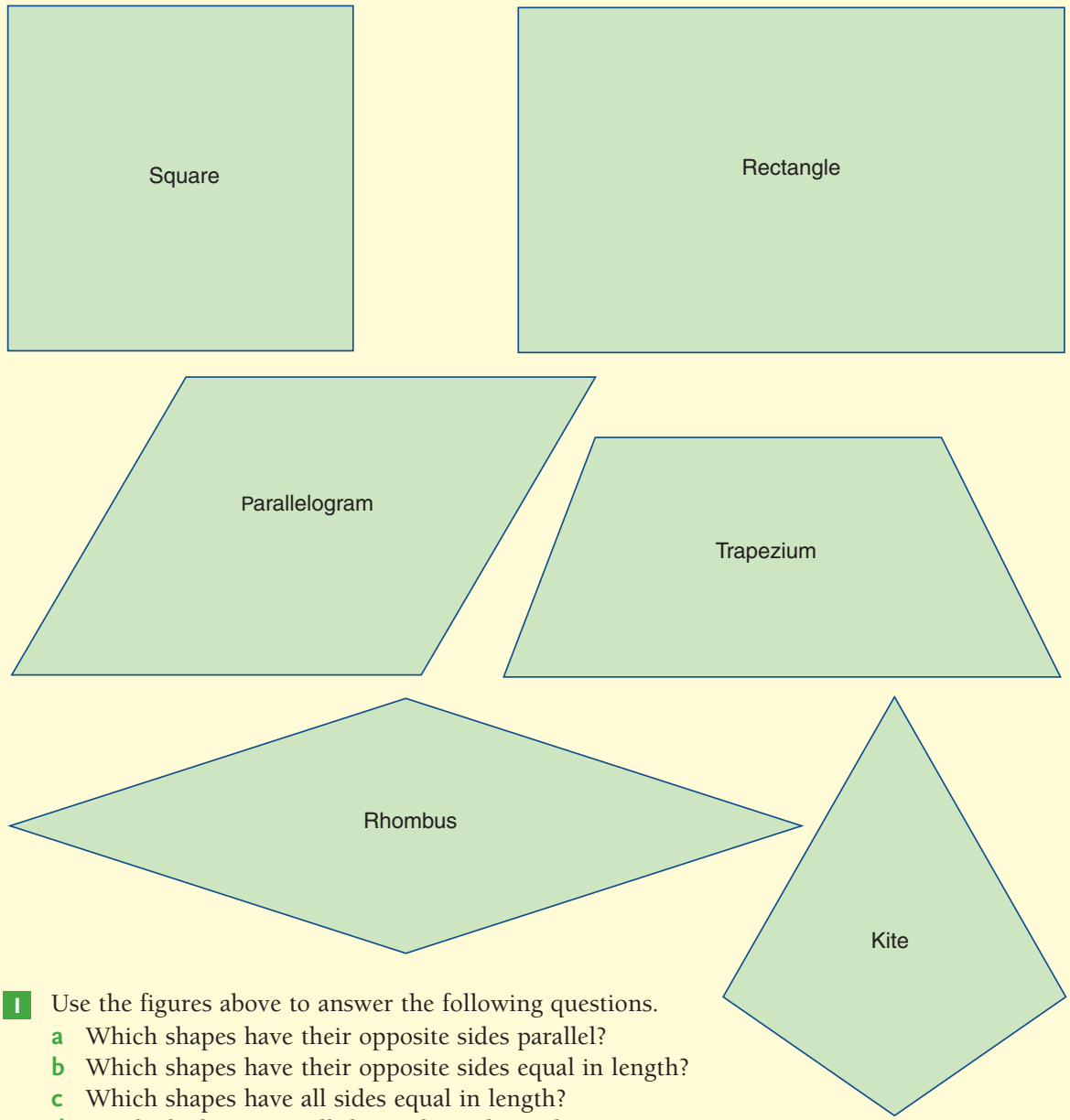


11:03 | Properties of Quadrilaterals

The square, rectangle, parallelogram and rhombus are special types of quadrilaterals. These shapes have many special properties that are related to their sides, angles and diagonals. In the following exercise we will establish many of these properties.

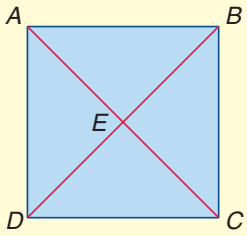
Exercise 11:03

Where necessary use your ruler, protractor or compasses to answer the questions.



I Use the figures above to answer the following questions.

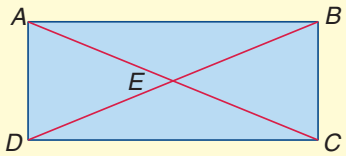
- **a** Which shapes have their opposite sides parallel?
- **b** Which shapes have their opposite sides equal in length?
- **c** Which shapes have all sides equal in length?
- **d** In which shapes are all the angles right angles?
- **e** Measure the angles of the parallelogram and rhombus. What can you say about them?

2 

The diagram shows a square *ABCD* with its diagonals coloured. By measurement find:

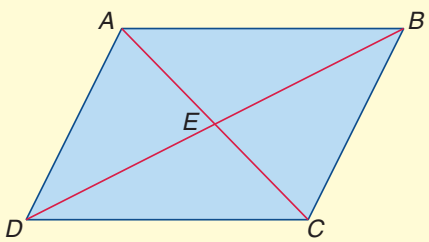
a if the diagonals *AC* and *BD* are equal in length
b if the diagonals meet at right angles
c if the diagonals cut each other in halves

3 Repeat question 2 for this rectangle.

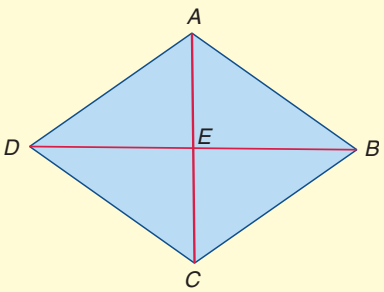




4 Repeat question 2 for this parallelogram.



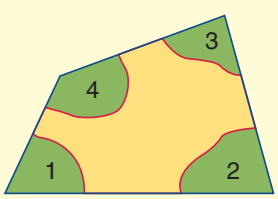
5 Repeat question 2 for this rhombus.

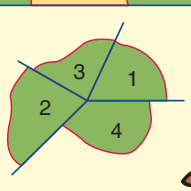


6 **a** In which of the four shapes drawn in questions 2, 3, 4 and 5 are the diagonals axes of symmetry?
b In which figures do the diagonals bisect the angles of the figure?

7 Draw any quadrilateral. Measure its angles. What is the sum of the angles?

8 Draw any quadrilateral and cut it out. Mark the angles 1, 2, 3 and 4 and cut them off. Rearrange these angles to form an angle of revolution. What does this tell you about the angles of a quadrilateral?



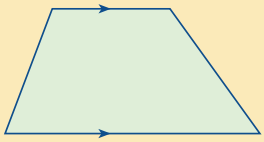
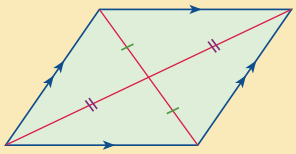
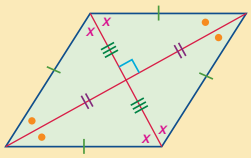
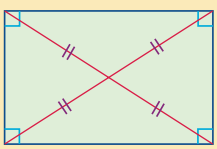
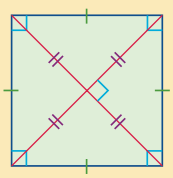
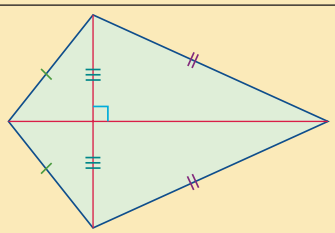




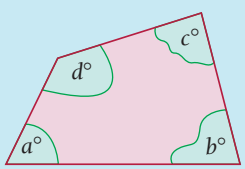
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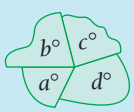
Refer to ID Card 4 on page xvi. Identify figures (1) to (14).
Learn the terms you do not know.

Summary of the properties of quadrilaterals

Quadrilateral	*Figure*	*Properties*
1 Trapezium		• **One pair of opposite sides parallel.**
2 Parallelogram		• **Two pairs of parallel sides.** • **Opposite sides are equal.** • Opposite angles are equal. • Diagonals bisect one another.
3 Rhombus		• A rhombus has all the properties of a parallelogram and . . . • **All sides are equal.** • Diagonals bisect each other at right angles. • Diagonals bisect the angles through which they pass.
4 Rectangle		• A rectangle has all the properties of a parallelogram and . . . • **All angles are right angles.** • Diagonals are equal.
5 Square		• A square has all of the properties of a rhombus and a rectangle. • **Four sides equal.** • **Four right angles.**
6 Kite		• **Two pairs of adjacent sides equal.** • Diagonals are perpendicular. • One diagonal is an axis of symmetry.



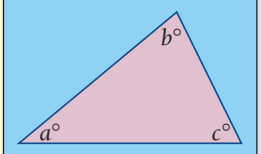
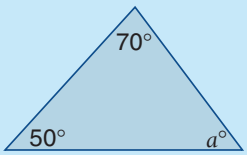
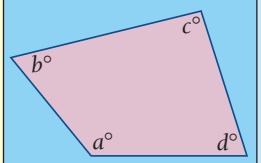
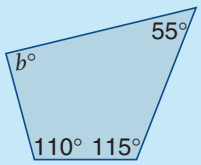
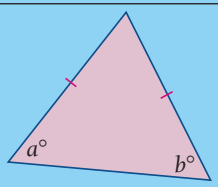
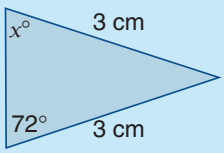
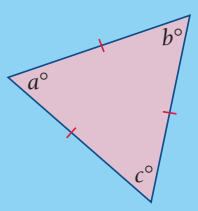
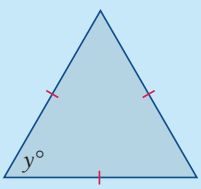
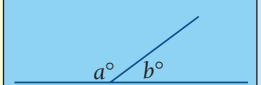
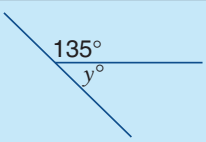
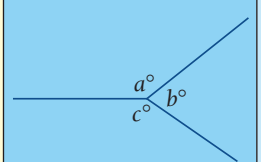
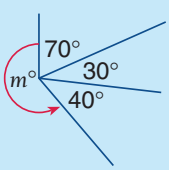




The angle sum of any quadrilateral is 360°.
$a° + b° + c° + d°$ makes a revolution.

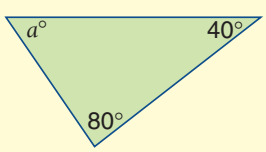
$$a° + b° + c° + d° = 360°$$

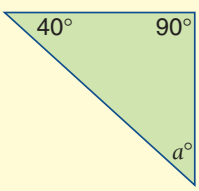
11:04 | Finding the Size of an Angle

Figure	Rule	Example
	triangle $a° + b° + c° = 180°$	 Find the value of a. $a + 50 + 70 = 180$ $a + 120 = 180$ $\therefore a = 60$
	quadrilateral $a° + b° + c° + d° = 360°$	 Find the value of b. $b + 55 + 115 + 110 = 360$ $b + 280 = 360$ $\therefore b = 80$
	isosceles triangle $a° = b°$	 Find the value of x. This is an isosceles triangle since two sides are equal. $\therefore x = 72$
	equilateral triangle $a° = b° = c°$	 Find the value of y. This is an equilateral triangle as all sides are equal. $\therefore$ All angles are equal. $3y = 180$ $y = 60$
	straight angle $a° + b° = 180°$	 Find the value of y. $135 + y = 180$ $\therefore y = 45$
	angles at a point $a° + b° + c° = 360°$	 Find the value of m. $m + 70 + 40 = 360$ $m + 140 = 360$ $\therefore m = 220$

Exercise 11:04

1 Find the value of each pronumeral.

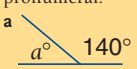
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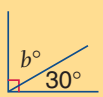
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Foundation Worksheet 11:04

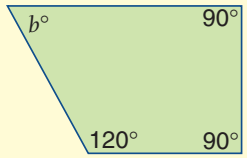
Finding the size of an angle

1 Find the value of each pronumeral.

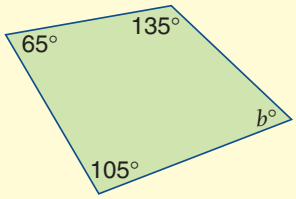
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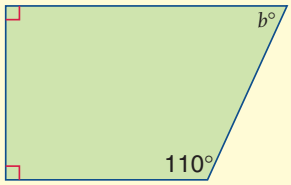
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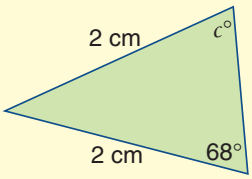
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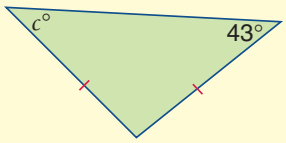
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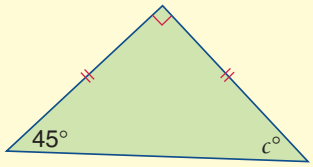
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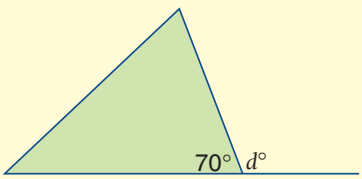
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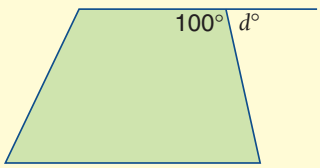
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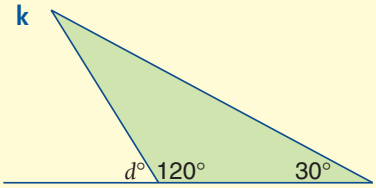
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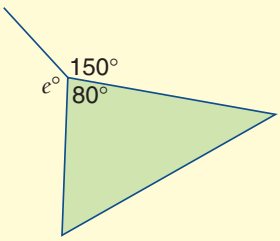
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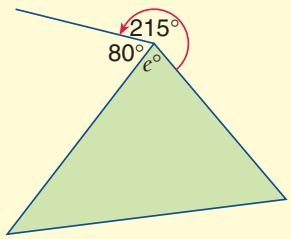
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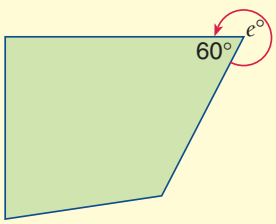
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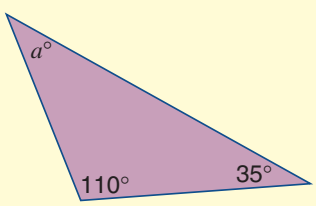


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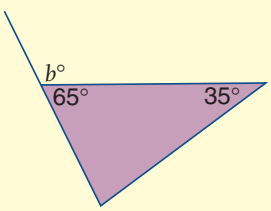


2 Find the value of the pronumeral in each case.

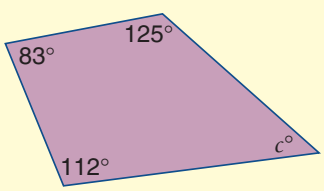
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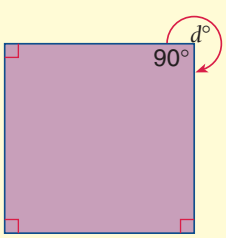
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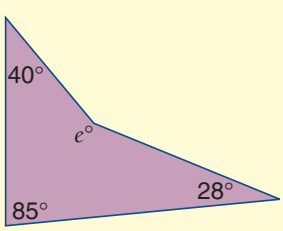
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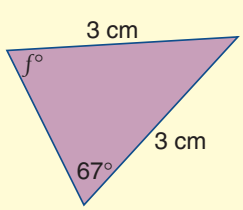
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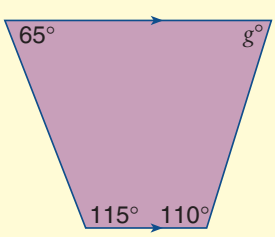
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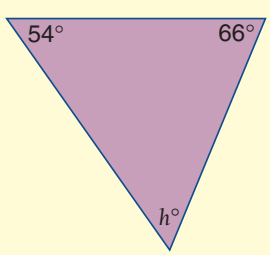
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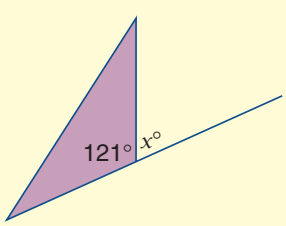
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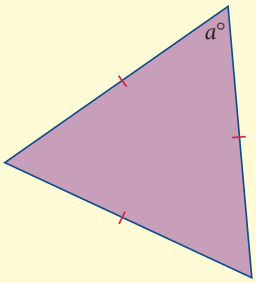
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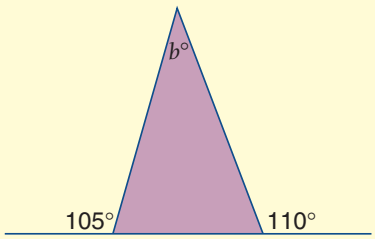
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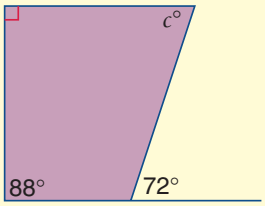
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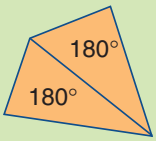
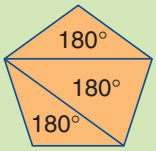
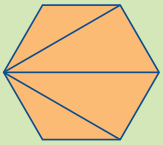
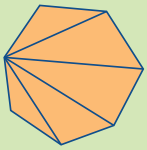
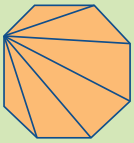
11:05 | Angle Sum of a Polygon

Finding the angle sum of a polygon

- Start from one corner and divide the polygon into triangles.
- The angles in each triangle add up to 180°.

Polygons that have all sides equal and all angles equal are called **regular polygons**.



Polygon					
Number of sides	4	5	6	7	8
Number of triangles	2	3	4		
Angle sum of polygon	$2 \times 180°$ $= 360°$	$3 \times 180°$ $= 540°$	$4 \times 180°$ $=$		

Exercise 11:05

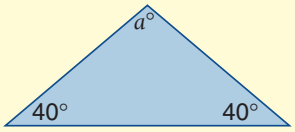
1 Look at the table above. Can you see a connection between the number of sides on the polygon and the number of triangles that would be drawn?

- **a** How many triangles have been drawn in the 6-sided polygon?
- **b** How many triangles have been drawn in the 8-sided polygon?
- **c** How many triangles would be drawn in a 12-sided polygon? What is the angle sum of a 12-sided polygon (*dodecagon*)?
- **d** How many triangles would be drawn in a 9-sided polygon? What is the angle sum of a 9-sided polygon (*nonagon*)?

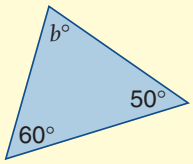
Angle sum of a polygon = (number of sides −2) × 180°.

2 Find the value of the pronumeral in each of the following.

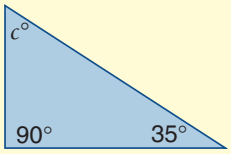
a



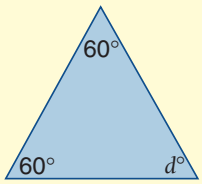
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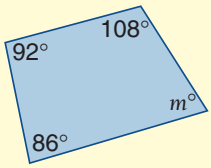
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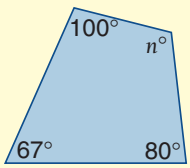
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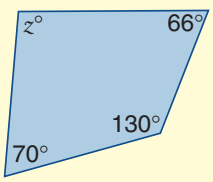
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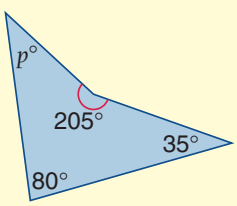
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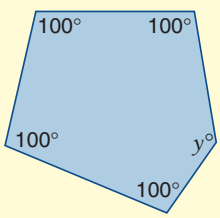
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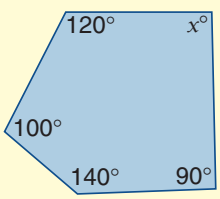
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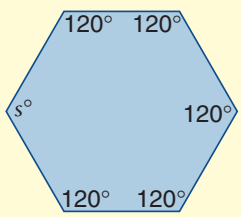
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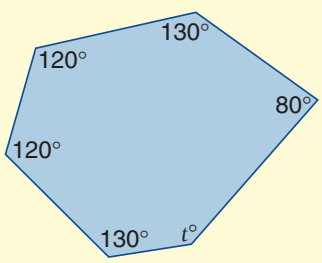
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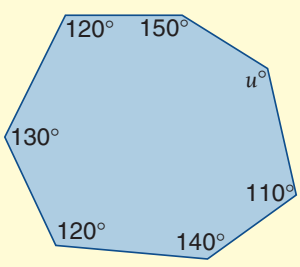
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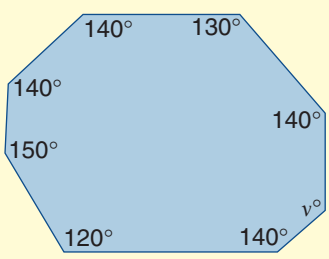
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Appendix F

F:02 Plane shapes and patterns

F:03 Transformations: reflections, translations and rotations

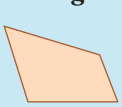


11:05A Triangles

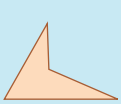
11:05B Quadrilaterals



- **A quadrilateral that has all angles less than 180° is called convex.**
- **If one angle is greater than 180° it is non-convex (or concave).**



This quadrilateral is convex.

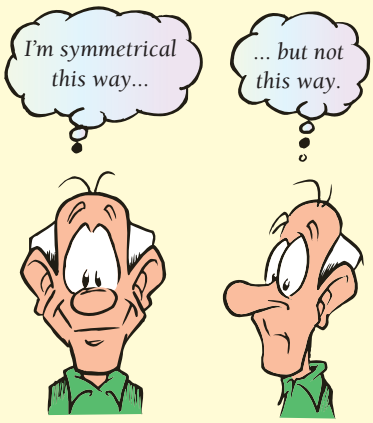


This quadrilateral is non-convex.

Which of the quadrilaterals in question 2 above is non-convex?

11:06 | Symmetry

Many shapes are *symmetrical*. This means that they seem well balanced and in the right proportion. In maths there are two types of symmetry: **line symmetry** and **rotational symmetry**.

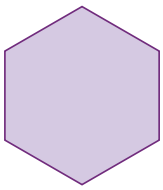


A shape has *line symmetry* if it can be divided by a line into two identical parts that are mirror images of one another. The dividing line is called an *axis of symmetry*. (*Note:* the plural of 'axis' is 'axes', pronounced 'axe-ease'.)

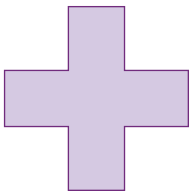
worked examples

1 Mark in the axes of symmetry of the shapes below.

a



b

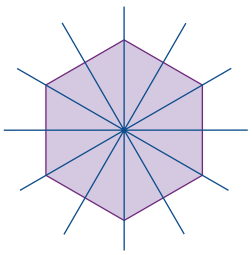


2 If the blue line is an axis of symmetry of the shape, complete the drawing.

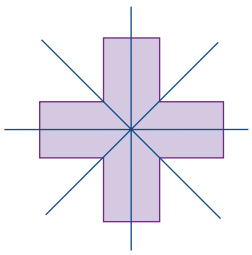


Solutions

1 a



b



2



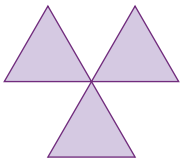
Some shapes that do not possess line symmetry still appear to have some symmetrical properties.

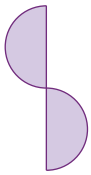
A shape has *rotational symmetry* if it can be spun about a point so that it repeats its shape more than once in a rotation. If it repeats its shape after half a turn, it is said to have *point symmetry*. The point about which the shape spins is called the *centre of symmetry*.

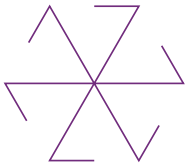
- When investigating rotational symmetry, it is often useful to make a tracing of the figure. You can then place the tracing over the original figure and rotate it.
- Shapes with point symmetry (half-turn symmetry) have a special property. Every point on the figure has a matching point on the figure. These points are the same distance from the centre of symmetry, and if they are joined by a line, the line passes through the centre of symmetry.

examples

Do these shapes have point symmetry?

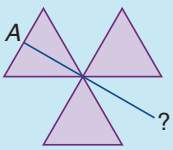
1 

2 

3 

Solution

1 No 2 Yes 3 Yes

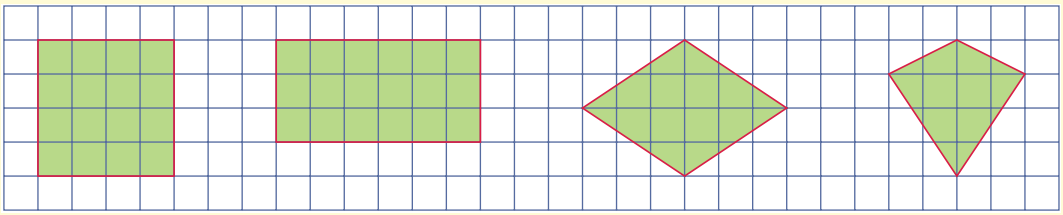


Draw a line from any point through the centre. Is there a matching point on the other side of the centre the same distance away? If the answer is yes for every point on the figure, it has point symmetry.

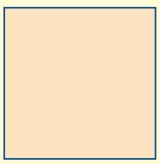
Exercise 11:06

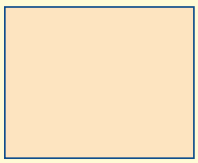
1 Copy the following shapes and mark in all the axes of symmetry. Do all of these shapes have **line symmetry**?

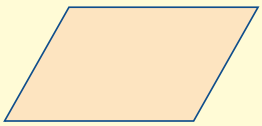
a b c d

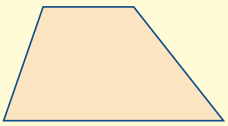


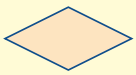
2 How many axes of symmetry has each shape below?

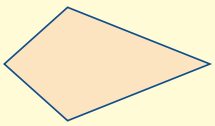
a 
Square

b 
Rectangle

c 
Parallelogram

d 
Trapezium

e 
Rhombus

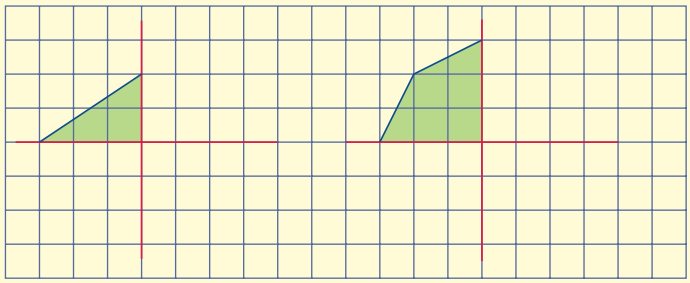
f 
Kite

- **You can fold along an *axis of symmetry* and the two halves overlap.**
- **You can rotate a figure with *point symmetry* 180° about the *centre of rotation*, and the figure will look the same.**

3 Which of the shapes in question 2 have point symmetry?

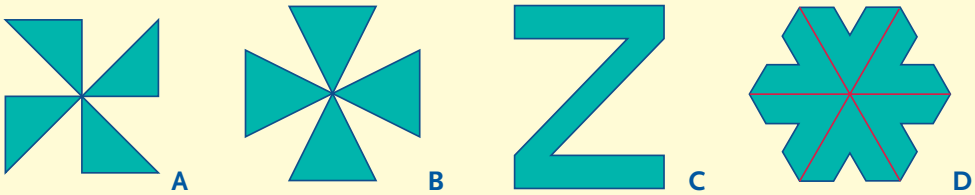
4 If the red lines are axes of symmetry, copy and complete the figures.

a b

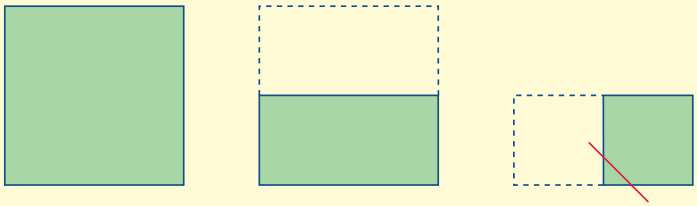




5 a Which of the shapes below have point symmetry?
b Which shapes also have line symmetry?



6 Take a piece of square paper and fold it in half and in half again.



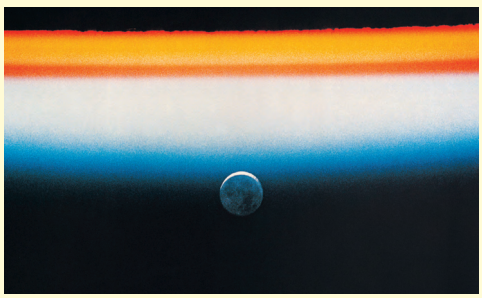
Draw a line on the folded paper and cut along it. Experiment by putting the line in different places. See if you can predict the shape that will result.

7 Discuss the aspects of symmetry in each picture.

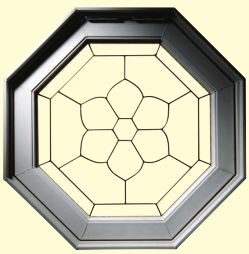
a



b

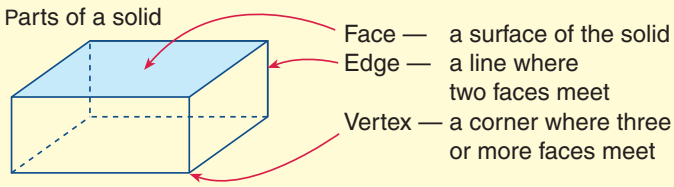


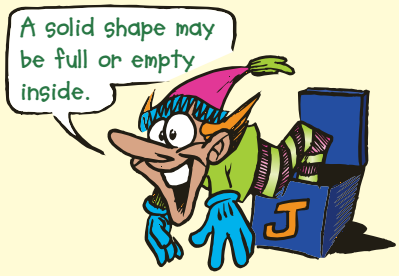
c



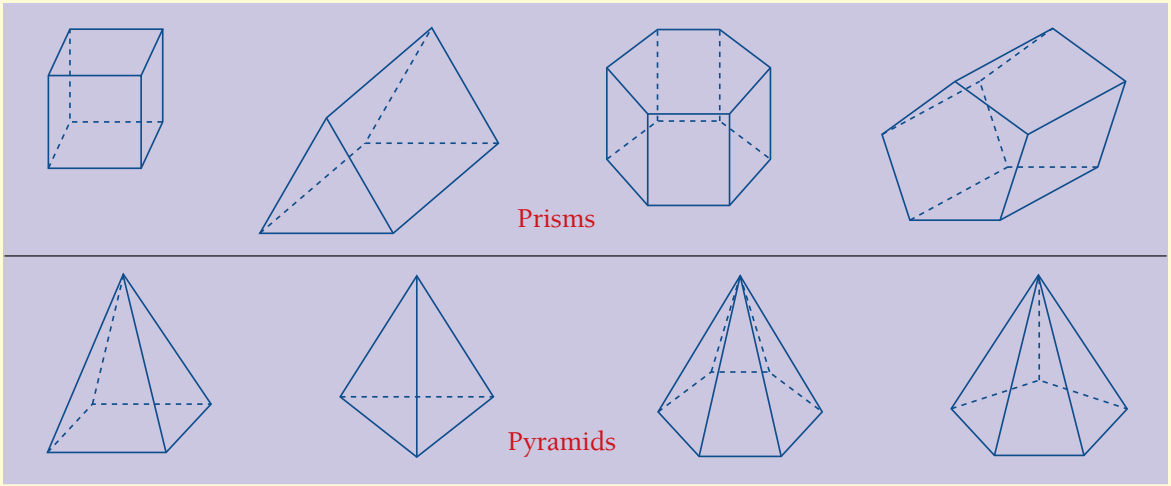
11:07 | Solids

Solid shapes have thickness as well as length and breadth.
We say that they are **three-dimensional** (3D). Our world is filled with solid shapes.

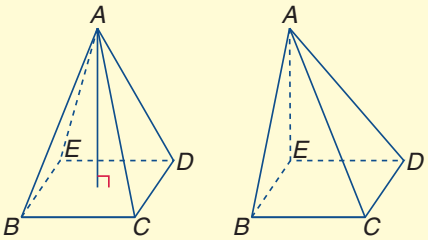




There are two main families of solids, the **prisms** and the **pyramids**. Look carefully at the two family groups and see what each family has in common. Not all members of each family are shown.



Definitions



Right pyramid Oblique pyramid

- A **right pyramid** has its axis perpendicular to its base. (Its top point is above the centre of the base.)
- An **oblique pyramid** is one in which the axis is *not* perpendicular to the base.
- **Polyhedra** are 3D figures whose faces are plane shapes with straight edges. (The faces are flat.)

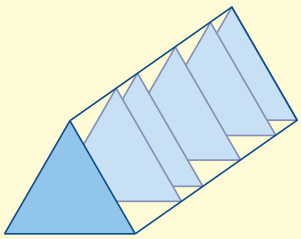
■ *Polyhedra* is the plural of *polyhedron*.

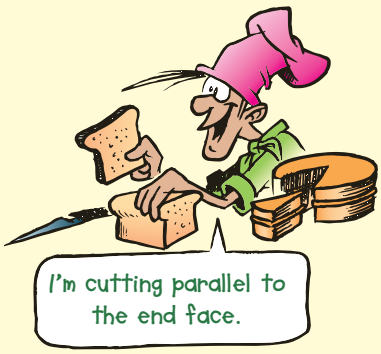
- **Intersecting lines** are in a common plane and meet at one point, eg *BC* and *AC*.
- **Parallel lines** are in a common plane and will never meet, eg *BC* || *ED*.
- **Skew lines** do not meet because they are *not* in the same plane, eg *AB* and *CD*.

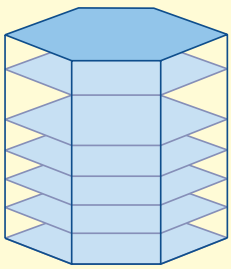
Prisms

All prisms have a special pair of parallel faces. These faces are the only two faces that need not be rectangular in shape.

If a prism is 'sliced' parallel to these faces, the same shape always results. This shape is called the **cross-section**.







A *prism* is named according to the shape of its cross-section.

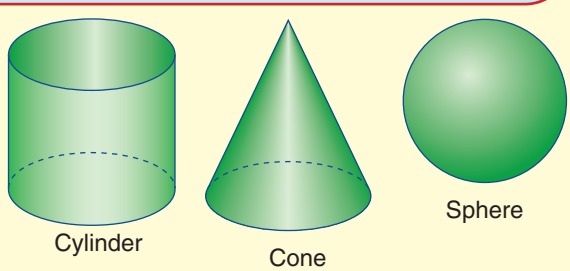
Pyramids

All pyramids have one face that need not be triangular. This face is used to name the pyramid. All the other faces of a pyramid *are* triangular. Pyramids cannot be sliced like prisms so that identical shapes always result.

A *pyramid* is named according to the shape of its non-triangular face. If all its faces are triangles, the pyramid is a triangular pyramid.

Other solids

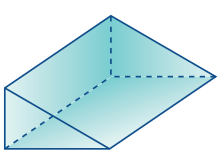
Some solids are neither prisms nor pyramids. The most common of these are the **cylinder**, **cone** and **sphere**.



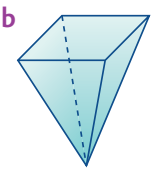
examples

Name the following solid shapes.

a



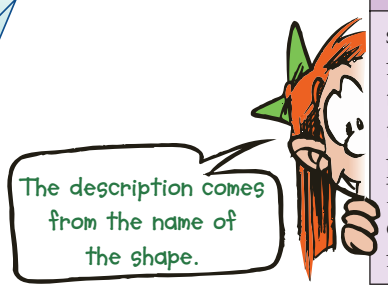
b



Solutions

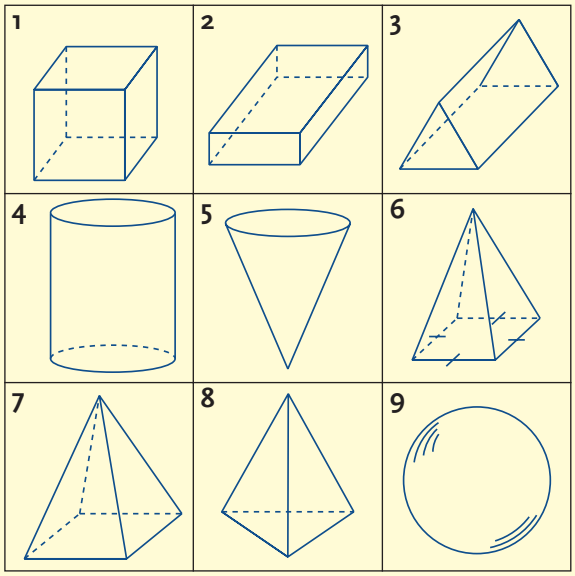
a triangular prism

b square pyramid



Shape	*Description*
square	square
rectangle	rectangular
triangle	triangular
trapezium	trapezoidal
pentagon	pentagonal
hexagon	hexagonal
octagon	octagonal
rhombus	rhombic

Exercise 11:07



Questions 1 to 11 refer to the grid at left.

1 Give the number of each of the following.
- **a** cube
- **b** cone
- **c** sphere
- **d** cylinder

2 What is the name of:
- **a** shape 4
- **b** shape 2
- **c** shape 9
- **d** shape 5

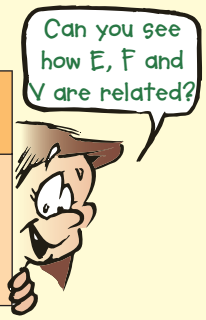
3 Which of the solids have a surface that is curved?

4 Which shapes are prisms?

5 Which shapes are pyramids?

6 Which of the solid shapes are neither prisms nor pyramids?

7 Copy and complete the following table.



Solid	*Number of faces* (F)	*Number of vertices* (V)	*Number of edges* (E)	F + V
cube triangular prism rectangular pyramid triangular pyramid				

Questions 8 to 11 refer to the shapes at the top of the page.

8 Which solid shapes have no vertices?

9 Which solid shapes can roll?

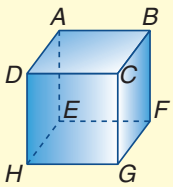
10 What are the shapes of the faces on:
- **a** shape 3?
- **b** shape 6?

11 What is the name of:
- **a** shape 2
- **b** shape 3
- **c** shape 4
- **d** shape 8



Dotted lines represent edges that are hidden from view. Hidden edges are not always shown in diagrams.

12

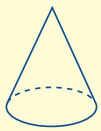


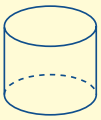
Euler's theorem: **For any convex polyhedra, $F + V = E + 2$ where F = number of faces, V = number of vertices and E = number of edges.**

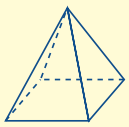
- **a** Which edges intersect *AB*?
- **b** Which edges are parallel to *AB*?
- **c** Which edges are skew to *AB*?
- **d** Which face is parallel to *ABCD*?

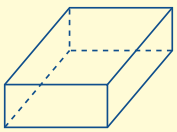
13 From the list of names below, choose the correct name for each of these solid shapes.

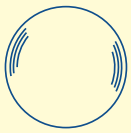
Names: hexagonal prism, square pyramid, sphere, pentagonal pyramid, triangular prism, cone, triangular pyramid, rectangular prism, cylinder

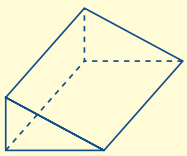
a 

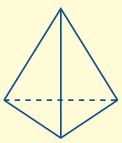
b 

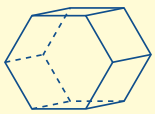
c 

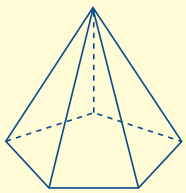
d 

e 

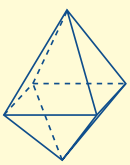
f 

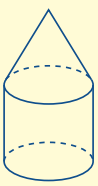
g 

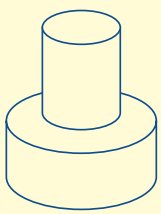
h 

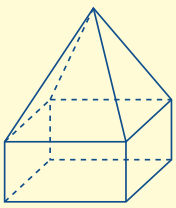
i 

14 Which solids have been joined to form the solids pictured?

a 

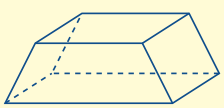
b 

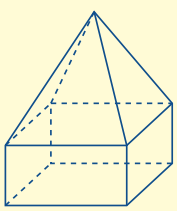
c 

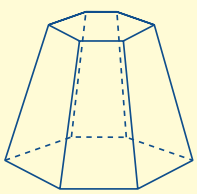
d 

15 For each of the solids below, show that Euler's theorem holds (see page 188):

a triangular prism
b square prism
c triangular pyramid
d square pyramid

e 

f 

g 

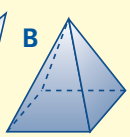
16 Write true or false for:

a A polyhedron is a solid whose faces are all flat.
b A prism has a uniform polygonal cross-section.
c A cylinder has a uniform circular cross-section.
d A pyramid has a polygonal base and one further vertex (the apex).
e A cone has a circular base and an apex.
f All points on the surface of a sphere are a fixed distance from its centre.
g An oblique pyramid has its apex above the centre of the base.
h A non-convex solid turns inward to give a concave section.

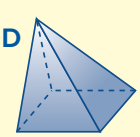
17 Which of the solids to the right is:

a a right pyramid?
b an oblique pyramid?
c a right cone?
d an oblique cone?

A 

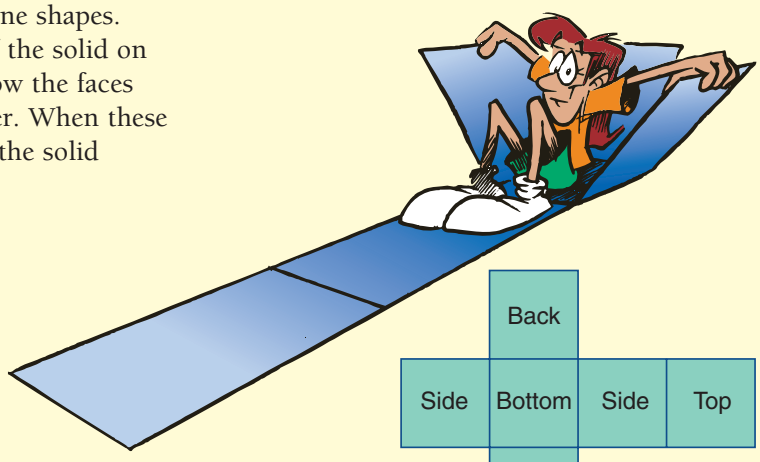
B 

C 

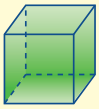
D 

11:08 | Nets of Solids

Solid shapes can be made from plane shapes. This is done by drawing the **net** of the solid on a piece of paper. The net shows how the faces of the solid are joined to each other. When these faces are folded along their edges, the solid is formed.



Some nets are quite easy to make, while others are quite difficult. The net of a cube is shown to the right.



When trying to draw a net, you need to answer the following questions:

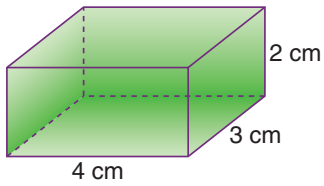
- What types of faces does the solid have?
- How many of each type are there?
- Which faces are joined to each other?

Then select a face of the solid and draw it. Look at the faces that join this face and then draw them. Continue in this manner until all the faces have been drawn.

This procedure is shown in the examples that follow.

worked example 1

Draw the net of this rectangular prism on square grid paper.

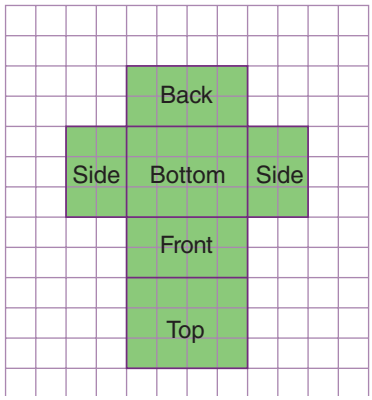


Solution

The solid has six rectangular faces. There are three pairs of different-sized rectangles.

Step 1 Draw the bottom. This is 4 cm long and 3 cm wide.
Step 2 Add the two sides that join it.
Step 3 Add the front and back that join it.
Step 4 Finally add the top, which can be joined to either the back, front or side.

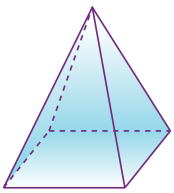
In the solution given, the top has been joined to the front.



worked example 2

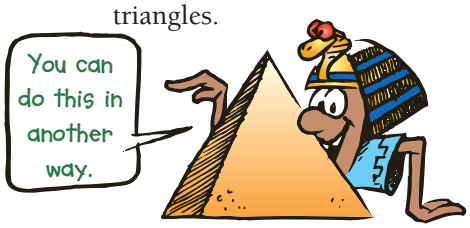
Draw the net of a square pyramid on a sheet of square grid paper.

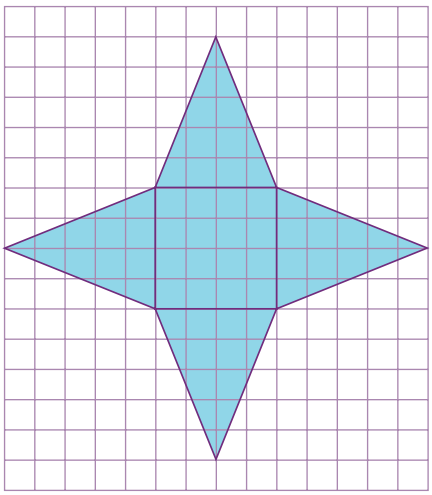
Solution



A square pyramid has four identical triangular faces joined to a square base, so:

Step 1 Draw a square base.
Step 2 Add the four adjoining triangles.

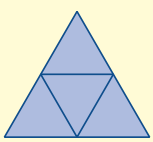




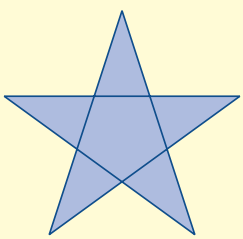
Exercise 11:08

1 Match the nets with the solids A, B and C.

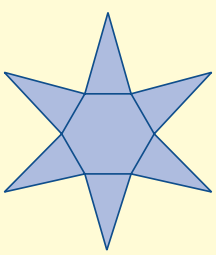
a

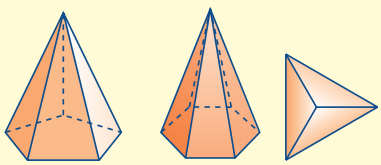


b



c

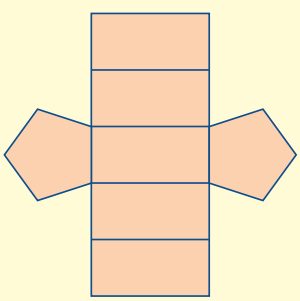




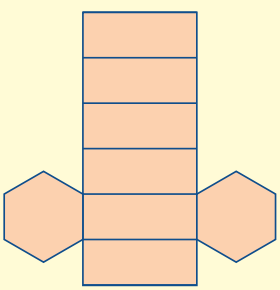
A B C

2 Match the nets with the solids A, B and C, below.

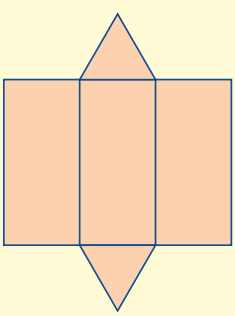
a

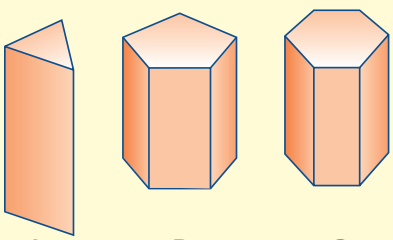


b



c

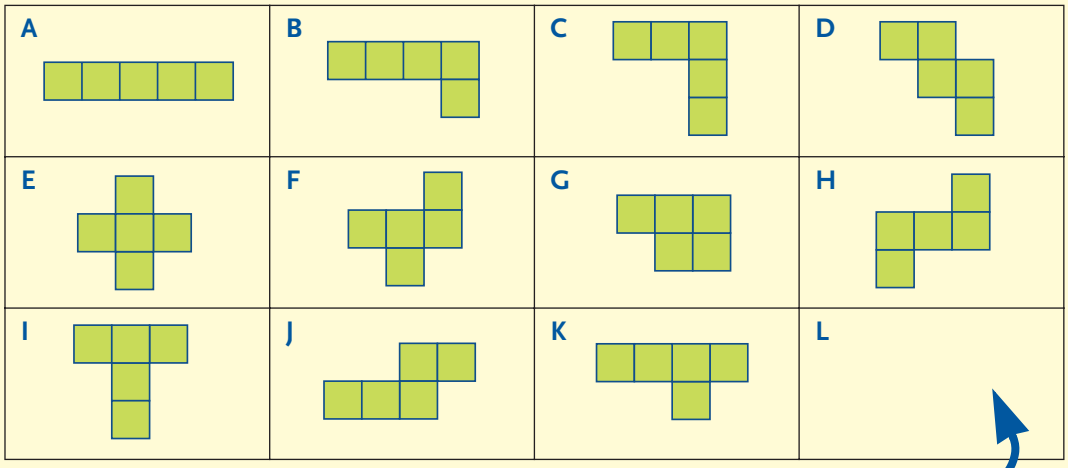




A B C

3 Five connected squares form a pentomino.

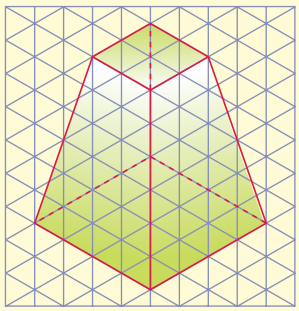
a Which of the pentominoes below form the net of an open box?



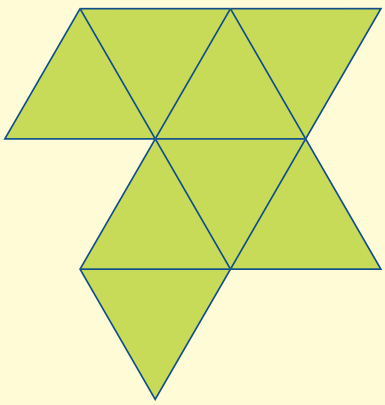
b Two arrangements that match exactly when turned or flipped over are considered to be the same. Draw the pentomino that is not shown above.

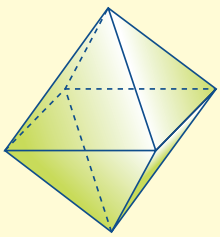
4 Make an accurate model of a rectangular prism that is 5 cm long, 3 cm wide and 2 cm high by drawing the net on square grid paper.

5 The diagram at right shows a square pyramid that has had the top part removed. Construct the net of this solid on square grid paper.



6 The diagram below is an attempt to draw the net of the solid shown, which is an octahedron. By copying the figure and assembling the solid, find the error in the net and draw the correct version.



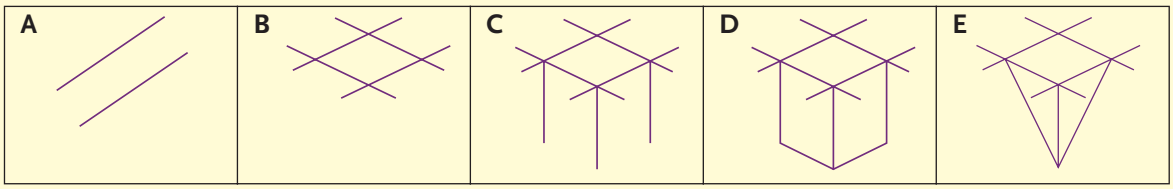


11:09 | Drawing Pictures of Solids

Drawing pictures of solids can be made easier by using one of the methods below.

Method 1

Some prisms and pyramids can be drawn using parallel lines.



- Draw two pairs of parallel lines that cross.
- Draw three lines of equal length straight down from the corners. Join the ends to get a prism.

OR

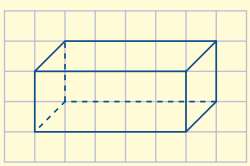
- Choose a point in the middle, below (or above) the figure. Draw lines from three corners to this point to get a pyramid.



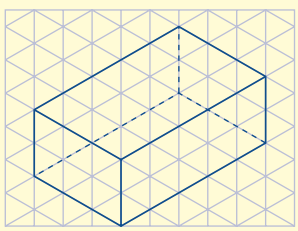
Method 2

To draw three-dimensional objects on paper, artists use different techniques called **projections**. Below, a cube and a rectangular prism have been drawn using two different projections.

Rectangular prism

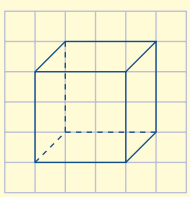


1 Using square grid paper

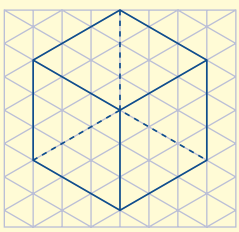


2 Using isometric grid paper

Cube



1 Using square grid paper



2 Using isometric grid paper

Notice that in both projections, deliberate distortions have been made to make things 'look right'. For instance, the square and rectangular faces have not all been drawn as squares and rectangles.

Also, in the drawings made on the square grid, the face that appears to be going backwards into the paper is drawn less than the correct distance. This is because of an optical illusion that results if the edge is drawn the correct length.

Notice also that in both projections, edges that are hidden from view and cannot be seen are shown by dotted lines.



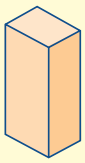
It is important that:
- **you are able to draw pictures of different solids, and**
- **you can understand a picture of a solid.**

Exercise 11:09

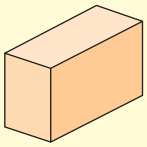
Equipment needed: rulers, square and isometric grid paper, cubes

1 Use Method 1 on the previous page to draw these solids.

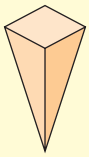
a



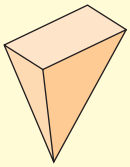
b



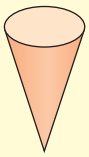
c



d



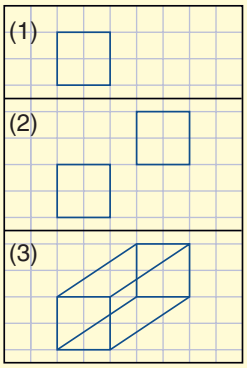
e

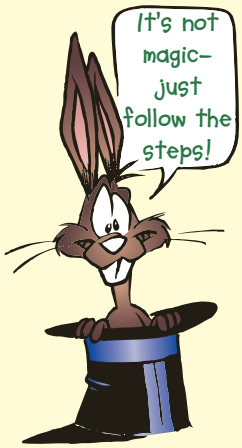


2 Follow these steps to draw a certain solid.

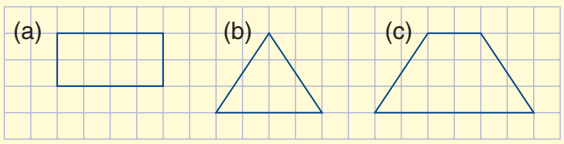
1. Draw a square on square grid paper.
2. Now translate the square sideways and upwards and draw the square again.
3. Join the corresponding corners of the squares.

Name the solid you have drawn.





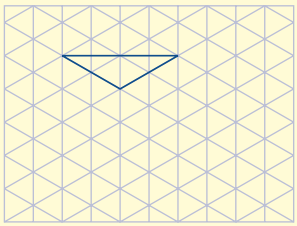
3 Repeat the steps in question 2 with each of the shapes drawn below. In each case, name the solid you have drawn.



4 Follow these steps to draw a certain solid.

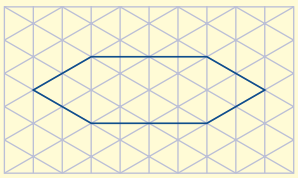
1. On a piece of isometric grid paper, draw the triangle given.
2. Now translate the triangle downwards and draw it in its new position.
3. Join the corresponding corners of the two triangles.

Name the solid shape you have drawn.

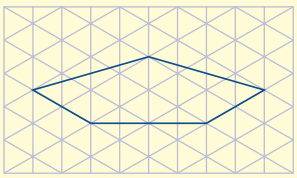


5 Repeat the steps in question 4, using the shapes drawn below. In each case, name the solid shape you have drawn.

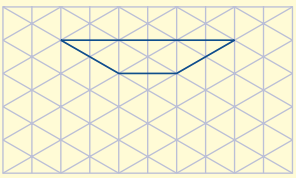
a



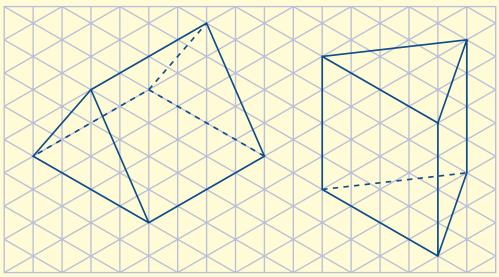
b



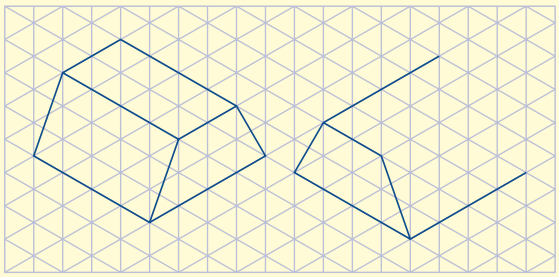
c



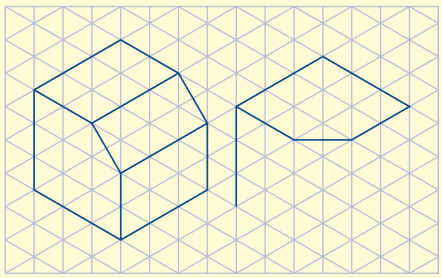
6 The diagrams to the right show two drawings of the same solid in different orientations. Complete the drawing of each of the solids given below in its new orientation.



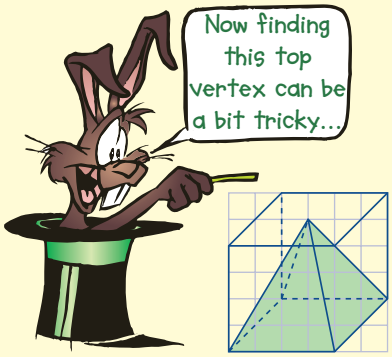
a



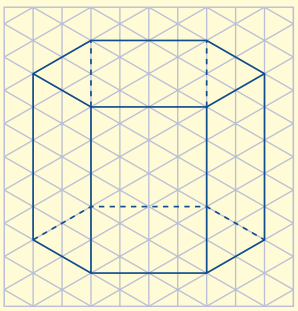
b



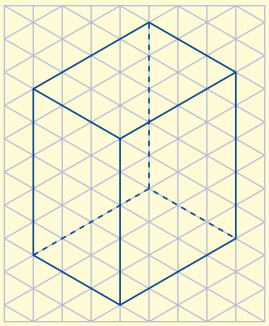
7 Sketching a pyramid can be done by first making a prism as shown to the right. Copy each of the prisms below in pencil on isometric grid paper and use it to complete the pyramid. When the pyramid has been drawn, erase the prism.



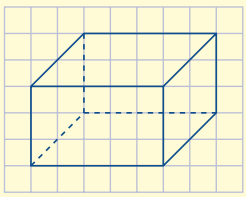
a



b

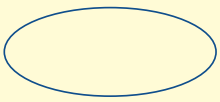


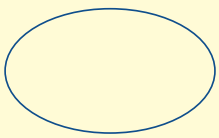
c

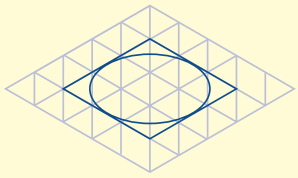


8 Find how many different solids can be built from 3 cubes. Sketch each solid on isometric grid paper.

9 When drawing cylinders and cones in three dimensions, the circles are represented by ellipses. Some ellipses are drawn below, including one on isometric grid paper. Practise drawing ellipses and use them to draw pictures of cones and cylinders.





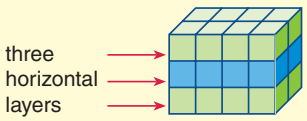


11:10 | Building Solids from Diagrams

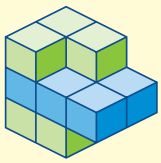
It is important to be able to 'read' three-dimensional diagrams. When looking at such diagrams, consider the following questions.

1 How many horizontal layers does it have?
2 How many blocks are in each layer? How are the blocks arranged?
3 Is each layer identical?

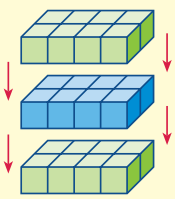
Once these questions have been answered, the solid can be built. Consider these questions for the solids drawn below.



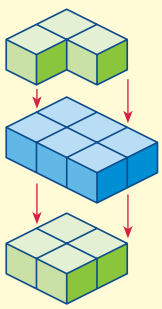
The solid on the left has 3 horizontal layers with 8 blocks in each layer. Each layer is identical.



The solid on the left has 3 horizontal layers. The layers are not identical, and contain 3, 6 and 4 blocks respectively.

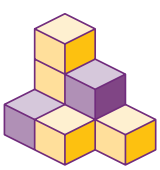


This diagram shows how the solid has been made by putting three identical layers of 8 blocks on top of each other.



This diagram shows how the solid above has been made by placing the three layers on top of each other.

worked examples

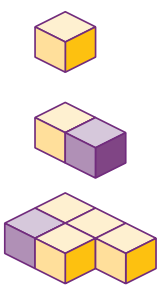


For the solid pictured:

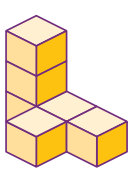
1 find the number of horizontal layers
2 find the number of cubes in each of the layers
3 sketch the solid that is formed if the coloured cubes are removed.

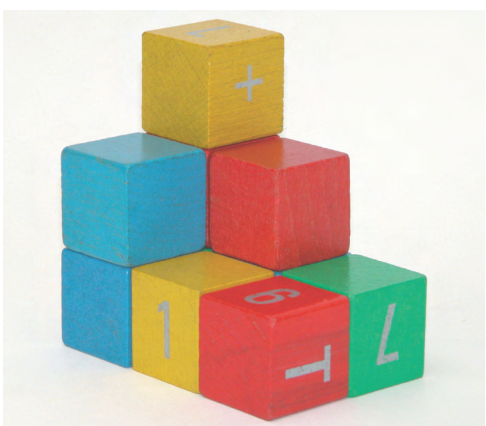
Solutions

1 The solid has three horizontal layers.
2 As the diagram shows, the layers are made of 1, 2 and 5 cubes respectively.



3



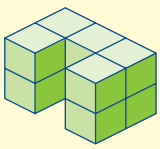


Exercise 11:10 (Practical)

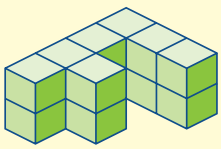
Equipment needed: cubes, isometric grid paper, square grid paper

1 Construct each of the following solids from cubes.

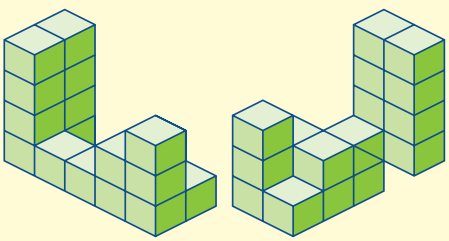
a 

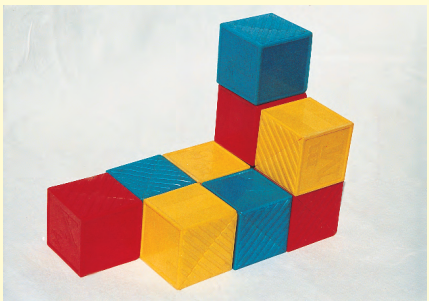
b 

c 

d 

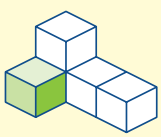
2 Sometimes one diagram is unable to give us enough information to build the solid. Use the two diagrams below to build the solid and to work out how many cubes are needed.





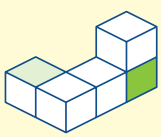
- Would we need a second diagram to build this solid?

3 Build each solid shown and then remove the coloured cubes. Draw the solid that remains on isometric grid paper.

a 

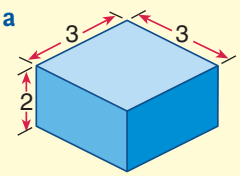
b 

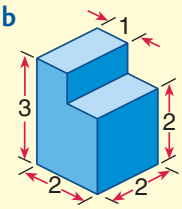
4 Build each solid shown and then add a cube to each coloured face. Draw the resulting solid on isometric grid paper.

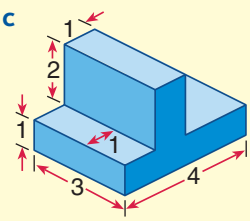
a 

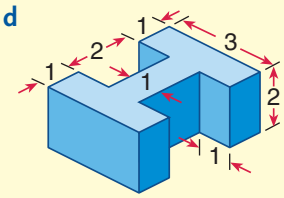
b 

5 Build the solids shown. (The numbers indicate how many cubes are needed for that length.)

a 

b 

c 

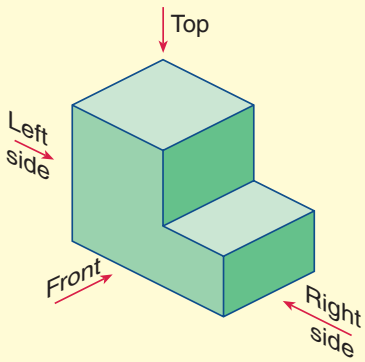
d 



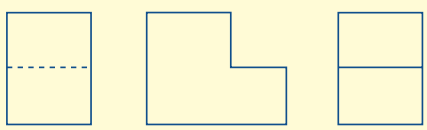
Appendix F F:05 Building solids using blocks (extension)

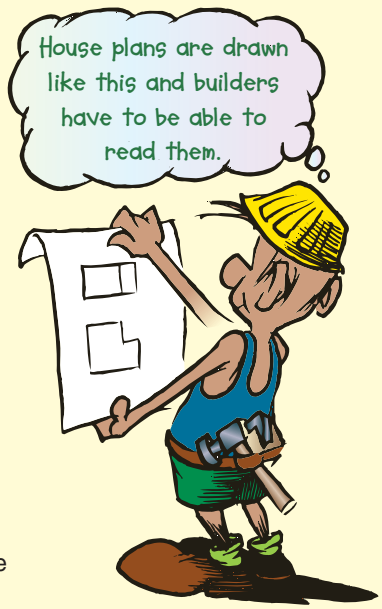
11:11 | Looking at Solids from Different Views

In many technical situations a solid is represented by drawing it from different views. It is most often drawn looking at it from the front, the top and the sides. When used together these drawings can be used to 'describe' the solid. This is illustrated below.





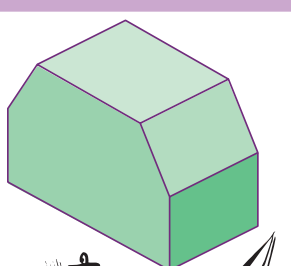




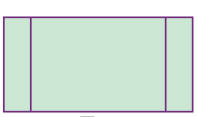
- From each view you can obtain two of the three dimensions of the solid.
- From the front view you can get the length and height of the solid.
- From the top view you can get the length and width of the solid.
- From the side views you can get the width and height of the solid.
- Dotted lines are used to show any hidden edges. This has been done in the left side view of the solid above.

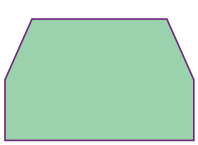
worked example 1

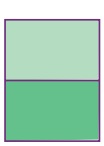
Draw the front view, top view and right side view of the solid pictured.

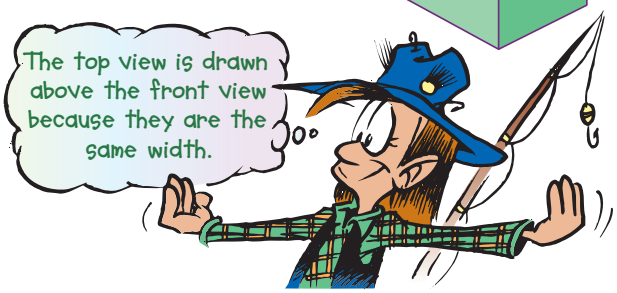


Solution



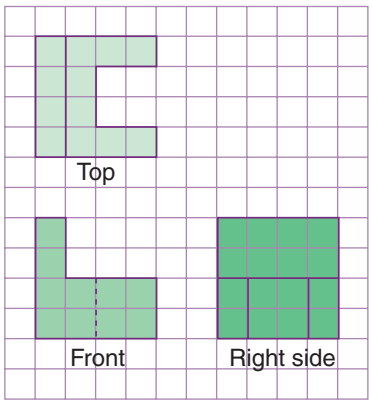


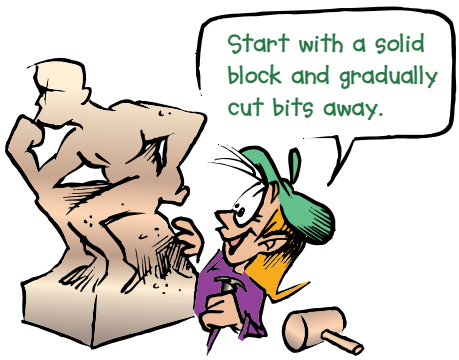




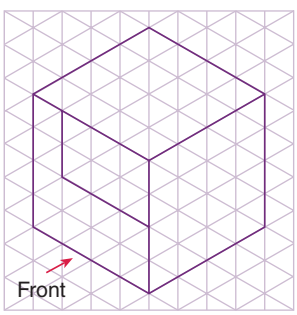
worked example 2

Using the views given in the diagram, draw a picture of the solid on isometric grid paper. Use the grid to work out the size of your drawing.



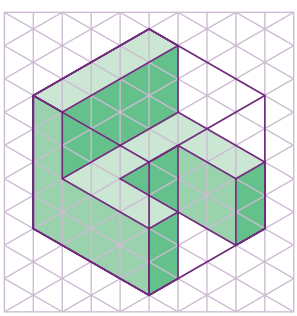


Solution



Step 1 Determine the length, width and height from the views and construct the prism that encloses the solid. Draw in the front face.

Step 2 Use the other two views to complete the drawing.



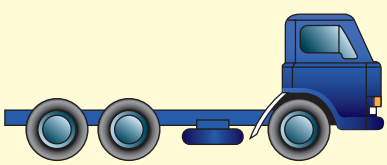
Exercise 11:11 (Some practical activities)

1 Sketch the following simple solids as viewed from above and from the side.

a matchbox **b** glass **c** cylindrical can **d** ice-cream cone

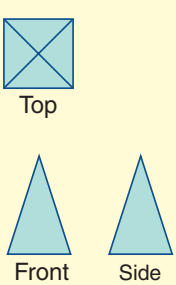
2 The diagram shows the side view of a truck. Which of the following dimensions could be worked out from this drawing?

A length **B** width **C** height

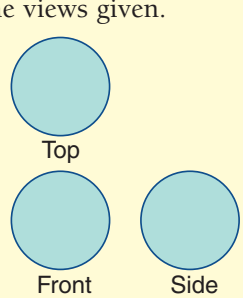


3 Name each of these solids from the views given.

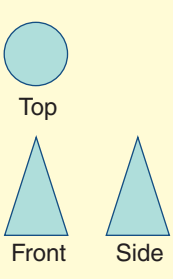
a



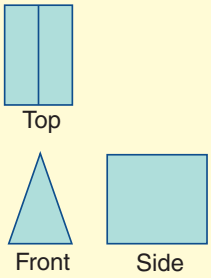
b



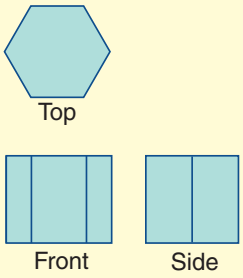
c



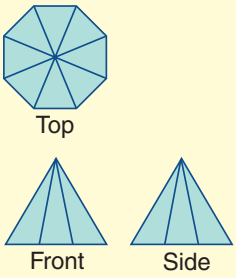
d



e

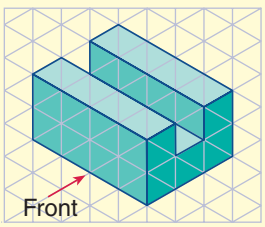


f

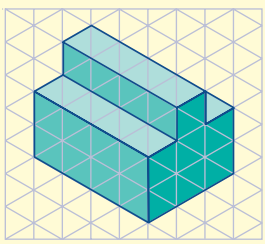


4 Each of the solids pictured would have the same top views. Draw the front and side views of each.

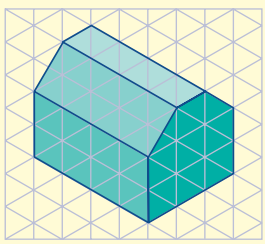
a



b

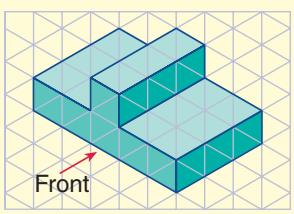


c

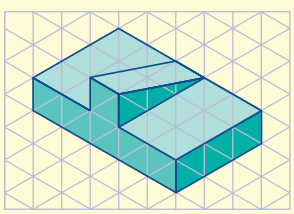


5 Each of the solids pictured would have the same front view and top view. Sketch the side view of each.

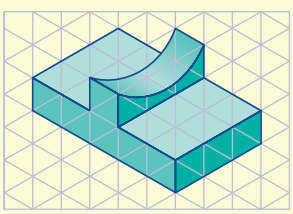
a



b

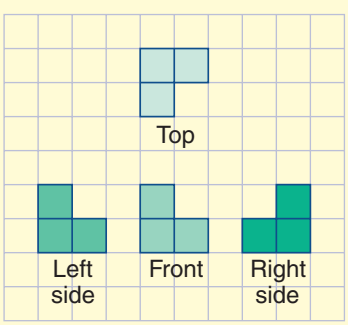


c

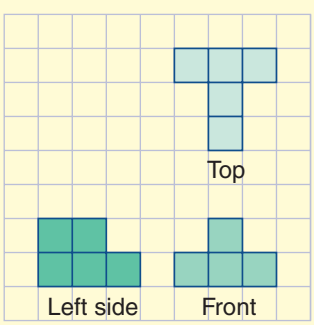


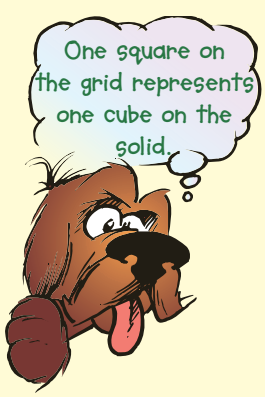
6 Below are shown different views of solids built from cubes. Build each solid and sketch it on isometric grid paper.

a



b





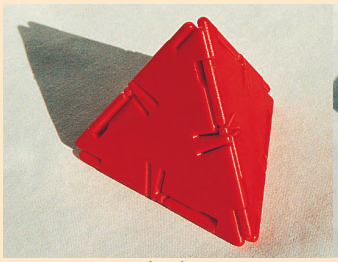
Appendix F
F:05 Fun Spot
Making a pop-up dodecahedron (extension)

Reading Mathematics 11:11 | The Platonic solids

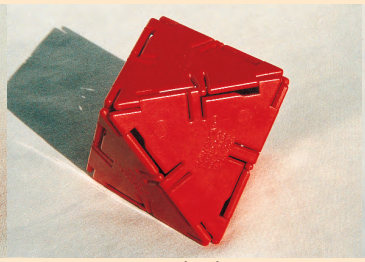
- Polyhedra are three-dimensional figures whose faces are plane shapes with straight edges. This means that the solids we have been studying so far that have only flat surfaces are called polyhedra.

> *Polyhedra* is the plural of *polyhedron*.

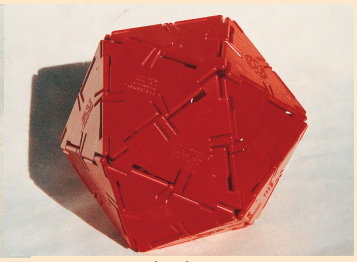
- The ancient Greeks realised that there are only five solids that can be made using regular figures such as the equilateral triangle, the square and the regular pentagon. These are the tetrahedron, octahedron, icosahedron, cube and dodecahedron.



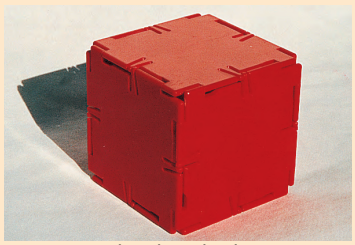
Tetrahedron



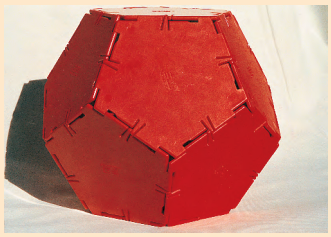
Octahedron



Icosahedron



Cube (hexahedron)



Dodecahedron

- The table below shows the properties of the Platonic solids.

Platonic solid	*Shape used*	*Number of shapes at each vertex*	*Number of:*		
			faces	*vertices*	*edges*
tetrahedron	 triangle	3	4	4	6
octahedron	 triangle	4	8	6	12
icosahedron	 triangle	5	20	12	30
hexahedron (cube)	 square	3	6	8	12
dodecahedron	 pentagon	3	12	20	30

Exercise

1 How many faces do these solids have?
a hexahedron **b** octahedron **c** tetrahedron **d** dodecahedron

2 Show that Euler's theorem holds for each of the Platonic solids.

3 Make models of at least two of the Platonic solids.

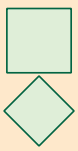
Mathematical terms 11

2D SHAPES

diagonal
- A line drawn from one corner of a polygon across the polygon to another corner.



orientation
- The way in which a figure is drawn.
- Two different orientations of a square are shown here.



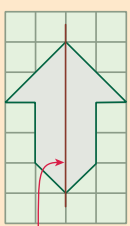
plane shape
- A shape that lies in a flat surface.
- A two-dimensional (2D) shape.

polygon
- A 2D shape that has only straight sides.

symmetry
- A balanced arrangement.

line symmetry
- A property of a figure where one half is the mirror image of the other.

axis of symmetry
- A line that divides a figure into two parts that are mirror images of each other.



rotational symmetry
- A property of a figure where it can be spun about a point so that it repeats its shape more than once in a rotation.



point symmetry
- A property where the figure repeats itself after half a turn.

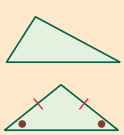
centre of symmetry
- The point about which the figure is spun.



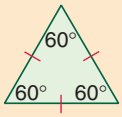
triangles
- A plane figure that has three straight sides.

scalene triangle
- Has no equal sides or angles.

isosceles triangle
- Has two sides equal and angles opposite the equal sides are equal.



equilateral triangle
- Has three sides equal and three angles equal to 60°.



acute-angled triangle
- Has three acute angles.



right-angled triangle
- Has one right angle.

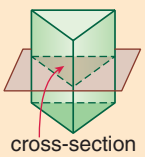


obtuse-angled triangle
- Has an obtuse angle.



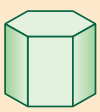
3D SHAPES

cross-section
- The shape on the face where a solid has been sliced.



cube
- A prism with six square faces.

prism
- A solid that has two identical ends joined by rectangular faces.



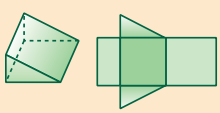
pyramid
- A solid that has a base from which triangular faces rise to meet at a point.



cylinder
- A prism-like solid that has a circle for a cross-section.



net of a solid
- A 2D drawing of the surfaces of a solid that can be folded to make the solid.

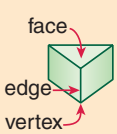


Euler's theorem: $F + V = E + 2$
- This relates the number of faces (F), vertices (V) and edges (E) of solids whose faces are flat.

face
- A flat surface.

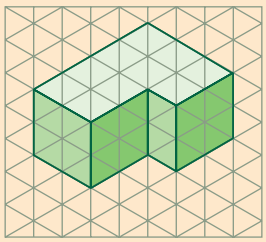
vertex
- A corner.

edge
- A line where two faces meet.



isometric projection

- A way of drawing 3D pictures of solids using isometric grid paper.
- The shape is drawn as if observed from above with sides receding to right and left.



polyhedra

- Three-dimensional figures whose faces are plane shapes with straight edges.
- The singular of *polyhedra* is *polyhedron*.

Platonic solids

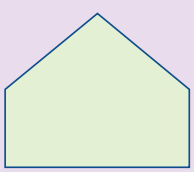
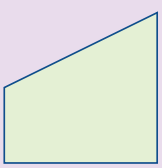
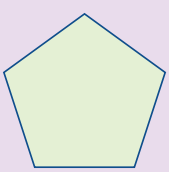
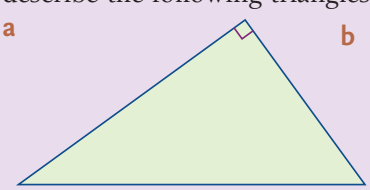
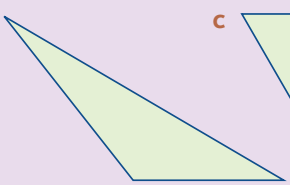
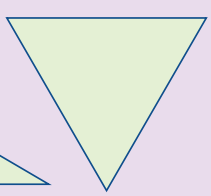
- These are the regular polyhedra: tetrahedron, cube (hexahedron), octahedron, dodecahedron, icosahedron.

diagnostic test

11

Diagnostic Test 11: | Shapes

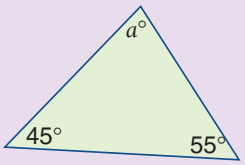
- Each section of the test has similar items that test a certain type of question.
- Errors made will indicate areas of weakness.
- Each weakness should be treated by going back to the section listed.
- You will need a ruler and compasses.

	Section
1 Show how each shape can be divided up into the pieces written underneath it.	11:01
a  a rectangle and triangle	
b  two triangles	
c  a trapezium and triangle	
2 Use one of the terms 'acute-angled', 'obtuse-angled' or 'right-angled' to describe the following triangles.	11:02
a 	
b 	
c 	

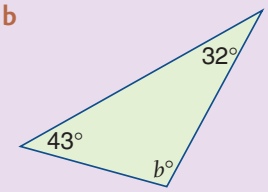
Section

3 Find the value of the pronumeral in each figure. 11:02

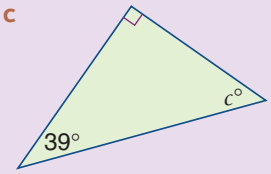
a



b

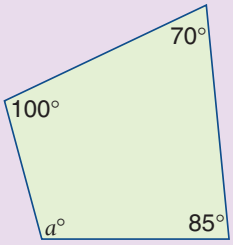


c

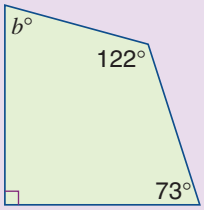


4 Find the value of the pronumeral in each figure. 11:03 11:04

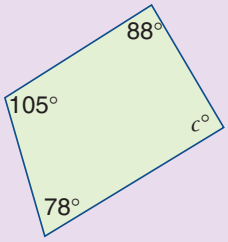
a



b



c

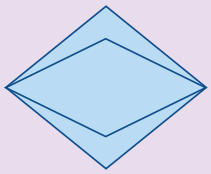


5 How many axes of symmetry does each of the following shapes have? 11:06

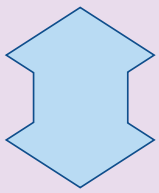
a



b



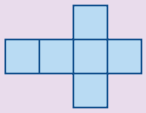
c



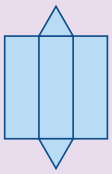
6 Which of the shapes in question 5 have point symmetry? 11:06

7 What solids are formed from the following nets? 11:08

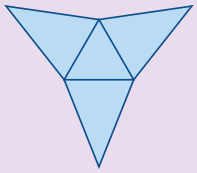
a



b



c



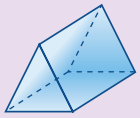
8 Using isometric grid paper, make drawings of: 11:09

a a rectangular prism **b** a triangular prism

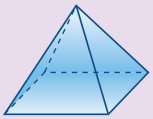
c a hexagonal prism

9 Draw the top view, front view and side view of these shapes. 11:11

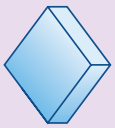
a



b



c



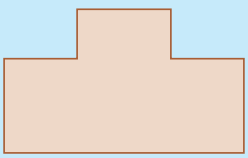
assignment 11A

Chapter 11 | Revision Assignment

1 In each of the following, write as a numeral the number written in words.
- **a** six thousand and thirty-five
- **b** seven hundred thousand
- **c** twelve thousand, six hundred and twenty-five
- **d** thirteen million, four hundred thousand and eighty-nine

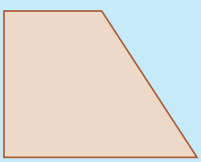
2 Sketch the following shapes and show how each can be divided into the two shapes named.

a



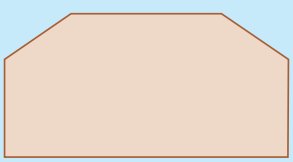
two rectangles

b



rectangle and triangle

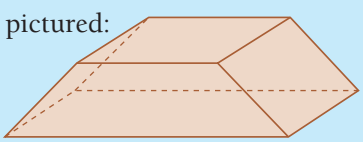
c



trapezium and rectangle

3 Simplify:
- **a** $1{\cdot}5 \times 10$
- **b** $1{\cdot}5 \times 100$
- **c** $15{\cdot}6 \div 10$
- **d** $15{\cdot}6 \div 100$

4 For the solid pictured:



- **a** find the number of faces
- **b** find the number of edges
- **c** find the number of vertices
- **d** name the shape of the cross-section
- **e** name the solid

5 Find:
- **a** $-4 + 8$
- **b** $5 - -8$
- **c** $-30 \div 6$
- **d** $(-4)^2$

6 Copy and complete:
- **a** 4 cm = . . . mm
- **b** 3·5 km = . . . m
- **c** 25 000 mm = . . . m
- **d** 420 m = . . . km

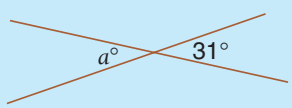
7 Find the perimeter of:
- **a** a square of side 200 m
- **b** a square of size 6·3 cm
- **c** a rectangle 6·5 m long and 2·5 m wide
- **d** a rectangle 1·5 km long and 800 m wide

8 If it is 10:15 am now, what time:
- **a** was it 1 h 10 minutes ago?
- **b** will it be in 2 h 30 min?
- **c** was it 2 h 40 minutes ago?
- **d** will it be in 4 h 55 min?

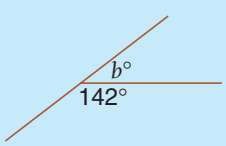
9
- **a** Calculate the distance travelled if a car travelled for 3 hours at an average speed of 60 km/h.
- **b** How long would it take a woman walking at 7 km/h to walk 21 km?
- **c** Calculate the average speed of a boat if it takes 4 hours to travel 60 km.

10 For each, find the value of the pronumeral.

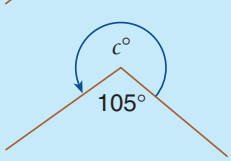
a



b



c



Drag and Drop CW Interactive
1 Recognising plane shapes
2 Properties of 2D shapes
3 Recognising 3D shapes

Review CW Questions

Chapter 11| Working Mathematically

1 Use ID Card 4 on page xvi to identify:

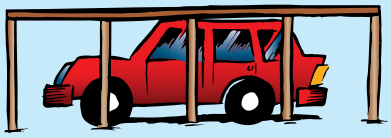
a	15	b	16
c	17	d	18
e	19	f	20
g	21	h	22
i	23	j	24

2 It takes me 25 minutes to drive to work. At what time must I leave to reach work 10 minutes early, if I start work at 7:25 am?

3 Beads of four different colours are being threaded onto a string in the order red, blue, yellow, green. What would be the colour of the 182nd bead?

4 You go to the post office and buy 10 stamps. The stamps come in one long strip. You fold them along the perforations to form a single stack. What is the least number of folds needed to do this?

5 The length of a carport is 10 000 mm. On one side, the carport is supported by five posts, each post being 100 mm wide. How far apart must the posts be if they are evenly spaced?



6 By referring to the graph below, answer the following questions.

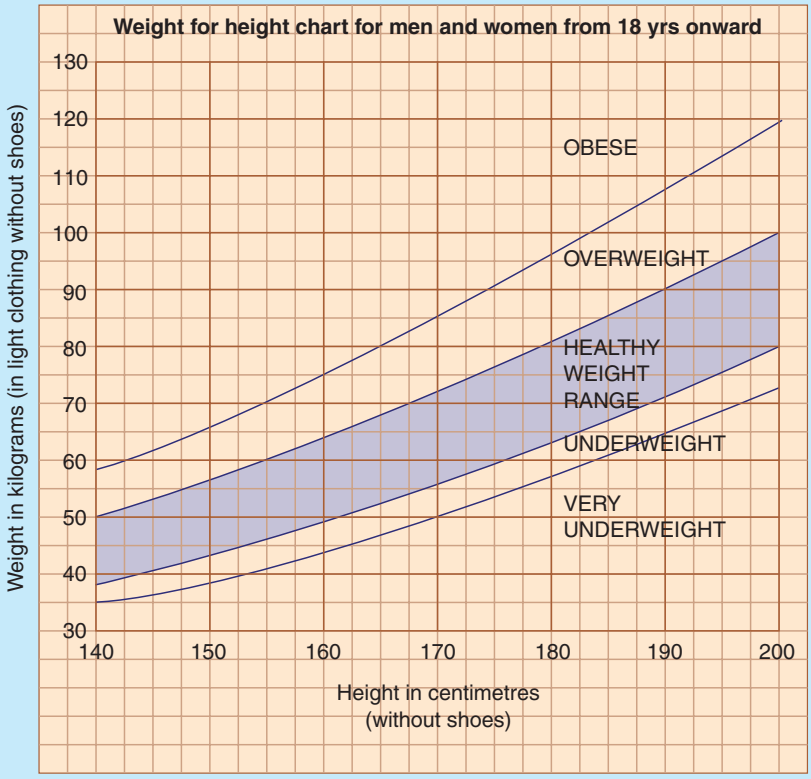
a Bill is 170 cm tall and weighs 70 kg. Does this put him in the healthy weight range?

b Jill is 150 cm tall. What is the lightest weight she can be and still be in the healthy weight range?

c If a man weighs 70 kg, between what heights would he have to be to be in the healthy weight range?

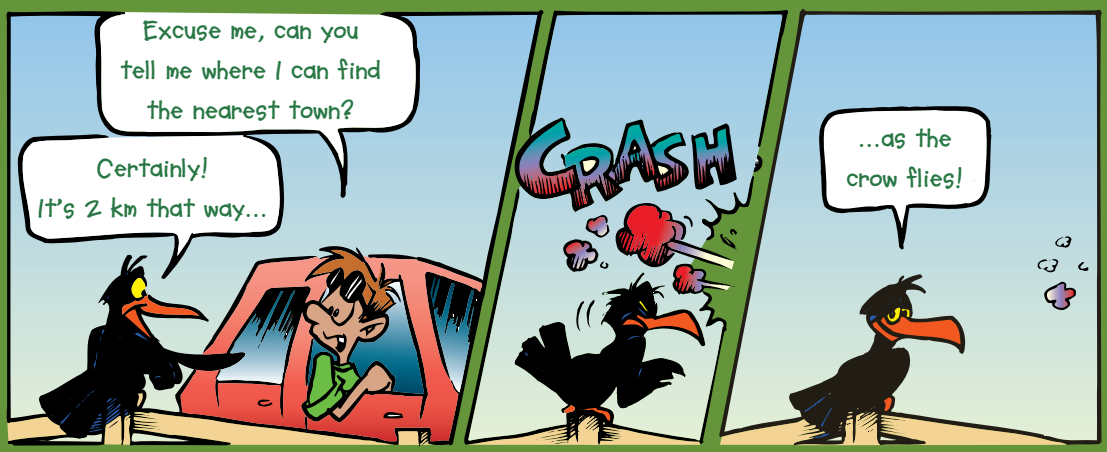
d If a woman is 180 cm tall, between what weights must she be to be in the healthy weight range?

e I am 175 cm tall and weigh 80 kg. How much weight must I lose to be in the healthy weight range?



12

Measurement: Length and Time



Chapter Contents

12:01 Measuring instruments
ID Card
12:02 Units of length
Investigation: Other units of length
12:03 Measuring length
Investigation: Measuring length
12:04 Estimating length
Practical Activity: Estimating length
12:05 Perimeter
12:06 The calendar and dates
12:07 Clocks and times
12:08 Operating with time
12:09 Longitude and time (extension)
12:10 Timetables
ID Card
Fun Spot: And now for something light
Investigation: Distance, speed and time
Mathematical Terms, Diagnostic Test, Revision Assignment, Working Mathematically

Learning Outcomes

Students will:

- Use twenty-four hour time and am and pm notation in real-life situations, and construct timelines.
- Use formulae to calculate perimeter.
- Perform calculations of time that involve mixed units.

Areas of Interaction

Approaches to Learning, Homo Faber, Environment

12:01 | Measuring Instruments

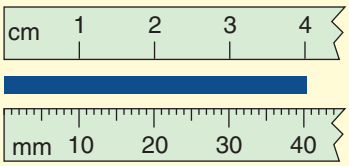
Refer to ID Card 1 (Metric Units) on page xiv. Identify figures (1) to (24).
Learn the units you do not know.

It's amazing how often we read measurements on a scale or digital readout. Speedometers, thermometers, digital watches, clocks, microwave ovens, weighing scales, measuring jugs and videos are all examples of these.

Exercise 12:01

Foundation Worksheet 12:01
Measuring instruments
1 Measure each length in centimetres.
2 What time is shown?
a b

1 Give the length of the coloured rod in both centimetres and millimetres.



2 Give the depth of the flood waters in each case.

a



b



3 Match each picture with one of the times given in the box below the clocks.

a



b



c



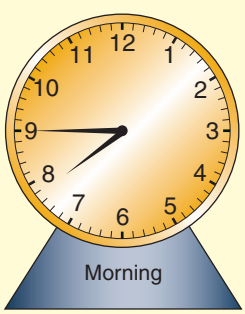
d



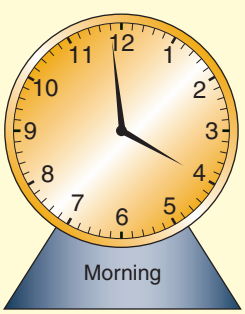
e



f



g



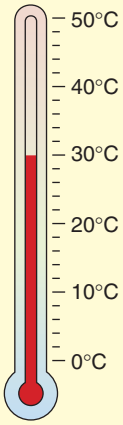
h



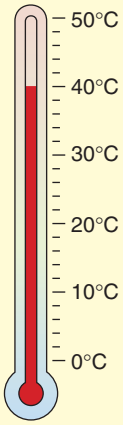
Seven minutes to nine, before noon	Seven minutes to nine, after noon
A quarter past eight, before noon	A quarter past eight, after noon
A quarter to eight, before noon	A quarter to eight, after noon
One minute to four, before noon	One minute to four, after noon

4 Write down the temperature shown on each thermometer.

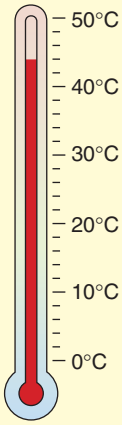
a



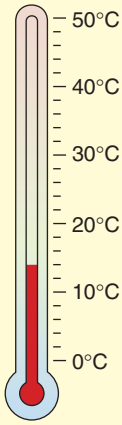
b



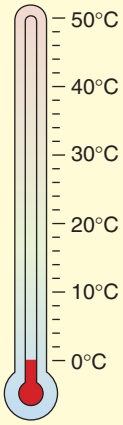
c



d

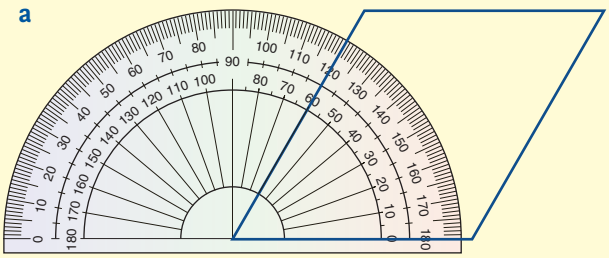


e

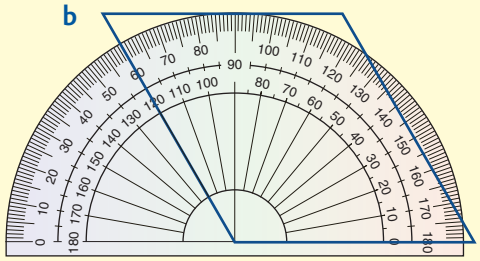


5 Write the measurement shown in each, including the unit used.

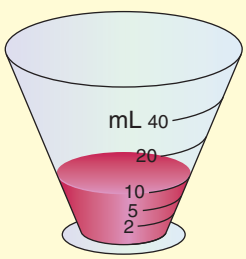
a



b

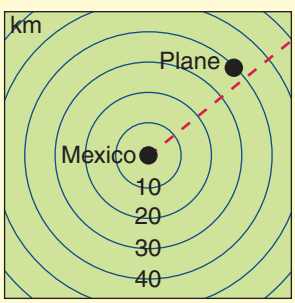


c



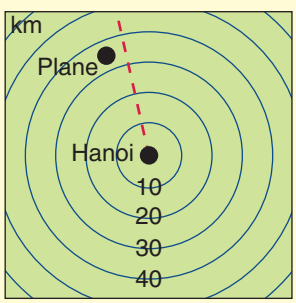
How much medicine has been prepared?

d



How far is the plane from Mexico?

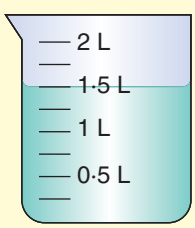
e



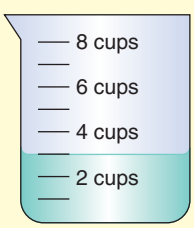
How far is the plane from Hanoi?

6 Give the volume of liquid in each jug.

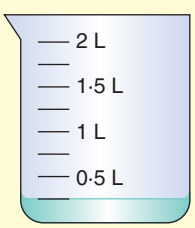
a



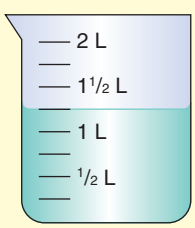
b



c



d



7 On the odometer of a car, the coloured figure at the end measures tenths of a kilometre. What measurement is shown on each odometer below?

a



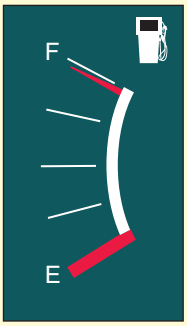
b



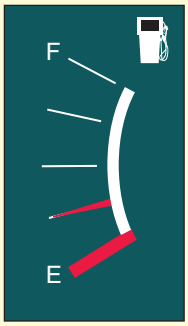
8 What is the speed shown on each of the speedometers in question 7?

9 Describe the measurement shown on each scale below.
Units are not always given, and you may need to make up the unit yourself, eg half a tank, warm.

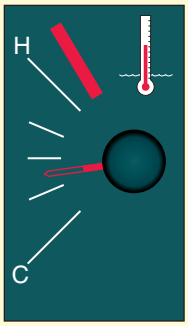
a



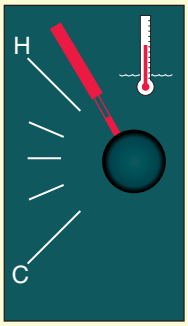
b



c

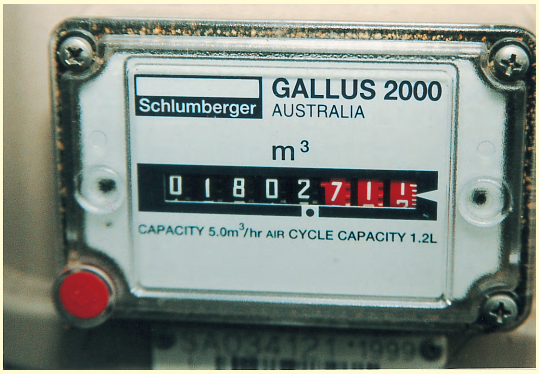


d



10 For each photograph give the reading on the scale. (Units of measurement are not required.)

a



b



12:02 | Units of Length

prep quiz 12:02

Answer the following.

1 $6{\cdot}2 \times 100$	**2** $2{\cdot}34 \times 1000$	**3** $670 \div 100$	**4** $5250 \div 1000$
5 $1{\cdot}35 + 2{\cdot}19$	**6** $4{\cdot}8 + 0{\cdot}69$	**7** $9{\cdot}3 - 4{\cdot}6$	**8** $10 - 4{\cdot}63$

Measure:

9 the length of this page, to the nearest centimetre

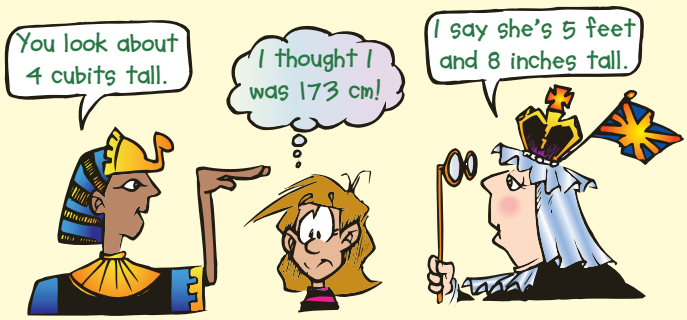
10 the distance from *A* to *B* to the nearest millimetre

A ——————— *B*

When doing questions 9 and 10 in the Prep Quiz:

- you needed to know what the units were (ie centimetres or millimetres)
- you needed an instrument to do the measuring (eg a ruler)

Even though many countries now use the **SI** or **metric system** of measurement, which originated in France, others use their own traditional systems of units. Many systems of measurements are no longer in use.



investigation 12:02

Investigation 12:02 | Other units of length

Please use the Assessment Grid on the following page to help you understand what is required for this Investigation.

Some ancient units were the *cubit*, *span* and *palm*. In Australia the *imperial system* was used before the *metric system*.

Some of the imperial units were the *foot*, *inch*, *yard*, *mile*, *furlong*, *rod*, *chain* and *fathom*.

a Find out about these units and any others you come across.

b Discover relationships between the old imperial units, eg 12 inches = 1 foot.

In the metric system, the basic unit of length is the **metre**. The prefixes **kilo**, **centi** and **milli** are then added to give the other most common units as shown in the table below.

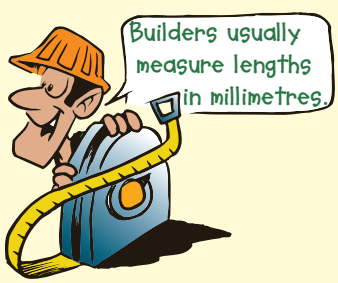
Unit	*Symbol*	*Meaning*
kilometre	km	1000 m
metre	m	1 m
centimetre	cm	$\frac{1}{100}$ m
millimetre	mm	$\frac{1}{1000}$ m

This means that:

100 cm = 1 m

1000 mm = 1 m

(also 10 mm = 1 cm)



Assessment Grid for Investigation 12:02 | Other units of length

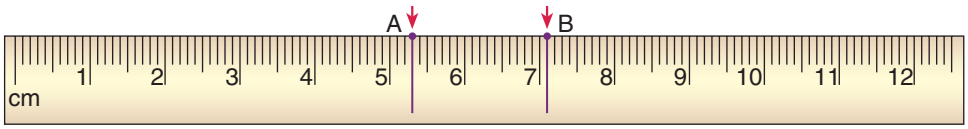
The following is a sample assessment grid for this investigation. You should carefully read the criteria *before* beginning the investigation so that you know what is required.

Assessment Criteria (B, C, D)				Achieved ✓
Criterion B Application & Reasoning	a	No real organised approach has been used to find out or arrange answers.	1	
			2	
	b	An organised approach has been used and all given units are discussed.	3	
			4	
	c	An organised approach has been used and further imperial units are discussed.	5	
			6	
	d	The answers to parts c, d and e demonstrate a good understanding of the practical use of measurement.	7	
			8	
	e	The answers to parts c, d and e demonstrate an excellent understanding of the practical use of measurement.	9	
			10	
Criterion C Communication	a	No working out is shown and presentation is poor.	1	
			2	
	b	Presentation is very good and answers are easy to find.	3	
			4	
	c	Parts c, d and e are discussed in some depth using correct mathematical terminology.	5	
			6	
Criterion D Reflection & Evaluation	a	The discussion of units is very superficial.	1	
			2	
	b	The given units are discussed at a satisfactory level.	3	
			4	
	c	The discussion of parts c, d and e shows that some reflection of measurement has been made.	5	
			6	
	d	The discussion of parts c, d and e demonstrates reflection of the answers to a and b and the practical use of measurement.	7	
			8	

You need to be able to read measurements in centimetres and millimetres, using rulers or tape measures. Make sure you can change from one unit to another.

worked example 1

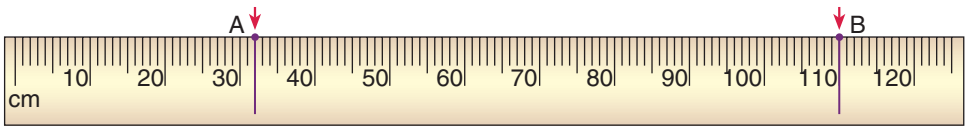
Read the measurements indicated on the ruler below.



To the nearest centimetre A would be 5 cm.
To the nearest millimetre A would be 5 cm and 3 mm. This is 53 mm and 5·3 cm.
Likewise, B, to the nearest mm, would be written as 71 mm or 7·1 cm.

worked example 2

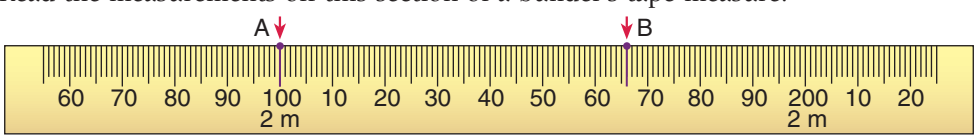
Read the measurements off this ruler, which is marked in millimetres.



To the nearest mm, A = 32 mm or 3·2 cm; B = 110 mm or 11·0 cm.

worked example 3

Read the measurements off this section of a builder's tape measure.



For measurement A, we can see that the line is on the 100 mm mark, but below it we are told that we are already 2 metres along the tape, so A = 2 m plus 100 mm. This should be written as 2100 mm or 2·100 m.

For B we must note three parts to the measurement: B = 2 m + 100 mm + 66 mm.
This should then be recorded as 2166 mm or 2·166 m.
(Both A and B have been written correct to the nearest millimetre.)

worked example 4

Convert each measurement to the units indicated.

a 5 m to cm **b** 1·2 km to m **c** 2·45 m to mm
d 420 cm to m **e** 850 m to km **f** 763 mm to cm

Remember!
- **To convert small units into large units, you divide!**
- **To convert large units to small units, you multiply!**

Solutions

a Since 1 m = 100 cm,
5 m = 5 × 100 cm
= 500 cm

b Since 1 km = 1000 m,
1·2 km = 1·2 × 1000 m
= 1200 m

continued →→→

c Since 1 m = 1000 mm,
2·45 m = 2·45 × 1000 mm
= 2450 mm

d Since 100 cm = 1 m,
420 cm = 420 ÷ 100 m
= 4·2 m

e Since 1000 m = 1 km,
850 m = 850 ÷ 1000 km
= 0·850 km

f Since 10 mm = 1 cm
763 mm = 763 ÷ 10 cm
= 76·3 cm

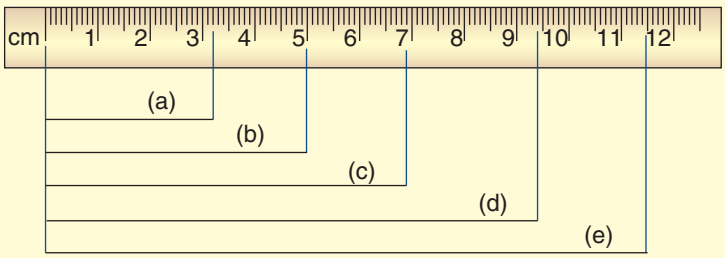
Note: Zeros like this one on the end of a decimal fraction can be omitted, depending on how accurate the answer needs to be, eg 850 m = 0·85 km.
However, if the answer had to be written to the nearest metre, it would have to be written as 0·850 km.

You only have to know how to multiply decimals by 10, 100 or 1000.

Exercise 12:02

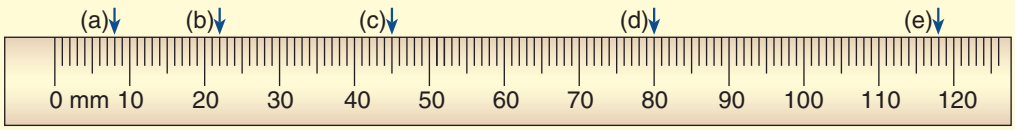
Foundation Worksheet 12:02
Units of length
1 Questions like Q1.
2 **a** 8 cm = . . . mm
b 300 cm = . . . m
3 **a** 3 min = . . . s
b 2 h = . . . min

1 Write down the length of each interval, to the nearest centimetre.



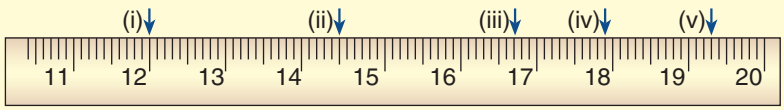
2 Write down the length of each interval in question 1 correct to the nearest millimetre.

3 Write down each measurement indicated, correct to the nearest millimetre.

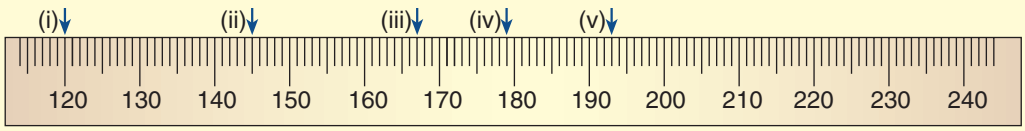


4 Rewrite each of your answers for question 3 in centimetres (eg 32 mm = 3·2 cm).

5 **a** Write down the measurement, in centimetres, indicated by each arrow on this section of a centimetre rule.

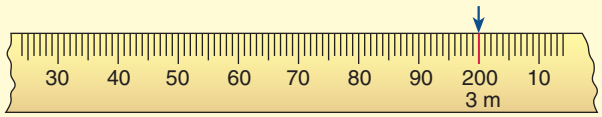


b Write down the measurement, in centimetres, indicated by each arrow on this section of a millimetre rule.

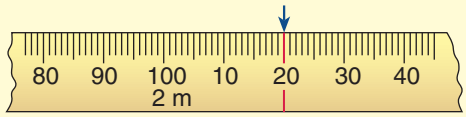


6 These sections of a builder's tape measure are marked off in millimetres.
Write down the measurements indicated, in millimetres.

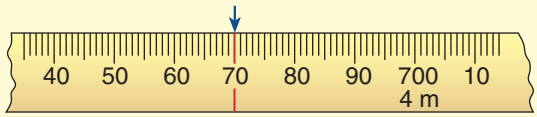
a



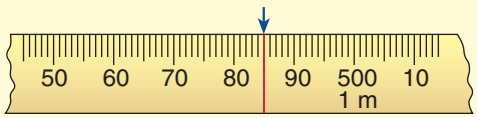
b



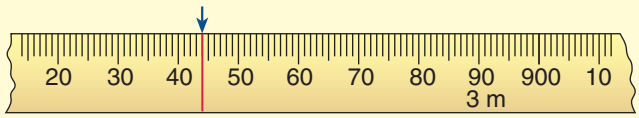
c



d

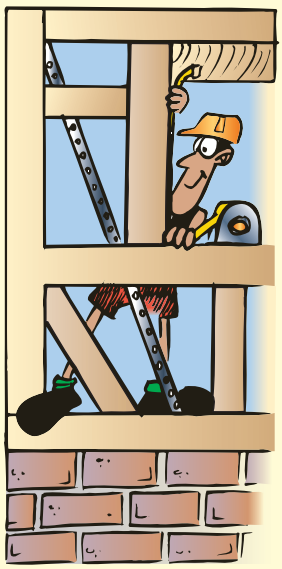


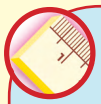
e



f







Important notice!
- **An *interval* is part of a line with a definite length.**
- **A *line* goes on forever in both directions.**

7 Convert each of these measurements into the smaller units indicated.

a 5 cm = . . . mm	**b** 17 cm = . . . mm	**c** 6·2 cm = . . . mm
d 25·6 cm = . . . mm	**e** 6 m = . . . cm	**f** 23 m = . . . cm
g 1·6 m = . . . cm	**h** 2·35 m = . . . cm	**i** 5 km = . . . m
j 67 km = . . . m	**k** 7·3 km = . . . m	**l** 9·32 km = . . . m
m 9 m = . . . mm	**n** 35 m = . . . mm	**o** 2·9 m = . . . mm
p 8·471 m = . . . mm	**q** 3 km = . . . cm	**r** 4·36 km = . . . cm
s 6 km = . . . mm	**t** 9·217 km = . . . mm	

8 Convert each of these measurements into the larger units indicated.

a 300 cm = . . . m	**b** 1200 cm = . . . m	**c** 60 cm = . . . m
d 537 cm = . . . m	**e** 5000 mm = . . . m	**f** 25 000 mm = . . . m
g 2500 mm = . . . m	**h** 630 mm = . . . m	**i** 2000 m = . . . km
j 17 000 m = . . . km	**k** 6700 m = . . . km	**l** 580 m = . . . km
m 40 mm = . . .cm	**n** 260 mm = . . . cm	**o** 65 mm = . . . cm
p 7 mm = . . . cm	**q** 10 000 cm = . . . km	**r** 27 000 cm = . . . km
s 9000 cm = . . . km	**t** 200 000 mm = . . . km	

9 Convert these measurements to the units indicated.

a	200 cm = . . . m	**b**	3000 m = . . . km
c	6 m = . . . mm	**d**	50 mm = . . . cm
e	25 km = . . . m	**f**	15 m = . . . cm
g	2·3 m = . . . cm	**h**	5200 mm = . . . m
i	6700 m = . . . km	**j**	1·7 km = . . . m
k	2·6 m = . . . mm	**l**	750 mm = . . . cm
m	635 cm = . . . m	**n**	1·95 m = . . . cm
o	1960 m = . . . km	**p**	93 600 mm = . . . m
q	7·63 km = . . . m	**r**	935 mm = . . . cm
s	75 cm = . . . m	**t**	870 m = . . . km
u	620 mm = . . . m	**v**	3·2 km = . . . cm
w	73 000 cm = . . . km	**x**	0·17 km = . . . m



10
a If I swim 20 laps of a 50-metre pool, how many kilometres will I swim?
b A cyclist rides three legs of a race, which measure 9·5 km, 6·7 km and 8·2 km. What is the total length of the race?
c I am 1·8 m tall. How many centimetres is this?
d Which total length is the longer, 8 pieces of timber each 1·2 m long or 6 pieces of timber each 1·5 m long?
e Ribbon is 60 cents a metre. How much would 750 cm of ribbon cost?
f How many pieces of timber 30 cm long can I cut from a length of timber that is 4 m long?

11 How many:

a	cm in $1\frac{1}{2}$ m?	**b**	cm in $3\frac{1}{10}$ m?	**c**	cm in $5\frac{1}{4}$ m?
d	mm in $3\frac{1}{2}$ m?	**e**	mm in $\frac{1}{4}$ m?	**f**	mm in $2\frac{1}{5}$ m?
g	m in $5\frac{1}{2}$ km?	**h**	m in $2\frac{1}{4}$ km?	**i**	m in $1\frac{3}{10}$ km?



12 A road sign in front of my house shows the distance to Buckley's Falls to be 6·3 km.
a If I travel to Buckley's Falls and back 5 times each week, what total distance is this?
b If my car can travel 9 km on 1 litre of petrol, how many litres of petrol will I use each week, travelling to Buckley's Falls and back?

13 There are other units of length in the metric system that are used less frequently, as indicated in the table. Use the table to complete these conversions.

a	3 hm = . . . m	**b**	7 dam = . . . m
c	50 dm = . . . m	**d**	9 m = . . . dm
e	60 m = . . . dam	**f**	700 m = . . . hm
g	4 dm = . . . cm	**h**	50 hm = . . . km

Unit	*Meaning*
hectometre (hm)	1 hm = 100 m
decametre (dam)	1 dam = 10 m
decimetre (dm)	1 dm = $\frac{1}{10}$ m

14 For very large or very small lengths we could use two further units:
1 megametre (Mm) = 1 million metres, and 1 micrometre (μm) = 1 millionth of a metre.

a	How many km in 1 Mm?	**b**	How many μm in 1 cm?
c	Convert 6000 km to megametres.	**d**	Convert 0·02 mm to micrometres.

12:03 | Measuring Length

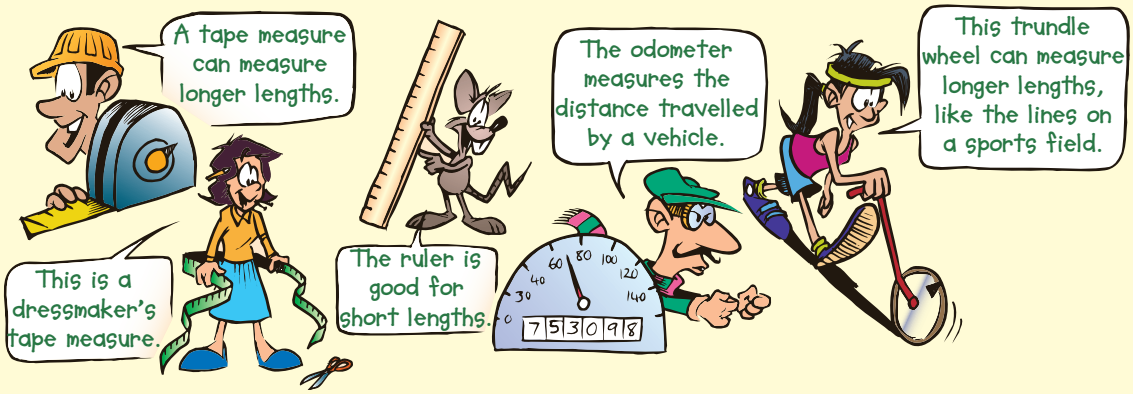
prep quiz

12:03

Complete:

1 1 km = . . . m	**2** 3 m = . . . cm	**3** 50 mm = . . . cm
4 700 cm = . . . m	**5** 60 000 m = . . . km	**6** 7 cm = . . . mm
7 5 m = . . . mm	**8** 7500 mm = . . . m	**9** 6·35 km = . . . m
10 765 cm = . . . m		

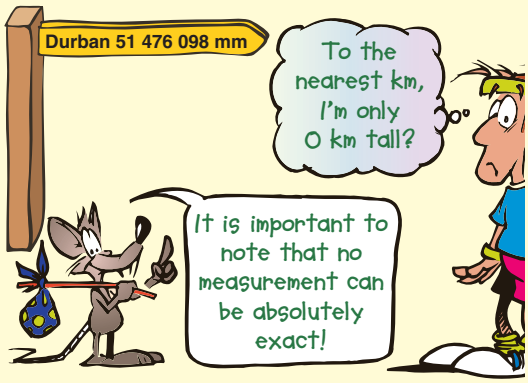
To measure a length we need an instrument that will do the job. Some of these are shown below.



Each instrument is marked off in units suitable to the lengths it is likely to be used to measure.

- **Rulers** and **tape measures** would be marked in centimetres or millimetres to measure shorter lengths accurately.
- A **trundle wheel** might only be used to measure a length to the nearest metre, and sometimes may have a counter attached to count the metres.
- The **odometer** on a car usually measures distances in tenths of a kilometre. The end digit (often in red) measures the tenths, so the remaining digits indicate the number of kilometres the car has travelled.

You can only measure a length correct to the nearest division on your measuring device, and as accurately as your eyesight will allow. Of course, the degree of accuracy needed will depend on what you are measuring. It would be silly to measure the distance between two towns in millimetres or your height in kilometres!



Exercise 12:03

Foundation Worksheet 12:03

Measuring length

1 Measure these lengths in centimetres.

a

2 Measure these lengths in millimetres.

a 

1 What units would you use to measure the following?

- **a** the height of a person
- **b** the distance from Madrid to Barcelona
- **c** the length of a football field
- **d** the width of your thumbnail
- **e** the dimensions of a milk carton
- **f** the length of a room
- **g** a person's waist
- **h** the length of a fly

2 Which of the instruments listed below might you use to measure each of the items in question 1? ruler, tape, trundle wheel, odometer

3 Measure each of these lengths to the nearest cm.

a

b

c

d

e

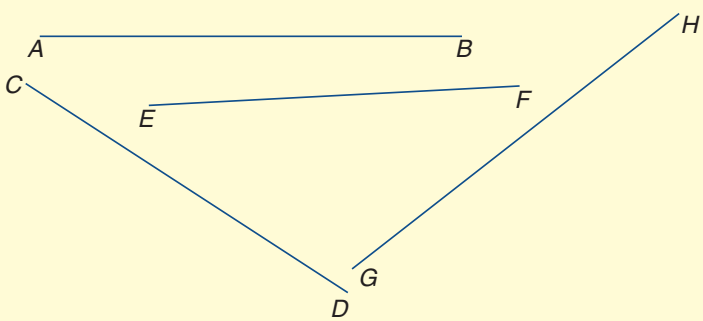
f

g

h

4 Now measure each interval in question 3 correct to the nearest mm.

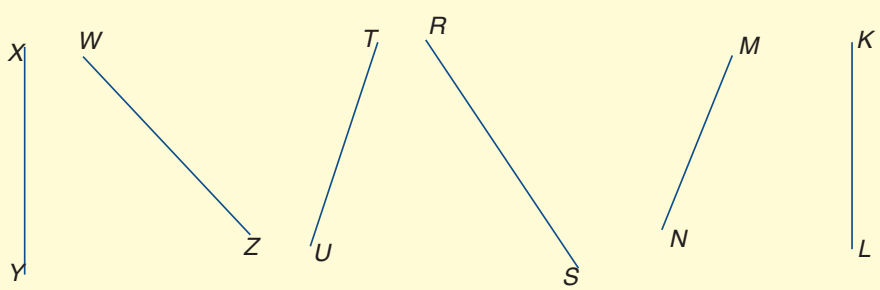
5 Arrange these intervals in order of length *from shortest to longest*.



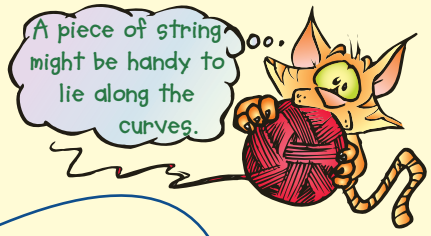
Notice

We name this interval *AB* (or *BA*).

6 List these intervals in order from longest to shortest.



7 Measure the length of these curves correct to the nearest cm.



a

b

c

d

8



a How many complete kilometres has this car travelled?
b How much further must the car travel for the reading to be 53 100?
c What will the odometer reading be after travelling a further $25\frac{1}{2}$ km.
d How many more kilometres must this car travel for the odometer to show 000000?
e According to the speedometer, at what speed is the car travelling?

Appendix G G:01 Investigation: Measurement extension

investigation 12:03

Investigation 12:03 | Measuring length

Please use the Assessment Grid on the following page to help you understand what is required for this Investigation.

Length	*Measurement*
height	
arm	
foot	
waist	

1 Complete this table for your own measurements.
Get a friend to check your accuracy.
(Measure to the nearest centimetre.)

2 Use a tape or metre rule to measure these lengths in your classroom to the nearest cm. Check your results with others in the class.
a the height of your desk
b the length of the board
c the length of the room
d the height of the ceiling
e the width of the doorway
f the length of the teacher's desk

3 Use a tape or trundle wheel to measure:
a the outside length of a school building
b the length of a football field
c the length of a basketball court
d the distance from the school gate to the door of your classroom

Assessment Grid for Investigation 12:03 | Measuring length

The following is a sample assessment grid for this investigation. You should carefully read the criteria *before* beginning the investigation so that you know what is required.

Assessment Criteria (B, C, D)				Achieved ✓
Criterion B Application & Reasoning	a	The way the measurements have been made does not ensure accuracy.	1	
			2	
	b	Measurements were taken accurately but not rounded off properly.	3	
			4	
	c	Measurements were rounded properly but the ceiling measurement was not organised properly.	5	
			6	
	d	All measurements were taken in an organised way and rounded properly.	7	
			8	
	e	Comments made on the tools used to take these measurements in question 4 demonstrate a good understanding of the exercise.	9	
			10	
Criterion C Communication	a	Presentation is poor and it is difficult to find the answers.	1	
			2	
	b	Presentation is good and answers are arranged in an organised way.	3	
			4	
	c	Presentation is good and the discussion in question 4 uses correct mathematically terminology.	5	
			6	
Criterion D Reflection & Evaluation	a	There has been an attempt to explain the methods used and to check the accuracy of answers.	1	
			2	
	b	The methods used are justified and the accuracy of the results have been checked.	3	
			4	
	c	The reliability of the findings and how to improve their accuracy are discussed.	5	
			6	
	d	The discussion in question 4 includes the reliability of the findings and possible alternatives that are more accurate.	7	
			8	

12:04 | Estimating Length

prep quiz 12:04

Arrange these measurements in order from shortest to longest.

53 cm	5·3 m	53 000 mm	0·53 km	5·3 cm

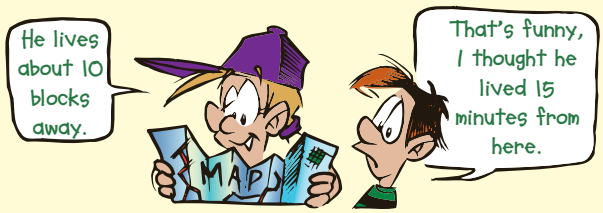
1 . . . . 2 . . . . 3 . . . . 4 . . . . 5 . . . .

Measure each interval on this line to the nearest cm.



6 AD = ... 7 AC = ... 8 BC = ... 9 BA = ... 10 DB = ...

Now that you have had practice in measuring lengths, let's see how good you are at estimating various lengths before you measure them.



People often use very general ways of estimating lengths or distances, and with practice it is possible to become quite good at it.

Obviously in this next exercise there is no correct answer for your estimates. You should, however, try to be as close to the exact measurements as possible.

Exercise 12:04

1 Estimate the length of each interval to the nearest cm, then measure each one to check your accuracy.

1 cm

10 cm

a _____ b _____

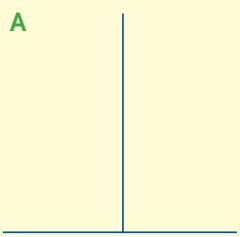
c _____

d _____ e _____

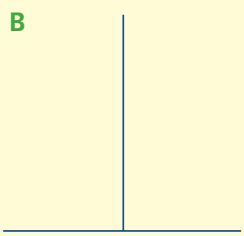
f _____ g _____

2 For which diagram do you think the horizontal and vertical lengths are the same? Check by measuring them.

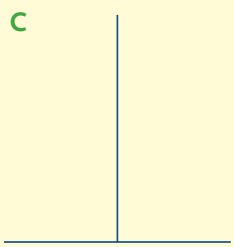
A



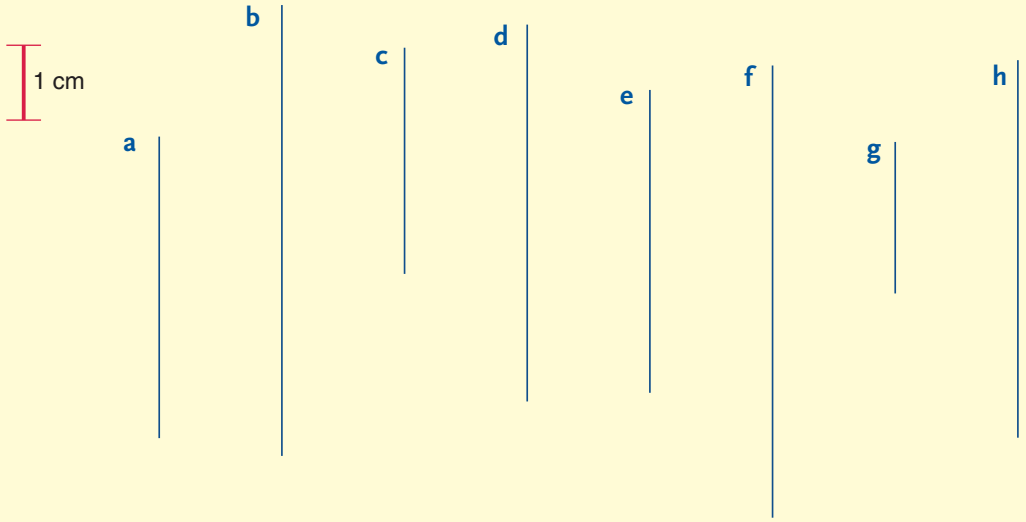
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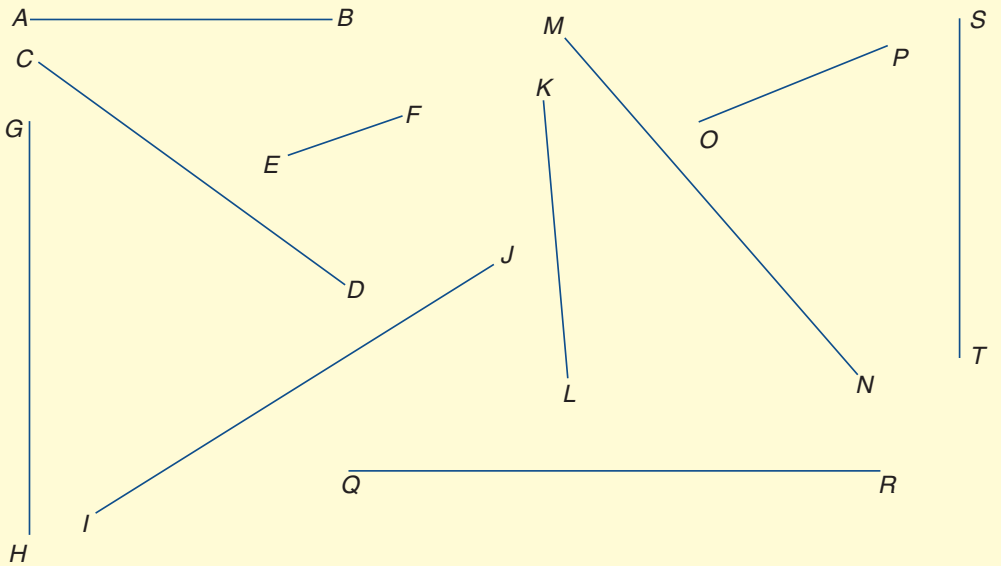
C



3 After doing questions 2 and 3, you may agree with the statement that most people are not good at estimating vertical lengths and usually think they are longer than they really are. Try estimating these lengths to the nearest cm. Check by direct measurement.



4 Now try estimating the lengths of these intervals to the nearest millimetre. Check by measuring and record your results. For each mm you are away from the exact measurement, give yourself a point. Compare your score with others. The lower the score, the better estimator you are.



Line	*Estimate*	*Length*	*Score*
AB			
CD			
EF			
GH			
IJ			

Line	*Estimate*	*Length*	*Score*
KL			
MN			
OP			
QR			
ST			

Practical Activity 12:04 | Estimating length

1 Estimate the following lengths and check by measuring.
   - **a** your forearm (elbow to wrist) (cm)
   - **b** from your knee to your ankle (cm)
   - **c** your index finger (mm)
   - **d** your nose (mm)

2 Select 5 students and estimate their heights correct to the nearest cm.

3 Choose 5 objects in the room, such as a book, pencil, etc, and estimate their lengths to the nearest mm.

4 Estimate the distance from one corner of the room to the corner diagonally opposite. Check with a tape measure.

5 Go outside and place pegs in the ground to indicate your estimates for lengths of 10 m, 20 m and 50 m. Check your accuracy by measuring with a tape or trundle wheel.

6 Walk 20 paces and measure the distance covered. Divide to find an approximation for the length of one of your paces. Use this knowledge to pace out lengths of 20 m, 50 m and 100 m. How do your estimates compare to your attempts in activity 5?



• Estimate the width of the bar.



• Estimate the width of the parachute.

12:05 | Perimeter

prep quiz 12:05

Add together the following measurements.

1 5·1 m + 6·9 m **2** 1·5 m + 2·3 m + 4·6 m **3** 1·2 m + 0·9 m + 5m

4 1·6 m + 73 cm **5** 80 cm + 60 mm **6** 2·5 km + 1750 m

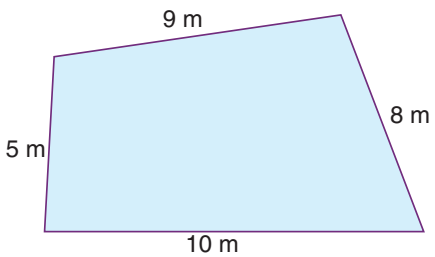
Find in metres:

7 4 × 2·5 m **8** 3 × 35 cm

9 2 × 1·2 m + 2 × 0·8 m **10** 2 × (1·2 m + 0·8 m)

The *perimeter* of a figure is the sum of the lengths of the sides of the figure. It is the distance around the figure.

worked examples

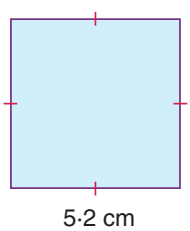


The perimeter P of this figure is given by:

$P = (5 + 10 + 8 + 9)$ m

$= 32$ m

Sometimes the lengths of some sides might be the same. This can be indicated by putting the *same mark* on equal sides.



For a square, all 4 sides are equal in length, so the perimeter P is given by:

$P = 4 \times 5{\cdot}2$ cm

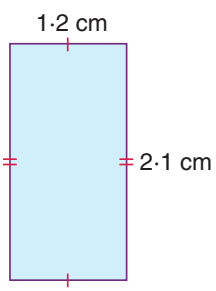
$= 20{\cdot}8$ cm

For a rectangle, two pairs of sides are equal in length, so the perimeter P is given by:

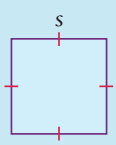
$P = (2 \times 2{\cdot}1 + 2 \times 1{\cdot}2)$ cm

$= (4{\cdot}2 + 2{\cdot}4)$ cm

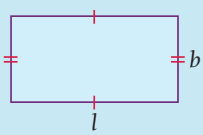
$= 6{\cdot}6$ cm



For a square, if s stands for the length of one side, then a formula for perimeter would be:
$P = 4 \times s$
(or simply $P = 4s$)



For a rectangle with a length of l and a breadth of b, the perimeter would be:
$P = 2 \times l + 2 \times b$
(or simply $P = 2l + 2b$)

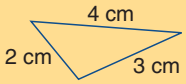


Exercise 12:05

Foundation Worksheet 12:05

Perimeter

1 Calculate each perimeter.

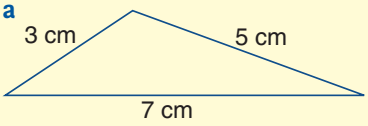


2 Find the perimeter of:

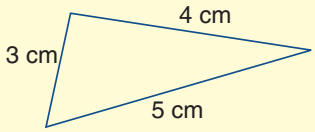
a a square of side length 10 cm.

1 Calculate the perimeters of these figures.

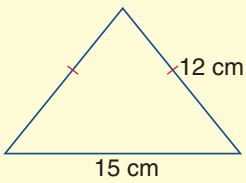
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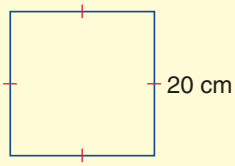
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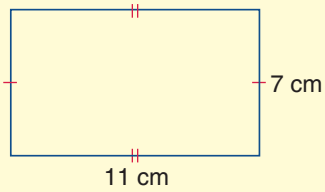
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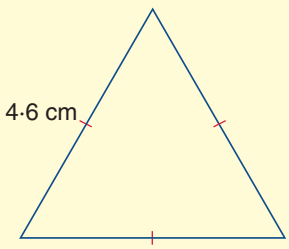
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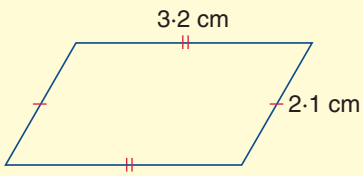
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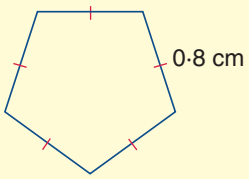
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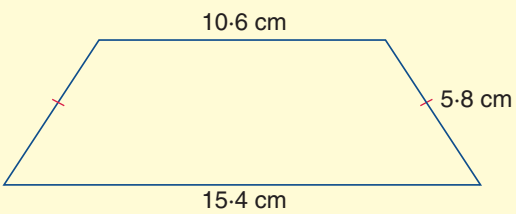
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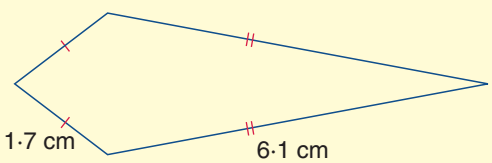
h



i



j





2 Find the perimeter of:

a a square with sides of length 3·2 cm

b a square with sides of length 56 mm

c a rectangle with a length of 2·4 cm and a breadth of 1·6 cm

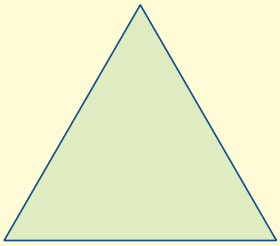
d a rectangle with dimensions of 5·2 cm and 3·6 cm

3 Measure the sides of these figures to the nearest centimetre and calculate their perimeters.

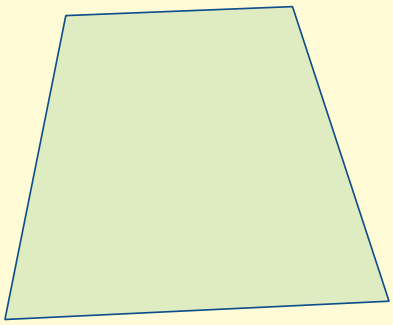
a



b



c



d



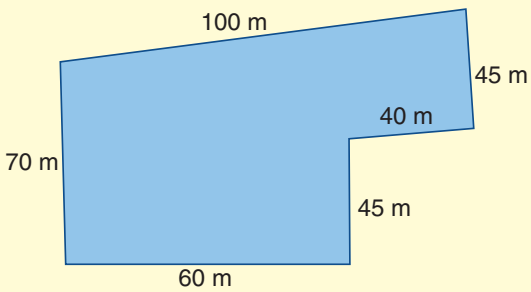
4 When measuring to the nearest centimetre, each measurement could be as much as 0·5 cm different from the real length. In question 3, we measured each side to the nearest centimetre before adding results to find the perimeter. If this method is used, what would be the greatest possible error in measuring a:

a triangle? **b** square? **c** pentagon? **d** hexagon?

5 Measure the perimeter of each part of question 3 in millimetres. Round off each of these answers to the nearest centimetre. Do you get the same answers that you got for question 3? Why or why not?

To find the *perimeter* of a figure by measurement to the nearest centimetre:
1 Measure each side to the nearest millimetre then add the sides.
2 Round off your final answer correct to the nearest centimetre.

6



A field has the dimensions as shown in the diagram.

a What is the perimeter of this field?

b How much would it cost to fence this field if the price of fencing is $75 for 10 metres?

7 A farmer has a rectangular paddock that is 350 m long and 190 m wide.

a Find the perimeter of this paddock.

b How much would it cost to fence the paddock at 85 cents per metre?

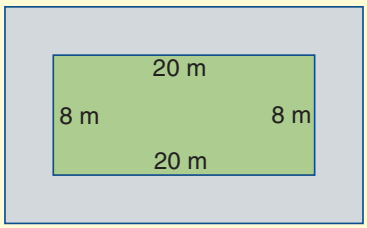
8 A jogger runs around a square park that has sides of length 145 m. How far will she jog if she runs around the park 10 times?

9 **a** The perimeter of a square is 14·8 cm. What is its side length?

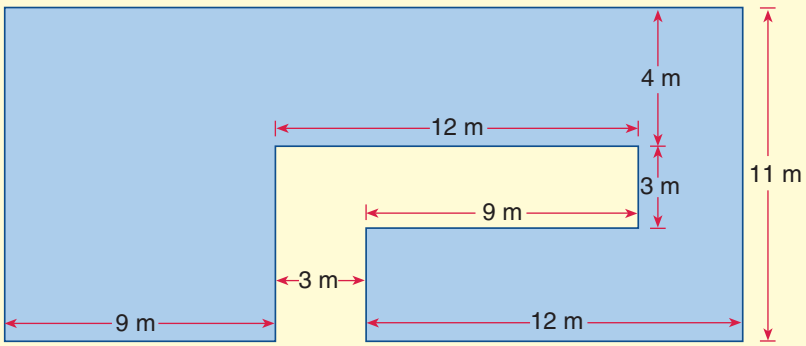
b The perimeter of a rectangle with a length of 5·6 cm is 14·4 cm. What is the width of the rectangle?

10 **a** A concrete path is constructed around a lawn as shown in the diagram. If the path is 1 m wide, what is the perimeter around the outside of the path?

b A similar 1 m wide path is placed around another rectangular lawn, which has a perimeter of 42 m. What would be the perimeter around the outside of this path?

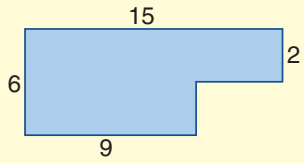


11 Find the perimeter of the building whose plan is shown below.

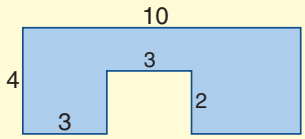


12 By determining the missing lengths (or otherwise), find the perimeter of each figure. (All angles are right angles and the measurements are in centimetres.)

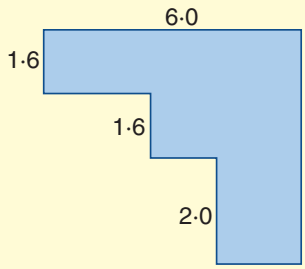
a



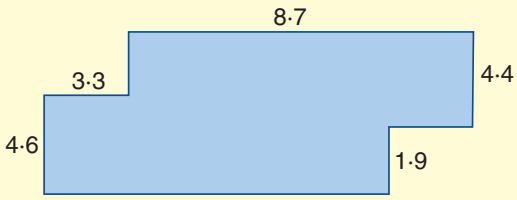
b



c



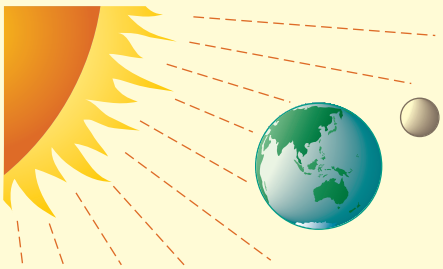
d



12:06 | The Calendar and Dates

People have always been concerned with time and its measurement. Our whole existence is locked into the passage of time.

> There is a time for everything and a season for every activity under heaven.
> Ecclesiastes 3:1, *NIV Bible*



In order to measure time, we must refer to something that is constant, ie doesn't change. Hence our main reference points are astronomical bodies such as the sun and stars.

- ***1 year*** **= the time taken for the Earth to revolve (ie travel) once around the sun**
- ***1 day*** **= the time taken for the Earth to rotate once on its axis**



Unfortunately the number of days in a year is not exact. The Earth actually takes approximately $365\frac{1}{4}$ days to travel around the sun. For convenience, we say that every four years a **leap year** occurs, which has one extra day. Therefore:

- 1 year = 365 days
- 1 leap year = 366 days



These years have, of course, been arranged into months and weeks to give us our calendar.

An important point to note is the identification or numbering of the years. There must be some starting point from which to count. When we say 1998, we mean AD 1998, where AD stands for the Latin words *Anno Domini*, indicating that the year is the one thousand, nine hundred and ninety-eighth year after the birth of Jesus Christ. Years before Christ's birth are indicated by the letters BC. For example, 520 BC means the 520th year before the birth of Christ.

Exercise 12:06

Foundation Worksheet 12:06
The calendar and dates
1 How many days in:
a 2 weeks?
2 How many days from:
a 13 June to 20 June?

1 How many:

a days in most years?
b days in a leap year?
c months in a year?
d complete weeks in a year?
e days in a week?
f weeks in a fortnight?
g days in a fortnight?
h complete fortnights in a year?
i years in a decade?
j years in a century?

2 Copy and complete the following table for a normal year and answer the questions.

Month	*Days*	*Month*	*Days*	*Month*	*Days*
January February March April		May June July August		September October November December	

a How many months have 31 days?
b How many months have 30 days?
c Where is the extra day added in a leap year?

For interest!
- How long is a millennium?
- How long is 'three score years and ten'?

3 The year is divided into 4 **seasons**. Name the seasons and the months in each one.

4 **a** If 1996 was a leap year, which other years will be leap years before the year 2020?
b What major sporting event takes place in every leap year?

5 We refer to a year such as 2005 as being in the twenty-first century. In what century would the following years fall?
a 1873 **b** 1592 **c** 1960 **d** 520 **e** 2150

6 When writing a date, we usually quote the day, month and year, such as 6 August 2003. This is often abbreviated to 6/8/03, which means:



Write out fully the dates abbreviated as:
a 3/1/56 **b** 15/4/64 **c** 31/7/80 **d** 27/11/84 **e** 10/4/02

In the United States, however, 6/8/03 would stand for 8 June 2003. The month is quoted first and then the day. Write out the following dates abbreviated using the US system.
f 5/13/84 **g** 12/3/47 **h** 1/30/51 **i** 9/15/80 **j** 8/5/03

7 It is important to be able to calculate using the calendar. Try these problems.
a If today's date is 3 October, what will be the date:
i in 1 week's time? **ii** in 3 weeks' time? **iii** in 5 weeks' time?
b How many days from:
i 10 March to 29 March? **ii** 10 March to 10 April?
c How many weeks between 19 May and 30 June?
d Kylie's birthday is on 21 October. How many days before Christmas is this?
e What will be the date 3 weeks after 20 February 2020?
f If 5 July is a Saturday, what day of the week will the following be?
i 10 July **ii** 30 November **iii** Christmas Day

8 If my birthday in 2003 fell on a Sunday, on what day of the week will my birthday fall in:
a 2011? **b** 2014? **c** 2016?

12:07 | Clocks and Times

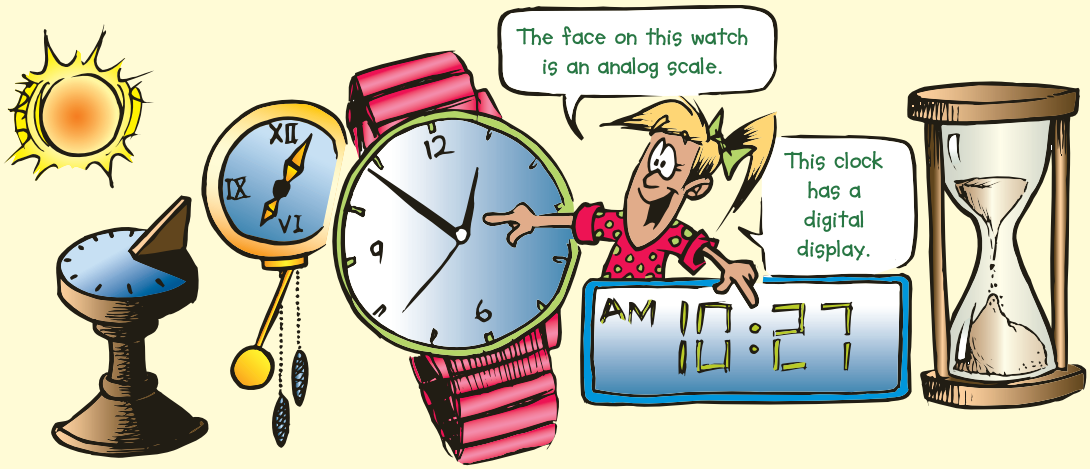
For convenience, each day is broken up into smaller units of time, so that we know what 'time of the day' it is. Without this division it would be difficult to coordinate our activities with other people.

In the metric system the basic unit of time is the **second**.

The table shows the other units of time and their abbreviations, with which you should be familiar.

To measure the time of day, different instruments have been used, some of which are shown below.

60 seconds (s) = 1 minute (min)
60 min = 1 hour (h)
24 h = 1 day



In recent years, due to developments in electronics, we have seen the widespread use of digital clocks and watches, and the greater use of 24-hour time scales on such things as video recorders and computers. The advantage of '24-hour time' is that there is no need to indicate am or pm.

- **24-hour time** is always given as a 4-digit number, the first two indicating the hour after midnight and the second two indicating the minutes past the hour, eg 09:45.
- **Digital time** does not use a zero as a first figure, eg 9:45. One disadvantage is that if am or pm is not used we would not know whether 10:25 referred to 24-hour time or digital time.

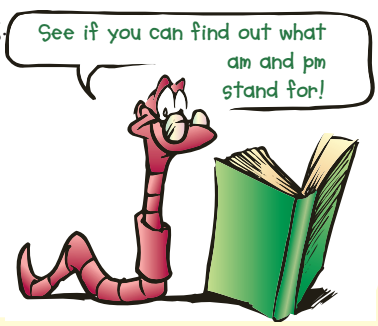
am – means between 12 midnight and 12 noon
pm – means between 12 noon and 12 midnight

examples

- 0520 would be 5:20 am or '20 minutes past 5' in the morning.
- 1730 would be 5:30 pm or 'half past 5' in the afternoon.

If this number is greater than 12, then subtract 12 (17 – 12 = 5) to find the time after 12 noon.

Likewise:
9:15 am would be 0915.
11:48 pm would be 2348.
(2348 can also be written as 23:48.)



Exercise 12:07

Foundation Worksheet 12:07

Clocks and times

1 How many minutes in:
a 1 hour **b** 2 hours

2 Write each time as 'minutes to' and 'minutes past'.

a 

1 Complete the following.

a 2 h = . . . min	**b** 3 min = . . . s	**c** 180 min = . . . h
d 240 s = . . . min	**e** $\frac{1}{4}$ h = . . . min	**f** $\frac{1}{2}$ min = . . . s
g 30 min = . . . h	**h** 15 s = . . . min	**i** 48 h = . . . days

2 **a** How many:
i seconds in an hour? **ii** minutes in a day? **iii** hours in a week?
b If Charlie works a 36 hour week, for how many hours a week is he *not* at work?

3 

The time shown by this clock is usually given as '3:40' (meaning 40 minutes past 3 o'clock) or '20 to 4' (meaning 20 minutes before 4 o'clock).
Write down two ways of expressing the time shown by the clocks below.

a 

b 

c 

d 

e 

f 

g 

h 

4 The following are written in 24-hour time. Rewrite these times in standard 12-hour time, indicating whether they are am or pm.

a 0520	**b** 1030	**c** 1310	**d** 1600
e 2240	**f** 0915	**g** 1205	**h** 0855
i 00:20	**j** 23:59	**k** 14:43	**l** 11:01

5 Write these times as they would appear on a 24-hour clock.

a 5:20 am	**b** 10:50 am	**c** 3:15 pm	**d** 9:20 pm
e half past 2 in the morning		**f** 10 past 5 in the afternoon	
g a quarter to 3 in the morning		**h** 12 noon	

6 The timers on video recorders often have a 24-hour display. The table below shows the starting and finishing times for various programs, and the timer settings that need to be made to record them. Complete the table.

Program	*Timer settings*
10:30 pm to 11:30 pm	22:30 to . . . . .
9:15 am to 10:45 am	. . . . . to . . . . .
7:45 pm to 9:10 pm	. . . . . to . . . . .
. . . . . to	05:30 to 06:40
. . . . . to	12:00 to 13:30
. . . . . to	17:40 to 19:20



12:08 | Operating with Time

60 min = 1 h **60 s = 1 min**

The ° ′ ″ **key can be used as** **h, min, s**.

Carefully work through the following examples.

worked example 1

I spent 2 h 40 min studying on Monday, 1 h 55 min on Tuesday and 3 h 40 min on Saturday. How long did I study altogether?

With pencil and paper

2 h	40 min
1 h	55 min
3 h	40 min
2	
8 h	15 min

40 + 55 + 40 min

= 135 = 2 h 15 min

With a calculator

2 [°′″] 40 [°′″] [+] 1 [°′″] 55 [°′″]

[+] 3 [°′″] 40 [°′″] [=] 8·25 h

(0·25 × 60 min = 15 min)

∴ I studied for 8 hours 15 minutes.

worked example 2

I've rented a boat for $6\frac{1}{2}$ hours. I've used it for 1 h 35 min. How much time is left?

With pencil and paper

	5	90
	~~6~~ h	~~30~~ min
−	1 h	35 min
	4 h	55 min

Trade 1 h for 60 min, then subtract.

With a calculator

6 [°′″] 30 [°′″] [−] 1 [°′″] 35 [°′″]

[=] 4·916666667 h

(0·916̇ × 60 min = 55 min)

∴ I have 4 hours 55 minutes left.

Using bridging strategies



worked example 3

It is 6:40 am now. How long before it is 8:15 pm?

6:40 am	to	7:00 am	is		20 min
7:00 am	to	noon	is	5 h	
noon	to	8 pm	is	8 h	
8 pm	to	8:15 pm	is		15 min
		Total	is	13 h	35 min

Exercise 12:08

Foundation Worksheet 12:08

Operating with time

1 2 h + 5 h 30 min
2 The difference between 10 am and 1 pm.
3 How long is it from 5:35 to 5:50?

1 Add these times together, expressing each answer in hours and minutes.

a 2 h 20 min + 1 h 30 min
b 5 h 19 min + 3 h 37 min
c 1 h 40 min + 2 h 30 min
d 7 h 34 min + 5 h 47 min

Remember: 60 min = 1 h

2 Subtract these times.

a 5 h 30 min − 2 h 10 min
b 3 h 55 min − 1 h 32 min
c 3 h 10 min − 1 h 40 min
d 9 h 20 min − 5 h 50 min
e 4 h 25 min − 1 h 47 min
f 7 h 13 min − 6 h 45 min

To subtract 1 h 40 min from 3 h 10 min, change 3 h 10 min to 2 h 70 min:

2 h 70 min
− 1 h 40 min
Answer = 1 h 30 min



3 Calculate the difference between each pair of times on the same day.

a 2:30 pm, 3:20 pm
b 5:20 am, 7:10 am
c 1:15 am, 7:30 am
d 3:40 pm, 9:50 pm
e 10:30 am, 1:30 pm
f 9:10 am, 2:30 pm
g 5:20 am, 4:15 pm
h 3:40 am, 10:50 pm



4 What is the length of time, in hours and minutes, from:

a 7:00 pm Tues to 5:00 am Wed?
b 9:15 pm Sat to 6:30 am Sun?
c 1:20 pm Fri to 12 noon Sat?
d 10:30 am Wed to 1:10 pm Thurs?
e 2:45 pm Tues to 5:30 am Thurs?

5 What is the time that is:

a $1\frac{1}{2}$ hours after 7:30 pm?
b $3\frac{1}{2}$ hours after 10:00 am?
c 1 h 20 min after 6:45 am?
d 2 h 40 min after 11:40 pm?
e $3\frac{1}{2}$ hours before 5:15 am?
f $1\frac{1}{4}$ hours before 12:30 pm?
g 2 h 15 min after a quarter to 7 in the morning?
h 7 h 45 min after 20 past 10 in the evening?

6 Round these answers correct to the nearest **(i)** hour **(ii)** minute.

a 6·5 h
b 3·75 h
c 4·25 h
d 7·333 h
e 8·6667 h
f 2·4 h
g 1·9 h
h 7·13 h
i 12·77 h
j 11·835 h

7 Sometimes the time in a certain place may be changed. This happens in Namibia during the hotter part of the year, when the time is put forward one hour, so that, for example, 4:00 pm becomes 5:00 pm. This is called 'Summer Time'. What time would the following become in Summer Time?

a 5:30 am **b** 11:15 am

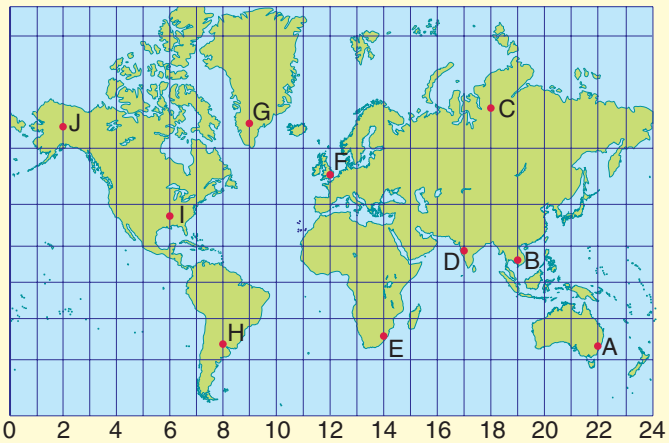


8 Because time is related to the rotation of the Earth, the time can be different in different places. In Canada, there are 6 *time zones*. The Atlantic Time Zone is 1 hour behind the Newfoundland Time Zone; the Eastern Time Zone is 2 hours behind the Newfoundland Time Zone; the Central Time Zone is 3 hours behind Newfoundland Time Zone, and so on.

What would be the time in:

a Yukon (YK) if it is 3:00 pm in Newfoundland (NF)?
b British Columbia (BC) if it is 1:00 am in Nova Scotia (NS)?
c Manitoba (MB) if it is 5:15 am in Ontario (ON)?
d Saskatchewan (SK) if it is 12:10 pm in Ontario (ON)?
e Prince Edward Island (PE) if it is 1:20 am in Alberta (AB)?
f Nunavit (NU) if it is 9:45 am in Northwest Territories (NT)?
g New Brunswick (NB) if it is 3:30 pm in Ontario (ON)?
h Manitoba (MB) if it is 1:30 am in British Columbia (BC)?





The map on the left shows how time differs around the world.

- B is 3 hours behind A (Sydney), as B is at 19 hours and A is at 22 hours.
  ie When it is 8 am in Sydney (A) it will be 5 am at B.
- H is 6 hours ahead of J, as H is at 8 hours as J is at 2 hours.
  ie When it is 10 am at J it will be 4 pm at H.

Each 15° difference in longitude makes a difference of 1 hour in time.

9 Use the map above to find the time at the places when it is noon in Delhi (at D).

a At B **b** At C **c** At E **d** At I **e** At J

10 **a** What is the time in Bangkok (B) when it is 3 pm in London (F)?
b What time is it in Tennessee (I) when it is 11:30 pm in Sydney (A)?

12:09 | Longitude and Time (Extension)

- The Earth rotates on its axis from the west to the east. Each rotation takes one day and it is this rotation that causes night and day and leads to time differences between different places on the Earth.
- The time difference between two places is related to the difference in their longitudes. Places with the same longitude have the same time.

We have said that it takes one day (24 hours) for the Earth to complete one revolution. In completing one revolution it spins through 360° of longitude.

The Earth takes:
- **24 hours to rotate through 360° of longitude**
- **1 hour to rotate through 15° of longitude.**

Hence every 15° difference in longitude results in a time difference of 1 hour.

- Even though local time varies by one hour for every 15° of longitude, in practice this would lead to ridiculous and confusing situations in places that are relatively close together. For instance, local time throughout the State of NSW in Australia could vary by up to 40 minutes. To avoid such problems the Earth is divided into zones and places within each zone use the same time. This is called **Standard Time**.

  In Australia there are three time zones. These are Eastern Standard Time (EST), based on the 150°E meridian, Central Standard Time (which is half an hour behind EST), and Western Standard Time (which is 2 hours behind EST).

  In summer, some states have daylight saving, in which the clocks are put forward one hour. In eastern Australia this is called Eastern Standard Summer Time or *daylight saving*.

worked examples

1 If A is on the prime meridian (0°) and B is on the 150°E meridian, find:
   a the time difference between A and B
   b the time at B when it is 12 noon at A
   c the time at A when it is 5 pm at B.

2 It is 3 pm at A, which has a longitude of 75°W, when it is 1 pm at B. Find the longitude of B.

3 How far ahead of Greenwich time (0°) is Australian Eastern Standard Time (AEST)? (*Remember:* this is based on the 150° meridian.)



Solutions

1 a A has longitude 0°
B has longitude 150°
∴ Difference in longitude = 150° − 0°
= 150°
∴ Difference in time = $\frac{150}{15}$ h
= 10 h
∴ Since B is east of A,
B is 10 h ahead of A.

b Time at A is 12 noon.
B is 10 hours ahead of A.
(It was 12 noon at B, 10 hours ago.)
∴ At B the time is 10 pm.

c Time at B is 5 pm.
B is 10 hours ahead of A.
∴ A is 10 hours behind B.
∴ Time at A is 10 hours before 5 pm.
∴ Time at A is 7 am.

2 Time at A = 3 pm
Time at B = 1 pm
∴ A is 2 h ahead of B
∴ A is 2 × 15° east of B
∴ A is 30° east of B
∴ B is 30° west of A
∴ Longitude of B is 105°W.

- If A's time is ahead of B's, then A is east of B.
- If A's time is behind B's, then A is west of B.

3 Difference in longitude = 150°
Difference in time = $\frac{150}{15}$ h
= 10 h
Hence AEST is 10 h ahead of Greenwich time.

Exercise 12:09 (Extension)

1 Find the difference in local times between A and B for each of the following. (State the difference and which place is ahead.)

	Longitude of A	*Longitude of B*
a	15°E	45°E
b	135°E	30°E
c	15°W	90°W
d	135°W	60°W

	Longitude of A	*Longitude of B*
e	30°E	30°W
f	60°W	120°E
g	150°E	150°W
h	105°W	15°E

2 Copy and complete the following tables.

(England) Greenwich

a

Longitude	105°W	90°W	75°W	60°W	45°W	30°W	15°W	0°	15°E	30°E	45°E	60°E	75°E	90°E	105°E
Time								noon							

b

Longitude	150°W	120°W	90°W	60°W	45°W	30°W	15°W	0°	15°E	30°E	45°E	60°E	90°E	120°E	150°E
Time								3 pm							

c

Longitude								0°							
Time	1 am	2 am	3 am	4 am	6 am	7 am	9 am	10 am	Noon	1 pm	3 pm	6 pm	7 pm	8 pm	9 pm

3 The local time at a city with a longitude of 150°E is 12 noon. What is the local time at cities with longitudes of:

a 135°E **b** 165°E **c** 90°E **d** 15°W **e** 75°W **f** 0° **g** 105°W

12:10 | Timetables

Timetables are part of everyday life. We use them to predict tides and to catch trains. The skills covered in the previous exercise should help you with the following questions.



Exercise 12:10

1 This timetable shows the TV programs on Star Sports for one day.

a At what time do the following programs begin?
- **i** Golf?
- **ii** Soccer/Futbol?
- **iii** Rugby?

b How many hours are there of:
- **i** tennis?
- **ii** surfing?
- **iii** sport news?

c I have just finished watching the first screening of Auto Racing. How long will it be before Auto Racing is on again?

d What was the most common sport on this day? Can you give a reason why this might be the case?

Time	
01:29	Sport News
01:30	Tennis
02:30	Auto Racing
03:30	Golf
04:00	Rugby
06:00	Auto Racing
07:00	Tennis
08:00	Tennis
14:30	Surfing
15:30	Extreme Sports
16:30	Tennis
20:00	Tennis
21:00	Wrestling
21:59	Sport News
22:00	Soccer/Futbol
22:30	Tennis

2

united cinemas

HARRY POTTER
& THE PHILOSOPHER'S STONE (PG)
Fri, Sat 10:10, 12:30, 2:40, 5:00, 7:15, 9:30, 11:45pm
Sun 10:10, 12:30, 2:40, 5:00, 7:15, 9:30pm

a How many times is *Harry Potter & the Philosopher's Stone* shown on Fridays?

b If I arrive at the cinema at 5:25 pm, how long must I wait before the next session begins?

c Estimate the finishing time of the last session on Sunday.

d What is the average time from one session to the next on Sunday.

3 Here is a timetable for the Oughtred Rd – Tartaglia Station bus route.

BUS TIMETABLE					
MONDAY TO FRIDAY					
• ROUTE 509 • OUGHTRED RD TO TARTAGLIA STATION					
DEPARTS	**ARRIVES**	**ARRIVES**	**ARRIVES**	**ARRIVES**	**ARRIVES**
Oughtred Rd Terminus	Cnr Samos Rd and Pythagoras Lane	Agnesi Rd Shops	Pascal High School	Cnr Tycho Ave and Brae St	Tartaglia Station
2:38 B	**2:47**	**2:52**	**2:58**	**3:01**	**3:04**
3:20	**3:29**	**3:34**	**3:40**	**3:43**	**3:46**
3:48	**3:57**	**4:02**	**4:08**	**4:11**	**4:14**
4:10	**4:19**	**4:24**	**4:30**	**4:33**	**4:36**
4:35	**4:44**	**4:49**	**4:55**	**4:58**	**5:01**
5:08	**5:17**	**5:22**	**5:28**	**5:31**	**5:34**

B – On school days, diverts to Oughtred Road School

a At what time does the bus that leaves Oughtred Road Terminus at 2:38 arrive at:
- **i** the corner of Samos Road and Pythagoras Lane?
- **ii** the Agnesi Road shops?
- **iii** Tartaglia Station?

b How long is the bus ride from:
- **i** Oughtred Rd Terminus to the corner of Tycho Avenue and Brae Street?
- **ii** the Agnesi Road shops to Tartaglia Station?
- **iii** the corner of Samos Road and Pythagoras Lane to Pascal High School?

c If the bus is 8 minutes late, at what time would the school bus reach the Agnesi shops?

d If an evening bus leaves Oughtred Road Terminus at 6:45 pm, at what time would it reach Pascal High School?

e A woman leaves her home on Samos Road at 3:45 pm and takes 9 minutes to walk to the bus stop. At what time could she expect to reach Tartaglia Station?

4 Study the train timetable on the next page and answer the following questions.

a How many trains stop at Lansdowne Road?

b What time would you catch a train from:
- **i** Sutton to be in Lansdowne Road just before 9:30 am?
- **ii** Dublin Pearse to be in Dun Laoghaire by noon?
- **iii** Kilbarrack to be in Shankill by 10:30 am?

c Looking at the times in the third column, how long does it take to get from:
- **i** Howth to Raheny?
- **ii** Blackrock to Killiney?
- **iii** Bayside to Bray?

HOWTH – DUBLIN – GREYSTONES
Monday to Friday

Howth	0700	0740	0850	0953	1030	1102	1115	1146
Sutton	0703	0744	0853	0956	1033	1105	1118	1149
Bayside	0705	0746	0856	0958	1035	1107	1120	1151
Howth Junction	0709	0750	0859	1002	1039	1111	1124	1155
Kilbarrack	0710	0751	0901	1003	1040	1112	1125	1156
Raheny	0712	0753	0903	1005	1042	1114	1127	1158
Harmonstown	0714	0755	0905	1007	1044	1116	1129	1200
Killester	0717	0758	0907	1010	1047	1119	1132	1203
Clontarf Road	0720	0801	0911	1013	1050	1122	1135	1206
Dublin Connolly arr	•	•	0914	•	•	1125	•	1210
dep	0724	0805	0925	1017	1054	1135	1139	•
Tara Street	0727	0808	0928	1020	1057	1136	1142	1212
Dublin Pearse arr	0728	0809	0921	•	1058	•	•	1214
dep	0734	0811	0931	1022	1106	1139	1144	•
Grand Canal Dock	0737	0813	0933	1024	1108	•	1146	1216
Lansdowne Road	•	0815	0935	1026	1110	•	1148	1218
Sandymount	•	0817	0937	1028	1112	•	1150	1220
Sydney Parade	0741	0819	0939	1030	1114	•	1152	1222
Booterstown	•	0822	0941	1033	1116	•	1155	1225
Blackrock	0746	0824	0944	1035	1118	•	1157	1227
Seapoint	•	0826	0946	1037	1120	•	1159	1229
Salthill	•	0828	0948	1039	1122	•	1201	1231
Dun Laoghaire	0752	0830	0951	1041	1125	•	1203	1233
Sandycove	•	0833	0954	1044	1128	•	1206	1236
Glenageary	•	0835	0956	1046	1130	•	1208	1238
Dalkey	0759	0838	0959	1049	1132	•	1211	1241
Killiney	0807	0842	1003	1053	1136	•	1215	1245
Shankill	•	0844	1006	1055	1139	•	1217	1247
Bray arr	•	0849	1010	•	•	•	•	1253
dep	0813	0909	1012	1101	1145	1214	1223	•
Greystones	0824	0918	1022	1110	1154	1224	1232	1302

5

Vegetable	*Quantity*	*Cooking Time*
Beans	500 g	15 min
Carrots	6	12 min
Corn	1 cob	4 min
Peas	500 g	10 min
Potatoes	500 g	12 min
Spinach	500 g	8 min

This chart shows the time taken to cook various vegetables in a microwave oven. Vegetables that have similar cooking times can be cooked at the same time, but the cooking time should be doubled.

a How long would it take to cook the following?
- **i** 1 kg beans
- **ii** 3 cobs of corn
- **iii** 250 g potatoes
- **iv** 4 carrots

b How long would it take to cook 6 carrots and $\frac{1}{2}$ kg of potatoes?

c What total time would it take to cook the following lists of vegetables?
- **i** 6 carrots, 4 cobs of corn, $\frac{1}{2}$ kg peas, $\frac{1}{4}$ kg spinach
- **ii** 250 g beans, 10 carrots, 750 kg potatoes, 1 kg spinach

6 The tides for a period of 3 days are given in this table. The times are obviously in 24-hour time.

Day	*Time*	*Tide (m)*
1	0414	0·3
	1012	1·4
	1607	0·4
	2227	1·7
2	0458	0·4
	1052	1·3
	1637	0·5
	2304	1·7
3	0544	0·5
	1135	1·2
	1709	0·6
	2343	1·6

a What is the lowest tide and at what time does it occur?

b How long is it between high tides on the first day?

c How long is it between low tides on the second day?

d How long is it between the first low tide and the first high tide on day 3?

e What is the difference in height between the lowest and highest tides on day 2?

f At what time during the three days did the highest tide occur?

g Barney believes the best fishing occurs half an hour after high tide. If this is so, what would be the best fishing times on day 3?

Refer to ID Card 1 (Metric Units) on page xiv. Identify the units (1) to (24). Learn the units you do not know.



12:10

12:10A: Tide times
12:10B: Flying and arrival times
12:10C: Fencing Australia



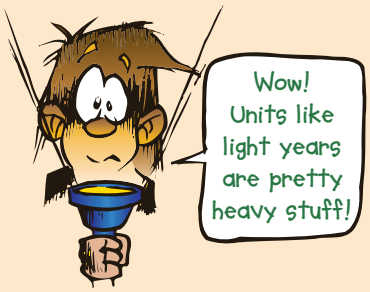
fun spot

12:10

Fun Spot 12:10 | And now for something light

A kilometre may seem long to us at times but, when we look beyond our Earth to the solar system and the stars, a kilometre is very, very small.

Astronomers need to use much greater units of distance than a kilometre. The average distance from the Earth to the sun is 150 million km, ie 150 000 000 km. For convenience astronomers call this **1 astronomical unit**. Another unit is the **light year**, which is the distance light can travel in 1 year.



1 Considering that light travels about 300 000 km in 1 second, calculate how far light would travel in:

a 1 minute
b 1 hour
c 1 day
d 1 year

This answer is the number of kilometres in 1 light year.



2 Using your answer to **1d**, calculate approximately how many astronomical units equal 1 light year. (Give your answer to the nearest thousand.)
Note: This answer means that travelling at the speed of light, you could travel to the sun from the Earth this many times in one year.

3 This table gives the distances from the sun of some of the planets, in astronomical units. Calculate each distance in kilometres.

Planet	*Astron. units*	*Kilometres*	*Planet*	*Astron. units*	*Kilometres*
Mercury	0·39		Mars	1·52	
Venus	0·72		Saturn	9·54	
Earth	1·00	150 000 000	Pluto	39·4	

4 If you think Pluto is far away, the distance to the nearest star is 4·35 light years.
a How many km is this? **b** How many astronomical units is this?

5 A further unit astronomers use is the **parsec**.
1 parsec = 206 265 astronomical units.
a How many km is this? **b** How many light years is this?

Appendix G G:02 Distance, speed and time

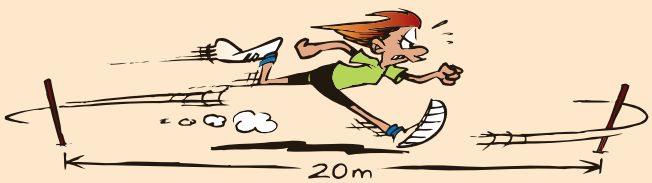
Investigation 12:10 | Distance, speed and time

Please use the Assessment Grid on the following page to help you understand what is required for this Investigation.

1 Use a tape measure or trundle wheel to measure:
   a how far you can walk in 10 seconds
   b how far you can run in 10 seconds

2 Use your measurements from question 1 to find:
   a your average speed when walking, in m/s
   b your average speed when running, in m/s
   (Use $S = D \div T$)

3



Set up two sticks 20 m apart. Run 80 m around these sticks while a friend measures the time taken in seconds. Calculate your speed using $S = D \div T$.

4 Measure out a 100 m section of a road. Find the speed of passing cars. How many cars exceeded the speed limit? How many did not? Draw a column graph of your results using such categories as: 50–55 km/h, 55–60 km/h, 60–65 km/h, 65–70 km/h etc.



- Estimate the height of the hurdles.
- Estimate the speed of the athletes in m/s.

Assessment Grid for Investigation 12:10 | Distance, speed and time

The following is a sample assessment grid for this investigation. You should carefully read the criteria *before* beginning the investigation so that you know what is required.

Assessment Criteria (B, C, D)				Achieved ✓
Criterion B Application & Reasoning	a	No organised approach has been used to answer the questions.	1	
			2	
	b	An organised approach has been used and most of the results are reasonable.	3	
			4	
	c	An organised approach has been used and the results are reasonable.	5	
			6	
	d	Question 4 has been completed satisfactorily using a reasonable number of vehicles.	7	
			8	
	e	Question 4 has been completed accurately using a reasonable number of vehicles.	9	
			10	
Criterion C Communication	a	Working out is not shown and answers are hard to find.	1	
			2	
	b	Presentation is good and includes tables and graphs and some correct mathematical terminology.	3	
			4	
	c	Presentation is good and includes tables and graphs and correct mathematical terminology throughout.	5	
			6	
Criterion D Reflection & Evaluation	a	Some attempt has been made to explain the processes involved in the investigation.	1	
			2	
	b	The methods and processes used are justified and results have been checked with some success.	3	
			4	
	c	Reasoned explanations are given to accompany the working out and there is comment on the reliability of the results.	5	
			6	
	d	There is comment on the accuracy of the results and some discussion on how to improve the accuracy of the results.	7	
			8	

Mathematical terms 12

measuring (to measure)
- The process of recording the size of a quantity by comparing it to a known amount of the quantity (called a unit).

measurement
- The result of measuring.
- Contains a numeral and a unit.
  eg 65 mm

unit
- A known amount of a quantity used to record measurements.

scale
- A set of markings on a measuring instrument that allow a measurement to be read using the instrument.

analog display
- Uses a scale and a pointer to give the size of a measurement.

digital display
- Gives the size of a measurement directly as a number.

metre, kilometre, centimetre, millimetre
- Metric units of length.
- For relationships between the units see section 9:02.

odometer
- An instrument that records the distance travelled by a vehicle.

trundle wheel
- An instrument comprised of a wheel and a counter that allows distances to be measured in metres.

perimeter
- The sum of the lengths of the sides of a figure.
- The distance around the figure.

dimension
- A measurement of the size of an object in a particular direction.

second, minute, hour, day, week, fortnight, month, year
- Commonly used units of time.
- See sections 9:06 and 9:07 for the relationship between the units.

leap year
- A year containing 366 days.
- Occurs every fourth year (except for century years whose number is not divisible by 400).

century
- A period of 100 years.

AD
- *Anno Domini:* 'In the year of our Lord'.

BC
- Before Christ.

midnight
- 12 pm, the 'middle' of the night.

noon
- 12 am, the 'middle' of the day.

am
- Means between 12 midnight and 12 noon.

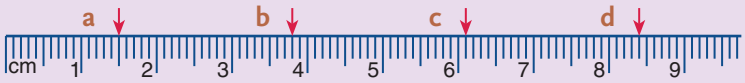
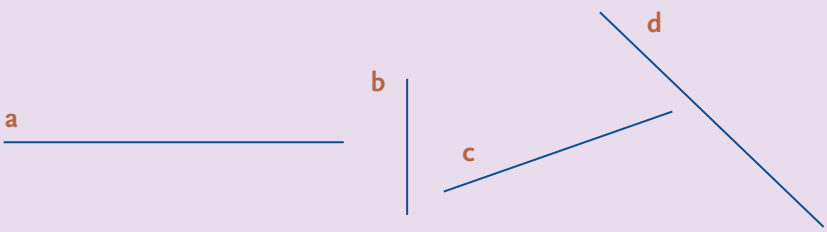
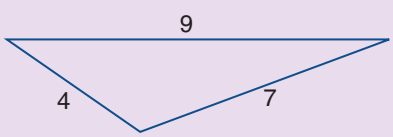
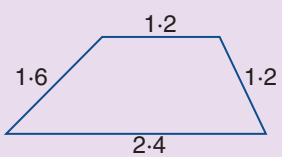
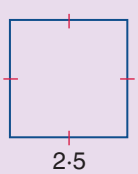
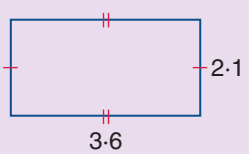
pm
- Means between 12 noon and 12 midnight.

average speed
- Obtained by dividing the distance travelled by the time taken.

24-hour time
- Time given as a 4-digit number, the first two digits indicating the hour, the second two indicating minutes.
  eg 13:20 (or 1320) means 20 minutes past 1 in the afternoon.

Diagnostic Test 12: | Measurement: Length and Time

- Each section of the test has similar items that test a certain type of example.
- Failure in more than one item will identify an area of weakness.
- Each weakness should be treated by going back to the section listed.

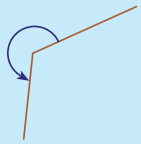
	Section
1 Write down each measurement to the nearest mm. Answer in centimetres. 	12:02
2 **a** 3 cm = . . . mm **b** 7 km = . . . m **c** 2·5 m = . . . cm	12:02
3 **a** 50 mm = . . . cm **b** 650 cm = . . . m **c** 7150 m = . . . km	12:02
4 Measure these lines to the nearest mm. 	12:03
5 Calculate the perimeter of each figure (all measurements are in cm). **a**  **b**  **c**  **d** 	12:05
6 Write these times in 24-hour time. **a** 5:30 am **b** 1:20 pm **c** 7:57 pm **d** 12:20 am	12:07
7 Find the difference from the first time to the next. **a** 8:30 am to 10:20 pm **b** 2:45 pm to 7:57 pm **c** 10:12 am to 1:05 pm **d** 6:47 pm to 12:05 am	12:08
8 If it is 1:30 pm (EST) in NSW, what time is it in: **a** WA? **b** SA? **c** Qld? **d** Vic?	12:08
9 **a** 3 h 45 min + 2 h 25 min **b** 2 h 40 min + 1 h 55 min	12:08
10 **a** 6 h 30 min – 1 h 35 min **b** 3 h 40 min – 2 h 55 min	12:08

Chapter 12 | Revision Assignment

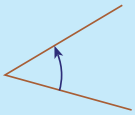
assignment 12A

1 Give the type of angle in each case.

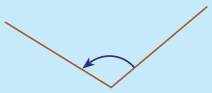
a



b

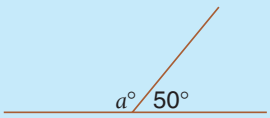


c

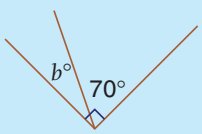


2 Find the value of the pronumeral in each.

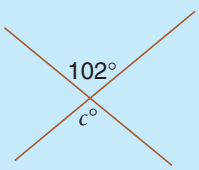
a



b

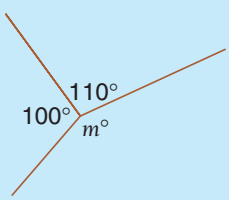


c

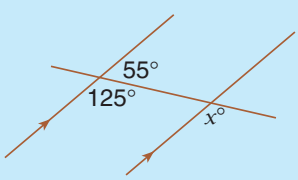


3 Find the value of each pronumeral.

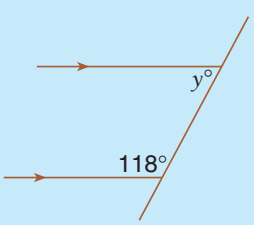
a



b

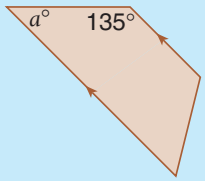


c

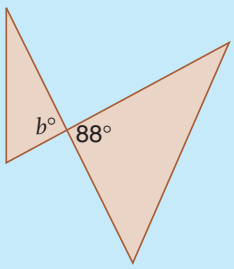


4 Find the value of each pronumeral.

a



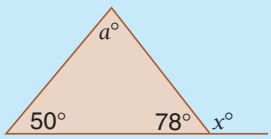
b



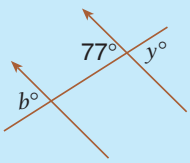
c



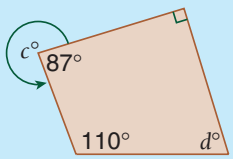
d



e



f



5 Complete the table of values using the rules shown at the top of each table.

a $L = M + 4$

M	1	2	5	10
L				

b $P = 2 \times B + 4$

B	0	1	2	4
P				

c $n = (m + 3) \times (m - 2)$

m	2	3	4	10
n				

d $y = x + 2$

x	1	2	3	4
y				

e $m + n = 6$

m	0	1	2	3
n				

f $P = 2 \times M + 1$

M	2	3	4	5
P				

6 Simplify:

a $\frac{1}{2}$ of 42 **b** $\frac{1}{4}$ of 72

c $\frac{1}{5}$ of 60 **d** $\frac{2}{3}$ of 48

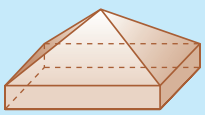
7 If $x = 7$, find the value of:

a $5 \times x$ **b** $5 \times x + 3$

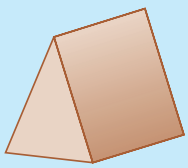
c $5 \times x - 4$ **d** $5 \times x - x$

8 How many vertices has each solid?

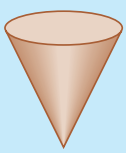
a

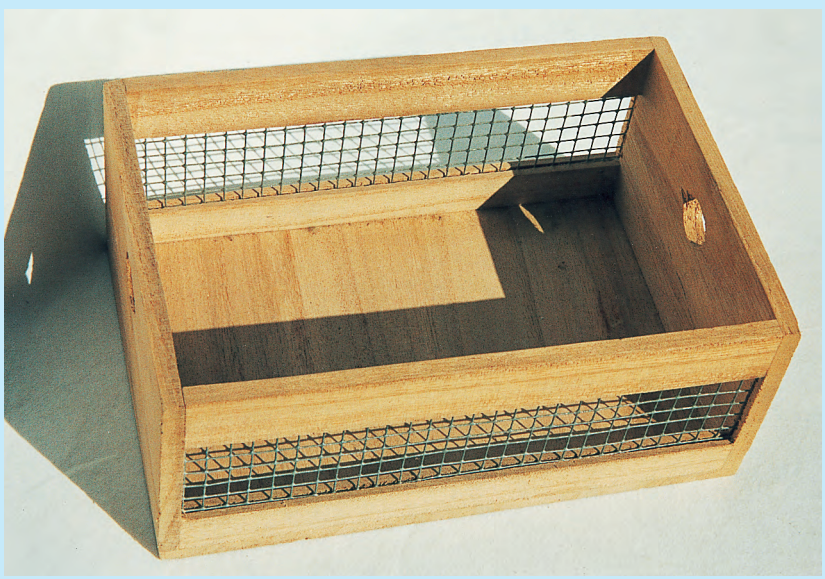


b



c





- If the height is 10 cm, estimate the volume of this rectangular prism.

Drag and Drop Interactive
1 Units of length
2 Clocks
3 Perimeter of shapes

Teacher's Resource Chapter Test

Review Questions

Chapter 12| Working Mathematically

assignment 12B

id 12

1 Use ID Card 7 on page xix to identify:

a 1 b 2 c 3 d 4 e 5
f 6 g 7 h 8 i 17 j 18

2 Two fruit growers send their fruit to market. One grower sends 70 cases and the other sends 96 cases. They receive $1660 altogether. How much should each person get?

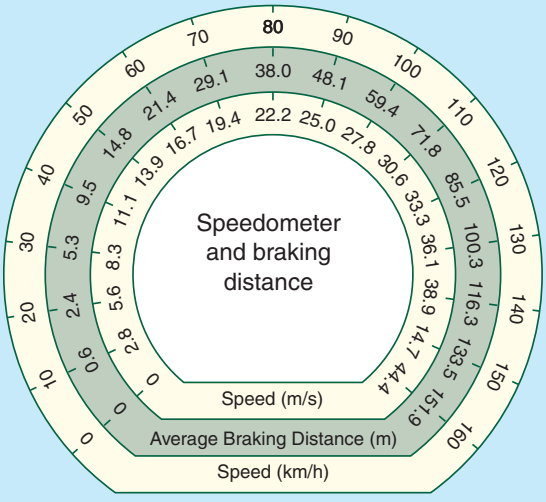
3 My car averages 9 km per 1 litre of petrol. How much will it cost for petrol if I drive from Bega to Bombala, a distance of 126 km, if petrol costs 89 cents per litre?

4 A block of land is rectangular in shape. Its perimeter is 360 m and it is known to be 8 times longer than it is wide. Find its length and width.

5 While waiting to go on a long train trip I decide to buy some magazines for the trip. On the news-stand I see five magazines priced at $2.20, $3.40, $1.20, $1.60 and $2.95. What possible combinations of magazines could I buy for exactly $5.

6 When a truck is loaded with 500 boxes, each of mass 5·8 kg, the total mass shown on a weighbridge is 8·1 tonnes. What is the mass of the truck when it is unloaded?

7 The speedometer below shows an estimate of the braking distance of a car for speeds up to 160 kilometres per hour. The innermost scale shows the speed in metres per second.





a What is the average braking distance if the speed is:
i 30 km/h? ii 100 km/h? iii 25 m/s? iv 2·8 m/s?

b At what speed (in km/h) is the braking distance:
i 21·4 m? ii 48·1 m? iii 85·5 m? iv 38·0 m?

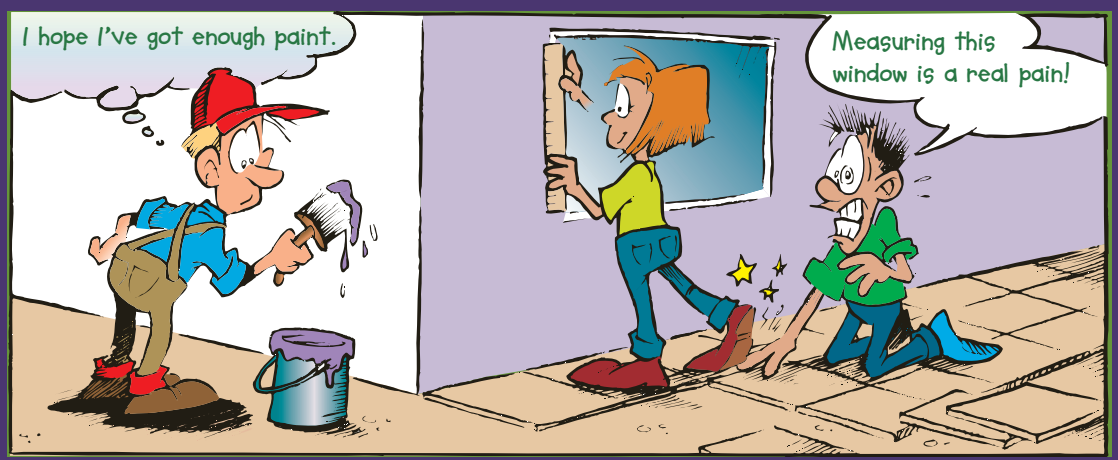
c At what speed (in m/s) is the braking distance:
i 2·4 m? ii 14·8 m? iii 9·5 m? iv 151·9 m?

d Use the speedometer to change 60 km/h into m/s.

e Use the speedometer to change 22·2 m/s into km/h.

13

Area and Volume



Chapter Contents

13:01 The definition of area
Investigation: Finding area
13:02 Area of a rectangle
Investigation: Area of a rectangle
13:03 Area of a triangle
13:04 Area problems
ID Card
13:05 Measuring 3D space (Volume)
Investigation: Measuring 3D space
13:06 Volume of a rectangular prism
13:07 Capacity
Reading Mathematics: Capacity
Practical Activity: Estimating capacity
Fun Spot: What makes money after 8 pm?
Challenge: The fantastic Soma cube
Mathematical Terms, Diagnostic Test, Revision Assignment, Working Mathematically

Learning Outcomes

Students will:

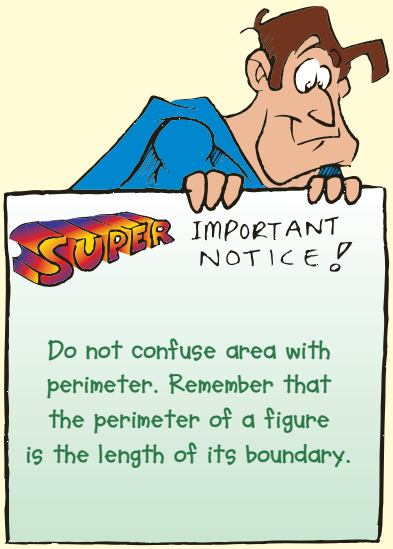
- Use formulae to calculate the perimeter and area of circles and figures composed of rectangles and triangles.
- Calculate surface area of rectangular and triangular prisms and volume of right prisms.

Areas of Interaction

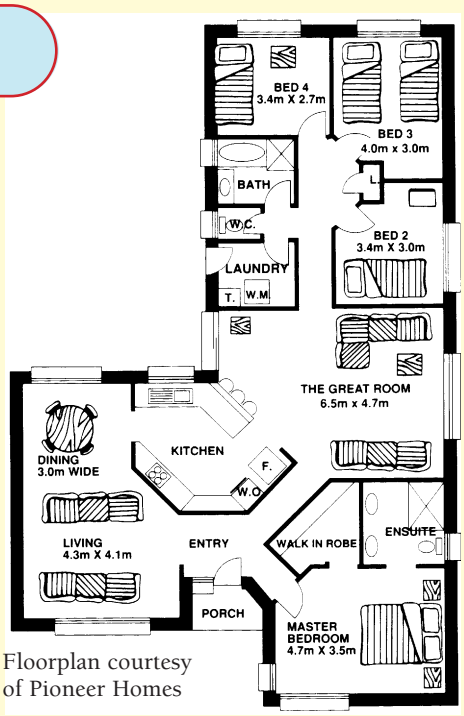
Approaches to Learning, Homo Faber, Environment

13:01 | The Definition of Area

When we measure the *area* of a shape, we are measuring the amount of space inside that shape.



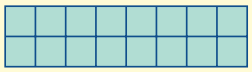
The living area in a home is measured in square metres.



Floorplan courtesy of Pioneer Homes

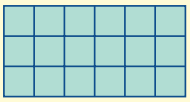
We measure area by dividing it into *square units*, and counting how many there are inside the figure.

A

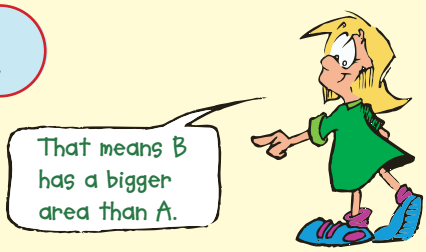


Number of squares = 16

B



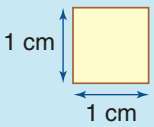
Number of squares = 18



We say that the area of figure A is 16 square units, and the area of figure B is 18 square units.

Obviously, if we are going to be consistent, we need a standard unit of area. A convenient unit is the square centimetre.

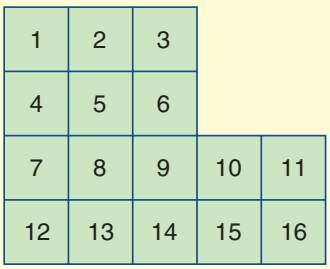
The area of a square with a side of 1 cm is 1 square centimetre.



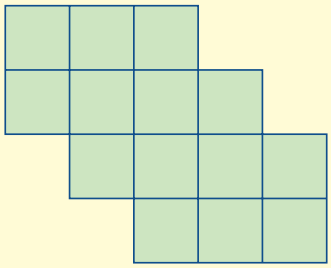
This can be written as

$1\ cm^2$.

The areas of these figures would be:



Area = 16 units^2



Area = 14 units^2

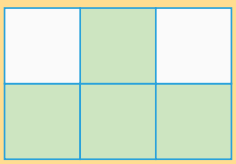


Exercise 13:01

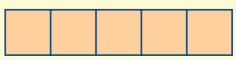
Foundation Worksheet 13:01

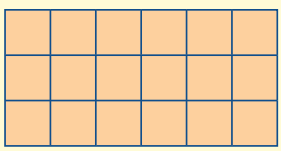
The definition of area

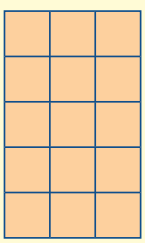
1 These shapes are drawn on centimetre grid paper. Find the area of each shape.

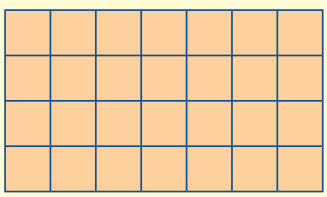
a 

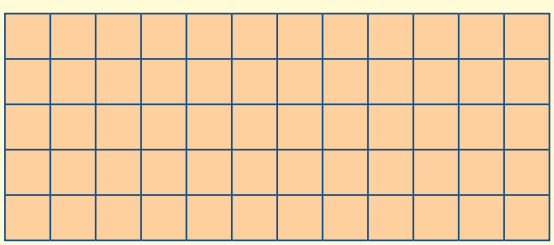
1 Find the area, in square units, of each figure. ('Square units' can be written as 'units^2'.)

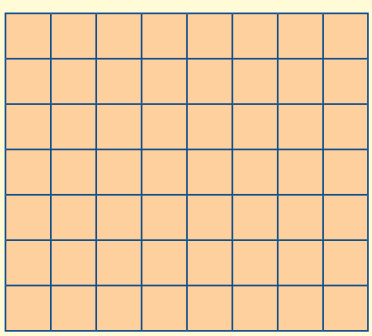
a 

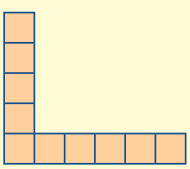
b 

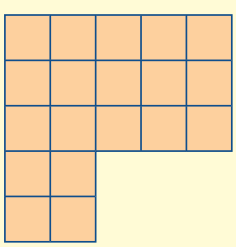
c 

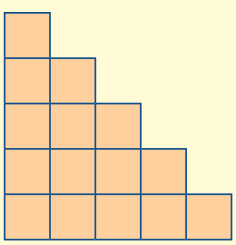
d 

e 

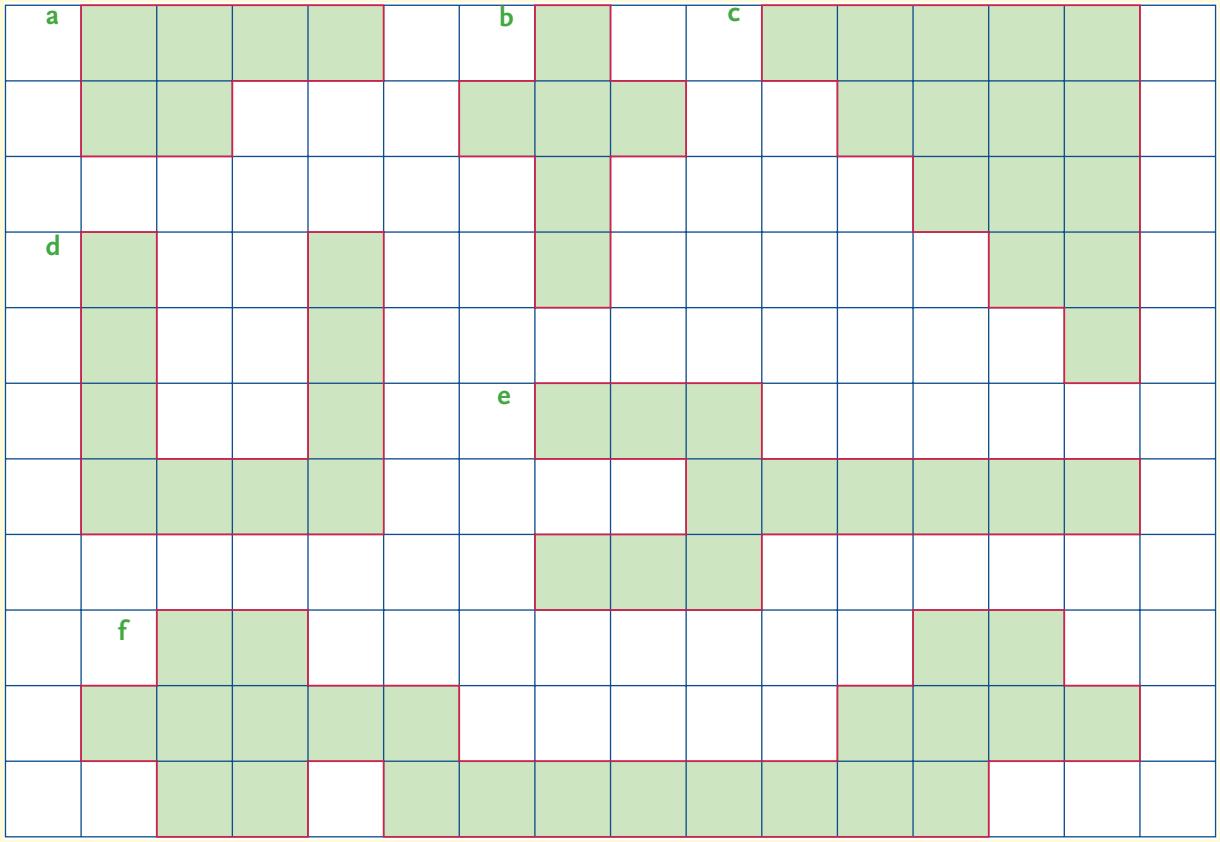
f 

g 

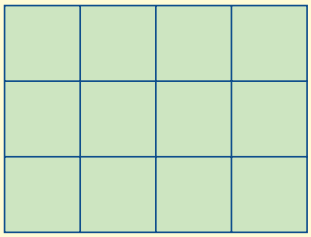
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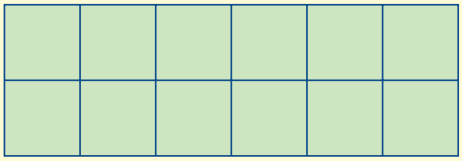
i 

2 Each of these shapes is drawn on centimetre grid paper. Find the area of each shape in cm^2.



3 **a** What is the area of each shape?





b What is the perimeter of each shape?

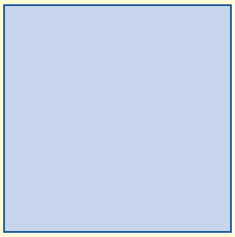
c Do you think shapes with the same area will always have the same perimeter?

4 Use a plastic centimetre grid sheet, or a square with a side of 1 cm cut from a grid sheet, to find the areas of these shapes in square centimetres.

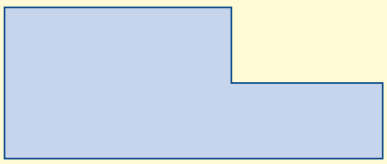
a



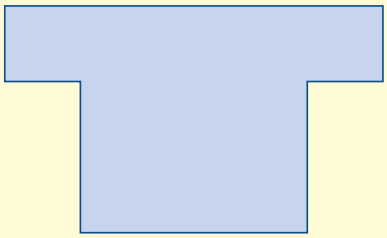
b



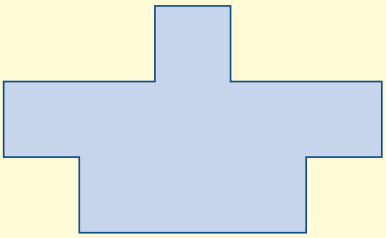
c



d



e



5 Not all shapes are made up of whole unit squares. What fraction of a square centimetre would these shaded areas be?

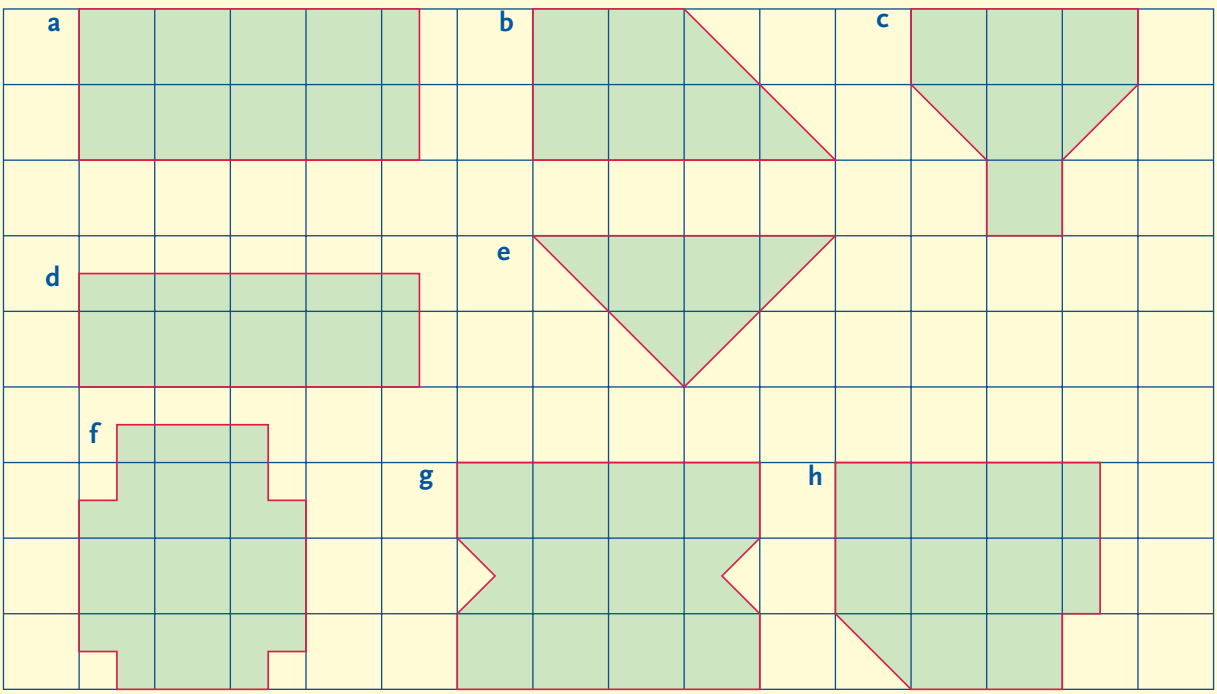
a 

b 

c 

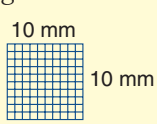
d 

6 Use your answers to question 5 to find the area, in cm^2, of these shapes.



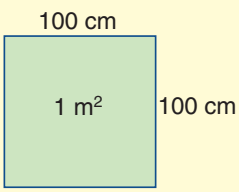
7 Square centimetres are not the only units of area used. Smaller areas might be measured in square millimetres (mm^2). Larger areas would be measured in square metres (m^2).



a 

Each side of this 1 square centimetre is, of course, 10 mm. How many square mm are in this square cm?

b Of course a square that has sides of 1 m would measure 100 cm on each side. How many square cm would fit into 1 square m?



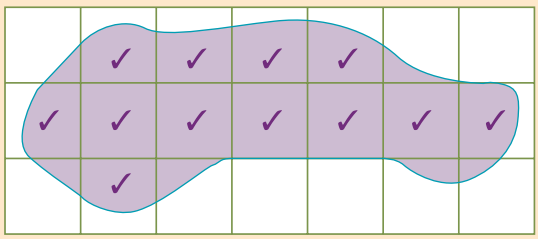
Investigation 13:01 | Finding area

Please use the Assessment Grid on the following page to help you understand what is required for this Investigation.

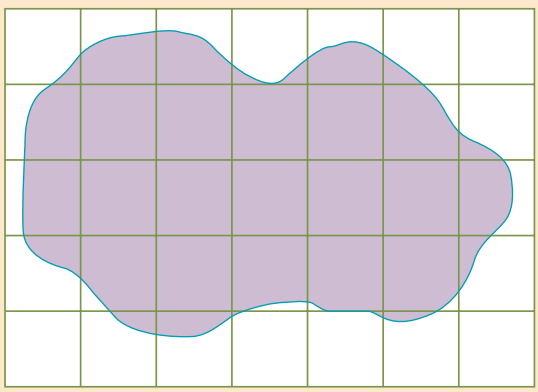
Discuss how you could find the area of a figure with curved sides.

1 An approximation for the area inside a curved boundary can be found by placing a grid over it and counting squares. If more than $\frac{1}{2}$ of a square is included in the figure, count it. If less than $\frac{1}{2}$ of a square is included in the figure, don't count it. Find the approximate areas of these shapes.

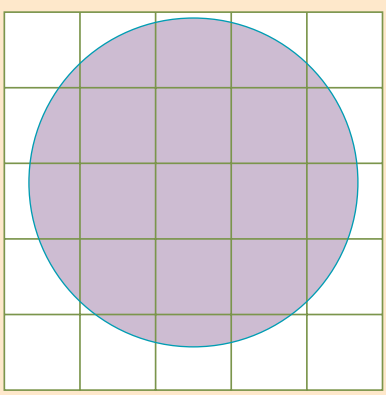
a



b

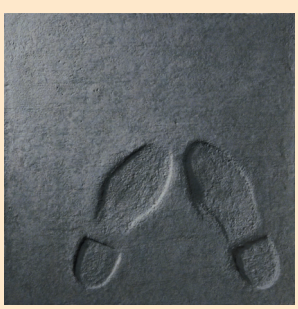


c









2 Try to estimate the areas of various shapes and then trace the shapes on grid sheets. Calculate their areas by counting squares.

3 Use the method above to find the area of your hand or your shoe.

4 Tape or glue sheets of paper together to make a square with sides of 1 metre. The area of this square will, of course, be 1 m^2. Use this square metre to find the area of some large shapes.

5 Draw several rectangles on grid paper, each with a perimeter of 24 cm. Then find the area of each. Do you notice any pattern?

Assessment Grid for Investigation 13:01 | Finding area

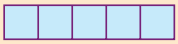
The following is a sample assessment grid for this investigation. You should carefully read the criteria *before* beginning the investigation so that you know what is required.

Assessment Criteria (B, C, D)				Achieved ✓
Criterion B Application & Reasoning	a	No systematic or organised approaches have been used to find the areas.	1	
			2	
	b	An organised approach has been used with some success.	3	
			4	
	c	An organised approach has been used successfully.	5	
			6	
	d	Organised approaches have been successfully used, and an attempt has been made to describe the patterns in question 5.	7	
			8	
	e	Organised approaches have been used successfully. There is a successful discussion of question 5.	9	
			10	
Criterion C Communication	a	Very little working out is shown.	1	
			2	
	b	Working out is shown; some diagrams and mathematical terminology have been used.	3	
			4	
	c	Presentation is good and working out is easy to follow. Diagrams and mathematical terminology are well used.	5	
			6	
Criterion D Reflection & Evaluation	a	Some attempt has been made to explain the method used and to check the results.	1	
			2	
	b	The method used is justified and the results have been checked with some success.	3	
			4	
	c	The method is justified and some comment is made on the reliability of the results.	5	
			6	
	d	There is some discussion on the method used and its reliability.	7	
			8	

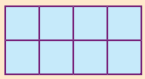
13:02 | Area of a Rectangle

prep quiz
13:02

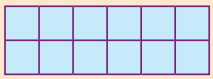
Write the area and perimeter of each rectangle.



1 $A =$ . . . units^2
2 $P =$ . . . units

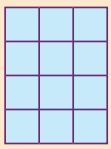


3 $A =$ . . . units^2
4 $P =$ . . . units

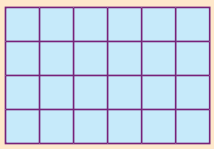


5 $A =$ . . . units^2
6 $P =$ . . . units

7 $A =$ . . . units^2
8 $P =$ . . . units

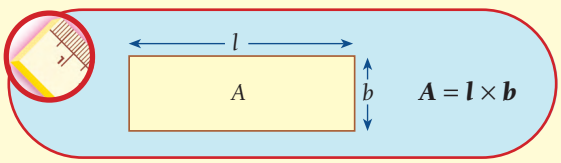


9 $A =$ . . . units^2
10 $P =$ . . . units



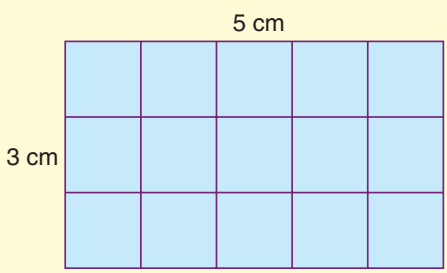
If you didn't know it already, you may have seen in the Prep Quiz a quicker way of finding the area of a rectangle than counting the squares.

If we know the number of units in both the length and breadth of the rectangle, we can simply use the following rule or formula.

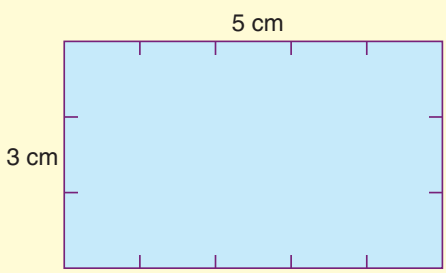




This can be seen from the following diagrams.



Number of squares = 15
∴ Area = 15 cm^2



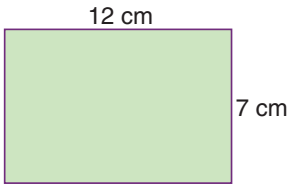
$A = l \times b$
$= 5 \times 3$
$= 15$
∴ Area = 15 cm^2

Important note!
To use this rule we must be careful. ***The length and breadth must be measured in the same units.*** If not, then one dimension must be converted.

worked examples

1 Calculate the areas of these rectangles.

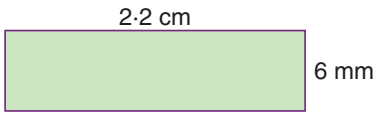
a



Here the units are both cm,

so: $A = 12 \times 7$
$= 84$
$\therefore$ Area $= 84\ cm^2$

b



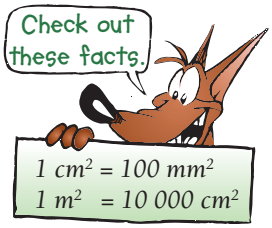
The units are different

2·2 cm could be written as 22 mm. } OR { 6 mm could be written as 0·6 cm.

Then $A = 22 \times 6$
$= 132$
$\therefore$ Area $= 132\ mm^2$

Then $A = 2{\cdot}2 \times 0{\cdot}6$
$= 1{\cdot}32$
$\therefore$ Area $= 1{\cdot}32\ cm^2$

(*Note:* $132\ mm^2 = 1{\cdot}32\ cm^2$)



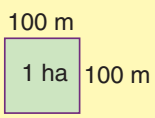
2 A rectangular paddock measured 250 m by 100 m.
Its area would therefore be given by:

$A = 250 \times 100$
$= 25\,000$
$\therefore$ Area of land $= 25\,000\ m^2$

But land is usually measured in units called **hectares** (see box).

$\therefore$ Area of this paddock in hectares would be 2·5 ha.

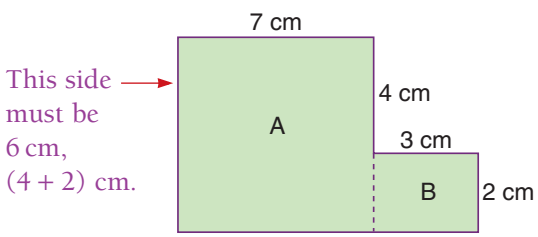
LAND AREAS
A square with sides 100 m long has an area of 1 *hectare* (ha).



$1\ ha = 10\,000\ m^2$

3 The areas of some shapes can often be found by dividing them into smaller rectangles. Before finding the area of each smaller rectangle, you may need to find some unknown lengths.

a



Area of A
$= (7 \times 6)\ cm^2$
$= 42\ cm^2$

Area of B
$= (3 \times 2)\ cm^2$
$= 6\ cm^2$

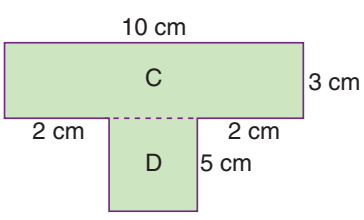
$\therefore$ Total area of figure
$= 42\ cm^2 + 6\ cm^2$
$= 48\ cm^2$

b Area of C
$= (10 \times 3)\ cm^2$
$= 30\ cm^2$

Area of D
$= (5 \times 6)\ cm^2$
$= 30\ cm^2$

$\therefore$ Total area of figure $= 30\ cm^2 + 30\ cm^2$
$= 60\ cm^2$

(Note: The figures are not drawn to scale.)



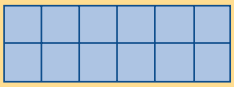
Exercise 13:02

Foundation Worksheet 13:02

Area of a rectangle

Find the area in square units:

1 a

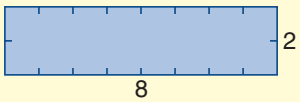


2 a

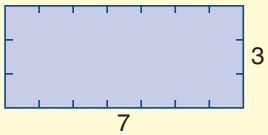


1 Find the area of each rectangle in square units.

a

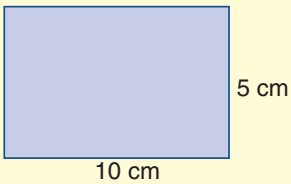


b

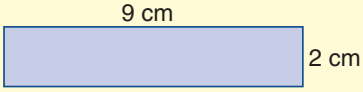


2 Determine the area of each rectangle using the dimensions given.

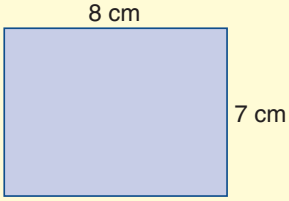
a



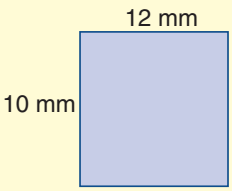
b



c



d



e

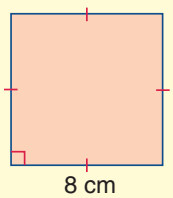


f

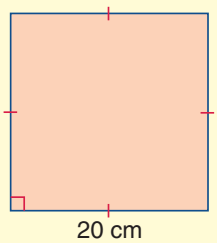


3 A square is, of course, a rectangle whose length is equal to its breadth. Hence if a square has sides of s cm, then its **area** $= s \times s$ **cm**^{2}. Find the areas of these squares.

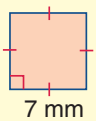
a



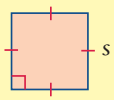
b



c



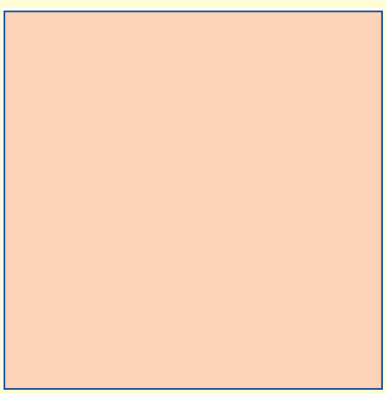
Area of a square



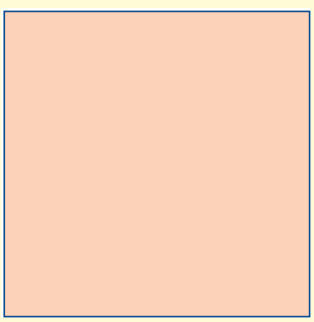
$A = s^2$

4 Measure these squares and calculate their areas in cm^2.

a

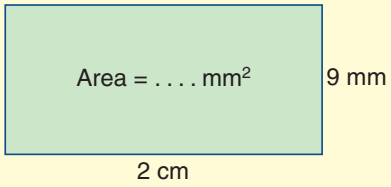


b

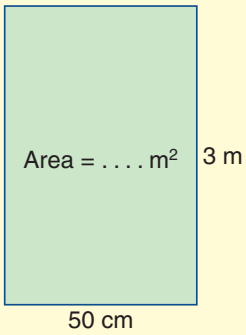


5 Calculate the area of each rectangle in the units indicated.

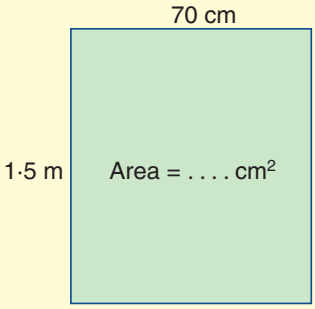
a



b

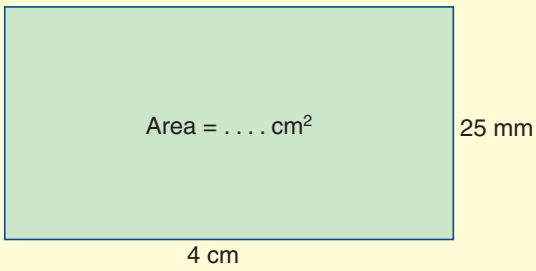


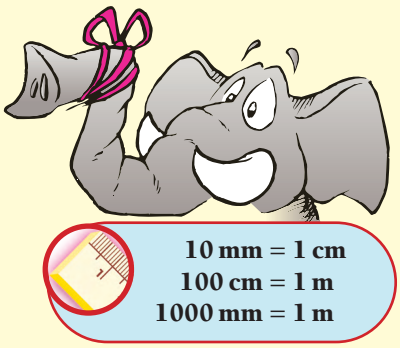
c



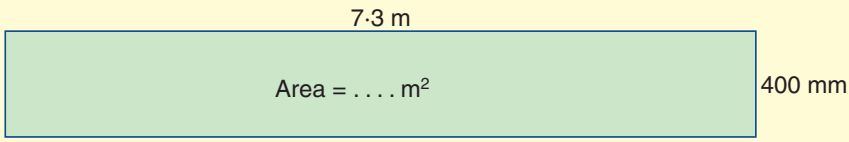
■ To calculate the area, both sides must be given the same units.

d



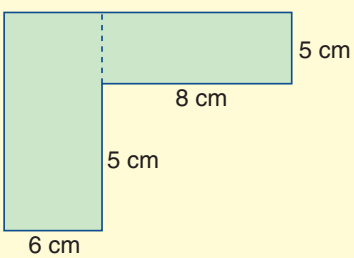


e

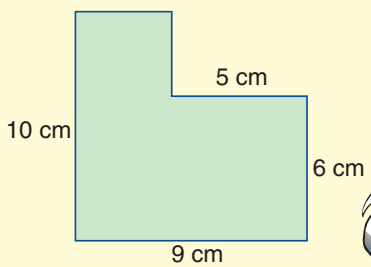


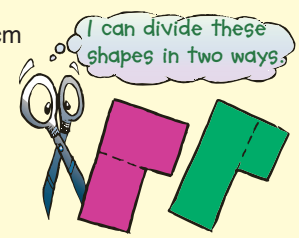
6 Calculate the areas of these figures by dividing them into rectangles.

a

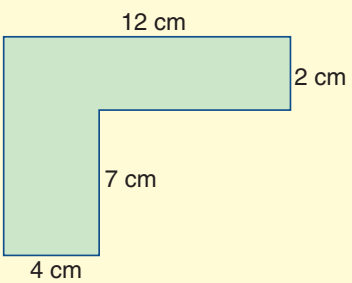


b

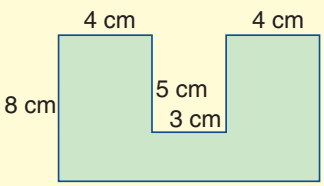




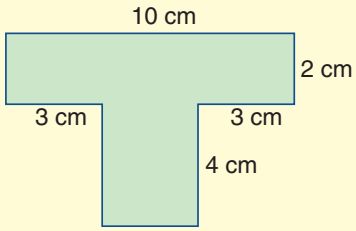
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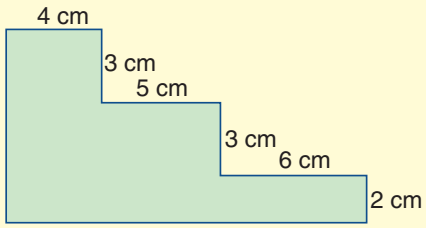
d



e

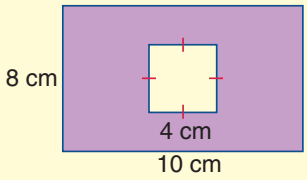


f

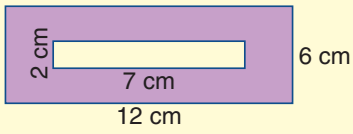


7 Determine the coloured area in each figure.

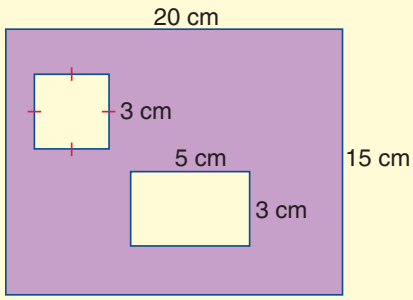
a



b



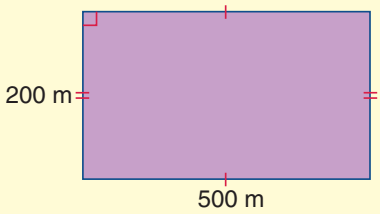
c



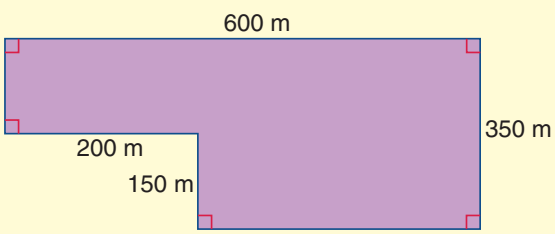
Note: We can take the smaller yellow areas away from the large area to get the coloured area.

8 Find the area, in hectares, of these fields. (*Remember:* 1 h = 10 000 m^2)

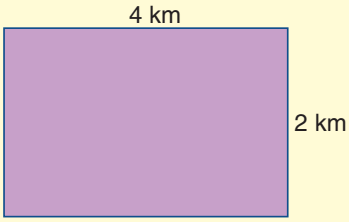
a



b

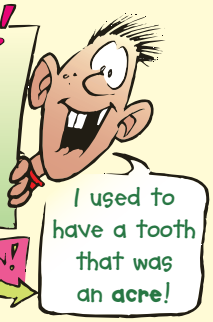


c



FOR INTEREST!

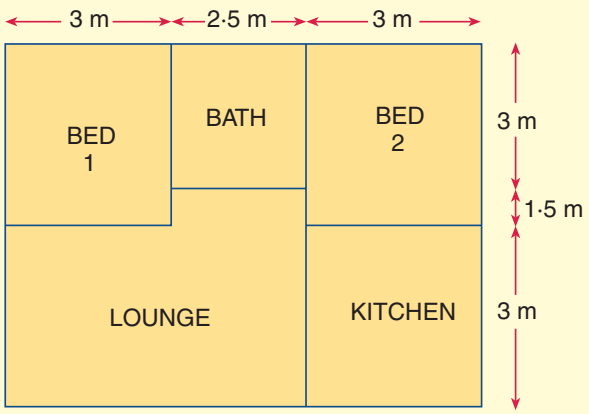
A common unit for land areas was the ACRE. Find out how big this was.



9 Select an area on the right to match the area of each part.

a A ruler
b A school
c A table
d A house block
e A postage stamp
f An Australian rules football field
g A road 10 km long

6 cm^2
2 ha
800 m^2
8 ha
10 ha
90 cm^2
1 m^2

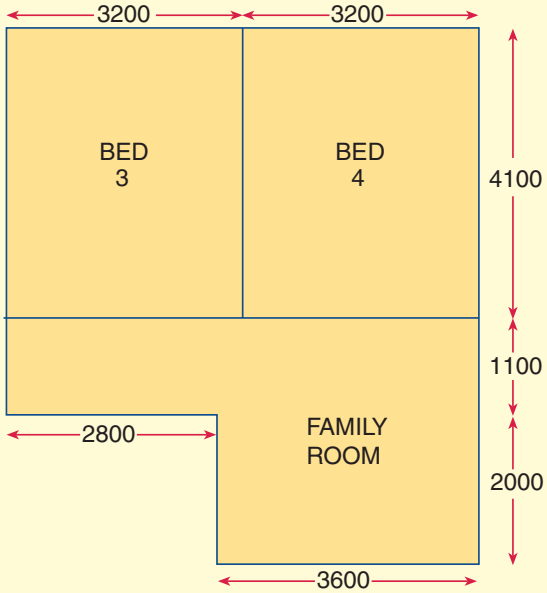
10 

This basic holiday house has the dimensions shown. Find the floor area, in m^2, of the:

- **a** kitchen
- **b** bathroom
- **c** lounge
- **d** two bedrooms
- **e** whole house

11 The plans shown are for an extension to a house. Measurements on plans are normally given in millimetres. Find the area, in square metres, of:

- **a** bedroom 4
- **b** the family room
- **c** the whole extension



investigation

13:02

Investigation 13:02 | Area of a rectangle

Please use the Assessment Grid on the following page to help you understand what is required for this Investigation.

1. Advertisements in newspapers cost more if they take up more area on the page. Take a page from a newspaper and find the areas in cm^2, taken up by advertisements, articles and photographs.
2. Calculate the area of the face of a brick. Use this to determine the number of bricks needed to build a wall 10 m long and 2 m high.
3. Measure the area of a paving brick and calculate the number needed to pave a rectangular driveway 3 m by 6 m.
4. Mark out an area 100 m by 100 m. This area is 1 hectare.
5. Determine the area of the school grounds in hectares. How many housing blocks would fit into it?

Assessment Grid for Investigation 13:02 | Area of a rectangle

The following is a sample assessment grid for this investigation. You should carefully read the criteria *before* beginning the investigation so that you know what is required.

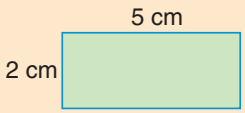
Assessment Criteria (B, C, D)				Achieved ✓
Criterion B Application & Reasoning	a	No systematic approach has been used to find the areas.	1	
			2	
	b	A systematic approach has been used and some questions are answered correctly.	3	
			4	
	c	A systematic approach has been used and most of the questions are answered correctly.	5	
			6	
	d	The exercises are answered correctly and most have been worked through in an organised way.	7	
			8	
	e	All exercises have been completed correctly and instructions have been carried out efficiently.	9	
			10	
Criterion C Communication	a	No working out is shown and no explanations are given.	1	
			2	
	b	Working out is shown and is easy to follow. Diagrams and some correct mathematical terminology have been used.	3	
			4	
	c	Presentation is easy to follow. Diagrams are used effectively and correct mathematical terminology is used throughout.	5	
			6	
Criterion D Reflection & Evaluation	a	Some attempt has been made to explain the methods used in the questions.	1	
			2	
	b	The methods used are justified and the answers have been checked for reasonableness with some success.	3	
			4	
	c	Reasoned explanations are given for the methods used and the answers have been checked successfully.	5	
			6	
	d	Methods are justified and answers have been checked successfully. There is some discussion on the reliability of the methods used.	7	
			8	

13:03 | Area of a Triangle

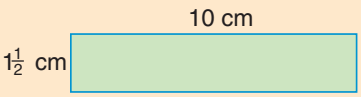
prep quiz 13:03

Calculate the areas of these rectangles.

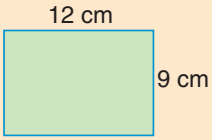
1



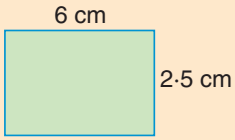
2



3

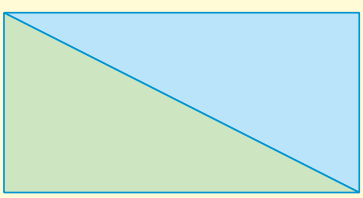


4

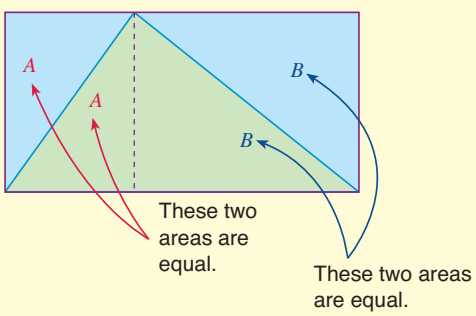


Find the area of: **5** a rectangle with dimensions of 8 mm and 5 mm
6 a rectangle with a length of 20 cm and a breadth of 15 cm
7 a square with a side length of 7 cm

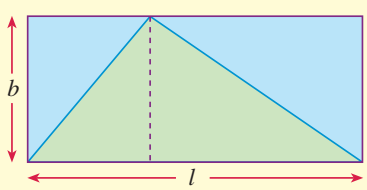
Complete: **8** $1\text{ cm}^2 = \ldots\text{ mm}^2$ **9** $1\text{ m}^2 = \ldots\text{ cm}^2$ **10** $1\text{ ha} = \ldots\text{ m}^2$







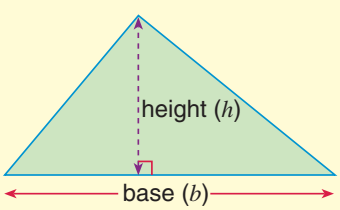
Looking at either of these two figures, you should be able to see that the area of the green triangle is half the area of the rectangle around it.



Knowing that the area of a rectangle is given by $l \times b$, we could say:

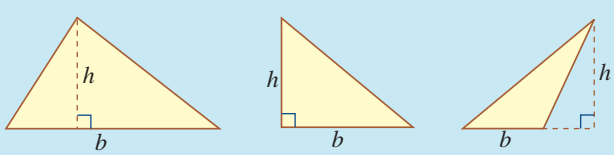
area of a triangle $= \frac{1}{2} l \times b$ units^2.

But we usually call the dimensions of a triangle its **base** and **height**. (*Note:* these two measurements *must* be perpendicular to one another.)



Area of a triangle is given by:

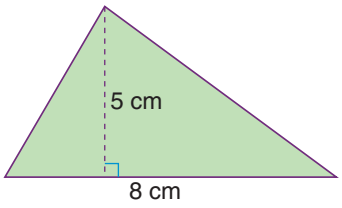
$$A = \frac{b \times h}{2} \quad \textbf{or} \quad \frac{1}{2}\boldsymbol{bh}$$



worked examples

1 Find the area of each triangle.

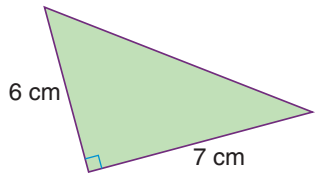
a



$$A = \frac{b \times h}{2}$$
$$= \frac{8 \times 5}{2}$$
$$= 20$$

$\therefore$ Area $= 20\ \text{cm}^2$

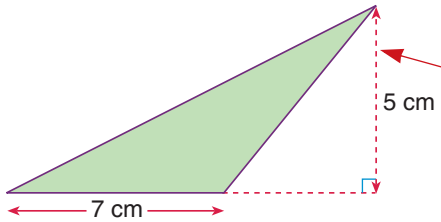
b



$$A = \frac{b \times h}{2}$$
$$= \frac{7 \times 6}{2}$$
$$= 21$$

$\therefore$ Area $= 21\ \text{cm}^2$

c



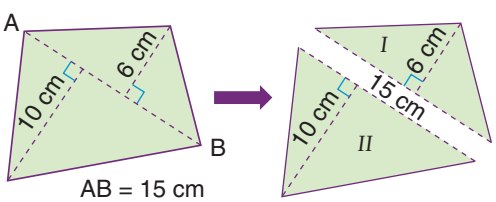
Note: Although this dimension is 'outside' the triangle, it is still the height of the triangle above the given base line.

$$A = \frac{b \times h}{2}$$
$$= \frac{7 \times 5}{2}$$
$$= 17\tfrac{1}{2} \text{ or } 17{\cdot}5$$

$\therefore$ Area $= 17{\cdot}5\ \text{cm}^2$

2 These shapes may be divided into triangles and rectangles to find their areas.

a



Area of triangle I

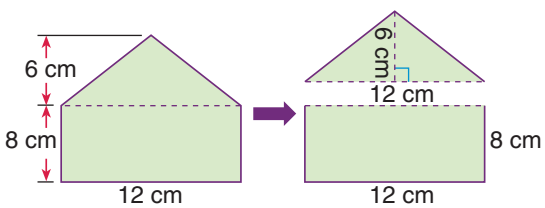
$$A = \frac{15 \times 6}{2}$$
$$= 45$$

Area of triangle II

$$A = \frac{15 \times 10}{2}$$
$$= 75$$

$\therefore$ Total area of figure $= 45\ \text{cm}^2 + 75\ \text{cm}^2$
$= 120\ \text{cm}^2$

b



Area of triangle

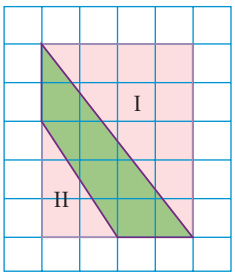
$$A = \frac{12 \times 6}{2}$$
$$= 36$$

Area of rectangle

$$A = 12 \times 8$$
$$= 96$$

$\therefore$ Total area of figure $= 36\ \text{cm}^2 + 96\ \text{cm}^2$
$= 132\ \text{cm}^2$

3



Areas like the green one shown can be calculated by completing a rectangle around the figure and subtracting the area of the resulting triangles from the area of the rectangle. The remainder must be the area of the shaded figure.



$$\text{Area of triangle I} = \frac{5 \times 4}{2}\ \text{cm}^2 = 10\ \text{cm}^2$$

$$\text{Area of triangle II} = \frac{3 \times 2}{2}\ \text{cm}^2 = 3\ \text{cm}^2$$

$$\text{Area of rectangle} = 5 \times 4\ \text{cm}^2 = 20\ \text{cm}^2$$

$$\therefore \text{Area of green figure} = 20 - (10 + 3)\ \text{cm}^2 = 7\ \text{cm}^2$$

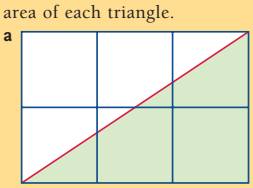
Exercise 13:03

Foundation Worksheet 13:03

Area of a triangle

1 Use the cm grid to find the area of each triangle.

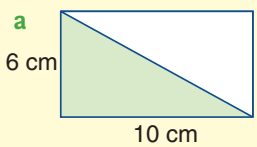
a



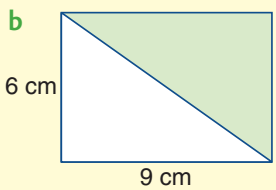
2 Other basic cases.

1 Calculate the area of the green triangle inside each rectangle.

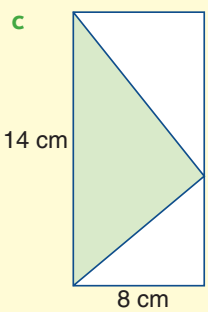
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b

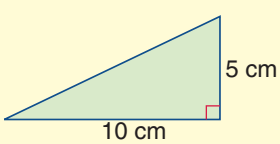


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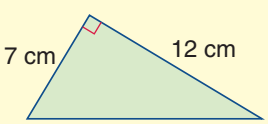


2 Calculate the area of each triangle.

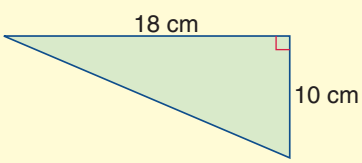
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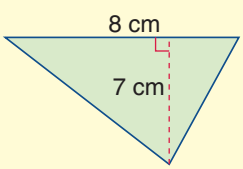
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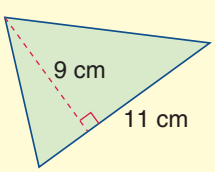
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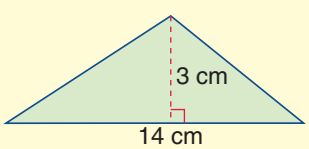
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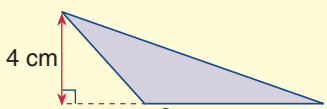


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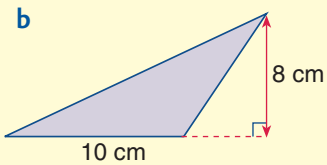


3 Calculate the areas of these triangles.

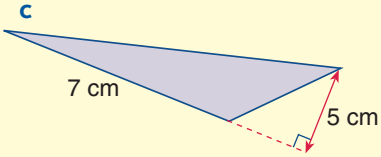
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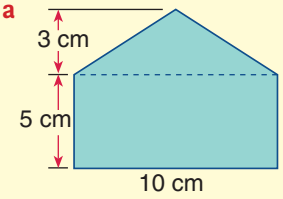
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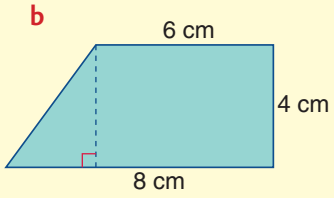
4 **a** A triangle has a base of 20 cm and a height of 12 cm. What is its area?
b A triangle has an area of 48 mm^2 and a height of 12 mm. What is the length of its base?
c A triangle has an area of 72 cm^2 and a base length of 6 cm. What is its height?

5 Determine the areas of these figures.

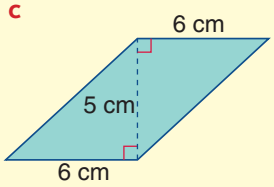
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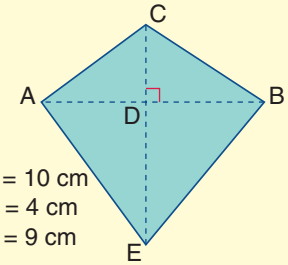
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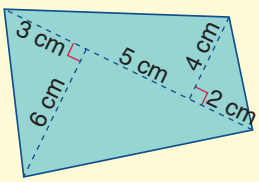


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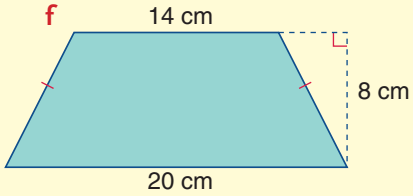


AB = 10 cm
CD = 4 cm
DE = 9 cm

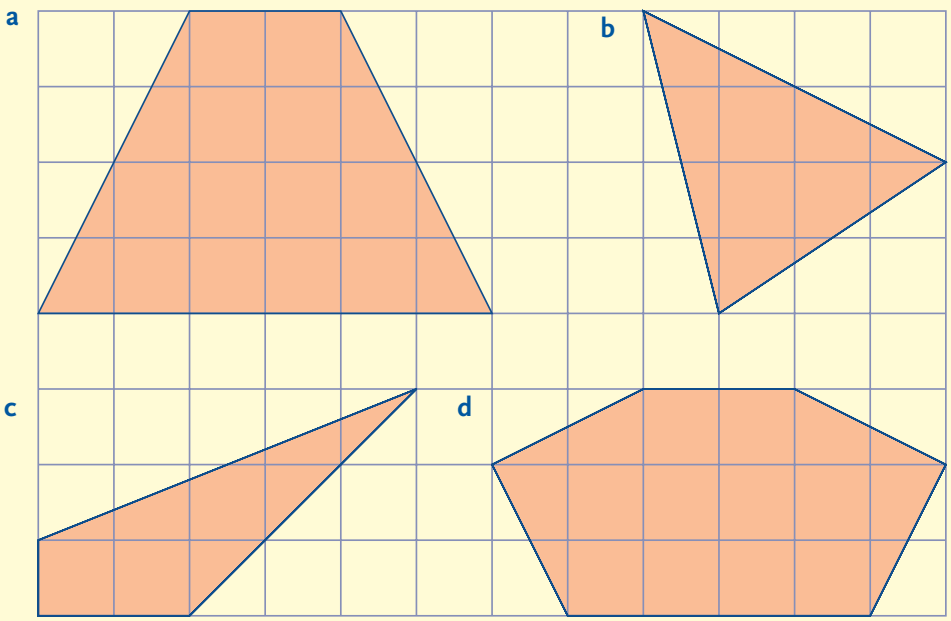
e



f



6 Determine the area of each shaded figure from the centimetre grid it is drawn on.

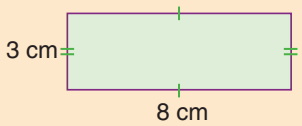


13:04 | Area Problems

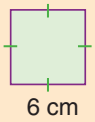
prep quiz 13:04

Calculate the area of each figure.

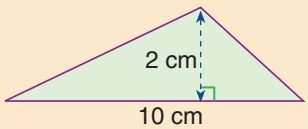
1



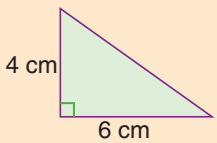
2



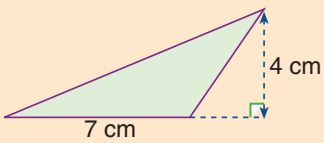
3



4



5



How many square tiles 10 cm by 10 cm will it take to cover a rectangular area:

6 $10\text{ cm} \times 80\text{ cm}$?

7 $10\text{ cm} \times 2\text{ m}$?

8 $50\text{ cm} \times 50\text{ cm}$?

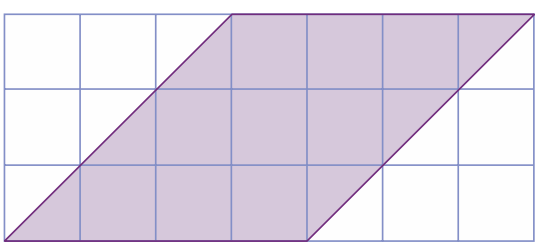
9 $50\text{ cm} \times 5\text{ m}$?

10 What is the perimeter of the rectangle in question 1?

Obviously the measurement of area can occur in a variety of situations. Some everyday examples would be tiling, wallpapering, making curtains, laying turf and painting.

The problems in this section ask you to apply your ability to find the areas of rectangles and triangles in various situations. It is also possible to find the areas of different common shapes from the area of a rectangle.

worked example 1



By cutting and pasting this parallelogram, form a rectangle and hence find its area.

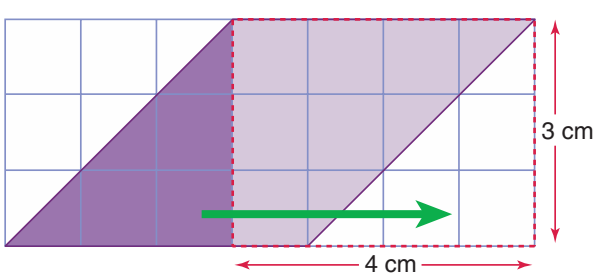


- Square tiles are often used to cover walls.

Solution

If we cut the parallelogram as shown, and slide the left triangle across to the right, we get a rectangle 4 cm by 3 cm.

$A = 4 \times 3$
$= 12$
$\therefore$ Area of rectangle $= 12\text{ cm}^2$
$\therefore$ Area of the parallelogram is 12 cm^2.

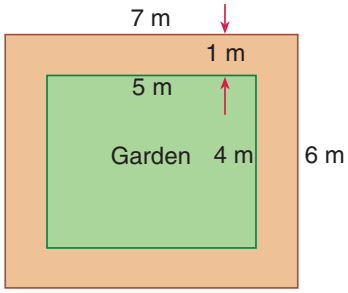


worked example 2

A rectangular garden bed is 5 m by 4 m. A path 1 m wide is to be put around the garden. What will be the area that needs to be paved?

Solution

The path and garden would look like the diagram below.



Because the path is 1 m wide, the outer dimensions would be 7 m by 6 m.

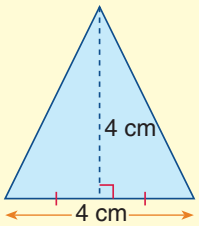
> ***Important Notice!***
> Always draw a diagram of the problem if there is not one with the question.

Area of path = area of outer rectangle − area of garden
$= (7 \times 6)\ m^2 - (5 \times 4)\ m^2$
$= 42\ m^2 - 20\ m^2$
$= 22\ m^2$
$\therefore$ Area of path will be $22\ m^2$.

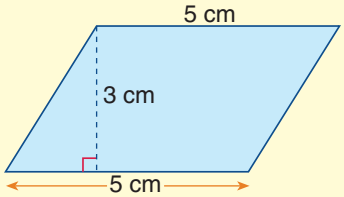
Exercise 13:04

1 Draw a diagram to show how each figure could be cut along the dotted lines and pieced together to form a rectangle. Thus find the area of each figure.

a

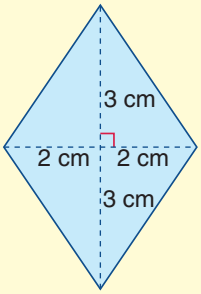


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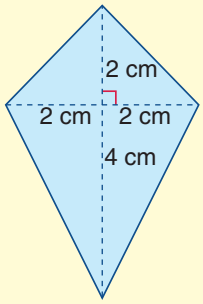




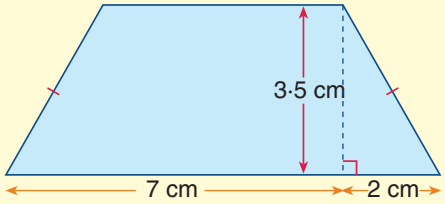
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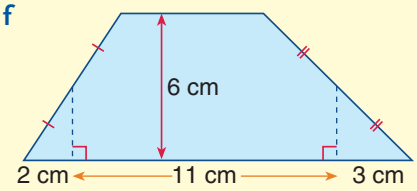
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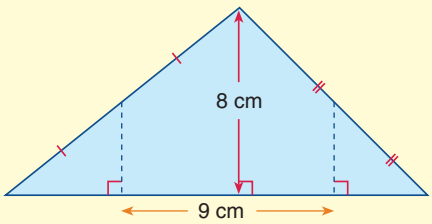
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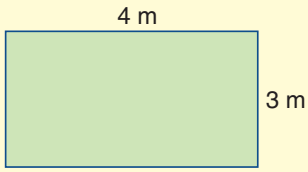
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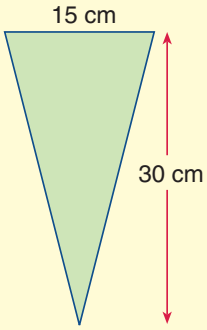


2 Tanith's parents are going to carpet her bedroom. Her bedroom is rectangular, measuring 4 m × 3 m. If the carpet costs $67 per square metre, what will be the cost of carpeting the room?



3 Jason's fence needs painting with paint that covers 16 square metres for each litre of paint. If the fence is rectangular in shape and is 80 m long by 2 m high, how many litres of paint will be needed?

4 An athletics club gave pennants to the winners of each final race. The pennants had the dimensions shown in the diagram. Find the area of each pennant.

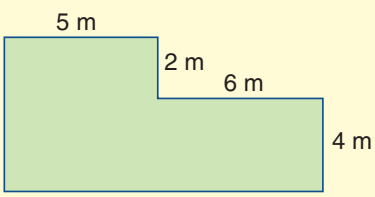


5 A garden 6 m by 2·5 m is to have a path 0·5 m wide around its border. If the garden is rectangular in shape, find:

a the area of the garden
b the total area of the garden and path
c the area of the path

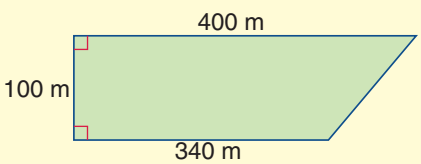
6 Brent needed to cover his Maths textbook because he wanted to take care of it. If the book measured 40 cm by 24·5 cm when opened out flat, what area of plastic would he need if he wanted an extra 2 cm around each edge to tuck in?

7 How much fertiliser would be needed to cover a lawn with the measurements shown if 1 kg will cover 10 m^2?



8 **a** Fred the farmer has a rectangular field measuring 80 m by 400 m. How many hectares of potatoes can he grow in this field?

b Fred's neighbour, Barney, has a field like the one shown in the diagram. Can he plant more potatoes than Fred?



9 A square piece of land is one kilometre on each side. How many hectares would this be?

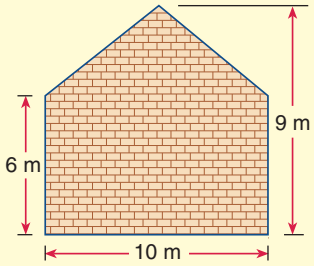
10 A curtain needs to be made for a window that is 2·5 metres wide. If you need a length of material that is three times the width of the window to allow for pleats and folds, and the curtain is to have a drop (length) of 2 m, what area of material is needed?

11 Wallpaper comes in rolls that are 50 cm wide. What length of wallpaper would be needed to cover two walls 4 m by 3 m high and one wall 2·5 m by 3 m high?

12 Floor tiles measure 300 mm by 300 mm. How many would be needed to cover a kitchen floor measuring 2·4 m by 3·6 m?

13 A wall is in the shape of a triangle on top of a rectangle with the measurements shown. Find the area in m^2 of the wall.

Go to a brick wall, mark an area of 1 square metre and count the number of bricks in that square metre. Allowing for some breakages, calculate the number of bricks required to build the wall.

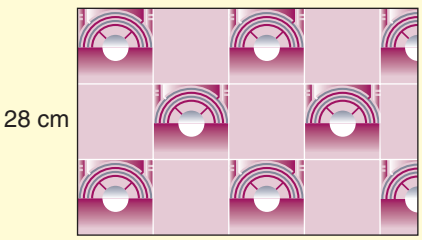


14 **a** A company supplies turf, which comes in rolls 40 cm wide and 2 m long. How many rolls must be ordered to cover an area 28 m by 24 m?

b If each roll costs $4.20, what is the cost of the turf?

c The same area could have been seeded. If a box of seed will cover 16 m^2 and costs $18.50, how much could have been saved on the cost of turf?

15 When laying tiles, an exact number may not cover an area, or a whole number may not lie along each edge. Look at this diagram.

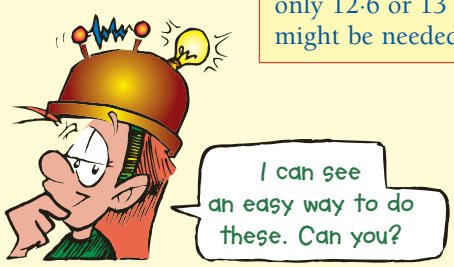


If the tiles are 10 cm × 10 cm, we can see that 15 tiles are needed, presuming that the pieces of tiles cut off are not good enough to be used elsewhere.

■ This is true even though the area is 28 cm × 45 cm, ie 1260 cm^2. Dividing this by 100 cm^2 (the tile area) would suggest that only 12·6 or 13 tiles might be needed.

a How many 10 cm × 10 cm tiles would be needed to cover an area 3·25 m by 2·17 m?

b How many 300 mm by 300 mm tiles would be needed to cover an area 2·5 m by 3·8 m?





- Estimate the area of the sail.

Refer to ID Card 4 on page xvi.
Identify figures (1) to (24).
Learn the terms you do not know.

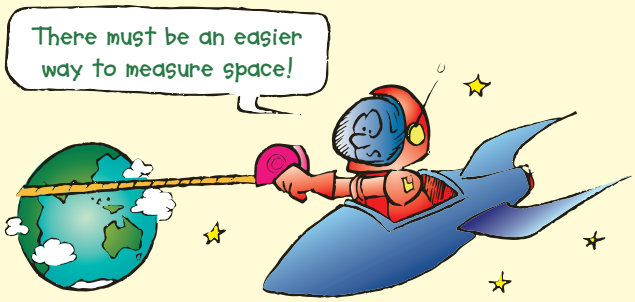
id 13:04

13:04A Areas of plane figures
13:04B Carla's chicken run

Activity CW

13:05 | Measuring 3D Space (Volume)

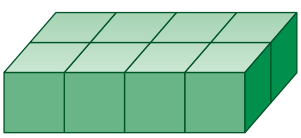
When we measure the space an object occupies, we are measuring its **volume**.



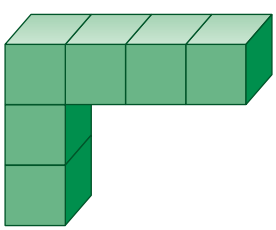
When we say that one object is bigger or smaller than another, we are usually comparing their volumes.

If we are going to measure the volume of an object, we need appropriate units. To measure area we used square units. To measure volume we use **cubic units**.

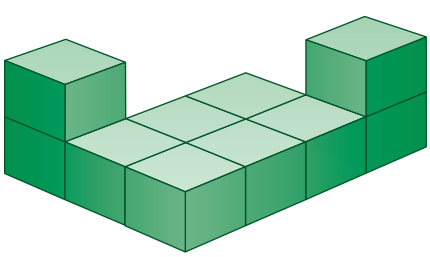
worked examples

1 If this is one cubic unit,  then the volume of this solid is 8 cubic units. 

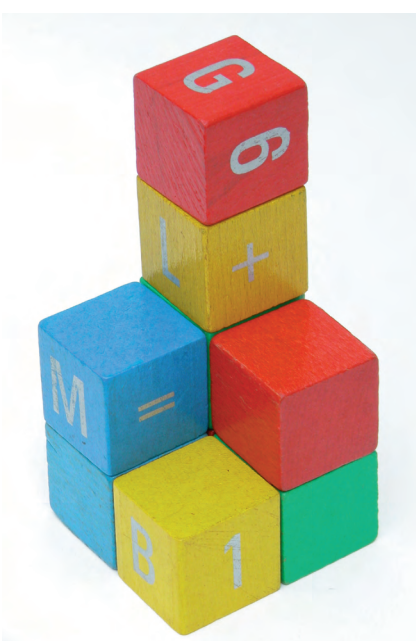
2 The volumes of these solids are found by counting the cubes. Check that you can get the answers shown below.



Volume = 6 cubic units



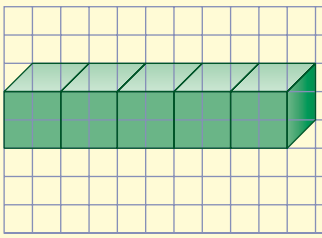
Volume = 10 cubic units



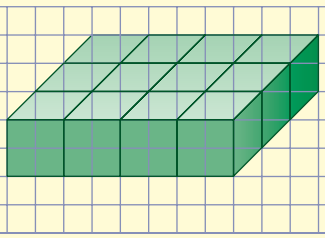
Exercise 13:05

1 Count the number of cubic units in each solid.

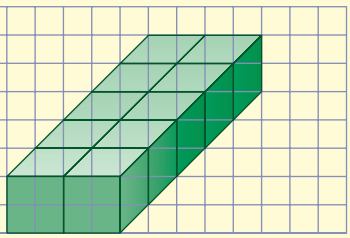
a



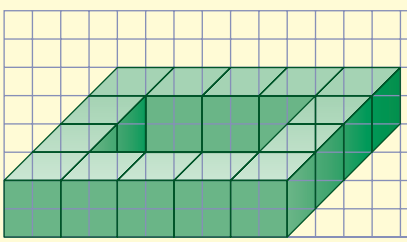
b



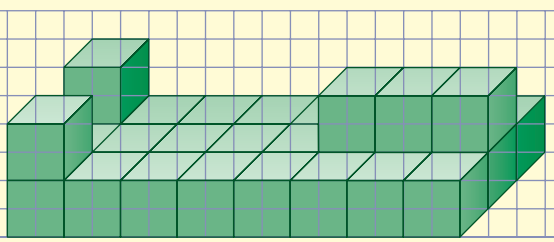
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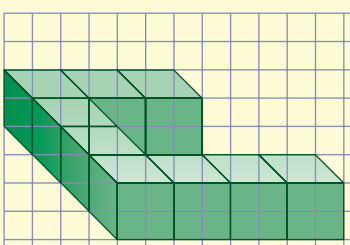
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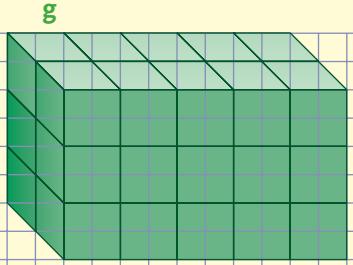
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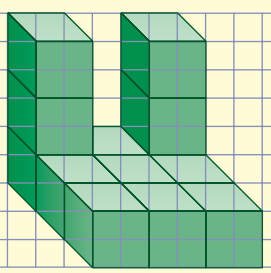
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g

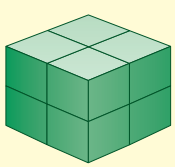


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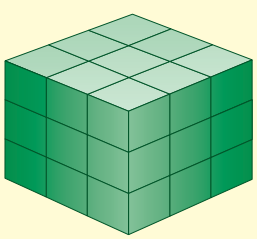


2 What is the volume, in cubic units, of each solid?

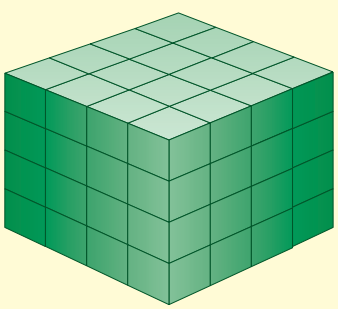
a



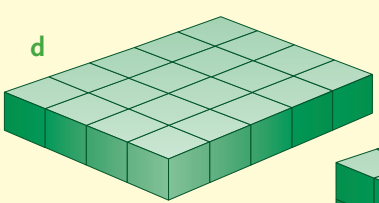
b



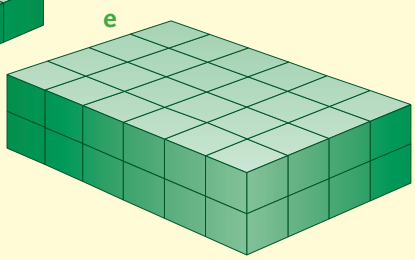
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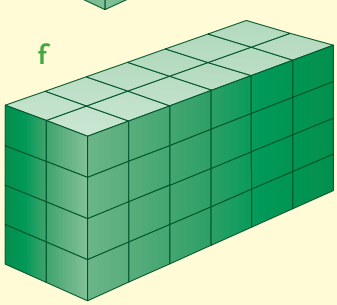
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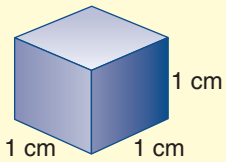


e

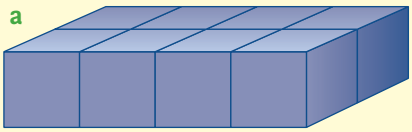


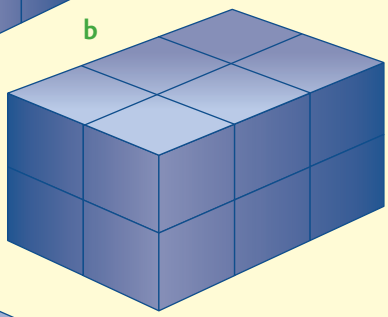
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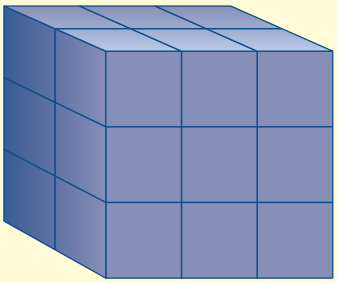


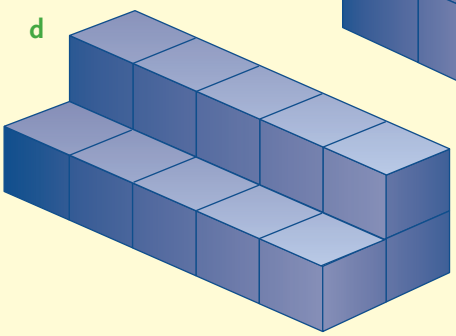
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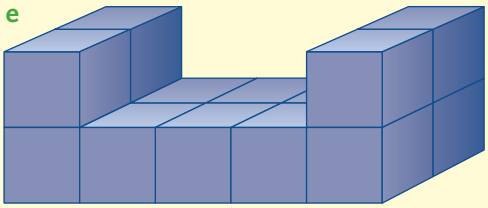
A cube that has edges 1 centimetre long has a volume of 1 cubic centimetre (1 cm^3). What is the volume, in cubic centimetres, of the following solids? (Each block represents 1 cubic centimetre.)

a 

b 

c 

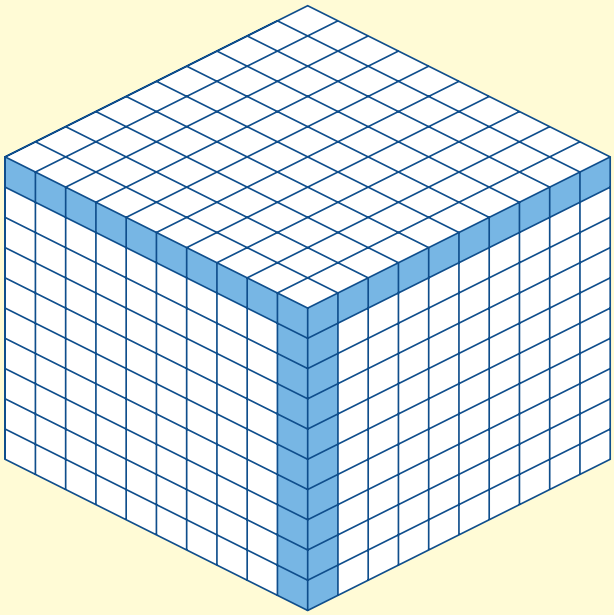
d 

e 

4 This large cube is made of smaller cubic units.

- **a** How many smaller cubes are there along each edge?
- **b** How many smaller cubes are there in the top layer?
- **c** How many smaller cubes are needed to make the large cube?



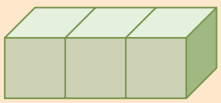


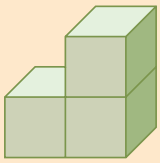
Investigation 13:05 | Measuring 3D space

investigation 13:05

Please use the Assessment Grid on the following page to help you understand what is required for this Investigation.

1 There are only 2 differently shaped solids that can be formed by joining three cubes together at their faces, as shown by these diagrams.





There are eight different solids that can be formed using 4 cubes. After stacking cubes together, draw all eight.

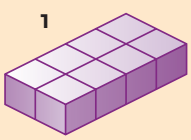
2 By stacking cubes together, find out how many *rectangular prisms* can be made that have different dimensions and a volume of:

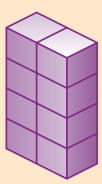
a 8 cubic units
b 20 cubic units
c 24 cubic units

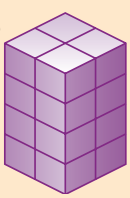
13:06 | Volume of a Rectangular Prism

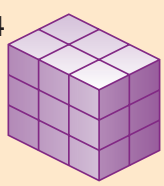
prep quiz 13:06

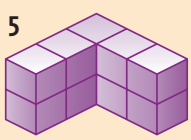
What is the volume, in cubic units, of each solid?

1 

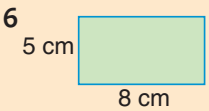
2 

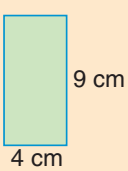
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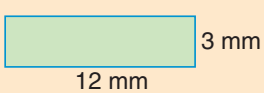
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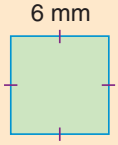
5 

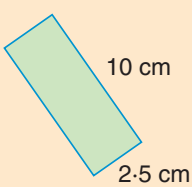
What is the area of each rectangle?

6 

7 

8 

9 

10 



Assessment Grid for Investigation 13:05 | Measuring 3D space

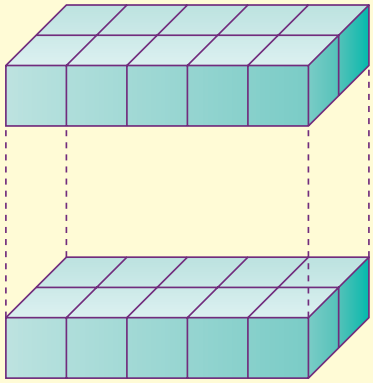
The following is a sample assessment grid for this investigation. You should carefully read the criteria *before* beginning the investigation so that you know what is required.

Assessment Criteria (B, C, D)				Achieved ✓
Criterion B Application & Reasoning	a	No organised approach has been used to make the solids.	1	
			2	
	b	An organised approach is used and some attempt has been made to describe or draw the shapes.	3	
			4	
	c	An organised approach has been used and the shapes are described or drawn in some way.	5	
			6	
	d	Most of the solids have been found in an organised way and have been described or drawn successfully.	7	
			8	
	e	All the solids are found and have been described successfully.	9	
			10	
Criterion C Communication	a	There is no working out, and there is little description of the solids.	1	
			2	
	b	Working out is shown with the aid of some diagrams or a description of the solids.	3	
			4	
	c	Presentation and working is easy to follow and the solids have been successfully described with the use of correct technology.	5	
			6	
Criterion D Reflection & Evaluation	a	Some attempt has been made to explain the method used to find the solids.	1	
			2	
	b	The method used is justified and the reasonableness of the findings checked.	3	
			4	
	c	A reasoned explanation of the method used is given and the findings are reasonable.	5	
			6	
	d	The method is justified and the findings are discussed. There is some discussion about the shapes formed and about the areas of these shapes.	7	
			8	



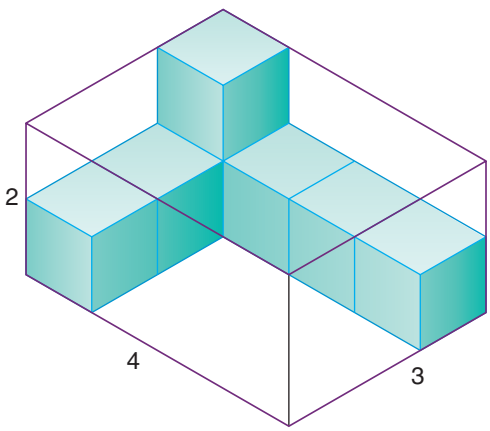
For this prism we can see there are 2 layers of 5×2 cubes.

To calculate the volume of a rectangular prism, we work out how many cubes are in one layer, and multiply by the number of layers.



worked examples

1



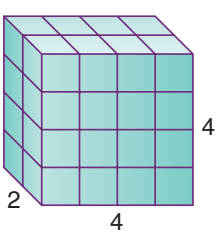
To determine the volume of this prism, we can see that there would be 4×3 cubes in the bottom layer, ie 12 cubic units.

Since there are two layers, the volume must be:

$V = (4 \times 3) \times 2$ cubic units
$= 24$ cubic units

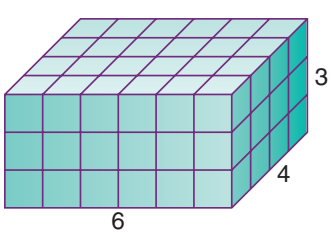
> *Notice*
> The number of cubes in each layer is the same as the area of the base rectangle in square units.

2 a



The volume of this solid would be given by:
$V = (4 \times 2) \times 4$
$= 32$ cubic units

b



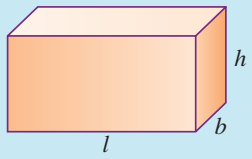
The volume of this rectangular prism would be given by:
$V = (6 \times 4) \times 3$
$= 72$ cubic units

To find the volume of a rectangular prism, use the rule:

$$V = l \times b \times h$$

or simply

$$V = lbh$$



Exercise 13:06

Foundation Worksheet 13:06

Volume of a rectangular prism

1 Find each volume in cm³.

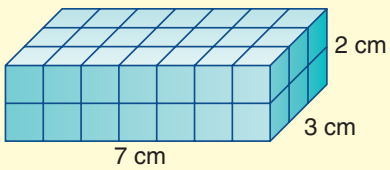
a



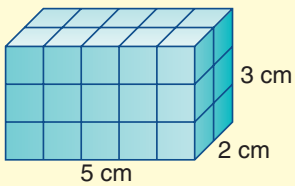
2 Examples like Question 1 of text.

1 Find the volume of each rectangular prism in cubic centimetres.

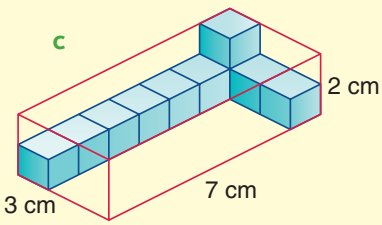
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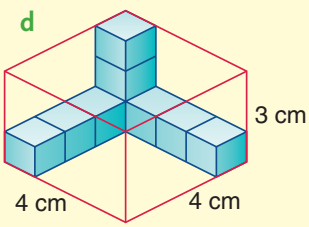
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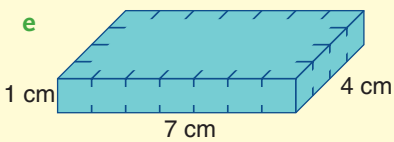
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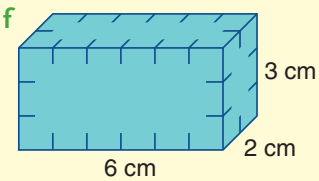
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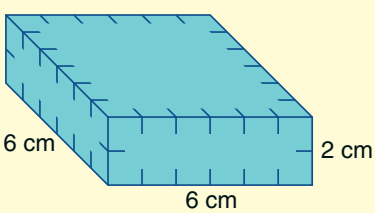
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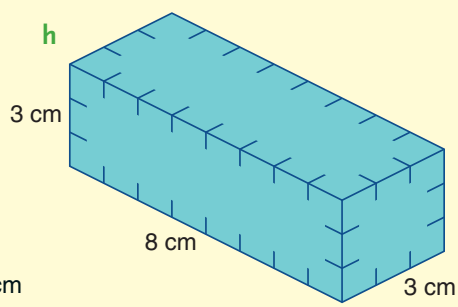
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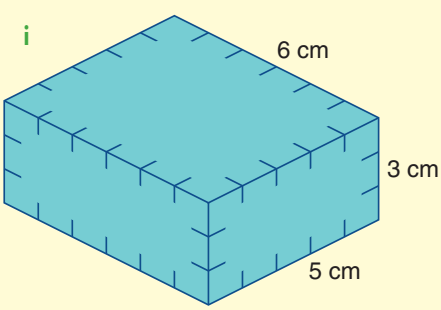
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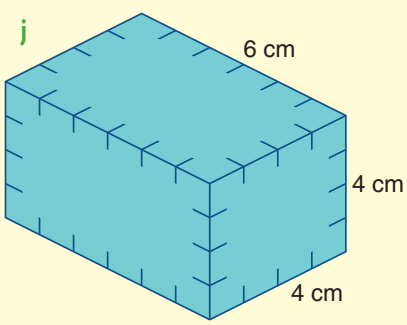
h



i

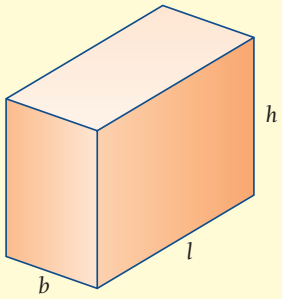


j

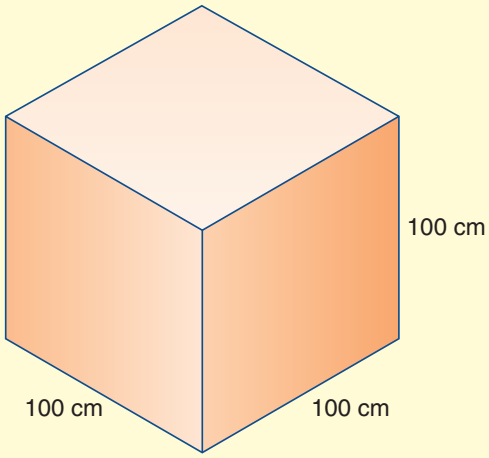


2 By using the formula $V = l \times b \times h$, calculate the volume of each rectangular prism described below.

a length = 3 cm, breadth = 2 cm, height = 1 cm
b length = 5 cm, breadth = 3 cm, height = 4 cm
c length = 2 cm, breadth = 1 cm, height = 6 cm
d $l = 9$ cm $b = 3$ cm $h = 2$ cm
e $l = 5$ cm $b = 5$ cm $h = 5$ cm
f $l = 10$ cm $b = 8$ cm $h = 4$ cm

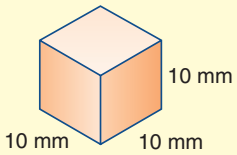


3



a A cube with a side length of 1 metre would have a volume of 1 cubic metre (1 m^3). What would the volume of this cube be in cubic centimetres (cm^3)?

b Similarly, a centimetre cube would have side lengths of 10 mm. What would the volume of this cube be in cubic millimetres (mm^3)?

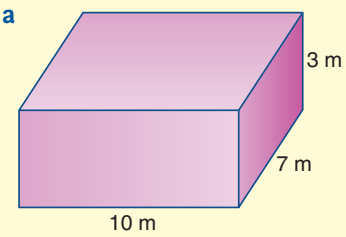


4 Calculate the volume of a cube that has a side length of:

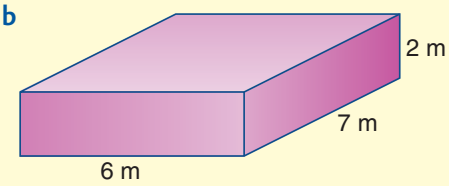
a 3 cm **b** 5 cm **c** 9 cm **d** 4 mm

5 Calculate the volume of each prism, in m^3.

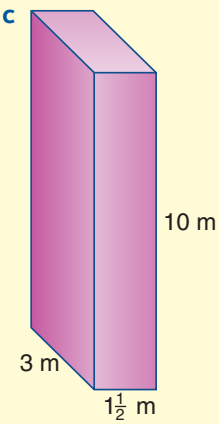
a



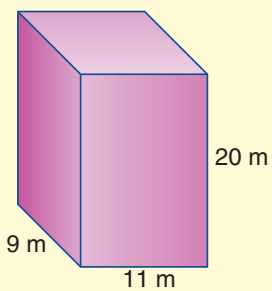
b



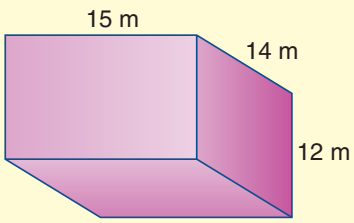
c



d

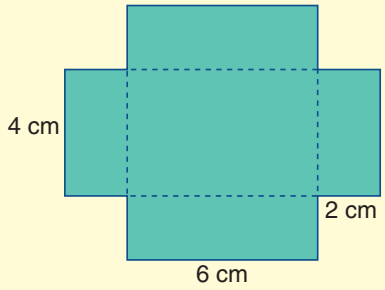


e

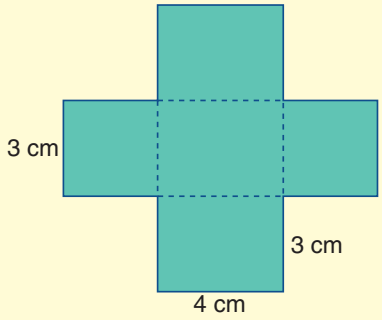


6 If open boxes were made by folding along the dotted lines, what would be the volume of each?

a



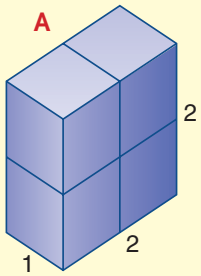
b

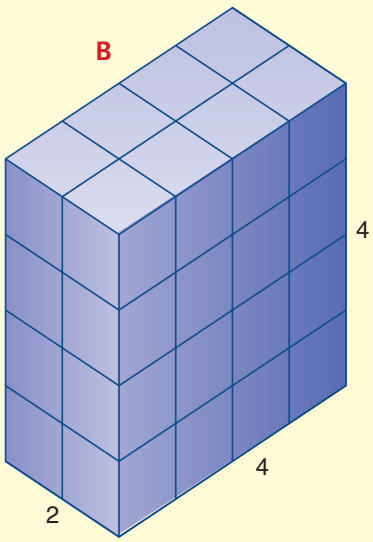


7 Look at the dimensions of these prisms. The dimensions of prism A have been doubled to give prism B.

a What is the volume of each prism?

b How many of prism A would be needed to make prism B?





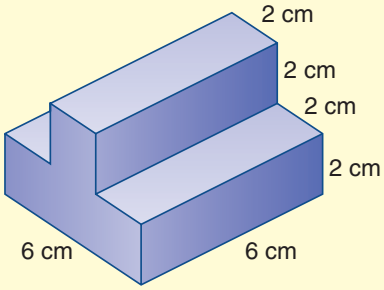
8 The dimensions of a rectangular prism are 5 cm × 4 cm × 2 cm.

a If these dimensions are doubled, what would the volume become?

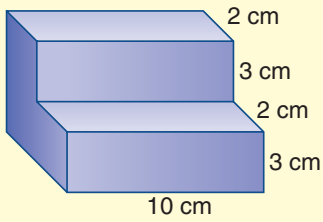
b How many of the original prisms would fit into the larger prism?

9 Find the volume of these solids formed by joining rectangular prisms together.

a



b

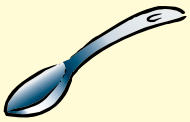


13:07 | Capacity

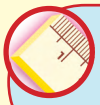
When we measure the amount of a liquid that a container can hold, we are measuring its **capacity**.



We might say that an amount of a liquid is so many cupfuls or spoonfuls, but as with the volume of solids, we need some standard units.

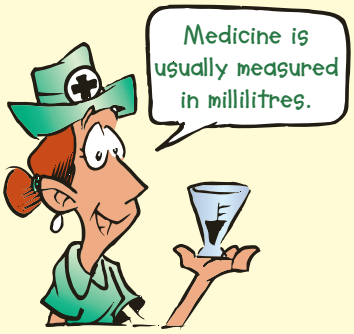


■ *Capacity* is the volume of *liquids*.



The basic unit in the metric system for capacity is the *litre* (L).
This means that:
1000 millilitres (mL) = 1 litre (L)
1000 litres (L) = 1 kilolitre (kL)







Exercise 13:07

Foundation Worksheet 13:07

Capacity

Complete:

1 a 2 L = . . . mL **b** 7 L = . . . mL
c 4000 mL = . . . L
2 a 3 kL = . . . L **b** 2000 L = . . . kL
3 a 3 cm^3 = . . . mL

1 How many millilitres would there be in:

a 2 L? **b** 5 L? **c** $\frac{1}{2}$ L? **d** 0·4 L?
e 3·5 L? **f** 100 L? **g** 0·05 L? **h** 1 kL?

2 How many litres would there be in:

a 3000 mL? **b** 9000 mL? **c** 7500 mL? **d** 8300 mL?
e 500 mL? **f** 300 mL? **g** 20 000 mL? **h** 50 mL?

3 Which measure of capacity, 50 mL, 500 mL or 50 L, would be most likely for:

a a bottle of drink? **b** the petrol tank of a car? **c** a medicine glass?

4 The capacity of a cup would be closest to 25 mL, 250 mL or 2500 mL?

5 How many 300 mL mugs could be filled completely from a 2 L kettle?

6 How many 375 mL cans of drink do I need to buy to have at least 5 L of drink?

7 Elizabeth's backyard pool contains 20 000 L of water. How many kL is this?

8 How many times would a 1·5 L jug need to be filled to pour out 50 drinks at a party, if the glasses hold 300 mL?

9 A 600 mL bottle of drink costs $1.20 and a 2 L bottle costs $4.20. Which is the better buy, and by how much per litre?

10 A leaking tap loses 1 mL of water every 10 seconds. How much water will be lost in:

a 10 minutes? **b** 1 hour?
c 1 day? **d** 1 week?
e 1 year?



There is a link between the capacity of liquids and the volume of solids.
A container with a volume of 1 cubic centimetre would hold 1 mL of liquid.
Also 1 m^3 = 1 kL.

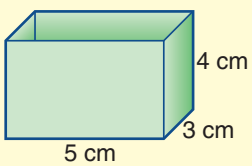
1 mL = 1 cm^3 **This also means that:** **1 L = 1000 cm^3** **and** **1 m^3 = 1 kL**.

11 Complete:

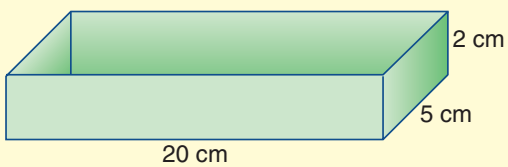
a 15 mL = ... cm^3 **b** 8 cm^3 = ... mL **c** 183 mL = ... cm^3
d 5000 cm^3 = ... L **e** 35 L = ... cm^3 **f** 2·5 L = ... cm^3
g 15 m^3 = ... kL **h** 4·5 kL = ... m^3 **i** 8·2 m^3 = ... kL

12 How many mL of liquid would each of these prisms hold?

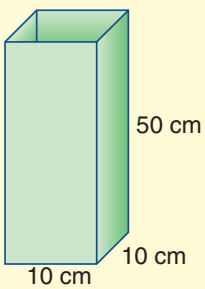
a



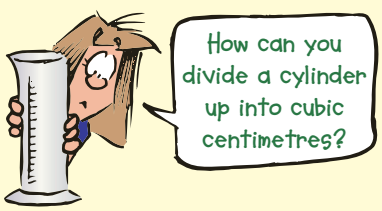
b



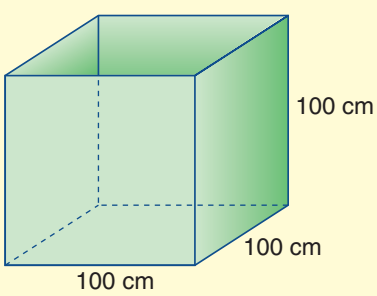
c



13 If this measuring cylinder has a capacity of 500 mL, what must its volume be in cubic centimetres?

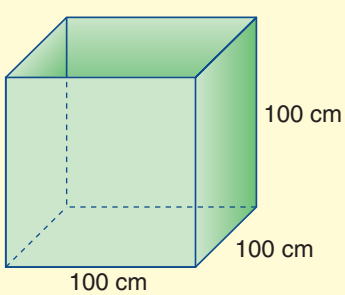


14



a What is the volume, in cm^3, of this cubic container?
b How many litres of water would fill this container?

■ 10 cm = 1 decimetre (dm)
1000 cm^3 = 1 dm^3 = 1 L



1 cubic metre = 1 kilolitre

15 **a** What is the volume, in cm^3, of a cubic metre?
b What is the capacity of a 1 m^3 container in:
i millilitres?
ii litres?
iii kilolitres?

16 A petrol can is in the shape of a rectangular prism with dimensions of 15 cm × 20 cm × 25 cm. How much petrol will it hold?

17 The capacities of car engines are sometimes quoted in litres and sometimes in cubic centimetres (often using the abbreviation cc).

- **a** A car's engine is said to be 2·4 L. How many cubic centimetres is this?
- **b** Another engine has a capacity of 1600 cc. What is this in litres?

'cc' stands for 'cubic centimetre'.

Reading mathematics 13:07 | Capacity

reading mathematics 13:07

Collect various drink containers and estimate their capacities. Check your accuracy by reading the labels.



If a container is not marked, the capacity can be measured by filling it with water and then pouring the water into a container whose capacity is known.

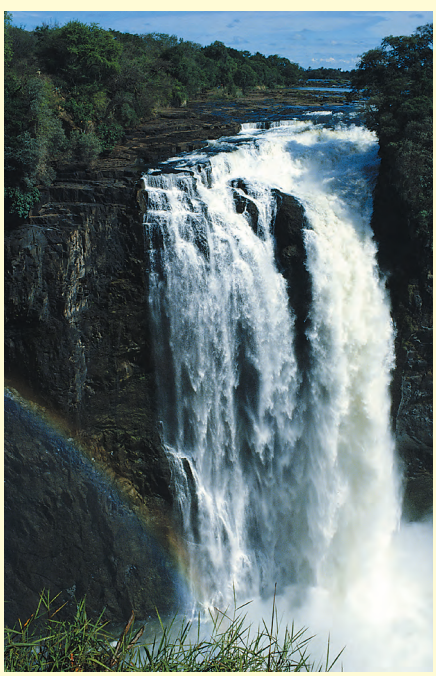








- Estimate the volume of each item.



- Estimate the flow of water each hour.

Practical Activity 13:07 | Estimating capacity

1 Work with a friend. You will need a large measuring cylinder (big enough to take your clenched fist) and a tray in which to stand the cylinder.

Begin by estimating the volume of your fist in cubic centimetres, then fill the measuring cylinder to the brim with water and gently plunge your clenched fist into the water. Measure how many millilitres of water have overflowed into the tray. Decide who has the bigger fist.

2 On average, each person in a town uses 30 litres of water each day. If there are 2500 people in the town and the town needs a water tank capable of holding 14 days' supply, find how many litres of water the tank should hold. Design a possible tank in the shape of a rectangular prism.

Appendix I Mass

fun spot 13:07

Fun Spot 13:07 | What makes money after 8 pm?

Work out the answer to each part and put the letter for that part in the box that is above the correct answer.

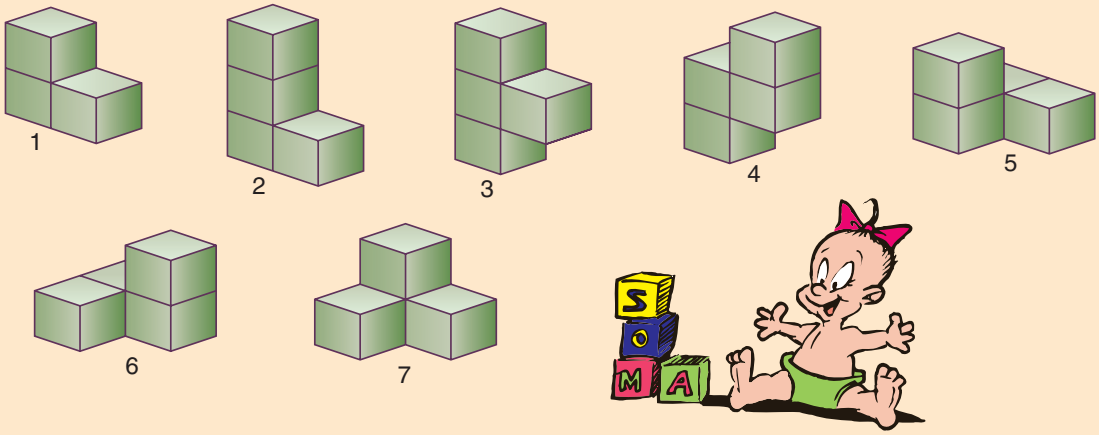
A 1 m = . . . cm
A 1 cm = . . . mm
D 5 kg = . . . g
E 1 L = . . . mL
E 3 h = . . . min
F 5 min = . . . s
I 3 km = . . . m
I 4 t = . . . kg
M 2 kL = . . . L
N 1 day = . . . h
N 6 kg = . . . g
N 5 cm^2 = . . . mm^2
N 4 ha = . . . m^2
R 4 km^2 = . . . ha
R 6 m^2 = . . . cm^2
T 4 000 000 cm^3 = . . . m^3
T 1·8 m = . . . mm

10	40 000	100	300	4	180	400	5000	4000	24	6000	1000	60 000	2000	3000	500	1800



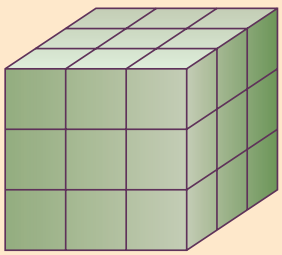
- Estimate the volume of the Parthenon in Athens.

Challenge 13:07 | The fantastic Soma cube



The Soma cube is formed by joining these 7 pieces together.

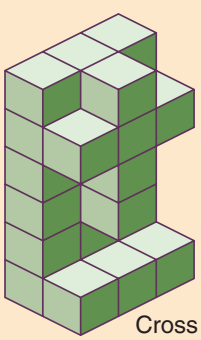
The first piece is formed by gluing 3 cubes together; the other six pieces are formed from 4 cubes.

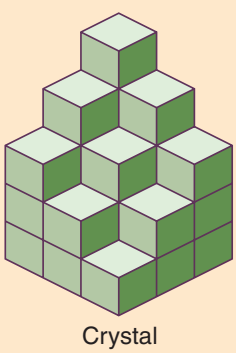


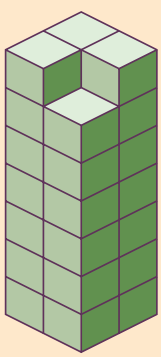
They represent all of the ways of joining 3 or 4 cubes together, having at least one corner in the construction.

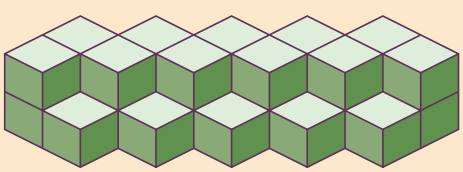
There are many, many ways of putting them together to make the Soma cube. See how many you can find.

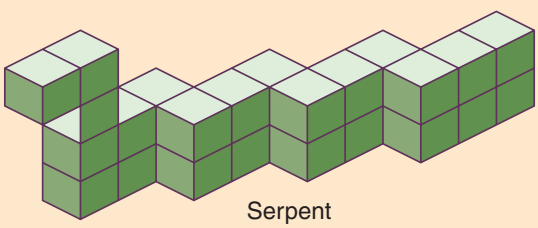
The pieces can also be put together to form other interesting shapes. Some are shown below. See if you can form them. Also try making some shapes of your own.


Cross


Crystal


Tower


Wall


Serpent

Mathematical terms 13

area
- The amount of space inside a two-dimensional shape.
- **Units of area**
  square millimetre, square centimetre, square metre, hectare, square kilometre

hectare
- An area of 10 000 m^2.
- 100 m × 100 m.

volume
- The amount of space inside a three-dimensional shape.
- **Units of volume**
  cubic millimetre, cubic centimetre, cubic metre

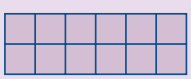
capacity
- The amount of fluid that can be held by a container.
- **Units of capacity**
  litre, millilitre, kilolitre

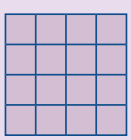
Diagnostic Test 13: | Area and Volume

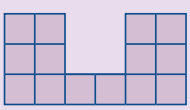
- Each section of the test has similar items that test a certain type of example.
- Failure in more than one item will identify an area of weakness.
- Each weakness should be treated by going back to the section listed.

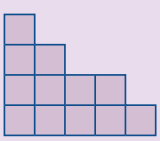
Section

1 What is the area of each figure, in square units? — 13:01

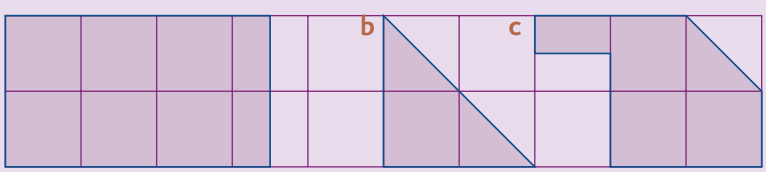
a 

b 

c 

d 

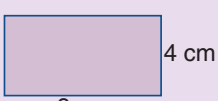
2 What is the area of each figure, in square centimetres? — 13:01

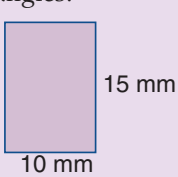
a 

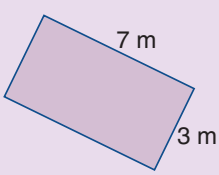
3 Complete: — 13:02

a 1 cm^2 = . . . mm^2
b 1 m^2 = . . . cm^2
c 1 ha = . . . m^2
d 50 000 cm^2 = . . . m^2

4 Find the areas of these rectangles. — 13:02

a 

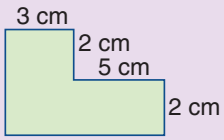
b 

c 

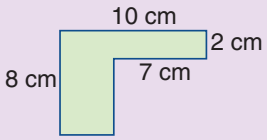
Section

5 Calculate the areas of these figures by dividing them into rectangles. 13:02

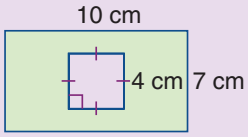
a



b

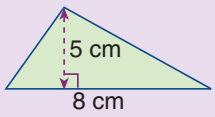


c

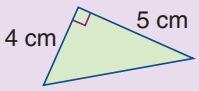


6 Find the areas of these triangles. 13:03

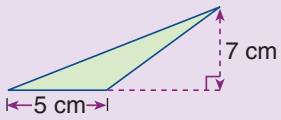
a



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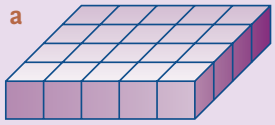


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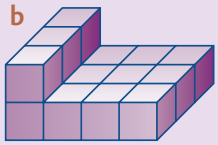


7 What is the volume of each figure, in cubic units? 13:05

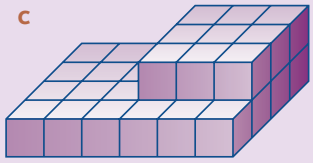
a



b

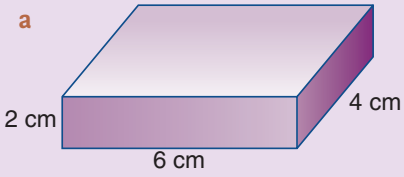


c

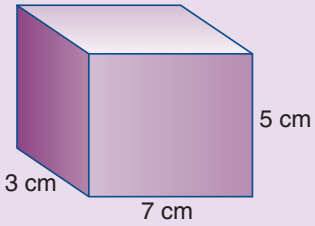


8 What are the volumes of these rectangular prisms, in cm^3? 13:06

a



b



9 What is the volume of: 13:06

a a cube with an edge length of 4 cm?

b a rectangular prism that has length 7 cm, breadth 3 cm and height 4 cm?

c a rectangular prism with dimensions of 2 m × 3 m × 5 m?

10 Complete the following. 13:07

a 1 L = . . . mL **b** 2000 L = . . . kL

c 3500 mL = . . . L **d** 1·5 kL = . . . L

11 Complete the following. 13:07

a 3 mL = . . . cm^3 **b** 50 cm^3 = . . . mL

c 1 L = . . . cm^3 **d** 1 m^3 = . . . kL

12 What would be the capacity of each prism in question 8, in millilitres? 13:07

1 Units of capacity and volume
2 Area puzzle

Drag and Drop CW Interactive

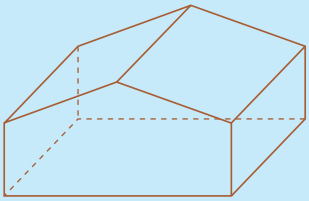
Teacher's Resource TR Chapter Test

Review CW Questions

Chapter 13 | Revision Assignment

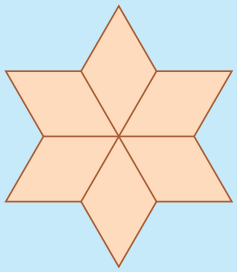
assignment 13A

1 For the solid shown:



- **a** Find the number of rectangular faces.
- **b** Find the number of non-rectangular faces.
- **c** What is the shape of the cross-sectional area?
- **d** Name the solid.
- **e** Sketch the net of the solid.

2



- **a** What shape has been used to make this star?
- **b** Are all stars the same shape?
- **c** Is the term 'star' a precise mathematical term like 'square' or 'rectangle'?
- **d** Does this star have more than one axis of symmetry?
- **e** If the bottom half of the star is folded onto the top half so that the two halves match exactly, what shape is formed?

3 In a normal year how many days are there from:

- **a** 1st February to 1st May?
- **b** 7th October to 16th December?
- **c** January 26th to April 25th?
- **d** May 21st to August 28th?

4 I am paid every fortnight. If I am paid on 7th January, give the dates of my next four paydays.

5 Rewrite the following 24-hour times as standard 12-hour times, indicating whether they are am or pm.

- **a** 1530
- **b** 0720
- **c** 0030
- **d** 2350
- **e** 13:40
- **f** 06:20
- **g** 17:15
- **h** 20:00

6 Copy and complete the table.

Name	*Number of faces*	*Number of vertices*	*Number of edges*
Triangular prism			
Triangular pyramid			
Cube			
Rectangular prism			

7 Simplify:

- **a** $\frac{3}{10} + \frac{7}{10}$
- **b** $1\frac{3}{5} + \frac{2}{5}$
- **c** $2 - \frac{3}{4}$
- **d** $8\frac{1}{4} - 3$

8 Simplify:

- **a** $0{\cdot}8 + 0{\cdot}01$
- **b** $2{\cdot}7 + 6$
- **c** $8 + 3{\cdot}1$
- **d** $12{\cdot}16 + 7{\cdot}8$

9 Simplify:

- **a** $1{\cdot}2 \times 0{\cdot}2$
- **b** $0{\cdot}12 \times 2$
- **c** $0{\cdot}12 \times 0{\cdot}2$
- **d** $1{\cdot}2 \times 20$

10 Simplify:

- **a** $6 - 1{\cdot}8$
- **b** $0{\cdot}8 - 0{\cdot}45$
- **c** $2{\cdot}1 - 1{\cdot}75$
- **d** $6{\cdot}85 - 2{\cdot}9$

Chapter 13 | Working Mathematically

assignment 13B

1 The Sydney–Melbourne ultramarathon is 860 km in length. When a runner had completed three-quarters of the race, how many kilometres did she still have to run?

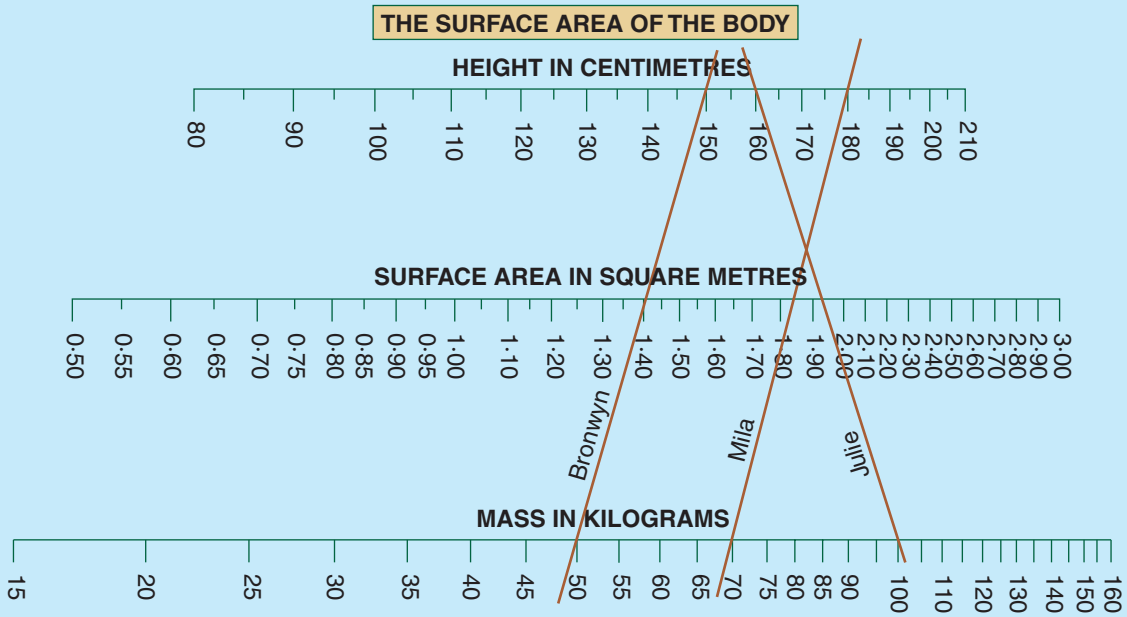
2 Approximately 55 bricks are needed to build 1 square metre of wall. How many bricks would be needed to build a wall 14 m long and 2·8 m high if the wall has two windows, each 2·4 m long and 1·2 m high?

3 Timber is bought by the *linear metre*. If the timber is 150 mm wide, how many metres would I need to buy to cover a rectangular area 3 m long and 1 m high?

4 The pages of a book are numbered and it is found that 495 digits are used. How many pages were there?

5 Twenty-seven one-centimetre cubes are used to build a three-centimetre cube. The cube is then painted red. How many of the one-centimetre cubes will have:
- **a** only 1 face painted?
- **b** only 2 faces painted?
- **c** 3 faces painted?

6 In the diagram below, the line connecting the height and weight intersects the middle line at the surface area of the body.

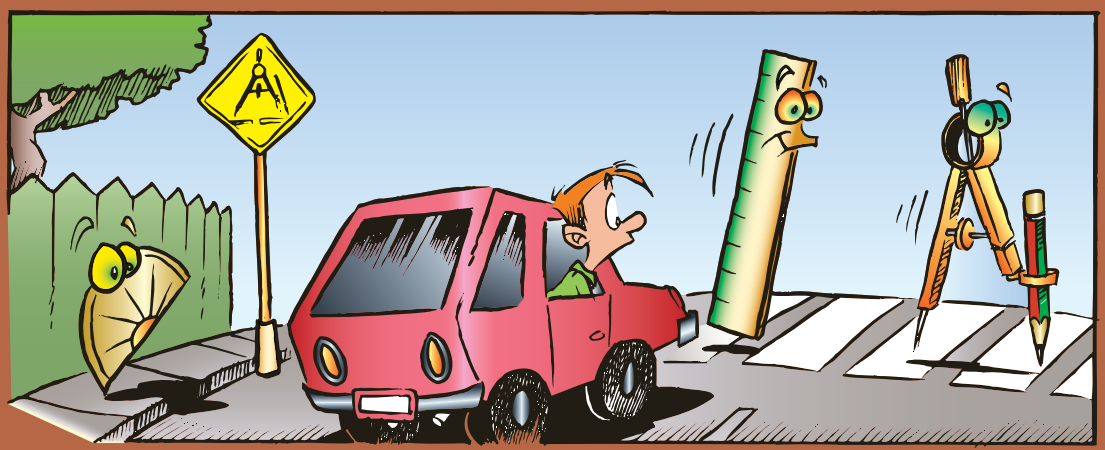


- **a** What is Mila's **i** height, **ii** mass, and **iii** body surface area?
- **b** What is Bronwyn's **i** height, **ii** mass, and **iii** body surface area?
- **c** What is Julie's **i** height, **ii** mass, and **iii** body surface area?

Find your own body surface area by drawing a straight line connecting your height and your mass. Do this for members of your family as well.

14

Using Geometrical Instruments



Chapter Contents

14:01 The ruler
14:02 The set square
14:03 The compasses
Challenge: The golden rectangle
14:04 Constructing triangles
14:05 Constructing quadrilaterals
Mathematical Terms, Diagnostic Test, Revision Assignment, Working Mathematically

Learning Outcomes

Students will classify and construct triangles and quadrilaterals using their properties.

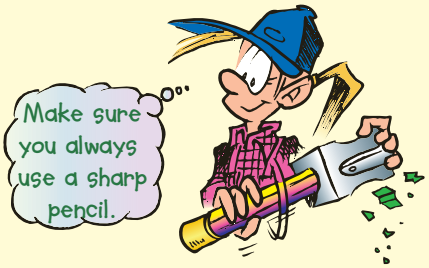
Areas of Interaction

Approaches to learning, Homo Faber, Environment

In this chapter we will look at three basic geometrical instruments: the **ruler** (sometimes called a rule), the **set square** and the **compasses**.

Throughout this chapter, and indeed throughout all your work, it is essential that your instruments are kept in excellent working order. If they are, it will be easier to keep your work neat and accurate.

Whether at school or in the workforce, you will be judged by the quality of your work.



14:01 | The Ruler

The ruler has two functions:
- It is used as a straight edge for drawing lines.
- It is used as a measuring instrument.

For both of these functions it is important to keep the edge of the ruler in good condition. When using the ruler to draw a straight line through two points, follow these steps.

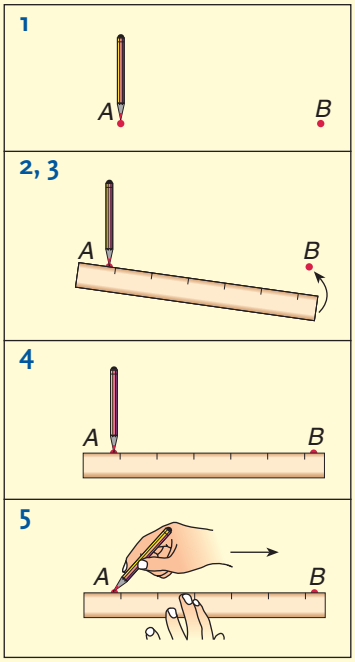
Step 1 Place the pencil on point A

Step 2 Move the edge of the ruler against the point of the pencil.

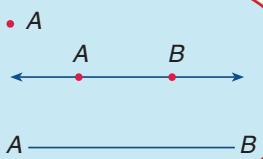
Step 3 Using the pencil point as a pivot, spin the ruler until the edge passes through the point B.

Step 4 Hold the ruler firmly with one hand and lean the pencil so that its point is against the edge of the ruler.

Step 5 Draw the line by pressing the pencil against the ruler and drawing the pencil towards you. Draw the line once only. Do not go over it.



Remember the following:
- **A point is named using a capital letter.**
- **A line is named using two points which lie on the line.**
- **A part of a line that has a fixed length is called an interval. The interval is named by giving its endpoints.**



When measuring with a ruler, follow these steps.

Step 1 Place the edge of the ruler against the interval and slide it so that the zero mark (or some other suitable mark) is against one end of the interval. Make sure your eyes are directly over the zero mark.

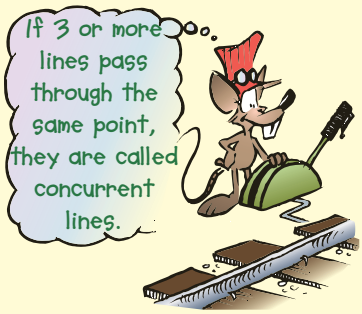
Step 2 Now move your head until your eyes are directly over the other end of the interval. This is important otherwise a *parallax error* will result.

Step 3 Calculate the length of the interval from the scale on the ruler. The length can be given in centimetres or millimetres, eg 7·6 cm or 76 mm.

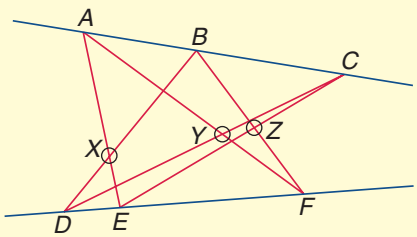
Exercise 14:01 (Practical)

1 Mark a point A on your page. Mark three other points B, C, D. Now draw the intervals AB, AC and AD.

2 Mark a point A and mark twenty points that are 5 cm from A. Join these points freehand to make a drawing of a circle.



3

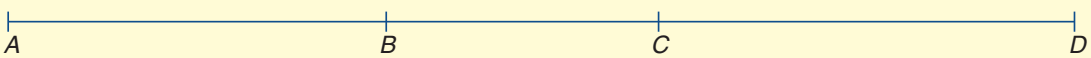


- Draw a line and mark three points A, B, C on it, as shown.
- Draw another line and mark three points D, E, F on it.
- Join A to E and A to F.
- Join B to D and B to F.
- Join C to D and C to E.
- Find the point where AE meets BD. Call it X.
- Find the point where AF meets DC. Call it Y.
- Find the point where BF meets EC. Call it Z.
- Use your ruler as a straight edge to check if X, Y and Z are in a straight line. (They should be!)

If 3 or more points lie on the same line, they are called *collinear points*.



4 Measure the intervals:



a AB **b** AC **c** AD **d** BC **e** BD **f** CD

5 Use the results of question 4 to answer these questions.

a Does $AB + BC = AC$? Should it?

b Does $AC + CD = AD$? Should it?

c Does $BD - CD = BC$? Should it?

6 Draw an interval AB of any length on your page. Use a ruler to find the point X that is halfway between A and B. X is called the *midpoint* of the interval AB.

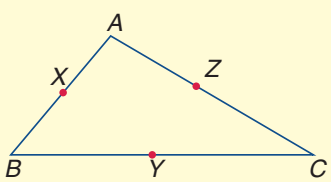
7 Draw any triangle ABC.

a Mark the midpoints of the sides AB, BC and AC.

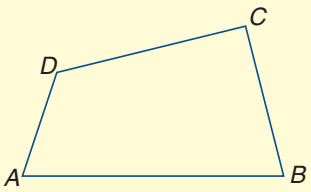
b Label them X, Y and Z.

c Join A to Y, B to Z and C to X.

d Are the lines AY, BZ and CX concurrent?

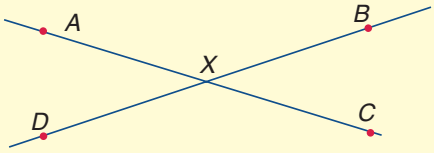


8 Mark four points A, B, C and D. (No three of these points are to be collinear.)



- **a** Join the points to form a quadrilateral.
- **b** Use your ruler to find the midpoint of each side.
- **c** Join the midpoints. What type of shape is formed?

9 Draw two lines that intersect at X.



- **a** From X, mark off equal intervals AX, BX, CX and DX of length 4 cm.
- **b** Join A, B, C and D. What shape is formed?

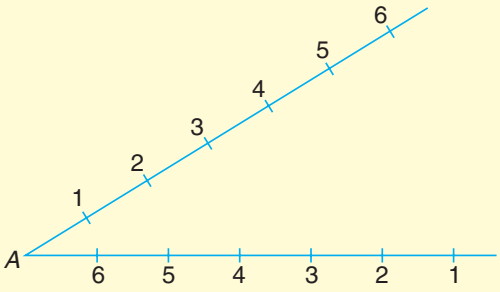
10 Draw an interval 60 mm long and divide the interval into:

a 3 equal parts **b** 4 equal parts **c** 5 equal parts **d** 10 equal parts

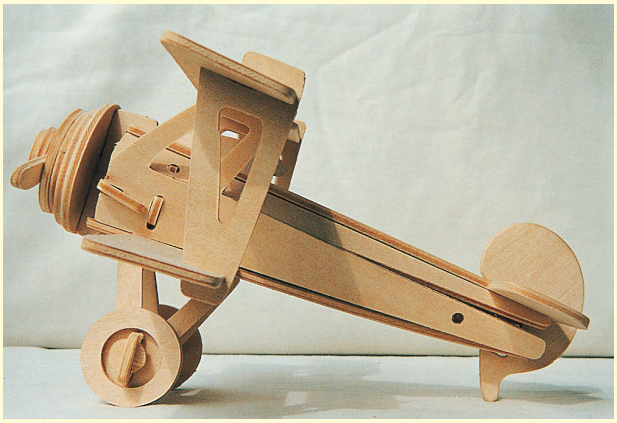
11
- **a** Draw an interval 45 mm long. Without measuring, try to mark the midpoint of the interval. Check your estimate by measurement.
- **b** Draw an interval 30 mm long and, without measuring, divide the interval into three equal parts. Check your estimate by measurement.
- **c** Draw an interval 60 mm long and, without measuring, divide the interval into 5 equal parts. Check your estimate by measurement.

12
- **a** Draw any triangle and find the midpoints of any two sides.
- **b** Join these midpoints and measure the distance between them.
- **c** Measure the length of the other side.
- **d** Is the distance between the midpoints half the length of the other side?
- **e** Repeat the above with three other triangles. Is the result the same?

13 Draw two intervals from the same point A and mark points off at 1 cm intervals as shown in the diagram. Join up the equal numbers. Do you see a curve emerging? This type of geometric design is called an *envelope*.



14 Draw three intersecting lines and use the ideas in question 13 to create a design.

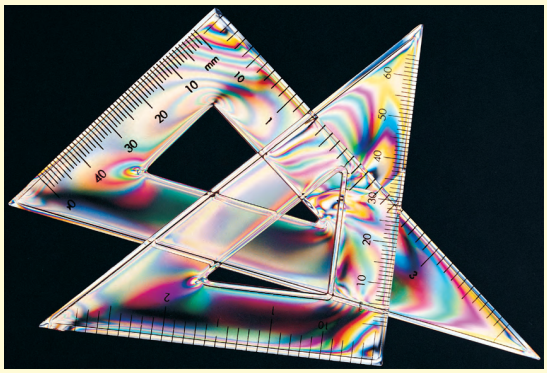


- Use a ruler and a pencil to copy this model plane.

14:02 | The Set Square

There are two types of set squares.
These are shown in the photograph.

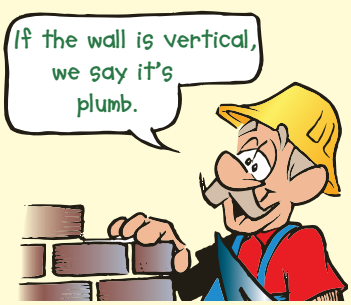
- Because of the angles present on each, they are often referred to as **a 60–30 set square** and a **45 set square**.
- The set square can be used to perform several functions. These are dealt with below.



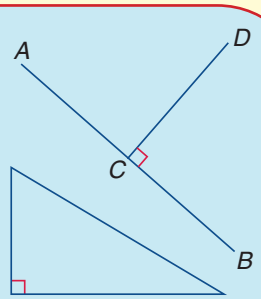
Drawing perpendicular lines

When two lines are at right angles to each other, we say they are **perpendicular**.

- Many people confuse the terms 'vertical' and 'perpendicular'. Perpendicular lines can be drawn in any position.
- A **vertical** line is perpendicular to **the horizontal**.
- Builders use *plumb bobs* and *spirit levels* to set out vertical and horizontal lines.
- Special symbols are used to show perpendicular lines.

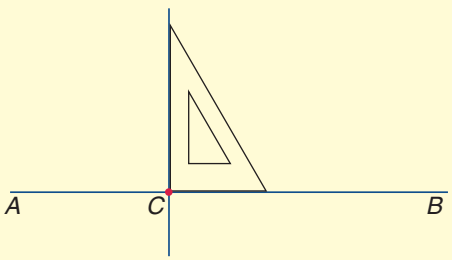


- **In the top diagram, *AB* is at right angles to *CD*. We say that '*AB is perpendicular to CD*', and we write *AB* ⊥ *CD*.**
- **In a diagram the right angle is shown by the symbol ⌞.**

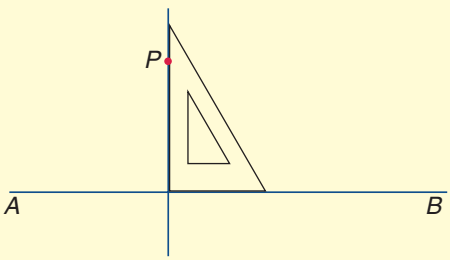


The set square can be used to draw:

1 a perpendicular to a line from a point on the line

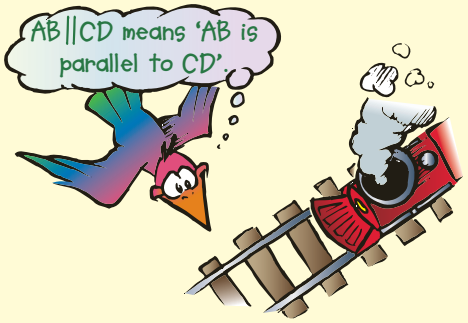


2 a perpendicular to a line through a point not on the line

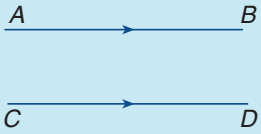


Drawing parallel lines

Parallel lines are lines (in the same plane) that do not meet. Special symbols are used to indicate that lines are parallel.

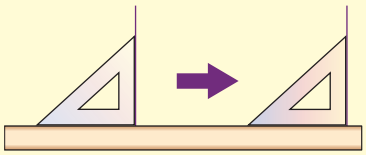


In the diagram, *AB* is parallel to *CD*. We write *AB* ∥ *CD*, and it is shown in the diagram by the use of similar arrows.



A set square and ruler can be used to:

1 construct parallel lines



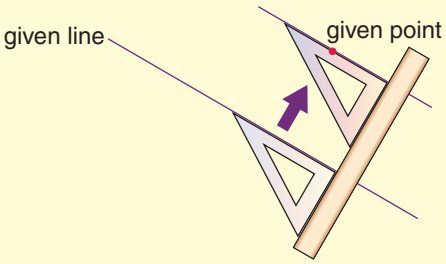
Constructing parallel lines

Step 1 Place an edge of the set square against the ruler and draw a line along one of the other edges.

Step 2 Hold the ruler still and slide the set square into a new position.

Step 3 Draw a line along the same edge that you used in step 1.

2 construct a line parallel to a given line through a given point



Drawing a line parallel to a given line through a given point.

Step 1 Place an edge of the set square on the given line.

Step 2 Place a ruler against one of the other edges.

Step 3 Slide the set square along the ruler until the edge you used in step 1 passes through the given point.

Step 4 Draw in the new line.

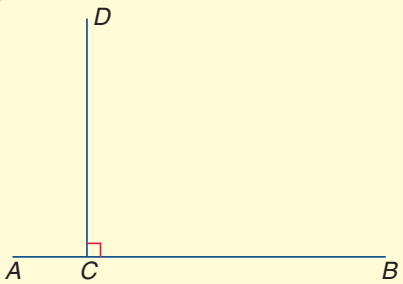
Exercise 14:02 (Practical)

1 Draw a line and mark a point on the line. Use a set square to draw a perpendicular through this point. Repeat this process with three other lines at different angles to the horizontal.

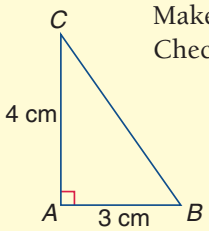
2 Draw an interval *AB* that is 5 cm long and mark a point *C* on the interval 1 cm from *A*. Use a set square to draw a perpendicular *CD* 3 cm long. Check your accuracy by answering these questions:

a How far is *D* from *A*?

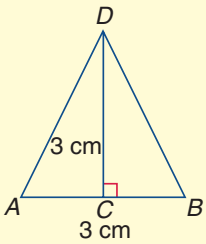
b How far is *D* from *B*?



3 Make an accurate drawing of the triangle shown. Check your accuracy by measuring BC.



4 Use a set square to make an accurate drawing of the triangle shown. C is the midpoint of AB. Check your accuracy by measuring AD and BD.



5 **a** Use a ruler and set square to construct a square of side 5 cm. Check your accuracy by measuring both diagonals. How long are they?

b Use a ruler and set square to construct a rectangle 5 cm long and 3 cm wide. Check your accuracy by measuring the lengths of the diagonals. How long are they?

6 The diagram represents a room that is shaped like a rectangular prism.

a In the room:

i is AB perpendicular to BC?

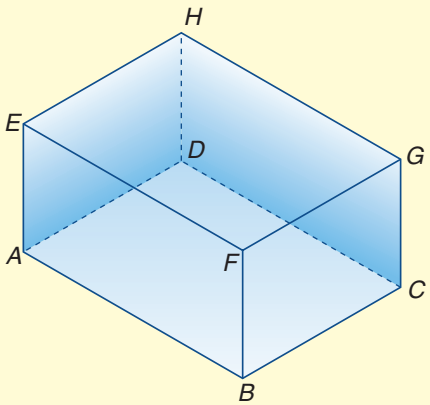
ii are AB and BC vertical?

iii is AB perpendicular to BF?

iv are AE and BF vertical?

b Name the intervals that are perpendicular to AD.

c Which of the intervals in part **b** are vertical?



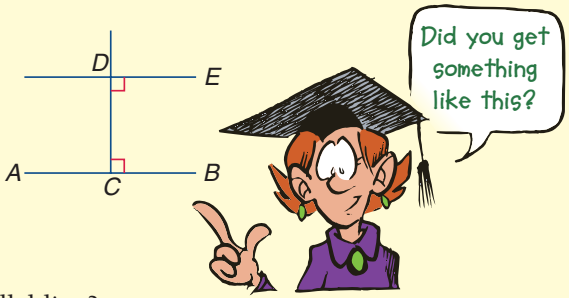
7 Draw a line on your page and then use a set square and ruler to draw another line parallel to it. Repeat this procedure, making sure that the lines are at different angles to the horizontal.

8 Draw a line AB and mark a point C on it.

a Draw a perpendicular to AB at C.

b Take any point D on the perpendicular and draw a perpendicular DE.

c Is DE parallel to AB?



9 Draw a pair of parallel lines.

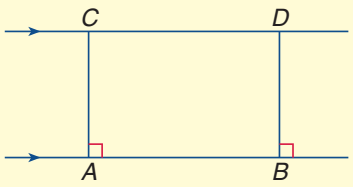
a Draw a line that is perpendicular to one of the parallel lines.

b Is this line perpendicular to the other parallel line?

Questions 8 and 9 demonstrate the following geometric fact:

If two lines are parallel, then a line that is perpendicular to one line is also perpendicular to the other line.

10



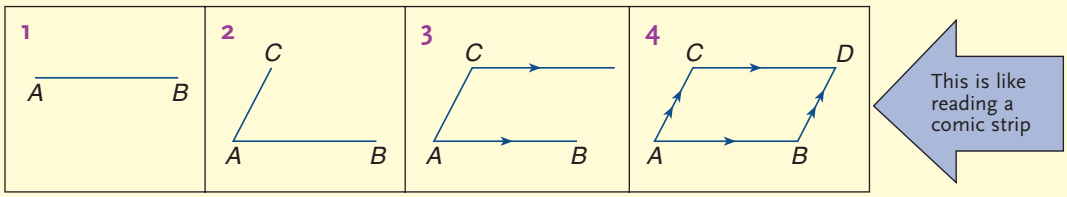
a Use a ruler and set square to draw a pair of parallel lines.
b Mark two points A and B on one of the lines and draw perpendiculars from this point to meet the other parallel line at C and D.
c Measure AC and BD. What do you find?

This question demonstrates the following geometric fact:

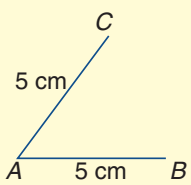
All the perpendiculars joining two parallel lines have the same length.

11 Draw a line and mark three points A, B and C on it.
a Construct perpendiculars of length 5 cm at the points A, B and C.
b Join the endpoints of these perpendiculars to form a line.
c Use your ruler and set square to check that this line is parallel to the line AB.

12 Draw an interval AB that is 5 cm in length. Choose a point C not on AB and join it to A.
a Draw a line parallel to AB through C.
b Draw a line parallel to AC through B.
c Mark the point where the two lines meet. Call this point D.
d Name the shape $ABDC$. (This is shown below.)

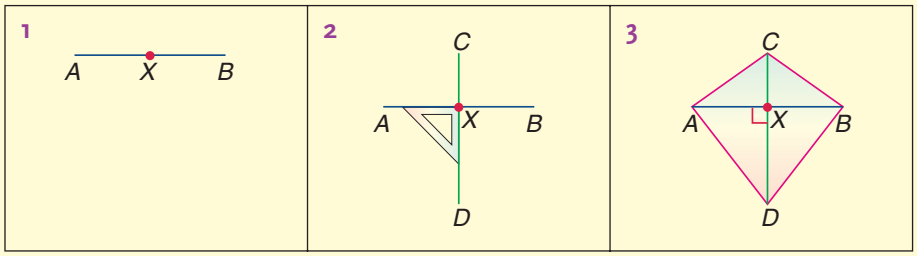


13 **a** Draw two intervals, AB and AC, each 5 cm long as shown in the diagram.
b Construct a line through C parallel to AB.
c Construct a line through B parallel to AC.
d Name the shape formed.

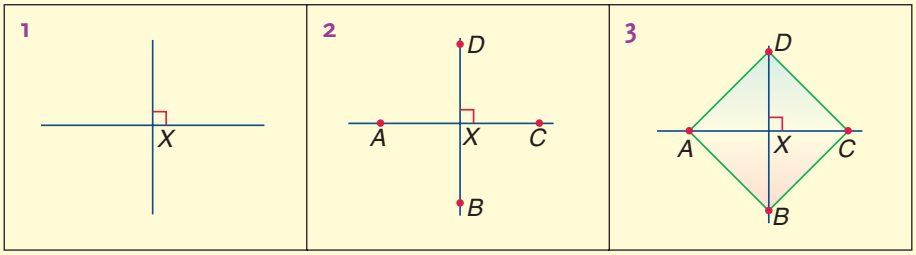




14 **a** Draw an interval AB and mark in the midpoint X.
b Construct a perpendicular to AB at X so that it cuts AB. Call this perpendicular CD.
c Join A, B, C and D. What shape is formed?

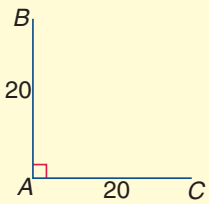


15 **a** Draw two lines that are perpendicular and label the point where they meet X. (See below.)
b From X, measure equal distances along each of the four rays and label these points A, B, C and D.
c What shape is formed by joining A, B, C and D?

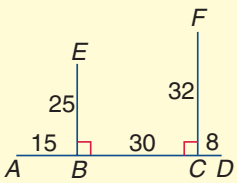


16 Make accurate full-sized drawings of the following shapes, using a ruler and set square (measurements in mm).

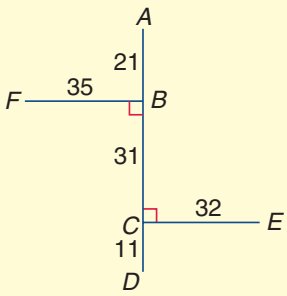
a



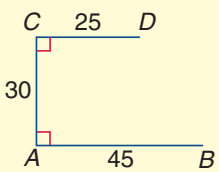
b



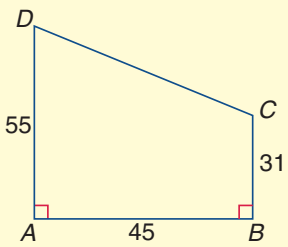
c



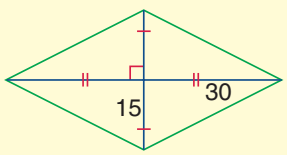
d



e



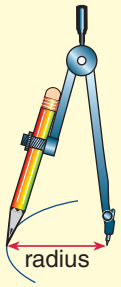
f





- We use parallel and perpendicular lines in the building of furniture

14:03 | The Compasses



Compasses are used to draw circles or parts of circles called **arcs**.

It is important to realise that all the points on a circle or arc are the same distance away from a fixed point called the **centre**. The distance from the centre to the circle is called the **radius**. Compasses mark out equal distances.

To find all the points that are 3 cm from a point A:

Step 1 Set the compasses to 3 cm.
Step 2 Place the fixed point of the compasses on A.
Step 3 Draw the circle.

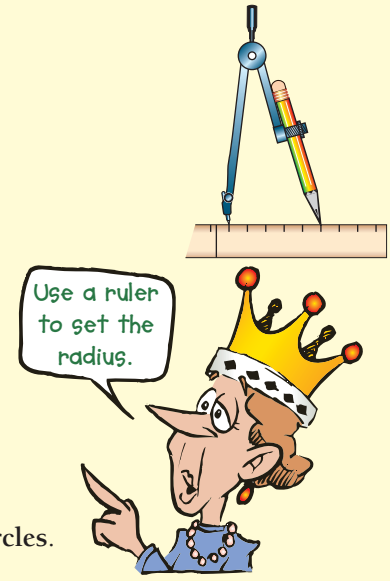
Some tips on using the compasses may be useful.

- Make sure that the hinge at the top of the compasses is tight and won't slip.
- Make sure that the needle of the compasses and the pencil (or lead) are aligned.
- Now, while pressing down on the needle, twist the knob on the top of the compasses to draw the circle.

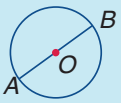
***A pair of compasses* is used to draw circles or arcs. The fixed point of the compasses marks the *centre* of the circle. The distance from the centre to the circle itself is called the *radius*.**

Exercise 14:03 (Practical)

1 **a** Draw a circle of radius 5.
b Select any point on the circle and draw a line from this point through the centre to the other side of the circle. Measure the length of this line. Is it 10 cm long? Should it be?



Any line joining two points on a circle that also passes through the centre is called a *diameter*, eg *AB* is a diameter.



2 **a** Draw a circle with a radius of 4 cm.
b Draw a circle with a diameter of 6 cm.

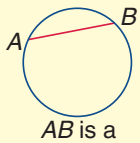
3 Using the same point as the centre, draw circles of radius:
a 3 cm **b** 45 mm **c** 1·8 cm
A set of circles with the same centre are called **concentric circles**.

4 **a** Draw a circle.
b From any point on this circle draw a circle of the same radius.
c Join the centres of both circles and join the points where the circles cross.
d Name the quadrilateral formed by joining the endpoints of these two intervals.

5 **a** Draw any two circles of the same radius so that they cross in two places.
b Join the centres.
c Join the points where the circles cross.
d Check with your set square to see if the two lines are perpendicular.

6 Repeat question 5 with two circles of different radii.

7 **a** Draw a circle and join any two points on the circle. This line is called a **chord**.
b Find the midpoint of the chord and construct a perpendicular to the chord at this point.
c Does the perpendicular pass through the centre?



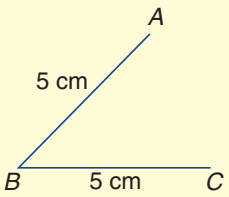
AB is a chord

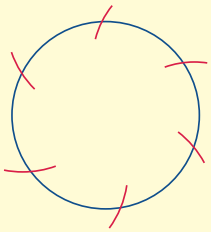


8 Mark two points *A* and *B* approximately 3 cm apart.
a With *A* as centre, draw a circle of radius 5 cm.
b With *B* as centre, draw a circle of radius 4 cm.
c Label the two points where the circles meet, *C* and *D*.
d How far is *C* from *A*? *C* from *B*?
e How far is *D* from *A*? *D* from *B*?

9 **a** Draw an interval *AB* that is 5 cm in length.
b Find the points that are 6 cm from *A* and 4 cm from *B*.

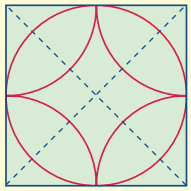
10 Use the compasses to find the fourth vertex of the rhombus *ABCD* on the right.



11 

a Draw a circle and use the compasses to step off the radius around the circle.
b Place the compasses at each of these points and draw a circle.
c Instead of placing the compasses at every point, place it at every second point. Draw the design.

12 Construct a square and use it to copy the design. (To find the centre of the circle, mark the point where the diagonals cross.)

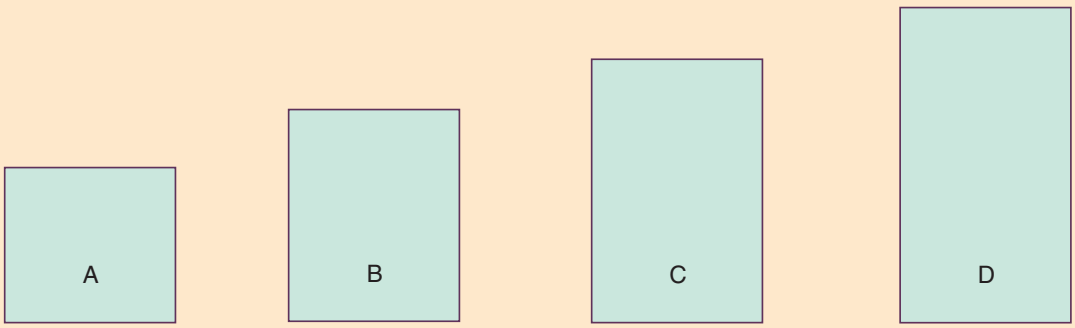




- Using concentric circles as a guide, draw this nautilus.

Challenge 14:03 | The golden rectangle

Which rectangle is the most appealing to you? Imagine it is a painting hanging on your wall. Which one do you think would look the best?



Of course the answer is a matter of opinion, but if you think C is the most appealing, then you agree with the ancient Greeks, who thought that rectangles with sides in this proportion were special. They called them **golden rectangles**. The sides were approximately in the ratio 1 to 1·6. That is, if the breadth were 1 m, then the length would be 1·6 m.

Follow these instructions to construct a golden rectangle.

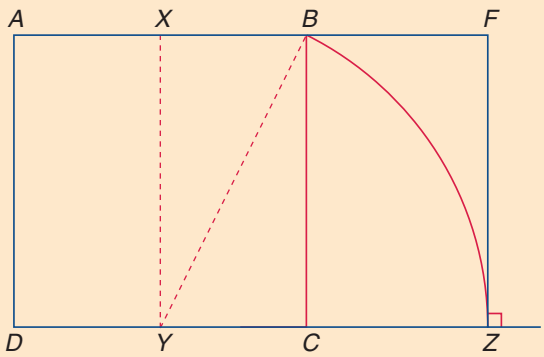
Step 1 Begin with any square *ABCD*.

Step 2 Draw the line *XY*, which divides the square in half.

Step 3 With its centre at *Y* and a radius of *YB*, draw an arc cutting the line *DC* at a point *Z*.

Step 4 From *Z* draw a perpendicular line.

Step 5 Extend side *AB* to meet this perpendicular line at *F*.



The rectangle *AFZD* is a golden rectangle. Measure the length of *DZ* and divide it by *AD* (you may need a calculator). You should get just over 1·6.

The rectangle *BFZC* is also a golden rectangle. Divide the length of *BC* by the length of *BF*. Do you get the same result?



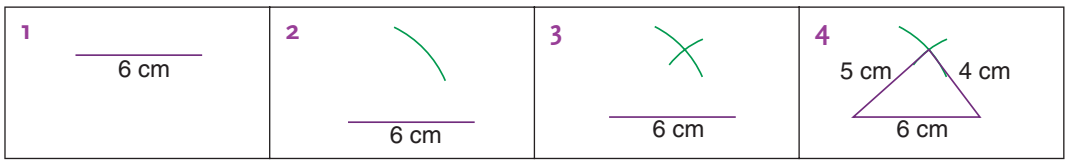
14:04 | Constructing Triangles

To construct a triangle, we must be given certain information about the lengths of its sides and the sizes of its angles. When we construct triangles, we make use of the ruler, compasses and protractor.

worked examples

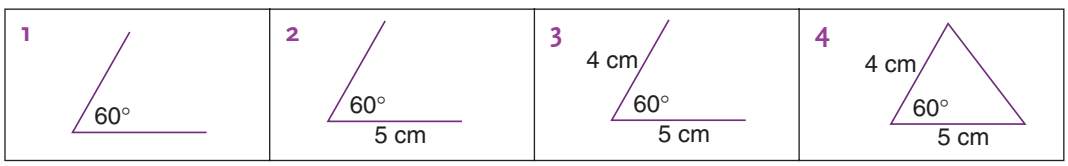
1 Construct a triangle that has sides of length 6 cm, 5 cm and 4 cm.

Step 1 Draw an interval 6 cm long.
Step 2 Set the compasses to 5 cm and from one end of the interval draw an arc.
Step 3 Set the compasses to 4 cm and from the other end of the interval draw an arc to cut the arc that has already been drawn.
Step 4 Join the point where the arcs meet to both ends of the interval to obtain the triangle.



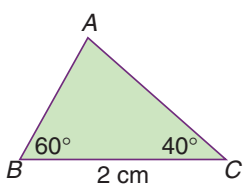
2 Construct a triangle that has sides of length 4 cm and 5 cm which form an angle of 60°.

Step 1 Use the protractor to draw a 60° angle.
Step 2 Measure 5 cm along one arm of the angle and mark a point.
Step 3 Measure 4 cm along the other arm of the angle and mark a point.
Step 4 Join the two points to form the triangle.



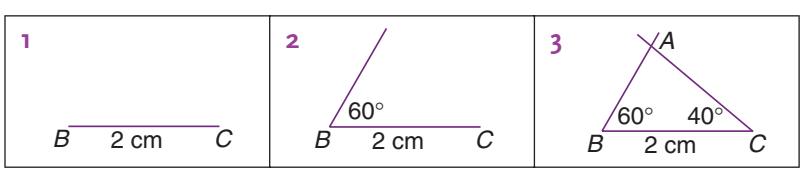
3 Make an accurate full-sized drawing of the triangle shown here.





This can be done in several ways. The following explains one of these.

Step 1 Draw the interval BC 2 cm in length.
Step 2 Use the protractor to draw a 60° angle with its vertex at B.
Step 3 Use the protractor to draw a 40° angle at C. Extend the arms to meet at A.



Exercise 14:04 (Practical)

1 Construct triangles that have the following side lengths.

a 4 cm, 5 cm, 5 cm
b 4 cm, 4 cm, 4 cm
c 3 cm, 4 cm, 5 cm
d 6 cm, 4·5 cm, 3·8 cm



2 **a** Can you construct a triangle that has sides of 6 cm, 3 cm and 2 cm?
b Can you construct a triangle that has sides 7 cm, 3 cm and 4 cm in length?

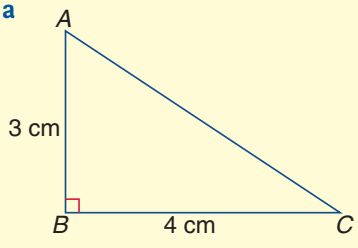
3 **a** Draw at least five different triangles.
b Measure the two shorter sides in each.
c Is the sum of the two shorter sides greater than the third side?
d You are told that a triangle has sides of 4 cm, 5 cm and 10 cm. Is this possible?

4 **a** Draw a triangle with sides of 6 cm, 5 cm and 4 cm.
b By measurement, find the size of the largest angle.
c Which side is opposite the largest angle?

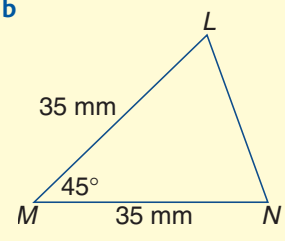
5 Using the triangles drawn in question 3, find:
a the size of the largest angle in each triangle
b which side is opposite the largest angle.

6 Make accurate full-sized drawings of the following triangles. In each case measure the length of the third side.

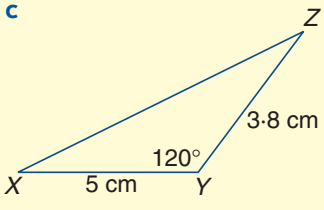
a



b

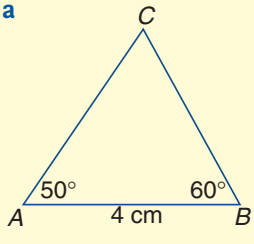


c

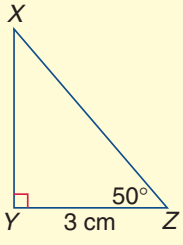


7 Make accurate full-sized drawings of the following triangles. In each, measure the size of the third angle.

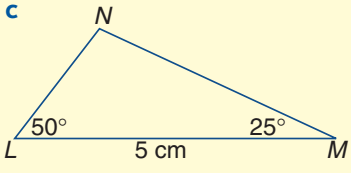
a



b



c



8 **a** Construct as many triangles as you can that have a 4 cm side, a 5 cm side and a 40° angle.
b Construct as many triangles as you can that have a 50° angle, a 60° angle and a 5 cm side.

14:05 | Constructing Quadrilaterals

Constructing special quadrilaterals such as squares and rectangles is relatively easy. However, some quadrilaterals are not so easy to construct.



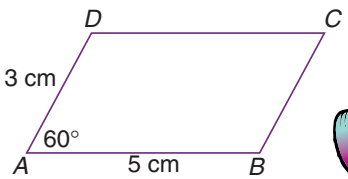
Before starting a construction, think about the information that has been given. Try to work out the steps that are to be taken. Often you can do the same construction in many different ways.

Make sure you know how to use your geometrical instruments, especially the protractor, and take care so that your work will be neat and accurate. It helps if you draw a sketch.

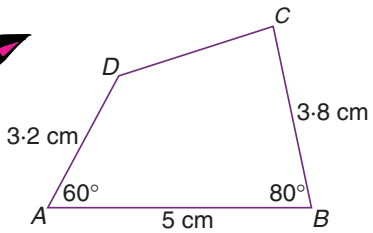
worked examples

1 Construct a square with a side of length 3 cm.

2 Construct the parallelogram drawn below.



3 Construct the quadrilateral drawn below.

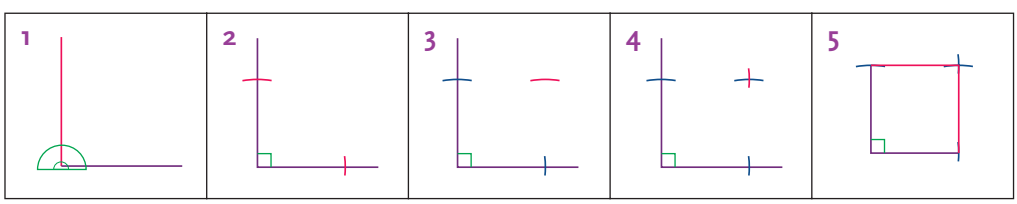




Solutions

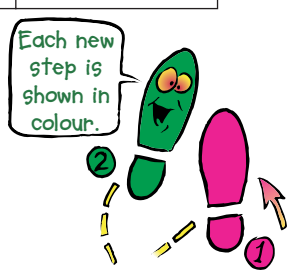
1 A square has four equal sides and four right angles. To construct a square, we could use the following steps. They are shown in the strip diagram below.

Step 1 Draw a right angle with the protractor or set square.
Step 2 Set the compasses to 3 cm and mark off the length of the sides along the arms of the right angle. Use this same radius for each arc.
Step 3 From the mark on one arm, draw an arc.
Step 4 From the mark on the other arm, draw an arc to cut the arc drawn in step 3.
Step 5 Join the point where the arcs meet to the mark on each arm to form the square.

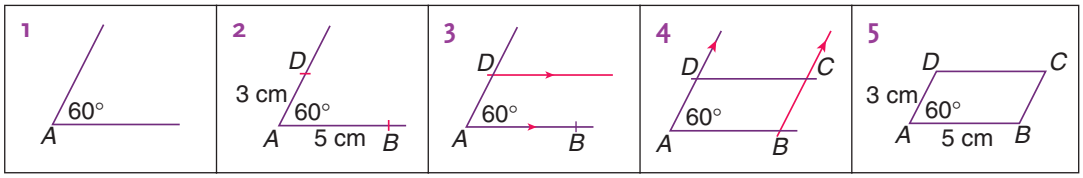


2 To construct the parallelogram, follow these steps.

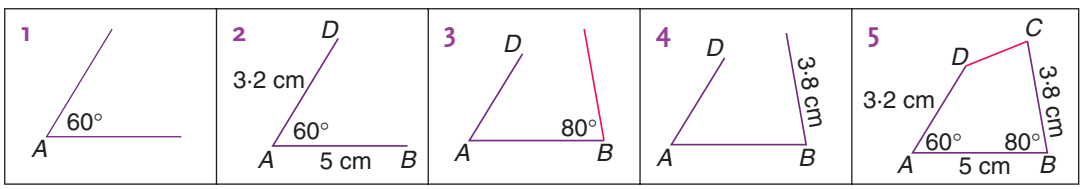
Step 1 Draw a 60° angle using a protractor.
Step 2 Mark off the sides *AB* and *AD* along the arms of the angle.
Step 3 Draw a line through *D* parallel to *AB*.



Step 4 Draw a line through B parallel to AD to cut the line already drawn in step 3.
Step 5 Erase the parts that are not needed.



3 To construct the quadrilateral, follow these steps.
Step 1 Draw a 60° angle with the protractor.
Step 2 Mark off the sides AB (5 cm) and AD (3·2 cm).
Step 3 Draw an 80° angle with its vertex at B.
Step 4 Mark off the side BC (3·8 cm).
Step 5 Join CD.

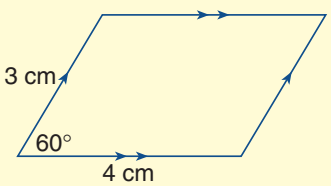


Exercise 14:05 (Practical)

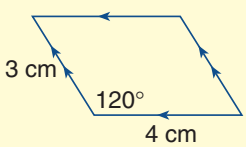
1 **a** Construct a square with sides of 4 cm. Check your accuracy by measuring all the sides and angles. Are all the sides 4 cm long? Are all the angles right angles?
b Construct a rectangle 5 cm long and 3 cm wide. Check your accuracy by measuring all the angles to see if they are right angles. Are the diagonals equal?

2 Construct the parallelograms shown.

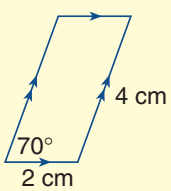
a



b

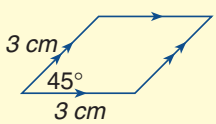


c

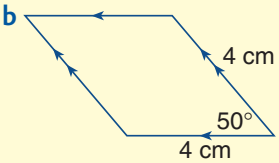


3 Using the steps for constructing a parallelogram as a guide, construct the following rhombuses.

a

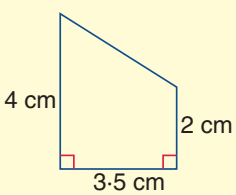


b

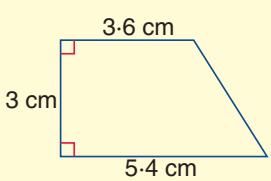


4 Construct the following trapeziums.

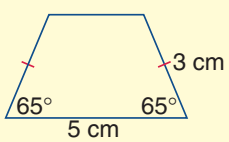
a



b

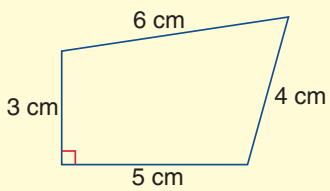


c

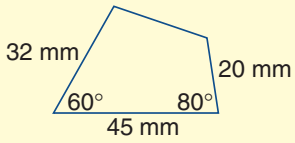


5 Construct the following quadrilaterals.

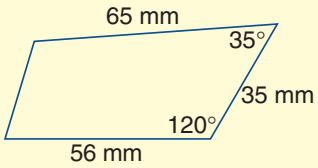
a



b

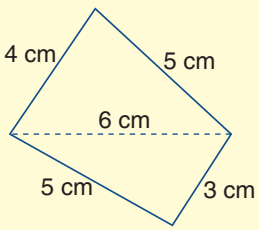


c

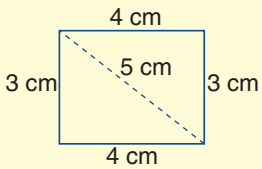


6 All quadrilaterals can be made from two triangles. Construct each quadrilateral by constructing the two triangles. (Start by drawing the diagonal which is dotted.)

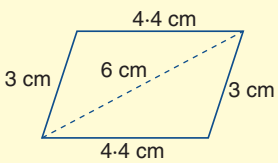
a



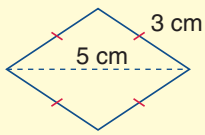
b



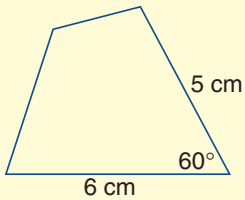
c



d

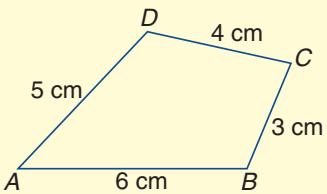


7 Sometimes it is possible to construct more than one quadrilateral from the information given. The diagram shows a quadrilateral. Sketch another quadrilateral with the same side and angle sizes.



- One angle is 105°.
- One side is 4 cm.

8



a How many quadrilaterals could you draw with sides of 3, 4, 5 and 6 cm?

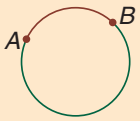
b What other piece of information would you need to know about the quadrilateral pictured to be able to copy it exactly?



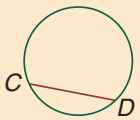
- Postage stamps are usually rectangular. How much did these stamps cost?

Mathematical terms 14

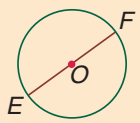
circle
- A set of points equidistant from a fixed point called the centre.
- **arc**
  A part of a circle
  eg *AB* is an arc.



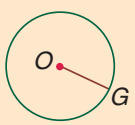
- **chord**
  An interval joining two points on a circle.
  eg *CD* is a chord.



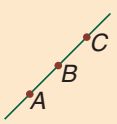
- **diameter**
  An interval joining two points on a circle that also passes through the centre.
  eg *EF* is a diameter.



- **radius (plural is radii)**
  An interval that joins the centre to a point on the circle.
  eg *OG* is a radius.



collinear points
- Points lying on the same straight line.



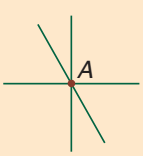
compasses (or pair of compasses)
- A geometrical instrument made from 2 arms that are joined at one end.



concentric circles
- Circles that have the same centre.



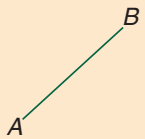
concurrent lines
- Lines that pass through the same point.



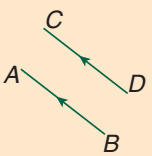
construct
- To draw accurately using instruments.

horizontal
- Parallel to the horizon.
- Level or flat.
- A direction at right angles to the vertical.

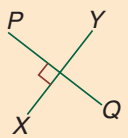
interval
- The part of a straight line from one point to another.
  eg *AB* is an interval.



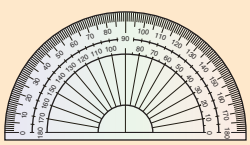
parallel lines
- Straight lines in the same plane that do not meet.
  eg *AB* is parallel to *CD*,
  AB || *CD*.



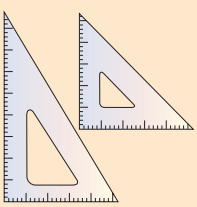
perpendicular lines
- Two straight lines that are at right angles to one another.
  eg *PQ* is perpendicular to *XY*,
  PQ ⊥ *XY*.



protractor
- A geometrical instrument used to measure angles.



set square
- A geometrical instrument in the shape of a right-angled triangle.
- There are two types, a 60–30 and a 45, named according to angles on the set square.



vertical
- The direction in which an object falls under gravity.
- A direction at right angles to the horizontal plane.

diagnostic test

14

Diagnostic Test 14: | Using Geometrical Instruments

- Each section of the test has similar items that test a certain type of example.
- Failure in more than one item will identify an area of weakness.
- Each weakness should be treated by going back to the section listed.

	Section
1 Construct the following triangles accurately. **a** 23 mm, 32 mm, 45 mm **b** 60°, 40°, 4·8 cm **c** 3·4 cm, 110°, 4 cm	14:04
2 **a** Construct an isosceles triangle. Mark the equal sides and equal angles. **b** Construct an equilateral triangle. Mark the equal sides and equal angles. **c** Construct a scalene triangle. Does it have any equal sides or angles?	14:04
3 Construct the following quadrilaterals accurately. **a** 2·5 cm, 5 cm **b** 4·8 cm, 55°, 5 cm **c** 4·8 cm, 4 cm, 3 cm, 70°, 4 cm	14:05

assignment

14A

Chapter 14 | Revision Assignment

1 Write each expression without multiplication signs.

a $5 \times y \times 3$ **b** $6 \times x \times y$

c $4 \times m - 7 \times n$ **d** $6 + 3 \times 4 \times x$

2 Find the value of each expression if $x = -2$ and $y = 3$.

a $y - x$ **b** $(x + y)^2$

c $x^2 + y^2$ **d** $(y - x) \times (y + x)$

3 Simplify:

a $\sqrt{25}$ **b** 6^2

c $15 \div 4$ **d** -8×-3

e $-7 - 7$ **f** $4 - -8$

g $(-3)^2$ **h** $-3 + 100$

i $(-1)^3$ **j** $6x + 3 - x$

k $x - y + x - y$ **l** $15 - x + 8$

m $x - x + x - x + x$

n $3 - (-x) - 3$ **o** $x^2 - x + x^2$

4 Find the area of each shape.

a 2 cm, 3·8 cm

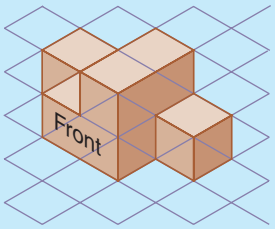
b 2·3 cm, 5·6 cm

c 2 cm, 1 cm, 6·5 cm, 1 cm, 2 cm, 12·5 cm

5 Complete the table, carrying out each operation on the last answer found.

Operation	Start	− 7	× 3	+ 11	− 35	÷ 6
Answer	5					

6 Draw the front view, side views and top view of the solid shown.



7 Write the following times as 24-hour times.

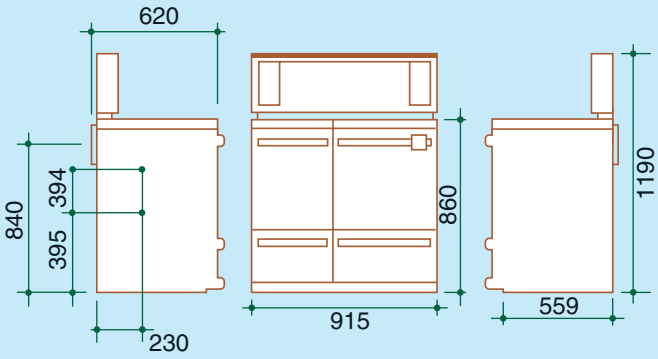
a 3:15 am **b** 3:15 pm
c 11:30 pm **d** 10:18 am

8 Add these times together, giving the answer in hours and minutes.

a 3 h 20 min + 1 h 40 min
b 2 h 27 min + 3 h 5 min
c 1 h 48 min + 1 h 42 min
d 2 h 28 min + 2 h 38 min

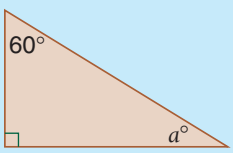
9 The diagram gives three views of an oven. Find:

a the maximum width of the oven
b the maximum height of the oven
c the maximum depth of the oven

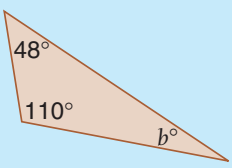


10 Find the value of the pronumeral in each figure.

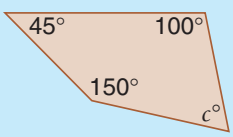
a



b



c





- Estimate and then count the number of soldiers.
- Work out your error as a percentage of the number of soldiers.

ie $\frac{\text{error}}{\text{number}} \times \frac{100}{1}\%$

assignment 14B

id 14

Chapter 14 | Working Mathematically

1 Use ID Card 6 on page xviii to identify:

a 1	b 2
c 3	d 4
e 21	f 22
g 23	h 24

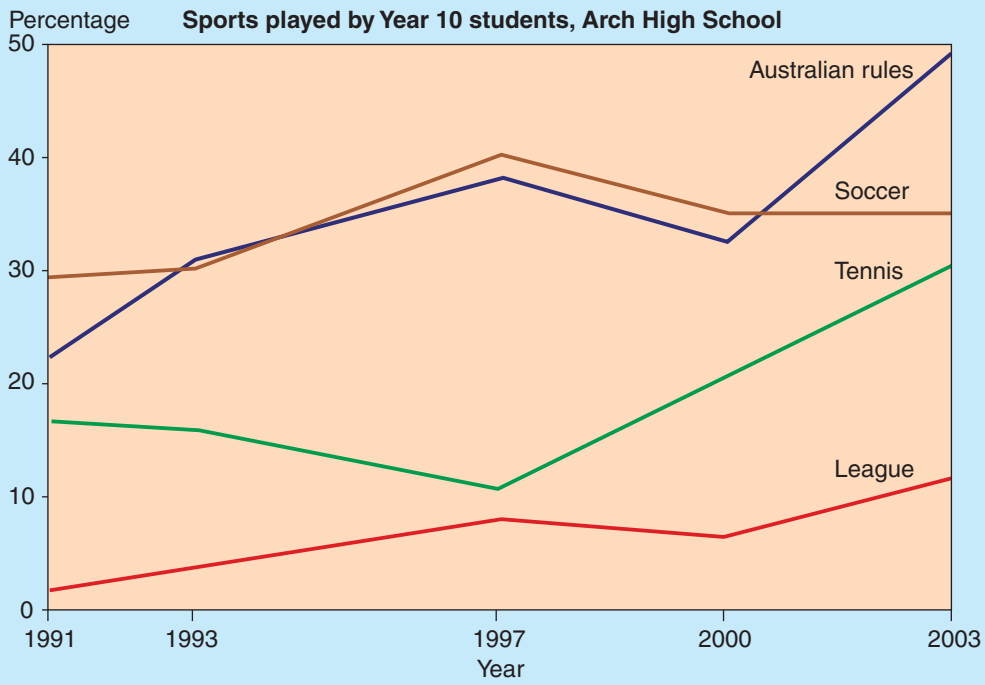
2 A spool of wire is 100 m long. How many pieces of wire 30 cm long could be cut from this spool?

3 A girl has 4 tops which can be worn with each of 5 skirts. How many different outfits can she wear?

4 If you have a square pyramid, what is the smallest number of colours needed to paint the pyramid so that no two adjoining faces have the same colour?

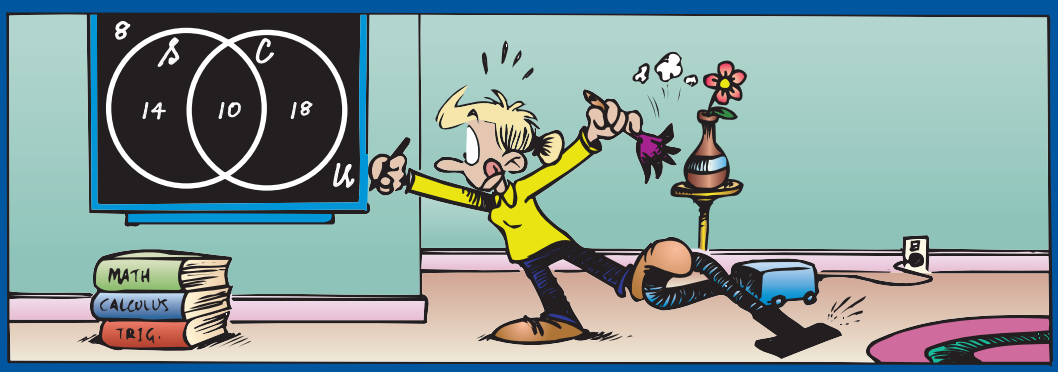
5 A jogger runs 5 km in the morning and 8 km in the evening of Monday to Thursday. On Friday he runs 10 km after work. He rests on Saturday and runs 15 km on Sunday afternoon. How far will he run in a week? How long will it take him to run 1000 km?

6



a What was the most commonly played sport in 1993?
b What was the most commonly played sport in 1997?
c Approximately what percentage of students played Australian rules in 2003?
d By what percentage has the playing of soccer increased from 1991 to 2003?
e If you add the percentages of students who played Australian rules, soccer and tennis in 2003, you get more than 100%. How do you explain this?

15 Sets



Chapter Contents

15:01 What is a set?
15:02 Intersections of sets, empty sets and Venn diagrams
Fun Spot: Coloured boxes
15:03 Unions of sets and subsets
15:04 Solving problems using sets and Venn diagrams
Investigation: World Cup mania
Mathematical Terms, Diagnostic Test

Learning Outcomes

Students will:

- Recognise that a set is a collection of elements with a common characteristic.
- Select and order elements of a set.
- Be able to identify and interpret the intersection and union of two sets.
- Use set theory to solve problems.

Areas of Interaction

Approaches to Learning, Homo Faber, Environment, Community

15:01 | What is a Set?

A **set** is a collection of objects or things. These objects or things that belong to the set are called **elements**.

For example, if A is the set of even numbers from 0 to 10 then the elements of set A are 2, 4, 6, 8, 10.

We say $A = \{2, 4, 6, 8, 10\}$ where 2 is an element of A $2 \in A$
and 4 is an element of A $4 \in A$
and 6 is an element of A $6 \in A$
and 8 is an element of A $8 \in A$
and 10 is an element of A $10 \in A$

worked example

Set M is the set of months of the year that end in "ber". Write the elements of the set M.

Solution

The following months are elements of the set M:

September $\in M$
October $\in M$
November $\in M$
December $\in M$

Therefore M = {September, October, November, December}

If something is not an element of the set, we write $\notin$.

For example, March is not an element of M, so **March $\notin M$**

Exercise 15:01

1 Write the elements of the following sets:

a The set of multiples of 3 less that 20
b The set of factors of 24
c The set of months of the year with 30 days or less
d The set of colours in the Olympic flag
e The set of continents of the world

2 Write true or false for the following statements:

a 8 $\in$ {factors of 84}
b quadrilateral $\in$ {polygons}
c Australia $\in$ {countries of Europe}
d bicycle $\in$ {four-wheeled vehicles}
e Paris $\notin$ {cities in Germany}
f Mekong $\in$ {rivers in Vietnam}
g U $\in$ {vowels}
h rhombus $\notin$ {quadrilaterals}
i 1998 $\in$ {years when the Olympic Games were held}
j 15 $\in$ {chapters in this book}

3 Write the sets to which these elements could belong:

a Mississippi River **b** aeroplane **c** keyboard
d sock **e** Pacific

4 The following questions refer to these sets:

A = {planets in our solar system}
B = {countries in South East Asia}
C = {capital cities of the world}
D = {animals with four legs}
E = {parts of a car}
F = {parts of a truck}
G = {countries in the southern hemisphere}
H = {mammals}
I = {cities in Africa}
J = {metals}

a To which set or sets do the following elements belong:

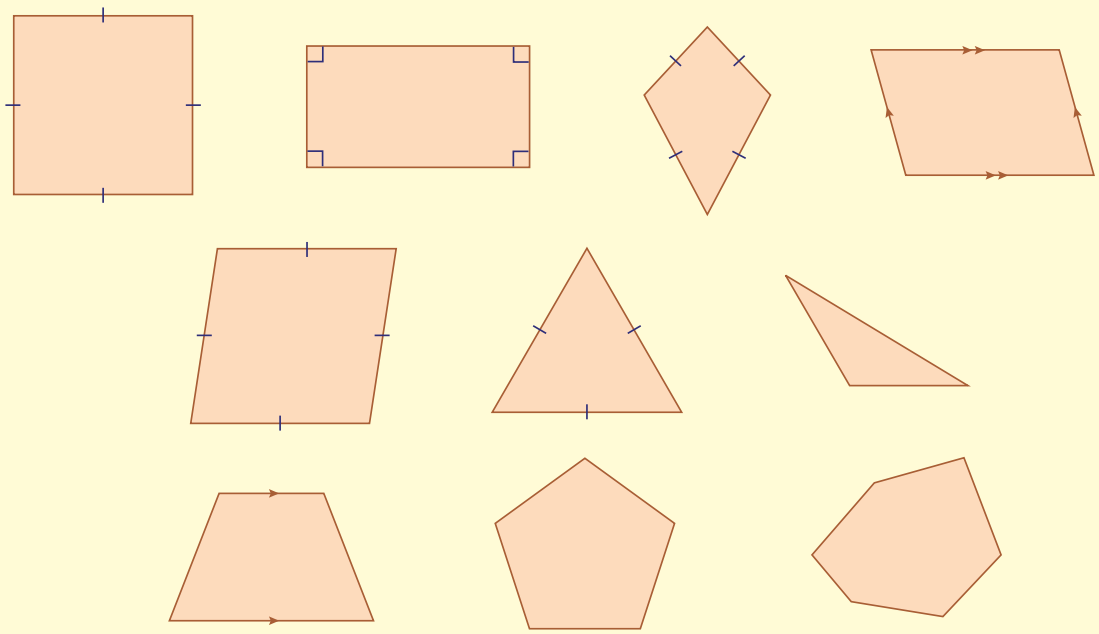
i steering wheel **ii** elephant
iii Nairobi **iv** Berlin
v mercury **vi** Indonesia
vii Timbuktu **viii** frog
ix Peru **x** trailer?

b If $x \in B$ but $x \notin G$, what is the element x?
c If $x \in C$ but $x \notin I$, what could the element x be?
d If $x \in H$ but $x \notin D$, what could the element x be?
e If $x \in D$ but $x \notin$ {animals that only live on land}, what could the element x be?

5 Write two descriptions of sets to which these elements could belong:
{Paris, Berlin, Madrid, Rome, Copenhagen, Athens, Bratislava}

6 Which of the shapes shown belong to the sets given?

A = {quadrilaterals}
B = {regular polygons}
C = {polygons}
D = {irregular polygons}



15:02 | Intersections of Sets, Empty Sets and Venn Diagrams

In the last exercise you found that some elements belonged to two sets.

Indonesia $\in$ B: {countries in South East Asia} and
Indonesia $\in$ G: {countries in the southern hemisphere}

If there are elements that belong to two sets at the same time then we say that those two sets *intersect*.
We say set B intersects set G or $B \cap G$.

worked examples

1 If set A = {factors of 24}
and set B = {factors of 18}
then the intersection of A and B
$A \cap B$ = {elements that belong to both set A and set B}
= {common factors of 18 and 24}
= {1, 2, 3, 6}
Also, the number of elements in this intersection is 4.
We write $n(A \cap B) = 4$

2 If set C = {quadrilaterals}
and set D = {shapes with equal sides}
then the intersection of C and D
$C \cap D$ = {elements that belong to both set C and set D}
= {quadrilaterals with equal sides}
= {square, rhombus}
Also $n(C \cap D) = 2$

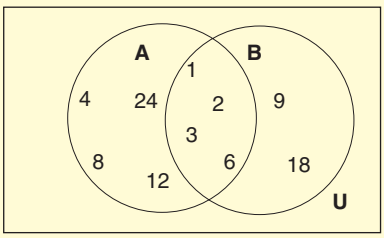
If a set has no elements then the set is empty. The empty set is written $\varnothing$.
eg
If X is the set of three-sided quadrilaterals then $X = \varnothing$ and $n(X) = 0$
If
S = {days of the week beginning with T}
P = {days of the week on the weekend}
then $S \cap P = \varnothing$

We can represent sets by the use of **Venn diagrams**.

From the example above, set A = {factors of 24} and set B = {factors of 18}

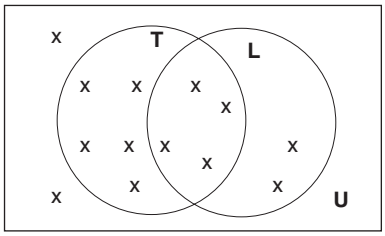
The intersection of the two sets is where the circles cross.

Both circles are contained within the **universal set** U that contains all numbers



worked examples

The Venn diagram shows the girls in a Grade 6 class of 24 students who play tennis (set T) and are left-handed (set L). Each x represents a student. The universal set represents all the girls in the class.



a How many girls are in the class?
There are 13 xs so there are 13 girls in the class.

b How many girls in the class play tennis?
There are 9 xs in the T circle so 9 girls play tennis.

c How many girls in the class are left-handed?
There are 6 xs in the L circle so 6 girls play tennis.

d How many girls in the class are left-handed and play tennis?
There are 4 xs where the L and T circles cross over so 4 girls are left-handed and play tennis.

e How many girls in the class are right-handed and play tennis?
There are 5 xs in the T circle but not in the L circle, so there are 5 girls who are right-handed and play tennis.

f How many girls in the class are left-handed and do not play tennis?
There are 2 xs in the L circle that are not in the T circle, so there are 2 girls who are left-handed and do not play tennis.

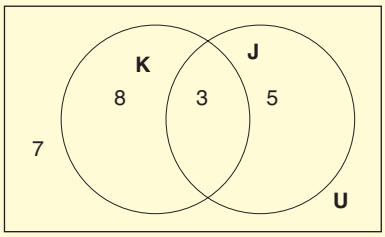
g How many boys are in the class?
If there are 24 students in the class and 13 of them are girls then there must be 11 boys in the class.

Exercise 15:02

1 Identify the elements of the following intersections:

a {factors of 25} ∩ {factors of 30}
b {cities in Thailand} ∩ {capital cities in Asia}
c {types triangles} ∩ {regular polygons}
d {multiples of 5} ∩ {even numbers less than 100}
e {students in your class} ∩ {people who have their birthdays in December}

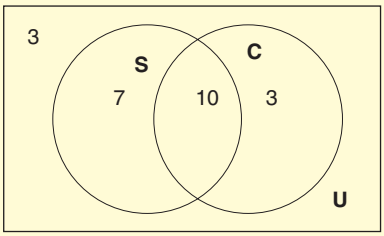
2 The Venn diagram shows the number of students in Mi U's class who speak Korean (K) and Japanese (J).
The universal set (U) is those students who can speak English and includes everyone in the class.



i In Mi U's class, how many students:
a speak Japanese?
b speak Korean?
c speak neither Japanese nor Korean?
d speak both Japanese and Korean?
e speak Japanese but not Korean?

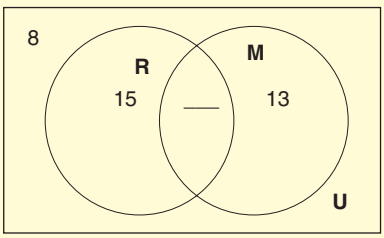
ii Why would it be wrong to say that 7 students speak only English?

3 People in Paul's class like to watch TV. In particular they watch cartoons (C) and sport (S). The Venn diagram shows how many watched these programmes on a Monday night. The universal set (U) includes everyone in Paul's class.



i How many students are in Paul's class?
ii How many students watched:
  a sport?
  b cartoons?
  c both sport and cartoons?
  d cartoons but not sport?
  e neither sport nor cartoons?
iii Why would it be misleading to say that 3 people don't like sport or cartoons?

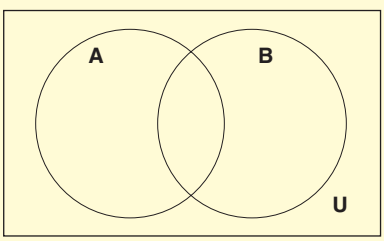
4 The Venn diagram indicates the number of people on a bus who are listening to music (M) or reading (R).



i What do you think the universal set (U) represents?
ii What is the value of $n(R \cap M)$ if there is a total of 40 people on the bus.
iii How many people on the bus are:
  a reading?
  b listening to music?
  c neither reading nor listening to music?
  d listening to music but not reading?

5 In Marc's class, 12 students have had a holiday in Europe (E), 14 have had a holiday in the Americas (A) and 8 have never been on a holiday in either of these places.
i If there are 24 students in Marc's class, how many must have been on holidays in both these places?
ii What do you think the universal set (U) would represent in this problem?
iii Draw a Venn diagram showing all this information.
iv How many students:
  a have had a holiday in Europe but not in the Americas?
  b have not had a holiday in the Americas?

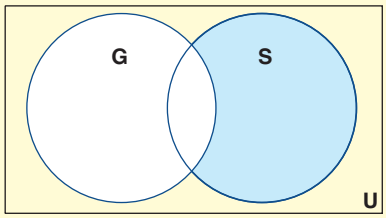
6 The Venn diagram shows two sets: set A which is the set of all boys at Carol's school who wear glasses and set B which is the set of all students in Carol's class.



a What might the universal set (U) represent in this case?
b What does this tell us about the boys in Carol's class?
c Describe the set of elements that are not in set B.
d Describe the set of elements that are not in set A.
e Does this tell us anything about the girls at Carol's school?
f Complete the statement: $A \cap B = ?$

7 Make up a Venn diagram for your Mathematics class. Let the universal set (U)be the set of students in your Mathematics class, let G be the set of girls in your class and S be the set of students wearing glasses.
Describe in words:

a $A \cap B$
b $n(A \cap B)$
c The set of elements not in G
d The set of elements not in S
e The set of elements not in G or S
f The shaded area

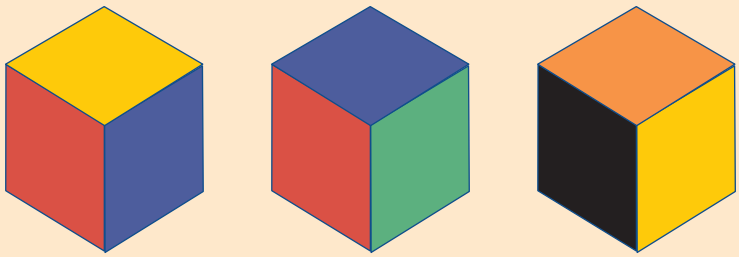


8 B is the set of brick houses and W is the set of wooden houses in Mara's street.
If $B \cap W = \varnothing$, what does this tells you about the houses in Mara's street?

Fun Spot 15:02 | Coloured boxes

fun spot 15:02

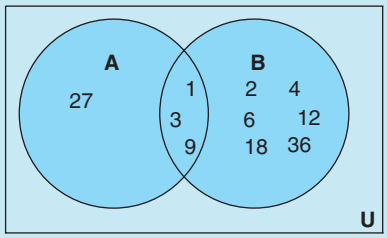
These are three different views of the same box.



What coloured face is opposite the orange face?

15:03 | Unions of Sets and Subsets

When the elements of two or more sets are combined, the result is the *union* of those two sets.
eg
If A is the set of factors of 27 then
$A = \{1, 3, 9, 27\}$
If B is the set of factors of 36 then
$B = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$
The union of A and B is written $A \cup B$ and includes all the factors of both sets:
$A \cup B = \{1, 2, 3, 4, 6, 9, 18, 27, 36\}$
This is shown in the Venn diagram where the universal set U includes all numbers and $A \cup B$ is shaded.



worked examples

B is the set of students in Se Young's class who wear braces.
G is the set of students in Se Young's class who wear glasses.

a Describe the set $B \cap G$.

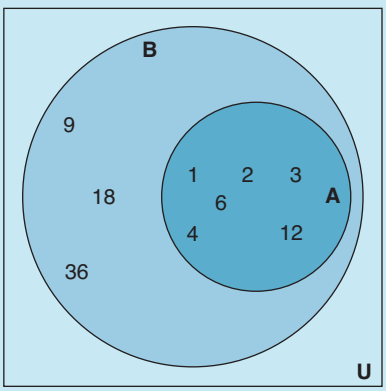
b Describe the set $B \cup G$.

Solutions

a $B \cap G$ = {students in Se Young's class who wear both braces and glasses}

b $B \cup G$ = {students in Se Young's class who wear braces or glasses or both}

When all the elements of one set (A) are entirely contained within another set (B), we say that set A is a *subset* of set B. This can be written as $A \subset B$.
eg
If A is the set of factors of 12 then
A = {1, 2, 3, 4, 6, 12}
If B is the set of factors of 36 then
B = {1, 2, 3, 4, 6, 9, 12, 18, 36}
Then $A \subset B$ since all the elements of A are contained in B.
This is shown in the Venn diagram



Exercise 15:03

1 A = {students at Duong's school}, B = {students in Duong's English class}, C = {students in Duong's English class who speak French}, D = {students in the city where Duong lives}. Write true or false for the following:

a $A \subset B$ **b** $C \not\subset B$ **c** $A \subset D$ **d** $C \subset B$ **e** $D \not\subset B$

2 Use the Venn diagram to answer true or false to the following:

a All the students in Debra's homeroom walk to school.

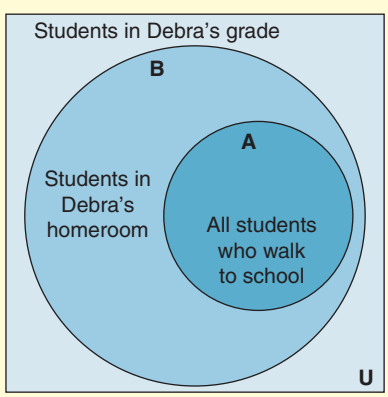
b All the students who walk to school are in Debra's homeroom.

c All the students in Debra's grade who walk to school are in her homeroom.

d $A \subset U$

e $B \not\subset U$

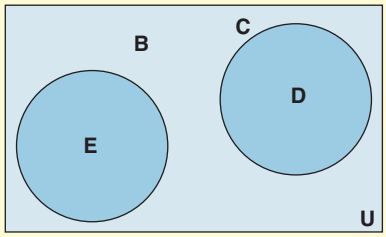
f $A \subset B \subset U$



3 L = {multiples of 8 from 50 to 100}
M = {multiples of 5 from 50 to 100}
N = {even numbers}
O = {multiples of 4}

a Write the list of elements of each set.
b Describe in words $L \cup M$.
c Answer true or false:
i $L \subset N$ **ii** $N \subset M$ **iii** $O \subset L$ **iv** $L \subset O$ **v** $O \subset N$

4 U = {cars parked in Ryosuke's street}
B = {blue cars}
C = {cars parked in garages}
D = {cars with windows open}
E = {cars with four doors}



Use the Venn diagram to answer the following.

a Insert the symbol $\subset$ or $\not\subset$ to make true statements.
i B ___ C **ii** E ___ C **iii** E ___ B **iv** D ___ C **v** D ___ C ____ U
b What does the diagram tell you about the cars with four doors in Ryosuke's street?
c What does the diagram tell you about the cars with open windows in Ryosuke's street?
d Are there blue cars parked in garages?
e Are there any four-door cars parked in garages?

5 V = {things most people don't like}
W = {things that taste sour}
X = {all things that taste good}
Y = {candy}
Z = {things that taste sweet}

Use the above information to describe the following statements:
a $Y \subset Z$ **b** $Z \subset X$ **c** $W \subset V$ **d** $Y \not\subset W$ **e** $V \not\subset X$

6 U (universal set) = {students in Grade 6}
H = {students who play hockey}
B = {students who play baseball}
S = {students who play soccer}
R = {students who play rugby}

Construct a Venn diagram to represent the information below.
$H, B, S, R \subset U$
$B \subset H$
$R \subset S$
$H \cap S = \varnothing$

15:04 | Solving Problems Using Sets and Venn Diagrams

worked example

In Tanzila's street there are 50 houses. Twenty-eight of these houses have satellite TV, 24 have cable TV and 8 have neither satellite nor cable.

How many houses have both satellite and cable?

Solution

If we want to represent this information in a Venn diagram:

The universal set U is the set of all houses in the street.
The set S can be those houses with satellite and the set C can be those with cable.

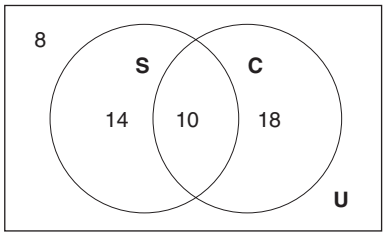
If we subtract the 8 houses with neither cable nor satellite from 50, we get 42 houses with either cable or satellite or both.

But 28 have cable and 24 have satellite and this makes 52. This means we have counted some houses more than once. These must be the houses with both cable and satellite TV.

There are 10 houses too many so these must have both.

So the Venn diagram looks like this:

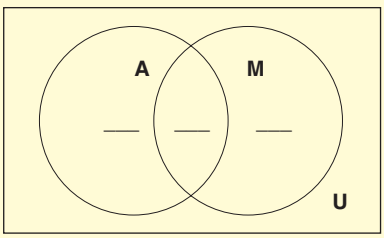
If 10 have both then 14 have satellite and no cable and 18 have cable and no satellite.



Exercise 15:04

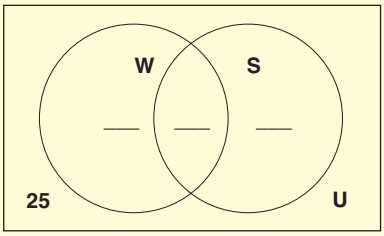
1 In Jen's class at school there are 24 students. All the students in her class study either Art or Music or both. If 18 study Art and 12 study Music:

- **a** Complete the Venn diagram.
- **b** How many study both Art and Music?
- **c** How many study Art but not Music?



2 In a car dealership there are 84 cars. Thirty of these cars are white and 45 are sedans. There are 25 cars that are neither white nor sedans.

- **a** What would the universal set (U) represent?
- **b** Complete the Venn diagram.
- **c** How many of the sedans are also white?
- **d** How many white cars are not sedans?



3 In a group of 30 high school boys, 15 play football, 19 play basketball and 6 do not play either sport.

Use a Venn diagram to answer the following questions.

a How many play both football and basketball?
b How many play basketball but not football?
c How many play only one of these sports?

4 Ms Poltze asked a group of students in her school in Berlin how many had been to two parks in the city in the past six months.

When she asked how many had been to Volkspark Friedrichshain, 17 students raised their hands.

When she asked how many had been to Volkspark Humboldthain, 21 students raised their hands.

When she asked how many had been to both parks, 9 students raised their hands.

When she asked how many had been to neither park, 4 students raised their hands.

A Venn diagram may be useful to answer these questions.

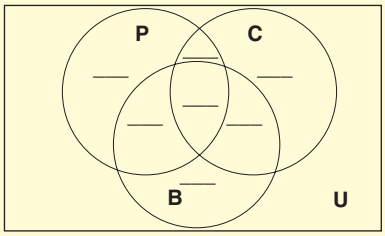
a How many students had been to only one of the parks?
b How many students were there in the group?

5 As part of a school project, Karl asked fellow students to tell him whether or not they had been to Frankfurt airport or Bangkok airport. Of the people he asked, 21 had been to Bangkok airport and 30 had been to Frankfurt airport, while 10 had not been to either of these. Eighteen had been to Bangkok airport, but not to Frankfurt airport.

A Venn diagram may be useful to answer these questions.

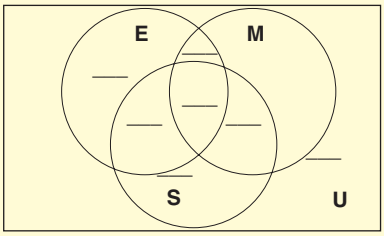
a How many had been to both airports?
b How many had been to only one of the airports?
c How many people did Karl ask?

6 At Vijay's school, 3 science subjects are offered: Physics, Chemistry and Biology. Of the 64 students in Vijay's year group, all must study at least one of these. Four study all three sciences, 6 study only Physics and Chemistry, 5 study only Physics and Biology and 9 study only Biology and Chemistry. Twenty-five study Physics, 31 study Chemistry and 36 study Biology.



a Complete the Venn diagram shown.
b How many students study only Physics?
c How many students study only one science?
d How many students study only two sciences?

7 Jakob's teacher recorded the results of the students in his class in English, Mathematics and Science.



Of the 18 students:

One student scored a grade of 7 in all three subjects.

Three students scored a grade of 7 in English and Mathematics only.

Two students scored a grade of 7 in Science and Mathematics only.

Two students scored a grade of 7 in English and Science only.

Altogether, 8 scored a grade of 7 in English.

Altogether, 7 scored a grade of 7 in Mathematics.

Altogether, 7 scored a grade of 7 in English.

a Complete the Venn diagram.
b How many students did not score any grade 7s?
c How many students scored at least one grade 7?
d How many students scored only two grade 7s?
e How many students scored only one grade 7?

Investigation 15:04

Investigation 15:04 | World Cup mania

Please use the Assessment Grid on the following page to help you understand what is required for this Investigation.

At the end of the Soccer World Cup in Germany in 2006, a number of fans were surveyed. They were asked which of the teams from Group F they had seen.

Four hundred had seen Australia, 650 had seen Japan, 550 had seen Brazil and 400 had seen Croatia.

Ninety-nine had not seen any of the 4 teams play.

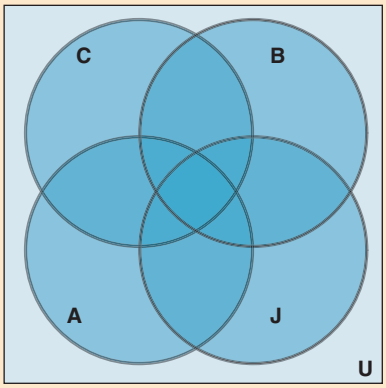
Twenty people managed to get tickets to see all four of the teams play.

Of the 63 who saw exactly three of the teams play:

12 would saw Croatia, Brazil and Australia

17 would saw Croatia, Brazil and Japan

15 would saw Brazil, Japan and Australia.



Altogether, 99 people saw Croatia and Brazil play.
Altogether, 122 people saw Brazil and Japan play.
Altogether, 114 people saw Croatia and Australia play.
Altogether, 131 people saw Japan and Australia play.

Use the Venn diagram on the right to find out:

1 How many people only managed to see one team play.
2 How many people were surveyed.

Assessment Grid for Investigation 15:04 | World Cup mania

The following is a sample assessment grid for this investigation. You should carefully read the criteria *before* beginning the investigation so that you know what is required.

Assessment Criteria (B, C, D)				Achieved ✓
Criterion B Application & Reasoning	a	No systematic or organised approach has been used.	1	
			2	
	b	An organised approach has been attempted.	3	
			4	
	c	An organised approach has been successfully used and most of the values in the Venn diagram are correct.	5	
			6	
	d	An organised approach has been successfully used and all of the values in the Venn diagram are correct.	7	
			8	
	e	An organised approach has been used, the results are correct and the correct answer has been obtained.	9	
			10	
Criterion C Communication	a	Little or no working out is shown and the presentation makes it hard to interpret what has been done.	1	
			2	
	b	Working out is shown and some explanation of how the numbers were obtained is given.	3	
			4	
	c	The results, working out and presentation are clear and it is easy to figure out how the solution was found.	5	
			6	
Criterion D Reflection & Evaluation	a	Some attempt has been made to explain the method used.	1	
			2	
	b	The method used is explained and shows how the results were checked.	3	
			4	
	c	The method used is clearly explained, as is the significance of the numbers in the Venn diagram.	5	
			6	
	d	A precise and reasoned evaluation of the method is given and perhaps an alternative approach to the solution is mentioned.	7	
			8	

Mathematical terms 15

element
- The things that belong to a set.

empty set
- A set that has no elements.

intersection of sets
- This contains the elements that belong to both sets.

set
- A collection of things with one characteristic in common.

subset
- A set is a subset when all of its elements also belong to another set.

union of sets
- This contains the elements that belong to one set or the other or both.

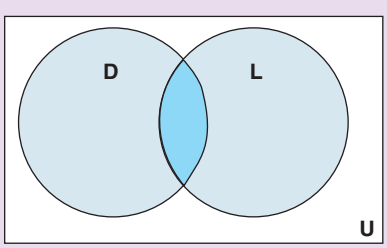
Venn diagram
- A diagram made up of circles and rectangles to help visualise the relationship between sets.

Diagnostic Test 15: | Sets

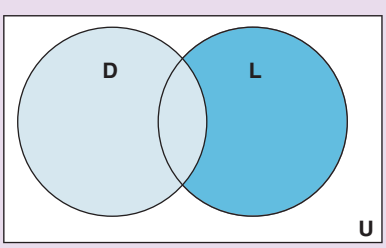
- Each section of the test has similar items that test a certain type of example.
- Failure in more than one item will identify an area of weakness.
- Each weakness should be treated by going back to the section listed.

	Section
1 Set A = {Factors of 18} Set B = {Factors of 36}.	15:01
i Write:	
a the elements of set A **b** the elements of set B	
c $n(A \cap B)$ **d** $n(A \cup B)$	15:02
ii Is $A \subset B$?	
2 Set P = {prime numbers less than 50}	
Set E = {even numbers less than 50}	15:03
a Find $P \cap E$ **b** Find $n(P)$ **c** Find $n(E)$	
d Find $n(P \cup E)$ **e** Find $n(P \cap$ {odd numbers})	
3 D = {Group D teams in the 2006 Soccer World Cup}	
L = {Teams in the second round of the 2006 Soccer World Cup}	15:04
Describe the shaded area in words	

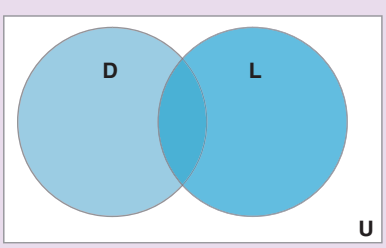
a



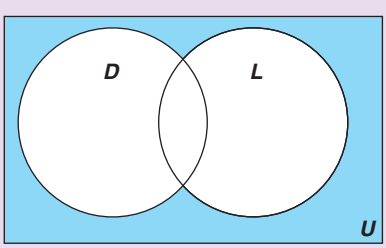
b



c

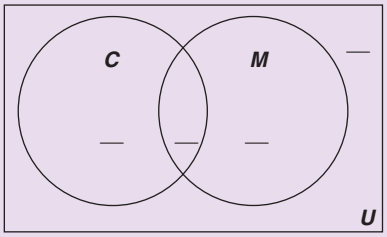


d



4 At Jeremy's school there are 55 teachers. Thirty-five teachers own a car, 15 teachers own a motorcycle and 18 teachers do not own any vehicles at all. 15:04

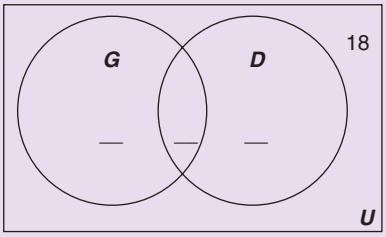
a Complete the Venn diagram shown, where:
U = {teachers a Jeremy's school}
C = {teachers that own cars}
M = {teachers that own motorcycles}



b How many teachers own both a car and a motor cycle?

5 U = {apartments in the building where Marco lives} 15:04
G = {apartments that get the *Geographic* magazine delivered}
D = {apartments that get the daily newspaper delivered}
If $n(G) = 25$, $n(D) = 35$ and $n(G \cap D) = 15$,

a complete the Venn diagram below



b How many apartments are there in Marco's building?

c How many apartments in Marco's building get only one of these publications delivered?

6 U = {students in Grade 6 at Ngoc's school} 15:04
N = {students in Grade 6 at Ngoc's school who play netball}
F = {students in Grade 6 at Ngoc's school who play football}
B = {students in Grade 6 at Ngoc's school who play basketball}
All the students at Ngoc's school who play football also play basketball.
There are no students who play both netball and basketball.
There are some students who do not play any of these sports.
Draw a Venn diagram to represent this information.

7 Draw a Venn diagram from the following information: 15:01
$A, B, C, D \subset U$ 15:02
$A \subset B$ 15:03
$C \subset D$
$B \cap D = \varnothing$

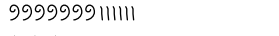
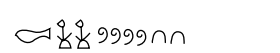
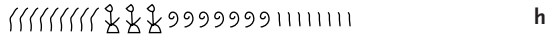
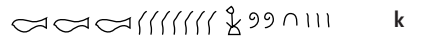
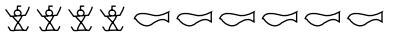
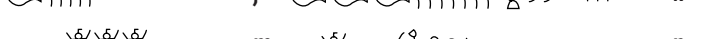
Answers

Chapter 1: Beginnings in Number

Exercise 1:01

1 **a** didn't need them **b** no **c** 10 **d** no
e subtract smaller unit from larger one **f** V formed by thumb and first finger **g** two Vs **h** India
i Arabs **j** Hindu–Arabic; to indicate an 'empty space'

2 **a** 35 **b** 71 **c** 1235 **d** 315 **e** 1 111 111 **f** 1032 **g** 131 200
h 12 007 **i** 11 000 000 **j** 13 100 000 **k** 2 230 000 **l** 1 001 312

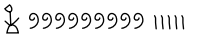
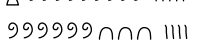
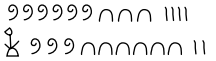
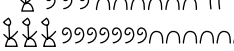
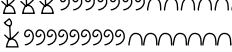
3 **a**  **b**  **c** 
d  **e** 
f  **g** **h** 
i **j**  **k**
l  **m**  **n** 
o 

4 **a** 315 **b** 34 **c** 128 **d** 682 **e** 709 **f** 1724 **g** 3400 **h** 548
i 947 **j** 808 **k** 1505 **l** 5555 **m** 52 500 **n** 499 **o** 999

5 **a** XXXVII **b** CCXIII **c** LXXXVI **d** DCXXXVII **e** DCLXXXIV **f** XLIX
g CCXC **h** DCXLV **i** MCDLII **j** DCCLXXVIII **k** $\overline{\text{C}}\text{M}\overline{\text{L}}\text{XXXVIII}$
l MCMLXXXIX **m** $\overline{\text{V}}\text{MMMCDLXXXIX}$ **n** $\overline{\text{V}}\text{CCCLXXXIV}$ **o** $\overline{\text{D}}\,\overline{\text{X}}\,\overline{\text{L}}\text{MMMDCXXVII}$

6 **a** 586 **b** 2107 **c** 432 **d** 8006 **e** 95 427 **f** 4330 **g** 1004 **h** 90 012
i 85 430 **j** 127

7

Hindu–Arabic numeral (AD 876)	*Egyptian numeral*	*Roman numeral*
		CCCXLII
		MCMIV
		DCXXXIV
		MCCCLXII
		MMMDCCLXXVI
		MCMLXXXVIII

8 It takes fewer symbols to write large numbers. **9** 999 **10** 1388

Exercise 1:02

1 **a** 1313 **b** 2801 **c** 14 926 **d** 50 380 **e** 33 709 **f** 93 165 **g** 3203 **h** 605 309

2 **a** 5815 **b** 324 679 **c** 806 050 **d** 1 927 463 **e** 56 002 914 **f** 83 907 201 **g** 37 070 847
h 927 100 070 **i** 204 042 420

3 **a** eight thousand, four hundred and five
b forty-three thousand, six hundred and twenty-seven
c ninety thousand and forty-six
d one hundred and thirty thousand, two hundred and fifteen
e nine hundred and twenty-seven thousand and four
f six million, three hundred and sixty thousand and sixty-four
g seventeen million, four thousand, nine hundred and eighty-eight
h nine million, three hundred and two thousand, eight hundred and fifty
i four hundred and forty-three million, two hundred thousand

4 **a** 500, 60, 7 **b** 5000, 600, 7 **c** 50 000, 60, 7 **d** 500 000, 70 000, 600
e 60 000, 70, 5 **f** 600 000, 5000, 700 **g** 6 000 000, 50 000, 70 **h** 500 000, 60 000, 7000
i 50 000 000, 600 000, 70 000

5 **a** 6000 + 400 + 20 + 1 **b** 80 000 + 0 + 100 + 70 + 9 **c** 700 + 80 + 6 **d** 90 000 + 1 000 + 0 + 30 + 2 **e** 6000 + 0 + 90 + 4 **f** 1 000 000 + 300 000 + 40 000 + 0 + 600 + 20 + 7 **g** 7 000 000 + 300 000 + 40 000 + 6 000 + 900 + 10 + 2 **h** 20 000 000 + 7 000 000 + 0 + 0 + 0 + 0 + 0 + 0 + 0 **i** 600 000 000 + 70 000 000 + 5 000 000 + 0 + 0 + 0 + 0 + 0 + 0

6 **a** 57 657 **b** 928 678 **c** 3 867 009 **d** 1 074 612 **e** 800 967 **f** 5 801 657 **g** 3 860 072 **h** 409 871

7 **a** 50 000 + 9000 + 600 + 70 + 5 **b** 800 000 + 6000 + 300 + 7

c 9 000 000 + 100 000 + 30 000 + 7000 + 800 + 20 + 6

8 **a** 1688, 9841, 47 341, 63 425, 120 070 **b** 1903, 24 106, 65 125, 91 000, 100 520

c 9999, 10 112, 86 314, 219 414, 635 188 **d** 8014, 58 096, 72 143, 88 000, 132 145

e 138 096, 365 214, 938 497, 1 090 040 **f** 8987, 77 717, 97 312, 123 000, 637 114

g 8914, 9060, 47 314, 100 000, 612 114 **h** 980 000, 1 500 000, 3 156 214, 5 937 193

Investigation 1:02 Estimation

1 **a** Count the grains in the thimble, then see how many thimbles of sand it takes to fill a cup and also how many cupfuls it takes to fill a bucket; then multiply.

b Count the number in, say, one column and then the number of columns to a page and the number of pages in the book; then multiply.

2 Estimates will vary.

Exercise 1:03

1 **a** 30, 30, 30 **b** 72, 72, 72 **c** 8, 8, 8 **d** 4, 4, 4

2

×	4	1	3	6	0	10	2	5	9	7	11	8
2	8	2	6	12	0	20	4	10	18	14	22	16
8	32	8	24	48	0	80	16	40	72	56	88	64
4	16	4	12	24	0	40	8	20	36	28	44	32
7	28	7	21	42	0	70	14	35	63	49	77	56

3 **a**

6
7
8
3
6

b

9
7
6
9
7

c

7
3
4
4
8

d

6
9
8
8
7

e

8
7
3
9
9

4 **a** 12 **b** \$14 **c** 11 kg **d** 15

5 **a** 156 **b** 409 **c** 1196 **d** 6784 **e** 86 **f** 1160 **g** 318 **h** 3156

6 **a** 12 **b** 183 **c** 100 **d** 11 **e** 20 **f** 46 **g** 4 **h** 9

7 **a** 4, 1 **b** 10, no **c** 5, 2 **d** 11

8 **a** 1 **b** 0 **c** 2

9 **a** $2\frac{2}{3}$ **b** $2\frac{3}{4}$ **c** $3\frac{7}{10}$ **d** $6\frac{3}{5}$ **e** $1\frac{4}{7}$ **f** $\frac{5}{6}$ **g** $2\frac{3}{5}$ **h** $2\frac{2}{3}$ **i** $121\frac{3}{5}$ **j** $93\frac{3}{4}$ **k** $10\frac{8}{10}$ or $10\frac{4}{5}$ **l** $84\frac{1}{9}$

10 **a** $108\frac{9}{10}$ **b** 3600 **c** $5108\frac{6}{10}$ or $5108\frac{3}{5}$ **d** $7105\frac{2}{10}$ or $7105\frac{1}{5}$ **e** $155\frac{1}{7}$ **f** 1827 **g** 9037 **h** $15\,260\frac{1}{4}$ **i** 6011 **j** 10 207 **k** $3001\frac{2}{5}$ **l** $30\,720\frac{1}{8}$

11 **a** 7 **b** 15 **c** $9\frac{2}{3}$ **d** 10 **e** 27 **f** 6 **g** 2 **h** 39

Exercise 1:04

1 **a** 7 **b** 7 **c** 5 r 3 or $5\frac{3}{12}$ or $5\frac{1}{4}$ **d** 7 r 2 or $7\frac{2}{11}$ **e** 14 r 4 or $14\frac{4}{15}$ **f** 11 r 9 or $11\frac{9}{17}$ **g** 13 r 4 or $13\frac{4}{18}$ or $13\frac{2}{9}$ **h** 16 **i** 28 r 3 or $28\frac{3}{23}$ **j** 23 r 3 or $23\frac{3}{24}$ or $23\frac{1}{8}$ **k** 66 **l** 60 r 13 or $60\frac{13}{16}$

2 **a** $88 = 2 \times 44$ **b** $708 = 30 \times 23 + 18$ **c** $182 = 14 \times 13$ **d** $654 = 18 \times 36 + 6$

3 **a** 3 r 3 or $3\frac{3}{31}$ **b** 3 r 11 or $3\frac{11}{39}$ **c** 4 r 11 or $4\frac{11}{22}$ or $4\frac{1}{2}$ **d** 5 r 17 or $5\frac{17}{18}$ **e** 3 r 48 or $3\frac{48}{99}$ or $3\frac{16}{33}$ **f** 5 r 46 or $5\frac{46}{47}$ **g** 5 r 7 or $5\frac{7}{29}$ **h** 4 r 40 or $4\frac{40}{62}$ or $4\frac{20}{31}$

Exercise 1:05

1 **a** 14 **b** 81 **c** 7 **d** 8 **e** 1400 **f** \$0.65 **g** \$6.45 **h** \$21 **i** 12 **j** \$4.20
2 **a** 12 **b** 48 **c** 7 **d** 6 **e** 1800 **f** \$0.15 **g** \$12.60 **h** \$30.60 **i** 10 **j** \$6.20
3 **a** 13 **b** 54 **c** 3 **d** 7 **e** 1700 **f** \$0.45 **g** \$5.75 **h** \$17.60 **i** 16 **j** \$3.10
4 **a** 15 **b** 49 **c** 7 **d** 6 **e** 1300 **f** \$0.35 **g** \$6.75 **h** \$13.60 **i** 14 **j** \$5.10

Prep Quiz 1:06

1 27 **2** 25 **3** 32 **4** 125 **5** 100 **6** 1000 **7** 10 000 **8** 100 000 **9** 700 **10** 4000

Exercise 1:06

1 **a** 16 **b** 100 **c** 4 **d** 1 **e** 49 **f** 81 **g** 8 **h** 625
i 27 **j** 1000 **k** 10 000 **l** 100 000
2 **a** 7^2 **b** 8^3 **c** 10^4 **d** 10^3 **e** 4^6 **f** 2^4 **g** 10^5
3 **a** 60 **b** 300 **c** 5000 **d** 20 000 **e** 7000 **f** 100 **g** 4000 **h** 900
i 80 000 **j** 9000 **k** 80 000 **l** 392

Prep Quiz 1:07

1 100 **2** 500 **3** 800 **4** 1000 **5** 2000 **6** 9000 **7** 63 **8** 251 **9** 7935 **10** 8132

Exercise 1:07

1 **a** 6459 **b** 28 630 **c** 900 874 **d** 4 380 000 **e** 74 586 **f** 1 101 010 **g** 5 060 842 **h** 48 390
2 **a** 8539 **b** 738 **c** 7304 **d** 9067 **e** 9003 **f** 8210 **g** 6 819 346 **h** 1 402 075
i 3 094 363 **j** 6 900 000
3 **a** 1 thousand + 3 hundred + 6 tens **b** 5 ten-thousand + 3 thousand + 2 hundred
c 1 million + 6 hundred-thousand **d** 9 thousand + 3 hundred + 8 tens + 6 units
e 4 million + 3 hundred-thousand + 7 thousand + 2 hundred
f 8 hundred-thousand + 3 hundred + 5 units **g** 1 thousand + 9 hundreds + 8 tens + 9 units
h 7 million + 3 hundred-thousand + 4 ten-thousand + 5 thousand + 8 hundred + 3 tens + 1 unit
4 **a** $(6 \times 1000) + (7 \times 100) + (0 \times 10) + (9 \times 1)$
b $(1 \times 10\,000) + (3 \times 1000) + (3 \times 100) + (0 \times 10) + (4 \times 1)$
c $(9 \times 1000) + (0 \times 100) + (8 \times 10) + (4 \times 1)$
d $(1 \times 100\,000) + (0 \times 10\,000) + (3 \times 1000) + (2 \times 100) + (4 \times 10) + (3 \times 1)$
e $(9 \times 10\,000) + (7 \times 1000) + (6 \times 100) + (7 \times 10) + (0 \times 1)$
f $(2 \times 100\,000) + (0 \times 10\,000) + (7 \times 1000) + (1 \times 100) + (1 \times 10) + (6 \times 1)$
g $(1 \times 1\,000\,000) + (3 \times 100\,000) + (4 \times 10\,000) + (4 \times 1000) + (5 \times 100) + (6 \times 10) + (2 \times 1)$
h $(9 \times 1\,000\,000) + (7 \times 100\,000) + (6 \times 10\,000) + (5 \times 1000) + (8 \times 100) + (1 \times 10) + (3 \times 1)$
5 **a** $(8 \times 10^3) + (5 \times 10^2) + (3 \times 10^1) + (9 \times 1)$ **b** $(7 \times 10^2) + (3 \times 10^1) + (8 \times 1)$
c $(7 \times 10^3) + (3 \times 10^2) + (0 \times 10^1) + (4 \times 1)$ **d** $(9 \times 10^3) + (0 \times 10^2) + (6 \times 10^1) + (7 \times 1)$
e $(9 \times 10^3) + (0 \times 10^2) + (0 \times 10^1) + (3 \times 1)$ **f** $(8 \times 10^3) + (2 \times 10^2) + (1 \times 10^1) + (0 \times 1)$
g $(6 \times 10^6) + (8 \times 10^5) + (1 \times 10^4) + (9 \times 10^3) + (3 \times 10^2) + (4 \times 10^1) + (6 \times 1)$
h $(1 \times 10^6) + (4 \times 10^5) + (0 \times 10^4) + (2 \times 10^3) + (0 \times 10^2) + (7 \times 10^1) + (5 \times 1)$
i $(3 \times 10^6) + (0 \times 10^5) + (9 \times 10^4) + (4 \times 10^3) + (3 \times 10^2) + (6 \times 10^1) + (3 \times 1)$
j $(6 \times 10^6) + (9 \times 10^5) + (0 \times 10^4) + (0 \times 10^3) + (0 \times 10^2) + (0 \times 10^1) + (0 \times 1)$
6 **a** $(1 \times 10^3) + (3 \times 10^2) + (6 \times 10^1) + (0 \times 1)$ **b** $(5 \times 10^4) + (3 \times 10^3) + (2 \times 10^2) + (0 \times 10^1) + (0 \times 1)$
c $(1 \times 10^6) + (6 \times 10^5) + (0 \times 10^4) + (0 \times 10^3) + (0 \times 10^2) + (0 \times 10^1) + (0 \times 1)$
d $(9 \times 10^3) + (3 \times 10^2) + (8 \times 10^1) + (6 \times 1)$
e $(4 \times 10^6) + (3 \times 10^5) + (0 \times 10^4) + (7 \times 10^3) + (2 \times 10^2) + (0 \times 10^1) + (0 \times 1)$
f $(8 \times 10^5) + (0 \times 10^4) + (0 \times 10^3) + (3 \times 10^2) + (0 \times 10^1) + (5 \times 1)$
g $(1 \times 10^3) + (9 \times 10^2) + (8 \times 10^1) + (9 \times 1)$
h $(7 \times 10^6) + (3 \times 10^5) + (4 \times 10^4) + (5 \times 10^3) + (8 \times 10^2) + (3 \times 10^1) + (1 \times 1)$

Prep Quiz 1:08

1 80 **2** 50 **3** 20 **4** 40 **5** 400 **6** 900 **7** 100 **8** 300 **9** 6000 **10** 10 000

Exercise 1:08

1 **a** E = 1500, A = 1491, D = 9
b E = 7000, A = 7590, D = 590
c E = 100 000, A = 96 792, D = 3208
d E = 9 000 000, A = 9 236 753, D = 236 753
e E = 500, A = 565, D = 65
f E = 5000, A = 5717, D = 717
g E = 1000, A = 518, D = 482
h E = 200 000, A = 212 322, D = 12 322
i E = 12 000, A = 11 439, D = 561
j E = 12 000, A = 10 296, D = 1704
k E = 160 000, A = 156 683·9, D = 3 316·1
l E = 2 100 000, A = 1 951 216·2, D = 148 783·8

m E = 2000
A = 2048
D = 48

n E = 20 000
A = 21 345
D = 1345

o E = 4000
A = 3096
D = 904

p E = 100 000
A = 99 865
D = 135

2 a E = 25 000
A = 25 303
D = 303

b E = 88 000
A = 86 045
D = 1955

c E = 630 000 s
A = 604 800 s
D = 25 200 s

d E = $10 000
A = $9120
D = $880

e E = 100
A = 138
D = 38

f E = $50 000
A = $55 451.69
D = $5451.69

g E = $40 000
A = $31 794.29
D = $8205.71

h E = $420 000
A = $394 205
D = $25 795

3

	Question	*Working*	*Estimate*	*Answer*	*Difference*
a	11 615 + 58 107	10 000 + 60 000	70 000	69 722	278
b	92 094 ÷ 6	90 000 ÷ 6	15 000	15 349	349
c	48 314 – 27 199	50 000 – 30 000	20 000	21 115	1115
d	37 118 × 7	40 000 × 7	280 000	259 826	20 174
e	3 416 809 + 4 369 170	3 000 000 + 4 000 000	7 000 000	7 785 979	785 979
f	567 180 ÷ 5	600 000 ÷ 5	120 000	113 436	6564
g	69 847 × 9	70 000 × 9	630 000	628 623	1377
h	486·76 ÷ 1·72	500 ÷ 2	250	283	33
i	735·15 – 275·35	700 – 300	400	459·8	59·8
j	2315 × 7·8	2000 × 8	16 000	18 057	2057

4 Question **2b** would be better suited to this method.

Fun Spot 1:08 What's the difference between a boxer and a telephone?

One boxes in a ring. The other rings in a box.

1:09A Operations Checkup

1 a 136 **b** 1901 **c** 65 435 **d** 334 975
2 a 454 **b** 2385 **c** 39 018 **d** 453 462
3 a 3954 **b** 20 048 **c** 7890 **d** 91 350
4 a 2496 **b** 177 351 **c** 389 385 **d** 274 340
5 a 110 **b** $130\frac{4}{5}$ **c** $339\frac{1}{7}$ **d** 406
6 a $447\frac{1}{2}$ or 447·5 **b** 107 **c** 17 **d** 1835

1:09B Fractions Checkup

1 a $\boxed{6}$ **b** $\boxed{2}$ **c** $\boxed{30}$
2 a $\frac{4}{5}$ **b** $\frac{3}{4}$ **c** $\frac{2}{5}$
3 a $\frac{1}{10}$ **b** $\frac{6}{10}$ **c** $\frac{2}{5}$
4 a $\frac{1}{10}, \frac{1}{5}, \frac{1}{2}$ **b** $\frac{1}{10}, \frac{3}{10}, \frac{7}{10}$
5 a $\frac{5}{2}$ **b** $\frac{53}{10}$ **c** $\frac{5}{4}$
6 a $1\frac{3}{4}$ **b** $8\frac{7}{10}$ **c** $4\frac{1}{2}$
7 a $\frac{7}{10}$ **b** $\frac{3}{5}$ **c** $1\frac{1}{10}$
8 a $\frac{1}{10}$ **b** $\frac{1}{2}$ **c** $\frac{1}{2}$
9 a $\frac{3}{4}$ **b** $\frac{8}{9}$ **c** $\frac{7}{10}$
10 a $\frac{7}{10}$ **b** $\frac{1}{6}$ **c** $\frac{3}{16}$
11 a 3 **b** $1\frac{1}{2}$ **c** 4
12 a 1 **b** 9 **c** 20

1:09C Decimals and Money Checkup

1 a 0·7 **b** 2·13 **c** 0·009
2 a $\frac{317}{1000}$ **b** $\frac{59}{100}$ **c** $\frac{1}{2}$
3 a 0·1 **b** 0·11 **c** 0·49
4 a 0·3, 0·8, 1 **b** 0·11, 0·51, 0·91 **c** 0·12, 0·509, 0·8
5 a 5·5 **b** 3·14 **c** 8·18
6 a 68·25 **b** 13·45 **c** 27·3
7 a 7·95 **b** 4·33 **c** 9·105
8 a 8 **b** 3 **c** 0·543
9 a 16·2 **b** 0·3 **c** 0·041
10 a 0·31 **b** 5 **c** 800
11 a $5.65 **b** $8.05 **c** $7.40
12 a $14.80 **b** $65.70 **c** $202.20
13 a $6.20 **b** $46.50 **c** $3.15
14 a $18 **b** $11.10 **c** $45.25
15 a $4.50 **b** $4.05 **c** $8.75

1:09D Percentages Checkup

1 **a** $\frac{7}{100}$ **b** $\frac{33}{100}$ **c** $\frac{15}{100}$ **2** **a** $\frac{1}{5}$ **b** $\frac{3}{10}$ **c** $\frac{3}{20}$

3 **a** 1 **b** 3 **c** 2 **4** **a** $2\frac{1}{2}$ **b** $1\frac{1}{20}$ **c** $1\frac{1}{4}$

5 **a** 8% **b** 37% **c** 50% **6** **a** 15% **b** 14% **c** 70%

7 **a** 400% **b** 100% **c** 300% **8** **a** 350% **b** 175% **c** 730%

9 **a** 0·93 **b** 0·07 **c** 0·1 **10** **a** 1·5 **b** 1·13 **c** 4·25

11 **a** 7% **b** 85% **c** 90% **12** **a** 315% **b** 108% **c** 590%

Fun Spot 1:09 Magic squares

1 **a**

8	1	6
3	5	7
4	9	2

b

4	18	8
14	10	6
12	2	16

c

6	11	4
5	7	9
10	3	8

2 **a**

4	14	12
18	10	2
8	6	16

9	7	17
19	11	3
5	15	13

3	10	5
8	6	4
7	2	9

6	7	2
1	5	9
8	3	4

15	5	7
1	9	17
11	13	3

10	15	14
17	13	9
12	11	16

10	8	18
20	12	4
6	16	14

10	3	8
5	7	9
6	11	4

5	10	9
12	8	4
7	6	11

b Every row, column and diagonal has a sum of 81.

Diagnostic Test 1: Beginnings in Number

1 **a** 1023 **b** 1 130 200 **c** 200 312

2 **a**  **b**  **c** 

3 **a** 364 **b** 2002 **c** 1999 **4** **a** CCCLXXII **b** CMXLVIII **c** MMMCDIX

5 **a** 70 000 **b** 70 **c** 7 000 000 **6** **a** 8^3 **b** 3^5 **c** 2^4

7 **a** 800 **b** 20 000 **c** 7 000 000 **8** **a** 7228 **b** 5080

9 **a** $(1 \times 10^3) + (8 \times 10^2) + (2 \times 10^1) + (4 \times 1)$ **b** $(4 \times 10^3) + (3 \times 10^2) + (0 \times 10^1) + (7 \times 1)$

c $(2 \times 10^6) + (4 \times 10^5) + (1 \times 10^4) + (5 \times 10^3) + (2 \times 10^2) + (8 \times 10^1) + (6 \times 1)$

10 **a** 1 100 000 **b** 1 600 000 **c** 10 000

1A Revision Assignment

1 **a** two hundred and seventy thousand, three hundred and seven

b two million, four hundred and fifty-six thousand, three hundred and twenty-five

c ninety-seven million, five thousand

d two hundred and ninety-four million, one hundred and sixty-seven thousand

2 **a** 700 **b** 70 000 **c** 7 000 000 **d** 700 000

3 **a** 59 376 **b** 806 545 **c** 3 215 000

4 **a** $(8 \times 1000) + (2 \times 100) + (4 \times 10) + (7 \times 1)$ **b** $(5 \times 10\,000) + (6 \times 1000) + (1 \times 10) + (5 \times 1)$

c $(7 \times 100\,000) + (1 \times 10\,000) + (4 \times 1000) + (8 \times 100) + (5 \times 10)$

d $(3 \times 1\,000\,000) + (4 \times 100\,000) + (2 \times 10\,000)$

5 **a** 60 000 **b** 50 000 **c** 210 000

6 **a** 25 **b** 16 **c** 10 000 **d** 8000

7 **a** 321 **b** 120 100 **c** 3123 **d** 1 040 101

8 **a** 928 **b** 649 **c** 2600 **d** 20 710

9 **a** CCXXVII **b** CDXIX **c** MMDLV **d** MCMXCI

10 **a** 6 **b** 60 **c** 65 **d** Woodwork **e** 2 **f** 410

1B Working Mathematically

1 **a** *metres* **b** *centimetres* **c** *millimetres* **d** *kilometres* **e** *hectares*
f *kilograms* **g** *tonnes* **h** *litres* **i** *millilitres* **j** *degrees Celsius*
2 **a** *multiplied by (or times)* **b** *divided by* **c** *is equal to* **d** *is not equal to*
e *4 squared* **f** *4 cubed* **g** *per cent* **h** *therefore* **i** *numerator* **j** *denominator*
3 **a** 4 minutes **b** 5 cats **c** 2 h **d** 4 boys and 3 girls are in the family.
e They are both the same distance from Berlin.
4 **a** black currants **b** cauliflowers **c** 200 g **d** 4 mg **e** 98 mg

Chapter 2: Problem Solving

Fun Spot 2:01 Can you play 31?

If you want to win, try to make the number of counters remaining after your turn either 29, 22, 15, 8 or 1. From each of these values it is impossible for your opponent to then leave one of these values after his or her choice. After their move you can then move down to the next number until you leave only one counter for your opponent.

Exercise 2:01

1 **a** 15 kg **b** \$3.45 **c** 3 **d** price per kg and total cost **e** 69c, \$10.35
2 **a** total spent **b** 10 **c** £3.50 **d** 14 **e** £28.35
3 77 **4** \$160.85 **5** 191 km **6** 18 **7** 8 cars, 20 marbles
8 9 **9** 158 549 **10** \$41.90

Exercise 2:03

1 13 **2** 12 and 8 **3** 35 **4** 67 and 73 **5** 3 and 24
6 13 and 57 **7** 8 **8** 5
9 **a** 43 and 5 **b** 3 and 45 **c** 5 and 34
10 **a** 13 and 24 **b** 43 and 52 **c** 13 and 54
11 7 **12** 18 **13** 8 **14** \$15 **15** 12, 31, 13, 21 or 21, 22, 22, 12 or 11, 41, 14, 11
16 {9, 10, 42, 51}, {18, 21, 25, 49} **17** 42 **18** 58 **19** 5 **20** 9, 10, 11 or 12

Exercise 2:04

1 1600 **2** 26 **3** 5
4 (The thickness of the posts has been ignored.) **a** 9 **b** 46 m **c** 16 m
5 5 **6** **a** 3 **b** 12 **7** 24 min **8** 9 s **9** 4
10 **a** 7 **b** 9 **c** 10
11 10 **12** 73 **13** 7th **14** 19 **15** 10 h 20 min

Investigation 2:04 Catching the crooks

He could compare the mass and volume of the three crowns with a mass of gold equal to that given to each crown maker. Many answers are acceptable.

Exercise 2:05

1 **a** 4 **b** 12 **2** 10 **3** **a** 3 **b** 4 **4** 19
5 **a** 12 **b** 16 **c** 9 **6** 10 **7** 9 **8** **a** 12 **b** 4
9 12 **10** 42 **11** **a** 12 **b** 36 **12** 260 **13** 24
14 **a** 28 **b** 168 **15** 52
16 20 if coins can be used only once in each group of three (49 if coins can be used more than once)

Exercise 2:06

1 7 **2** 15 **3** 100 **4** 37 **5** d **6** c
7 Kylie Conway, Bronwyn Wilkes, Jennifer Collison **8** G

Exercise 2:07

1 25 **2** 49 kg **3** 42 **4** \$52.20 **5** 20 **6** €5000

Exercise 2:08

1 **a** 2 **b** 5 **c** 7 **2** 45 cm **3** 14 **4** 6 **5** 28
6 two children across, one child back, one adult across, other child back, two children across, one child back, second adult across, other child back, two children across.

Challenge 2:08 Backtracking

1 74 **2** 45 **3** 53 **4** 51 **5** 13 **6** 27 **7** 10 **8** 70

Exercise 2:09

1 **a** 29 **b** 50 **c** $\frac{1}{2}$ **d** 8 **e** 70
2 **a** 80 200 **b** 500 500 **c** 5050 **d** 1275

Exercise 2:10

1 **a** 9 m **b** 18 m **c** 90 m **d** 900 m **e** 1800 m **2** less than 45 m
3 **a** 56 **b** 28 **4** **a** 1 **b** 2 **c** 3 **d** 4 **e** 97
5 45 bags of gold were stolen (assuming that each prospector owns the gold he digs, 54 if they are working for someone else).
6 35 bags of gold were stolen.
7 **a** 7 **b** 12 **c** 17 **d** 22 **e** 27 **8** 5, 502
9 **a** 0 **b** 1 **c** 2 **d** 3 **e** 4 **f** 5 **g** 6 **h** 7
10 **a** 1 **b** 3 **c** 6 **d** 10 **e** 15 **f** 21
11 The difference between the number of handshakes is greater by 1 each time, 190

Reading Mathematics 2:10 Let's post a parcel in Sydney (NI)

1 **a** \$5.20 **b** \$7.90 **c** \$3.75 **d** \$7.40 **e** \$9.40 **f** \$2.75 **2** Answers will vary.

2A Revision Assignment

1 **a** 10 **b** 1000 **c** 1000 **d** 100 000 **2** **a** 5410 **b** 1763 **c** 180 **d** 53
3 **a** $\frac{7}{10}$ **b** $\frac{1}{100}$ **c** $\frac{201}{1000}$ **d** $\frac{33}{1000}$ **4** **a** 0·3 **b** 0·17 **c** 0·045 **d** 1·03
5 **a** 0·71 **b** 7·37 **c** 13·5 **d** 20·6 **6** **a** 8·3 **b** 3·2 **c** 1·25 **d** 8·17
7 **a** 0·6 **b** 0·06 **c** 6·0 **d** 0·06 **8** **a** 6·3 **b** 0·52 **c** 1·1 **d** 4·31
9 **a** \$5.30 **b** \$7.10 **c** \$295.40 **d** \$4.80 **10** **a** $1\frac{1}{5}$ **b** $\frac{79}{100}$ **c** $\frac{2}{5}$ **d** 0

2B Working Mathematically

1 **a** *square* **b** *rectangle* **c** *parallelogram* **d** *rhombus* (or diamond)
e *trapezium* **f** *regular pentagon* **g** *regular hexagon* **h** *regular octagon* **i** *kite* **j** *circle*
2 **a** 10, 24, 63, 60, 60, 18, 0, 33, 100, 63 **b** 69, 54, 36, 106, 107, 36, 63, 76, 93, 61
3 5·6 m **4** 7200
5 You could count the number of households represented on one page of the phone directory and then multiply by the number of pages in the directory.
6 1 v 2, 3 v 4, 5 v 6 **7** 90 tickets **8** **a** 9 am and 10 am **b** 250 cm **c** 3 pm **d** 8 pm **e** water to a depth of 225 cm
1 v 3, 2 v 5, 4 v 6
1 v 4, 2 v 6, 3 v 5
1 v 5, 2 v 4, 3 v 6
1 v 6, 2 v 3, 4 v 5

Chapter 3: Number: Its Order and Structure

Exercise 3:01A

1 **a** 120 **b** 2 **c** 5 **d** 72 **e** 42 **f** 48 **g** 20 **h** 7 **i** 10 **j** 27
k 8 **l** 2 **m** 20 **n** 30 **o** 35 **p** 65 **q** 11 **r** 37 **s** 0 **t** 2
u 73 **v** 32 **w** 0 **x** 1
2 **a** 66 **b** 60 **c** 12 **d** 17 **e** 2 **f** 5 **g** 42 **h** 30 **i** 12 **j** 60
k 2 **l** 20 **m** 2 **n** 7 **o** 132 **p** 160 **q** 4500 **r** 560 **s** 3 **t** 10
3 **a** 6 **b** 10 **c** 48 **d** 0 **e** 40 **f** 20 **g** 66 **h** 20 **i** 40 **j** 4
k 2 **l** 1 **m** 120 **n** 354 **o** 11 **p** 1003
4 **a** 52 **b** 5 **c** 80 **d** 10 **e** 3 **f** 6 **g** 4 **h** 8 **i** 123
5 **a** 0·8 **b** 0·03 **c** 3·0 **d** 4·5 **e** 1·6 **f** 12 **g** 9 **h** 0·4 **i** 1·2 **j** 1
k 2·7 **l** 6 **m** \$3.50 **n** \$2.50 **o** \$180 **p** \$16.80 **q** \$71.80 **r** \$633.50
6 **a** $(5 \times 3) + 8 = 23$ **b** $5 \times (3 + 8) = 55$ **c** $10 \times (7 + 5) = 120$ **d** $6 - (3 \times 2) = 0$ **e** $(6 - 3) \times 2 = 6$
f $(10 \times 7) + 5 = 75$ **g** $40 \div (4 + 1) = 8$ **h** $(40 \div 4) + 1 = 11$ **i** $80 \div (8 + 2) = 8$ **j** $(29 - 15) - 6 = 8$
k $29 - (15 - 6) = 20$ **l** $18 - (8 + 2) = 8$ **m** $(40 \div 2) \div 2 = 10$ **n** $40 \div (2 \div 2) = 40$ **o** $144 \div (6 \div 3) = 72$
p $(6 + 2) \times (8 + 2) = 80$ **q** $6 + (2 \times 8) + 2 = 24$ **r** $(10 \times 3) - (16 - 2) = 16$

Exercise 3:01B

1 **a** 30 **b** 24 **c** 14 **d** 3 **e** 15 **f** 4 **g** 2 **h** 4
i 3 **j** 0 **k** 7 **l** 3
2 **a** 80 **b** 54 **c** 5 **d** 5 **e** 100 **f** 33 **g** 16 **h** 7
i 112 **j** 80 **k** 64 **l** 12

3 **a** 22 **b** 10 **c** 0 **d** 2 **e** 30 **f** 90 **g** 75 **h** 39
i 3 **j** 11 **k** 43 **l** 25
4 **a** 30 **b** 12 **c** 90 **d** 6 **e** 6 **f** 6 **g** 11 **h** 11
i 5 **j** 1 **k** 6 **l** 6 **m** 4 **n** 72 **o** 47 **p** 54
q 61 **r** 74 **s** 4 **t** 20 **u** 900 **v** 20 **w** 26 **x** 2
5 **a** 24 **b** 78 **c** 6 **d** 42 **e** \$102 **f** \$19 **g** 0·1 **h** 8
i 78 **j** 196 **k** 1

Exercise 3:02A

1

0	20	36	56
13×0	$(6 + 5) + 9$	4×9	$50 + 6$
8×0	$6 + (5 + 9)$	$(2 \times 6) \times 3$	$6 + 50$
$0 \times (8 \times 11)$	20×1	9×4	1×56
$(0 \times 8) \times 11$	1×20	$2 \times (6 \times 3)$	56×1
98×0	$20 + 0$	$0 + 36$	$39 + 16 + 1$
	$3 + 17$	$2 \times 9 \times 2$	$39 + 1 + 16$
	$17 + 3$	$2 \times 2 \times 9$	

2 **a** T **b** T **c** T **d** T **e** T **f** T **g** T **h** T **i** T **j** T
k T **l** T **m** F **n** F **o** F **p** F **q** F **r** F **s** F **t** F
3 **a** 0 **b** 416 **c** 0 **d** 365 **e** 1 **f** any **g** 20 **h** 18 **i** 15 **j** 8
k 7 **l** 9 **m** 0 **n** 0 **o** 0·9 **p** $\frac{4}{10}$ **q** 6 **r** 0
4 **a** 1342 **b** 2325 **c** 1342 **d** 2325 **5** **a** 20 398 **b** 21 840 **c** 88 907
6 **a** no **b** Heather **c** no **d** Robert **e** Peter **f** Jan
7 **a** T **b** T **c** T **d** T **e** F **f** F **g** F **h** F **i** T **j** T
8 **a** 100 **b** 100 **c** 80 **d** 80 **e** 1000 **f** 10 **g** $\frac{13}{100}$ **h** 1 **i** 100 **j** 100
9 **a** 0 **b** \$17 **c** \$126 **d** 2453 **e** 1276 **f** 566 **g** 1800 **h** 35 600 **i** 6300 **j** 170
k 1300 **l** 4800 **10** **a** 276 **b** 280 **c** 284
11 **a** yes **b** \$17 **c** same **d** 340 **12** **a** True **b** True **c** True **d** True

Prep Quiz 3:02B

1 9 **2** 7 **3** 1 **4** 0 **5** 0 **6** 9700 **7** 120 **8** 40 **9** 160 **10** yes

Exercise 3:02B

1 **a** 495 **b** 160 **2** b **3** **a** T **b** T **c** T **d** T **e** T **f** T
4 **a** 3 **b** 3 **c** 11 **d** 9 **e** 2 **f** 6 **g** 15 **h** 15 **i** 20 **j** 7
5 **a** 495 **b** 693 **c** 194 **d** 882 **e** 570 **f** 760 **g** 1140 **h** 1710 **i** 7171 **j** 2448
k 856 **l** 540 **m** 3570 **n** 4242 **o** 1560 **p** 1608 **q** 780 **r** 490 **s** 4218 **t** 44 100
6 **a** 900 **b** 80 **c** 99 **d** 66 **e** 140 **f** 800 **g** 480 **h** 700 **i** 120 **j** 24
k 28 **l** 146 **7** **a** T **b** T **c** T **d** T

ID 3:03

1 $6 - 2 = 4$ **2** $6 + 2 = 8$ **3** $6 \div 2 = 3$ **4** $6 - 2 = 4$ **5** $6 \div 2 = 3$
6 2 **7** 6 **8** $6 \times 2 = 12$ **9** $6 - 2 = 4$ **10** $6 \times 2 = 12$
11 $2 + 6 = 8$ **12** $6 - 2 = 4$ **13** $6^2 = 36$ **14** $\sqrt{36} = 6$ **15** $6 - 2 = 4$
16 $6 \times 2 = 12$ **17** $(6 + 2) \div 2 = 4$ **18** $6 + 2 = 8$ **19** $6^2 = 36$ **20** $6 - 2 = 4$
21 $6 - 2 = 4$ **22** $6 + 2 = 8$ **23** $6 \div 2 = 3$ **24** $6 + 2 = 8$

Exercise 3:03

1 **a** 42 **b** 68 **c** 51 **d** 39 **e** 26 **f** 488 **g** 8 **h** 17 **i** 11 **j** 77
2 **a** $15 < 105$ **b** $8{\cdot}8 \neq 8{\cdot}08$ **c** $0{\cdot}7 = 70\%$ **d** $5^2 \times 7^3$ **e** $6 \times 9 - 3$
f $12 \div 4 + 6$ **g** $\therefore \sqrt{9} = 3$ **h** ie $\sqrt[3]{8} = 2$ **i** $\pi \div 3{\cdot}142$ **j** $\square \leqslant 10$
k $0{\cdot}3 > 0{\cdot}03$ **l** eg $5{\cdot}1 \not< 5{\cdot}09$ **m** $12 \times 4 \geqslant 12 \div 4$
3 **a** 0·499 is less than $\frac{1}{2}$ **b** 3999 is approximately equal to 4000
c 61 000 is not equal to 61 times 100 **d** 6 plus $\square$ is greater than or equal to 9
e 0·1 is greater than 0·099 **f** 6 times 199 is equal to 6 times 200 minus 6 times 1
g 9·1 is not less than $9\frac{1}{10}$ **h** 5·1 plus 4 is equal to 4 plus 5·1
i $\square$ minus 3 is less than or equal to 6
4 **a** T **b** T **c** F **d** T **e** F **f** F **g** T **h** T **i** T **j** T
k T **l** F **m** T **n** T **o** F

Prep Quiz 3:04

1 10, 12, 14 **2** 9, 11, 13 **3** 5, 6, 7 **4** 25, 36, 49 **5** T **6** T **7** F **8** T **9** T **10** T

Exercise 3:04

1 **a** 7007 **b** 10 001 **c** 973 **d** 10 339 **2** **a** 87, 89, 91, 93, 95, 97 **b** 993, 995, 997, 999, 1001
3 **a** 8148 **b** 112 **c** 3194 **d** 9998 **4** **a** 74, 76, 78, 80, 82 **b** 1998, 2000
5 **a** 1, 3, 5, 7, 9 **b** 0, 2, 4, 6, 8 **6** **a** 84, 86, 88, 90, 92 **b** 199, 201, 203, 205, 207
c 995, 997, 999, 1001, 1003 **d** 1018, 1020, 1022, 1024, 1026
7 **a** even **b** odd **c** even **d** even **e** odd **f** even **g** odd **h** even **i** even **j** even

8

	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}	T_{11}	T_{12}	T_{13}	T_{14}	T_{15}
Counting	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Even	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30
Odd	1	3	5	7	9	11	13	15	17	19	21	23	25	27	29
Square	1	4	9	16	25	36	49	64	81	100	121	144	169	196	225
Triangular	1	3	6	10	15	21	28	36	45	55	66	78	91	105	120
Fibonacci	1	1	2	3	5	8	13	21	34	55	89	144	233	377	610

9 **a** 4 **b** 9 **c** 16 **d** 25 **e** 36 **f** 49 **g** 64 **h** 81
10 **a** T **b** T **c** T **d** T

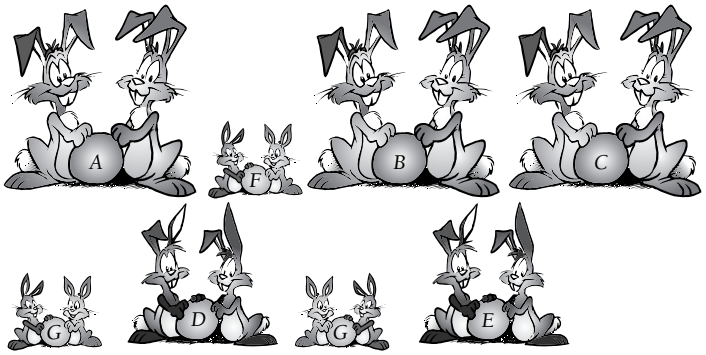
11

$3^2 + 7$	$9 + 7 = 16 = 4^2$
$4^2 + 9$	$16 + 9 = 25 = 5^2$
$5^2 + 11$	$25 + 11 = 36 = 6^2$
$6^2 + 13$	$36 + 13 = 49 = 7^2$
$7^2 + 15$	$49 + 15 = 64 = 8^2$
$8^2 + 17$	$64 + 17 = 81 = 9^2$
$9^2 + 19$	$81 + 19 = 100 = 10^2$
$10^2 + 21$	$100 + 21 = 121 = 11^2$
$11^2 + 23$	$121 + 23 = 144 = 12^2$

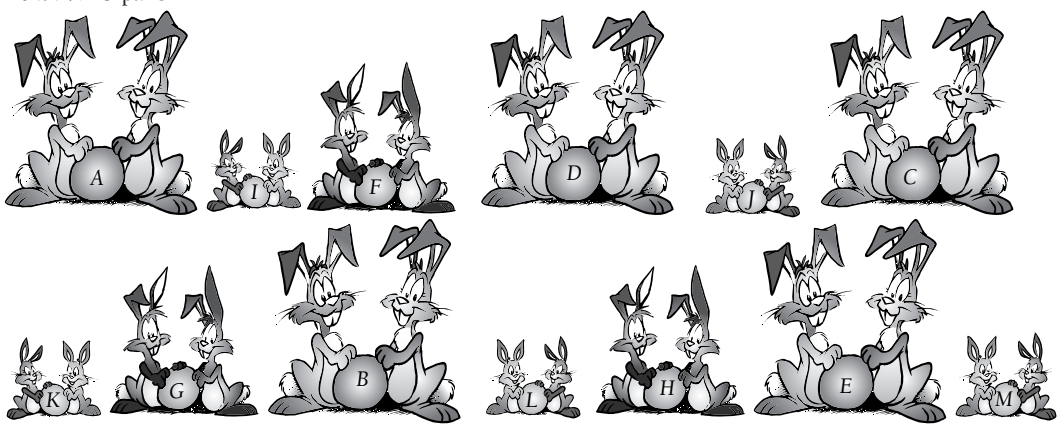
12 **a** 46, 100, 16, 200, 64, 144, 2, 4, 84, 66, 36, 122
b 37, 15, 25, 81, 33, 111, 1, 49, 77, 9, 71, 73
c 25, 100, 81, 16, 64, 144, 1, 49, 4, 9, 36 **d** 100, 16, 64, 144, 4, 36
13 **a** 15 **b** yes **c** 1, 3, 6 **14** **a** 28 **b** 45
15 **a** 10 **b** yes
16 1001, 1111, 1221, 1331, 1441, 1551, 1661, 1771, 1881, 1991, 2002, 2112, 2222, 2332, 2442, 2552, 2662, 2772, 2882, 2992
17 **a** 55, 78, 66, 21, 91, 10, 15 **b** 55, 21, 89, 610 **c** 55, 66, 707
18 Answers will vary.

Fun Spot 3:04 Fibonacci numbers

1 **a** *Month 6:* 8 pairs



Month 7: 13 pairs



b yes **2** 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610 **3** yes **4** yes **5** It is true.

Investigation 3:04 Pascal's triangle

1

1 (1)
1 1 (2)
1 2 1 (4)
1 3 3 1 (8)
1 4 6 4 1 (16)
1 5 10 10 5 1 (32)
1 6 15 20 15 6 1 (64)
1 7 21 35 35 21 7 1 (128)
1 8 28 56 70 56 28 8 1 (256)
1 9 36 84 126 126 84 36 9 1 (512)

2 The pattern made is of the powers of 2. $(1, 2^1, 2^2, 2^3, 2^4, 2^5, 2^6, 2^7, 2^8, 2^9)$. The rule is: Row total = $2^{\text{row number} - 1}$.

3 Answers will vary.

Prep Quiz 3:05

1 7 **2** 1 **3** 6 **4** 42 **5** 1, 2, 3, 6, 7, 14, 21, 42 **6** 36 **7** 36 **8** 36 **9** 36 **10** 36

Exercise 3:05

1

24	12	8	6	5	10	20	1	2	3	3	2	1	5	25

2 **a** 1, 2, 3, 4, 6, 8, 12, 24 **b** 1, 2, 4, 5, 10, 20 **c** 1, 2, 3, 4, 6, 12 **d** 1, 2, 3, 6, 9, 18 **e** 1, 5, 25

3 **a** 1, 2, 5, 10 **b** 1, 2, 4, 8 **c** 1, 3, 5, 15 **d** 1, 3, 9 **e** 1, 7
f 1, 2, 4, 5, 10, 20, 25, 50, 100 **g** 1, 3, 9, 27 **h** 1, 3, 5, 9, 15, 45
i 1, 2, 3, 4, 6, 9, 12, 18, 36 **j** 1, 2, 4, 5, 8, 10, 20, 40
k 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72, 144 **l** 1, 2, 4, 5, 8, 10, 16, 20, 32, 40, 80, 160
m 1, 2, 3, 4, 6, 11, 12, 22, 33, 44, 66, 132 **n** 1, 11, 121 **o** 1, 2, 4, 8, 19, 38, 76, 152

4 **a** F **b** T **c** T **d** T **e** T **f** T **g** T **h** T **i** T **j** F

5 **a** 3, 6, 9, 12, 15, 18, 21, 24, 27 **b** 5, 10, 15, 20, 25, 30, 35, 40, 45
c 2, 4, 6, 8, 10, 12, 14, 16, 18 **d** 10, 20, 30, 40, 50, 60, 70, 80, 90
e 4, 8, 12, 16, 20, 24, 28, 32, 36 **f** 11, 22, 33, 44, 55, 66, 77, 88, 99
g 6, 12, 18, 24, 30, 36, 42, 48, 54 **h** 9, 18, 27, 36, 45, 54, 63, 72, 81
i 7, 14, 21, 28, 35, 42, 49, 56, 63 **j** 8, 16, 24, 32, 40, 48, 56, 64, 72

6 **a** 12 **b** 45 **c** 116 **d** 15

7 **a** 2 **b** 15 **c** 20 **d** 14 **e** 6 **f** 50 **g** 48 **h** 1

8 **a** 35 **b** 60 **c** 30 **d** 150

9 **a** 12 **b** 200 **c** 36 **d** 28 **e** 40 **f** 120 **g** 40 **h** 66

10 **a** 65, 70, 75, 80, 85 **b** 1, 2, 4, 5, 8, 10, 20, 40 **c** 2, 4, 6, 8, 10, 12, 14, 16, 18 **d** 30, 60, 90
e 24, 30, 36, 42, 48

Prep Quiz 3:06

1 48 **2** 48 **3** 48 **4** 48 **5** 1, 2, 3, 4, 6, 8, 12, 16, 24, 48
6 6, 12, 18, 24, 30 **7** 6, 12, 18, 24, 30 **8** 1, 30, 2, 15, 3, 10, 5, 6
9 1, 48, 2, 24, 3, 16, 4, 12, 6, 8 **10** 1182

Exercise 3:06

1

Number	*Factors*	*Number*	*Factors*
1	1	11	1, 11
2	1, 2	12	1, 12, 2, 6, 3, 4
3	1, 3	13	1, 13
4	1, 4, 2	14	1, 14, 2, 7
5	1, 5	15	1, 15, 3, 5
6	1, 6, 2, 3	16	1, 16, 2, 8, 4
7	1, 7	17	1, 17
8	1, 8, 2, 4	18	1, 18, 2, 9, 3, 6
9	1, 9, 3	19	1, 19
10	1, 10, 2, 5	20	1, 20, 2, 10, 4, 5

a 2, 3, 5, 7, 11, 13, 17, 19 **b** 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20

2 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 91, 97, 101, 103, 107, 109, 113

3 **a** 2 **b** 7 **c** 9, 15 **d** 15
e $6 = 3 + 3$, $8 = 5 + 3$, $10 = 5 + 5$, $12 = 5 + 7$, $14 = 7 + 7$, $16 = 5 + 11$, $18 = 7 + 11$

4 **a** $3 \times 2 \times 2$ **b** $2 \times 2 \times 5$ **c** $2 \times 3 \times 3$ **d** $2 \times 2 \times 2 \times 2$

5 **a** $2 \times 3 \times 5$ **b** $2 \times 2 \times 3 \times 3$ **c** $2 \times 2 \times 5 \times 5$ **d** $2 \times 2 \times 2 \times 2 \times 3 \times 3$ **e** $2 \times 2 \times 2 \times 2 \times 2 \times 3$
f $3 \times 3 \times 5 \times 5$
6 **a** $2^3 \times 3^3$ **b** $3^3 \times 5^2$ **c** $3 \times 5 \times 7^2$ **d** $2^3 \times 3^2 \times 7$ **e** $3^2 \times 5^3$
f $2^4 \times 7 \times 11$ **g** $3^3 \times 7^2$ **h** $3^3 \times 5^3$

Prep Quiz 3:07

1 1, 2, 4, 5, 8, 10, 20, 40 **2** 70 **3** 18, 24, 30 **4** 998 **5** 1001 **6** 51 **7** 5 **8** 255 **9** 28 983 **10** 11 344 678

Exercise 3:07

1 3842, 5816, 9000, 8774, 8166, 7008 **2** 3006, 7110, 21 441, 8145, 211 002, 78
3 1004, 6124, 8156, 61 852 **4** 30 024, 81 810, 41 238, 765, 936
5 6633, 10 406, 92 180, 61 809
6 **a** 4, 5, 8, 11 **b** 11 **c** 4, 11 **d** 3, 5, 9, 11 **e** 3, 11
7 **a** 1078, 7600, 13 476 **b** 13 476, 33 885 **c** 7600, 13 476 **d** 13 476
e 33 885 **f** 7600 **g** 1078 **h** 7600 **i** 7600 **j** 7600
8 Test for divisibility of both 4 and 6. **9** **a** 3 and 5 **b** 2 and 9 **c** 3 and 4
10 **a** 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, 18, 20, 24, 30, 36, 40, 45, 48, 60, 72, 80, 90, 120, 144, 180, 240, 360, 720
b 1, 2, 173, 346 **c** 1, 2, 3, 5, 6, 9, 10, 15, 18, 27, 30, 45, 54, 90, 135, 270
d 1, 2, 3, 4, 5, 6, 7, 9, 10, 12, 14, 15, 18, 20, 21, 28, 30, 35, 36, 42, 45, 60, 63, 70, 84, 90, 105, 126, 140, 180, 210, 252, 315, 420, 630, 1260 **e** 1, 3, 7, 9, 21, 27, 63, 81, 189, 567
f 1, 3, 5, 7, 9, 15, 21, 25, 35, 45, 63, 75, 105, 175, 225, 315, 525, 1575 **g** 1, 3, 9, 27, 81, 243, 729
h 1, 2, 3, 4, 5, 6, 8, 10, 11, 12, 15, 16, 20, 22, 24, 30, 32, 40, 44, 48, 55, 60, 88, 96, 110, 120, 132, 165, 176, 220, 240, 264, 330, 352, 440, 480, 528, 660, 880, 1056, 1320, 1760, 2640, 5280
11 **a** 1002 **b** 1008 **c** 1008 **d** 1001 **e** 1025 **f** 1100 **g** 1005 **h** 1008
12 **a** 990 **b** 162 **c** 900 **d** 1, 3, 5, 7, 15, 21, 25, 35, 75, 105, 175, 525
e 1, 3, 9, 11, 27, 33, 99, 297

Prep Quiz 3:08

1 25 **2** 16 **3** 196 **4** 8 **5** 27 **6** 64 **7** 6 **8** 10 **9** 42 **10** 100

Exercise 3:08

1 **a** 7 **b** 11 **c** 13 **d** 5 **e** 9 **f** 12 **g** 16 **h** 21 **i** 14 **j** 7
k 4 **l** 9 **m** 18 **n** 21 **o** 99 **p** 30 **q** 1001 **r** 275
2 **a** 18 **b** 15 **c** 45 **d** 385 **e** 140 **f** 30
3 **a** 15 **b** 22 **c** 6 **d** 4 **e** 9 **f** 10
4 **a** 3 **b** 1 **c** 2 **d** 4 **e** 8 **f** 6 **g** 0 **h** 10 **i** 9 **j** 12
k 18 **l** 20 **m** 36 **n** 42 **o** 15
5 **a** 2 **b** 3 **c** 1 **d** 4 **e** 10 **f** 0 **g** 6 **h** 8 **i** 11 **j** 15
k 16 **l** 18
6 **a** 3, 4 **b** 8, 9 **c** 4, 5 **d** 6, 7 **e** 9, 10 **f** 1, 2 **g** 13, 14 **h** 5, 6 **i** 11, 12 **j** 4, 5
k 12, 13 **l** 10, 11
7 **a** 16 **b** 18 **c** 26 **d** 39 **e** 33 **f** 22
8 **a** 5 **b** 9 **c** 11 **d** 20 **e** 17 **f** 12

Prep Quiz 3:09

1 $(3 \times 10^4) + (1 \times 10^3) + (6 \times 10^2) + (2 \times 10^1) + (7 \times 1)$ **2** $(1 \times 10^4) + (5 \times 10^3) + (0 \times 10^2) + (3 \times 10^1) + (8 \times 1)$
3 63 789 **4** 10 520 **5** 2, 4, 8, 16, 32, 64 **6** 13 **7** 13 **8** 30 **9** 30 **10** 20

Exercise 3:09

1 **a** 2 **b** 3 **c** 4 **d** 5 **e** 6 **f** 7 **g** 8 **h** 9 **i** 10 **j** 11
k 45 **l** 24 **m** 35 **n** 63 **o** 13
2 **a** 10 001 **b** 10 100 **c** 1 000 000 **d** 100 011 **e** 101 000

3 **a**

81	1	0	1	0	0	0	1	$1\,010\,001_{\text{two}}$
107	1	1	0	1	0	1	1	$1\,101\,011_{\text{two}}$
71	1	0	0	0	1	1	1	$1\,000\,111_{\text{two}}$
80	1	0	1	0	0	0	0	$1\,010\,000_{\text{two}}$
100	1	1	0	0	1	0	0	$1\,100\,100_{\text{two}}$
97	1	1	0	0	0	0	1	$1\,100\,001_{\text{two}}$
125	1	1	1	1	1	0	1	$1\,111\,101_{\text{two}}$

Fun Spot 3:09 Making magic squares

1 a

6	11	4
5	7	9
10	3	8

b

20	45	10
15	25	35
40	5	30

c

27	12	33
30	24	18
15	36	21

2 a 21 **b** 75 **c** 72 **3**

8	256	2
4	16	64
128	1	32

, 4096

Diagonstic Test 3: Number: Its Order and Structure

1 a two million, four hundred and fifty-six thousand, three hundred and twenty-five
b ninety-seven million, five thousand
c two hundred and ninety-four million, one hundred and sixty-seven thousand
2 a 700 **b** 70 000 **c** 70 000 000 **3 a** 5496 **b** 80 659 **c** 3 210 000
4 a $(5 \times 10^4) + (6 \times 10^3) + (0 \times 10^2) + (1 \times 10^1) + (5 \times 1)$
b $(7 \times 10^5) + (1 \times 10^4) + (4 \times 10^3) + (8 \times 10^2) + (5 \times 10^1) + (0 \times 1)$
c $(3 \times 10^6) + (4 \times 10^5) + (2 \times 10^4) + (0 \times 10^3) + (0 \times 10^2) + (0 \times 10^1) + (0 \times 1)$
5 a 25 **b** 27 **c** 100 000 **6 a** 8^3 **b** 10^4 **c** 9^2
7 a 700 **b** 40 000 **c** 5000 **8 a** 28 **b** 31 **c** 10
9 a 11 **b** 15 **c** 50
10 a $6 \times 900 + 6 \times 8$ **b** $7 \times 600 - 7 \times 3$ **c** $4 \times 400 - 4 \times 2$
11 a 1, 4, 9 **b** 1, 3, 6 **c** 1, 3, 5
12 a 1, 2, 3, 4, 6, 8, 12, 24 **b** 1, 3, 7, 9, 21, 63 **c** 1, 2, 4, 5, 10, 20, 25, 50, 100
13 a 4, 8, 12, 16, 20 **b** 8, 16, 24, 32, 40 **c** 11, 22, 33, 44, 55
14 a 12 **b** 15 **c** 14 **15 a** 36 **b** 24 **c** 140
16 a 2, 3, 5, 7 **b** 4, 6, 8, 9 **c** 1 **17 a** $2 \times 2 \times 2 \times 3$ **b** $2 \times 3 \times 2 \times 3$ **c** $7 \times 2 \times 3$
18 a HCF = 4, LCM = 120 **b** HCF = 105, LCM = 4725 **c** HCF = 15, LCM = 630
19 a 5 **b** 77 **c** 12 **20 a** 3 **b** 11 **c** 6

3A Revision Assignment

1 a 3 **b** 4 **c** 63 **d** 29 **e** 48 **f** 17
2 15 700 **3 a** 15 072 **b** 16 328 **c** 13 816
4 a True **b** True **c** True **d** True **e** True **f** False
5 a 1846 **b** 0 **c** 8145 **d** 446 **e** 0 **f** 4186 **g** 11 **h** 9800 **i** 3700
6 a False **b** False **c** True **d** False **e** True **f** True **g** True **h** False **i** True
7 a 25, 27, 13 **b** 41 214 **c** 25 **d** 27 **e** 13 **f** 41 214
8 a 1, 100, 2, 50, 4, 25, 5, 20, 10 **b** 1, 125, 5, 25 **c** 1, 5, 25
9 a 8, 16, 24, 32, 40, 48, 56, 64, 72, 80 **b** 12, 24, 36, 48, 60, 72, 84, 96, 108, 120 **c** 24
10 a 2, 5, 7, 41 **b** 12, 15 36

3B Working Mathematically

1 a *oval (or ellipse)* **b** *cube* **c** *rectangular prism* **d** *triangular prism*
e *square pyramid* **f** *rectangular pyramid* **g** *triangular pyramid* **h** *cylinder*
i *cone* **j** *sphere* **2** 25, 64, 49, 144, 225, 324, 16, 9, 100, 81
3 270 **4** 9 **5** \$48; \$18 **6** 4 **7** 20
8 a i 2003 **ii** 2005 **b** 2000 **c** 800 **d** approximately 500; no
e no **f** no. You can't tell what fraction the parts of symbols represent.

Chapter 4: Decimals

Exercise 4:01

1 a true **b** true **c** true **d** true **e** true **f** true **g** false **h** true
i true **j** true
2 a 0·64 **b** 0·17 **c** 0·238 **d** 0·051 **e** 0·309 **f** 0·066 **g** 2·35 **h** 9·4
i 15·36 **j** 11·02 **k** 6·081 **l** 2·503
3 a $(7 \times 1) + (3 \times \frac{1}{10}) + (4 \times \frac{1}{100}) + (2 \times \frac{1}{1000})$ **b** $(3 \times 1) + (4 \times \frac{1}{10}) + (8 \times \frac{1}{100}) + (3 \times \frac{1}{1000})$
c $(4 \times 1) + (2 \times \frac{1}{10}) + (1 \times \frac{1}{100}) + (5 \times \frac{1}{1000})$ **d** $(4 \times 10) + (5 \times 1) + (3 \times \frac{1}{100})$
e $(3 \times 10) + (7 \times \frac{1}{10}) + (5 \times \frac{1}{100})$ **f** $(4 \times 10) + (7 \times \frac{1}{100})$

g $(2 \times 100) + (5 \times 10) + (6 \times 1) + (4 \times \frac{1}{100})$ **h** $(2 \times 10) + (4 \times 1) + (1 \times \frac{1}{10}) + (2 \times \frac{1}{100}) + (5 \times \frac{1}{1000})$

i $(6 \times 10) + (9 \times 1) + (3 \times \frac{1}{10}) + (4 \times \frac{1}{100}) + (5 \times \frac{1}{1000})$

j $(2 \times 100) + (4 \times 10) + (6 \times 1) + (3 \times \frac{1}{10}) + (7 \times \frac{1}{100}) + (9 \times \frac{1}{1000})$

k $(4 \times 100) + (8 \times 10) + (2 \times 1) + (3 \times \frac{1}{10}) + (7 \times \frac{1}{100}) + (5 \times \frac{1}{1000})$

l $(2 \times 100) + (3 \times 10) + (8 \times 1) + (6 \times \frac{1}{10}) + (6 \times \frac{1}{100}) + (7 \times \frac{1}{1000})$

4 **a** 0·7 **b** 0·05 **c** 1·2 **d** 0·05 **e** 0·69 **f** 0·03 **g** 0·1 **h** 2·8
i 0·612 **j** 0·5 **k** 0·049 **l** 0·29

5 **a** 0·3, 0·6, 0·8 **b** 1·2, 3·85, 7·5 **c** 0·55, 0·8, 1 **d** 0·03, 0·3, 3
e 0·03, 0·3, 0·33 **f** 0·39, 0·4, 0·41 **g** 0·792, 0·85, 0·9 **h** 0·09, 0·1, 0·8

6 **a** 1·36 **b** 1·84 **7** 2·152

8 **a** 2 **b** 3 **c** 1 **d** 1 **e** 2 **f** 3 **g** 3 **h** 2
i 1 **j** 2 **k** 3 **l** 1

9 **a** 0·23 **b** 0·12 **c** 0·007 **d** 0·125 **e** 0·015 **f** 0·604 **g** 2·93 **h** 15·103
i 10·09 **j** 7·999 **k** 11·033 **l** 8·005

10 **a** $\frac{7}{10}$ **b** $\frac{9}{10}$ **c** $\frac{3}{10}$ **d** $\frac{21}{100}$ **e** $\frac{77}{100}$ **f** $\frac{91}{100}$ **g** $\frac{447}{1000}$ **h** $\frac{601}{1000}$
i $\frac{853}{1000}$ **j** $2\frac{99}{100}$ **k** $8\frac{3}{10}$ **l** $9\frac{7}{1000}$

11 **a** 1·4 **b** 8·6 **c** 7·3 **d** 7·24 **e** 8·06 **f** 9·03 **g** 1·186 **h** 7·406
i 8·014 **j** 45·04 **k** 650·4 **l** 0·842

12 **a** $\frac{3}{10}$ **b** $\frac{3}{10}$ **c** $\frac{3}{10}$ **d** $\frac{7}{10}$ **e** $\frac{7}{10}$ **f** $\frac{7}{10}$ **g** $\frac{1}{100}$ **h** $\frac{1}{100}$
i $\frac{1}{100}$ **j** $\frac{9}{100}$ **k** $\frac{3}{100}$ **l** $\frac{1}{1000}$

13 **a** 500 **b** $\frac{5}{10}$ **c** $\frac{5}{1000}$ **d** $\frac{5}{100}$ **e** $\frac{5}{1000}$ **f** $\frac{5}{100}$ **g** $\frac{5}{10\,000}$ **h** $\frac{5}{100}$

14 **a** false **b** false **c** true **d** true **e** true **f** true **g** true **h** true

Exercise 4:02

1 **a** 5·86 **b** 16·05 **c** 23·235 **d** 35 466·4 **e** 66 673·5 **f** 13·0006 **g** 21·24 **h** 4·6
i 10 **j** 12·12 **k** 14·35 **l** 159·96 **m** 19·025 **n** 18·65 **o** 24·225 **p** 0·465

2 **a** 2·45 **b** 8·65 **c** 7·202 **d** 11·625 **e** 5·375 **f** 8·975 **g** 6·78 **h** 31·085
i 30·089 **j** 7·7 **k** 6·3 **l** 2·75 **m** 8·634 **n** 8·29 **o** 23·72 **p** 4·909
q 0·386 **r** 3·117 **s** 671·25 **t** 518·95 **u** 4718·155

Prep Quiz 4:03

1 0·8 **2** 2 **3** 2·4 **4** 1·8 **5** 3·2 **6** 2·5 **7** 1·2 **8** 3 **9** 2·1 **10** 3·2

Exercise 4:03

1 **a** 0·6 **b** 2·4 **c** 4·4 **d** 0·4 **e** 0·21 **f** 0·48 **g** 0·66 **h** 2·1 **i** 0·36
2 **a** 14·4 **b** 7·6 **c** 1 **d** 19 **e** 23·1 **f** 12 **g** 1·89 **h** 9·272 **i** 137·6
3 **a** 7·6 **b** 12·33 **c** 9·24 **d** 3 **e** 45·99 **f** 30·35 **g** 115·01 **h** 156·48 **i** 313·84
4 **a** 12·5 **b** 91·25 **c** 843·5 **d** 65 **e** 138 **f** 37 **g** 231·5 **h** 685 **i** 12
j 30 **k** 810 **l** 12·5 **m** 111·1 **n** 15 **o** 8100 **p** 750 **q** 30 050 **r** 6750

Exercise 4:04

1 **a** 0·21 **b** 2·1 **c** 0·004 **d** 6·03 **e** 3·03 **f** 3·09 **g** 9·2 **h** 0·002 **i** 1·218
2 **a** 0·07 **b** 4·9 **c** 0·9 **d** 0·01 **e** 0·025 **f** 0·003 **g** 1·26 **h** 30·4 **i** 0·126
3 **a** 0·25 **b** 0·35 **c** 0·15 **d** 0·26 **e** 0·35 **f** 0·335 **g** 0·365 **h** 0·1375 **i** 2·35
j 0·75 **k** 1·5 **l** 0·625

4 **a** 3·41 **b** 7·55 **c** 6·48 **d** 0·06 **e** 0·015 **f** 0·005 **g** 7·516 **h** 0·0038
i 67·845 **j** 0·0004 **k** 0·0035 **l** 0·35 **m** 0·8764 **n** 3·1564 **o** 3·437 **p** 0·0346
q 0·00875 **r** 0·0008

5 **a** 1·846 **b** 0·675 **c** 0·873 **d** 97·65 **e** 3·468 **f** 0·08 **g** 0·0346 **h** 0·04 **i** 7·58

Exercise 4:05

1 **a** \$18.97 **b** \$27.65 **c** \$4.10 **d** \$5.37 **e** \$13.28 **f** \$49.65 **g** \$44.70 **h** \$9.40
i \$11.70 **j** \$1.69 **k** \$0.76 **l** \$2.20

2 **a** 3·125 **b** 0 **c** 3·008 **d** 2·37 **e** 9·83 **f** 9 **g** 36·08 **h** 0 **i** 9·8

3 **a** true **b** true **c** false **d** false **e** true **f** true **g** true **h** true

4 **a** 8 **b** 5·8 **c** 225 **d** 4·5 **e** 0·5 **f** 16 **g** 4·3 **h** 14 **i** 9·79

5 **a** 128·7 km **b** 59·5 km **c** 8·9 km and 9 km. He may have entered the ground by different gates, parked in a different spot or travelled a different route.

6 80 seconds
7 **a** 4·2 L **b** 42 L
8 **a** FE and DC (2·225 m and 3·55 m) **b** 5·775 m **c** 15·55 m **d** 1·325 m

Prep Quiz 4:06

1 2·4 **2** 3 **3** 7·05 **4** 0·152 **5** 3·5 **6** 71 **7** 0·08 **8** 1·5 **9** 12·42 **10** 21

Exercise 4:06

1 **a** 2 **b** 1 **c** 3 **d** 4 **e** 2 **f** 1 **g** 2 **h** 1 **i** 3
2 **a** 0·24 **b** 0·088 **c** 0·07 **d** 0·62 **e** 4·2 **f** 0·09 **g** 0·028 **h** 0·024
i 0·0049 **j** 0·4 **k** 0·03 **l** 0·6 **m** 0·54 **n** 0·056 **o** 0·16
3 **a** 0·0375 **b** 1·284 **c** 2·68 **d** 2677·5 **e** 34·08 **f** 6840 **g** 6·5 **h** 0·024
i 43·4 **j** 13·6 **k** 28 080 **l** 270 **m** 3·467 **n** 52·32 **o** 8·041
4 **a** 29·76 **b** 0·9888 **c** 11·039 **d** 5·1114 **e** 26·52 **f** 2·856 **g** 117·72 **h** 0·0256
i 12·25 **j** 0·0625 **k** 511·14 **l** 51·114
5 **a** 0·04 **b** 0·0009 **c** 0·36 **d** 0·64 **e** 1·21 **f** 0·0025 **g** 0·000441 **h** 2·25
i 0·0289

Prep Quiz 4:07

1 0·44 **2** 5·95 **3** 6·34 **4** 15 **5** 0·04 **6** 2·7 **7** 1·04 **8** 1·045 **9** 5·5 **10** 0·0034

Exercise 4:07

1 **a** $83·5 \div 5$ **b** $34·4 \div 8$ **c** $651 \div 3$ **d** $0·49 \div 7$ **e** $8 \div 2$ **f** $712·5 \div 5$ **g** $3800 \div 4$ **h** $180 \div 9$
i $8000 \div 8$ **j** $270 \div 3$ **k** $47·2 \div 8$ **l** $220 \div 11$
2 **a** 16·7 **b** 4·3 **c** 217 **d** 0·07 **e** 4 **f** 142·5 **g** 950 **h** 20
i 1000 **j** 90 **k** 5·9 **l** 20
3 **a** 3 **b** 4 **c** 0·4 **d** 10 **e** 20 **f** 6000 **g** 9125 **h** 200
i 50 **j** 490 **k** 1600 **l** 2000
4 **a** 2·3 **b** 10·1 **c** 30 **d** 2 **e** 20 **f** 73 **g** 8·05 **h** 1·62
i 15·2 **j** 12·05 **k** 31 **l** 11·75 **m** 51 **n** 792·75 **o** 90·6

Fun Spot 4:07 Why did the orange stop in the middle of the road?

He wanted to play squash.

Prep Quiz 4:08

1 0·3 **2** 0·07 **3** 1·96 **4** 0·005 **5** 0·037 **6** 8 **7** 2 **8** 12 **9** 37 **10** 1663

Exercise 4:08

1 **a** terminating **b** repeating **c** terminating **d** repeating **e** terminating
f repeating **g** repeating **h** terminating **i** repeating **j** terminating
k repeating **l** repeating
2 **a** $0·\dot{9}$ **b** $3·1\dot{4}$ **c** $0·\dot{1}\dot{8}$ **d** $0·1\dot{6}$ **e** $0·\dot{9}\dot{0}$ **f** $6·\dot{7}4\dot{5}$ **g** $66·\dot{6}$ **h** $0·\dot{1}42857\dot{1}$
3 **a** 0·3 **b** 0·06 **c** 0·009 **d** 0·019 **e** 0·55 **f** 1·1 **g** 3·77 **h** 0·709
i 6·02 **j** 0·5 **k** 0·88 **l** 0·35 **m** 0·35 **n** 0·25 **o** 0·012
4 **a** 0·8 **b** 0·125 **c** 0·25 **d** 0·375 **e** 0·625 **f** 0·875 **g** 0·35 **h** 0·66
i 0·75 **j** $0·\dot{3}$ **k** $0·\dot{6}$ **l** $0·\dot{1}$ **m** $0·\dot{2}$ **n** $0·\dot{4}$ **o** $0·\dot{7}$ **p** $0·8\dot{3}$
q $0·1\dot{6}$ **r** $0·\dot{5}$
5 **a** $3·\dot{8}$ **b** 2·6 **c** $6·\dot{6}$ **d** $0·\dot{2}\dot{7}$ **e** $0·\dot{4}\dot{5}$ **f** $0·\dot{9}\dot{0}$ **g** $2·41\dot{6}$ **h** $0·7\dot{3}$
i 1·1875 **j** $0·\dot{5}$ 71428 **k** $0·\dot{4}$ 2857$\dot{1}$ **l** $0·07\dot{1}42857\dot{1}$
6 **a** 6·875 **b** $0·8\dot{1}$ **c** 12·25 **d** 0·3125 **e** $8·8\dot{3}$ **f** 2·62

Prep Quiz 4:09

1 20 **2** 150 **3** 200 **4** 4 **5** 5 **6** $7.70 **7** 7·7 **8** 3·11 **9** 14·6 and 14·7 **10** 6·75

Exercise 4:09

1 **a** 7900 **b** 1100 **c** 67 300 **d** 900 **e** 600 **f** 400 **g** 74 900 **h** 7900
2 **a** 9 **b** 80 **c** 45 **d** 3 **e** 2 **f** 18 **g** 237 **h** 100
3 **a** 243·13 **b** 79·66 **c** 91·35 **d** 9·81 **e** 0·30 **f** 0·09 **g** 0·10 **h** 1·99
4 **a** 6·7 **b** 8·5 **c** 2·1 **d** 6·1 **e** 0·1 **f** 246·7 **g** 29·9 **h** 10·0
5 $10.72
6 $712

7 **a** 0·4 **b** 0·44 **c** 0·444 **d** 0·4444
8 **a** 0·3 **b** 0·27 **c** 0·273 **d** 0·2727
9 **a** 1363c **b** 1941c **c** 6119c **d** 510c **e** 550c **f** 1768 050c
10 **a** \$14 **b** \$19 **c** \$61 **d** \$5 **e** \$5 **f** \$17 680

Prep Quiz 4:10

1 \$1 **2** \$1 **3** 1 **4** \$3.75 **5** 1·003 **6** \$18.83 **7** \$9.68 **8** \$20.70 **9** 0·06 **10** 2·1

Exercise 4:10

1 **a** \$34.25 **b** \$8.50 **c** \$1.00 **d** \$17.10 **e** \$85.45 **f** \$9.55 **g** \$103.35 **h** \$11.70
i \$27.15 **j** \$277.35 **k** \$108.10 **l** \$654.70
2 **a** \$7.05 **b** \$2.65 **c** \$5.25 **d** \$6.15 **e** \$8.80 **f** \$26.35 **g** \$17.60 **h** \$87.90
i \$23.75 **j** \$29.90 **k** \$45.70 **l** \$36.05 **m** \$22.00. This answer is half way between \$21.95 and \$22.00, so we take the highest amount. **n** \$16.70 **o** \$7.45 **p** \$0.45
3 **a** 17 **b** 836 **4** **a** 8 (of the 0·1 km), 3 (of the 0·01 km), 6 (of the 0·001 km); 17 altogether **b** 836 rolls
5 11·0, 11·06, 11·07, 11·08, 11·4 **6** **a** 99 **b** 9·9 **7** 3 m (ignoring the width of the nails)
8 1014·5 g **9** \$6.05 (rounded to the nearest 5 cents) **10** 2
11 **a** 0·58 s **b** 43·86 s **12** D **13** **a** 5 **b** \$137.75
14 **a** \$0.004 or 0·4 cents **b** 20·8 cents **c** \$260 **d** The employer or anyone who instructed the firm's computer to put these parts of one cent into their account. This would be a form of 'computer crime'.

Fun Spot 4:10 World championship diving

1 42 **2** 40 **3** 25 **4** 44 **5** 40 **6** 47 **7** 52 **8** 12 **9** 71 **10** 39

Diagnostic Test 4: Decimals

1 **a** $(3 \times 10) + (6 \times 1) + (1 \times \frac{1}{10}) + (5 \times \frac{1}{100}) + (2 \times \frac{1}{1000})$
b $(3 \times \frac{1}{10}) + (3 \times \frac{1}{100}) + (3 \times \frac{1}{1000}) + (3 \times \frac{1}{10\,000})$ **c** $(7 \times 1) + (3 \times \frac{1}{100}) + (9 \times \frac{1}{1000})$
2 **a** 0·3, 0·6, 0·9 **b** 0·11, 0·4, 0·6 **c** 0·077, 0·7, 0·77
3 **a** 16·75 **b** 1·058 **c** 1·455 **4** **a** 19·237 **b** 803·57 **c** 43·1
5 **a** 0·120 **b** 23·4 **c** 18·88 **6** **a** 1·07 **b** 1·24 **c** 1·8385
7 **a** 40 **b** 33·75 **c** 1560 **8** **a** 1·3465 **b** 0·009135 **c** 0·0045
9 **a** 0·004 **b** 1·232 **c** 0·0009 **10** **a** 0·34 **b** 4300 **c** 17 500
11 **a** terminating **b** repeating **c** repeating
12 **a** 3·77 **b** 3·77 **c** 6·007 **13** **a** 0·625 **b** $0{\cdot}1\dot{6}$ **c** $1{\cdot}\dot{9}\dot{0}$
14 **a** 78·48 **b** 0·67 **c** 9·30

4A Revision Assignment

1 **a** 1 326 763 **b** sixteen thousand and twenty **c** ten thousand
d $(4 \times 10\,000) + (2 \times 1000) + (3 \times 100) + (7 \times 10) + (5 \times 1)$
2 **a** 42 **b** 17 **c** 12 **d** 3 **3** **a** 19 **b** 22 **c** 155 **d** 3
4 **a** false **b** false **c** true **d** false **5** **a** $\square = 6$ **b** $\square = 73$ **c** $\square = 1$ **d** $\square = 2$
6 **a** 87 **b** 293 **c** 1664 **d** 1990 **7** **a** $4 \times 4 \times 4$, 64 **b** $3 \times 3 \times 3$, 27
c $10 \times 10 \times 10 \times 10 \times 10 \times 10$, 1 000 000 **d** $2 \times 10 \times 10 \times 10 \times 10$, 20 000
e $3 \times 3 \times 2 \times 2 \times 2 \times 2$, 144

4B Working Mathematically

1 **a** *point A* **b** *interval AB* **c** *Line* AB **d** *ray AB* **e** *collinear* points
f *midpoint* **g** *number line* **h** *diagonals*
2 379 **3** 30 rows **4** \$643.50 **5** 63 **6** 112
7 **a** 5% **b** 9% **c** 6% **d** Highland. Both graphs show identical information. **e** 2003

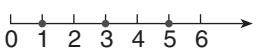
Chapter 5: Directed Numbers and the Number Plane

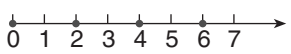
Prep Quiz 5:01

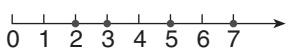
1 $3 + 7 = 10$ **2** $2 + 7 = 9$ **3** $4 + 3 + 3 = 10$ **4** $3 + 3 + 2 = 8$ **5** $10 - 4 = 6$ **6** $8 - 6 = 2$
7 $2 + 2 + 2 + 2 + 2 = 10$ or $5 \times 2 = 10$ **8** $3 + 3 + 3 = 9$ or $3 \times 3 = 9$
9 $10 - 2 - 2 = 6$ or $10 - 2 \times 2 = 6$ **10** $4 + 4 = 8$ or $2 \times 4 = 8$

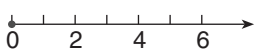
Exercise 5:01

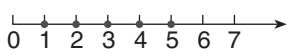
1 **a** {0, 2, 4} **b** {3, 4, 5, 6} **c** {9, 12, 15} **d** {11, 13, 17} **e** $\{\frac{1}{2}, 1\frac{1}{2}, 2\frac{1}{2}, 3\frac{1}{2}\}$
f $\{\frac{1}{4}, \frac{1}{2}, 1\}$ **g** {0·1, 0·5, 0·7} **h** {0, 0·4, 0·8, 1·2}

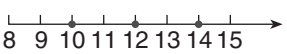
2 a


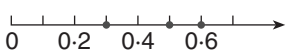
b


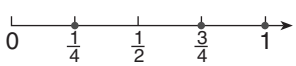
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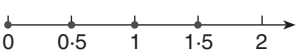
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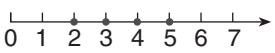
e


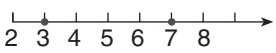
f


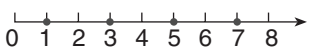
g


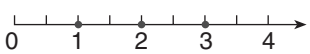
h


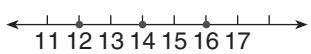
i


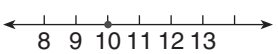
3 a


b


c


d


e


f


Exercise 5:02

1 a Bessemer St **b** Sunnyholt Rd **c** Byrne Blde **d** Noel St **e** Allawah St
f Tattersall Rd **g** Public School **h** Public School **i** Virginia St **j** Rosalind Rd
k Siebel St **l** Meroo St **m** Forbes Rd **n** Blacktown H.S. **o** Chester St
p Breakfast Rd or Richmond Rd **q** Vernon St or Noel St **r** Richmond Rd

2 a F4 **b** A1 **c** G7 **d** B2 or D3 **e** F7 **f** F9 **g** C4 **h** J7

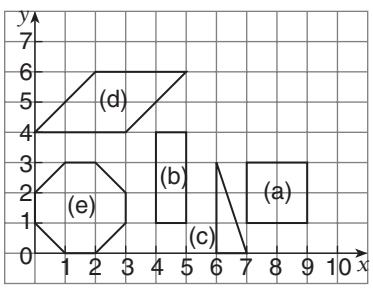
3 a B3, C3, D4 **b** Osborne Rd and Davis Rd **c** J7 or K7

4 a 500 m **b** 1500 m **c** 1000 m **d** 2·6 km

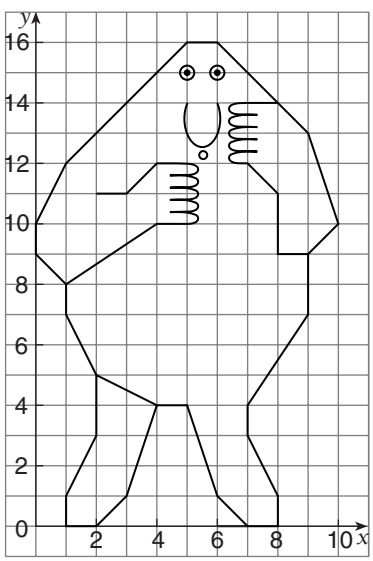
Exercise 5:03

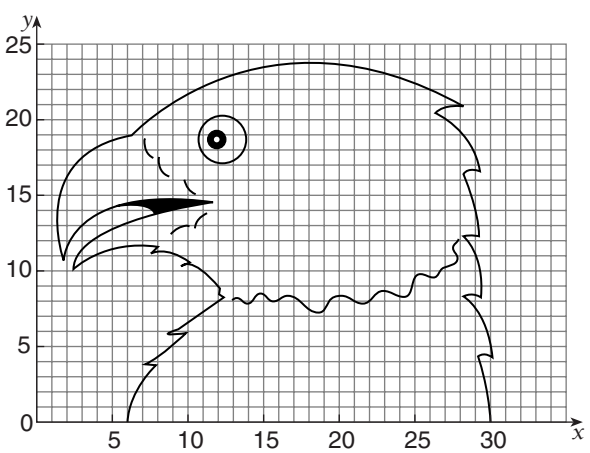
1 a G **b** E **c** U **d** V **e** O **f** Q **g** W **h** R **i** Z **j** F
k J **l** K **m** H **n** P **o** N **p** Y **q** I **r** A **s** C **t** S
u B **v** M **w** L **x** T **y** D **z** X

2 a (4, 4) **b** (0, 4) **c** (4, 2) **d** (1, 1) **e** (2, 2) **f** (0, 2) **g** (1, 3) **h** (4, 3) **i** (2, 1) **j** (3, 0)
k (5, 0) **l** (7, 0) **m** (6, 1) **n** (7, 2) **o** (0, 0) **p** (6, 4) **q** (7, 5) **r** (6, 6) **s** (7, 7) **t** (6, 7)
u (5, 6)

3


a square **b** rectangle **c** triangle
d parallelogram **e** octagon

4


5


Mastery Test 5:03

1 *x*-axis **2** *y*-axis **3** origin (0, 0) **4** (6, 3) **5** (3, 4) **6** (3, 0) **7** (0, 2) **8** B **9** A **10** 4 units

Exercise 5:04

1 a **A** 50 **B** −50 **C** −50 **D** 50 **E** 50 **F** 50 **G** −50 **H** 3 **I** 50 **J** 50 **K** 50 **L** 50 **M** −50 **N** 50 **O** −50 **P** 50 **Q** −50 **R** −50 **S** −50 **T** −50 **U** −50 **V** −50 **W** −3 **X** 50

b A and Q, B and X, C and P, D and R, E and V, F and T, G and N, H and W, I and U, J and O, K and M, L and S

2 a 8 **b** −4 **c** −14 **d** −15 **e** 12 **f** −2 **g** 0 **h** 1800 **i** −60 **j** −135

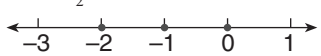
3 and **4** There are many possible answers.

5 a −11 **b** −7 **c** −6 **d** −4 **e** −2 **f** 7 **g** −11

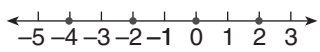
6 a 20° **b** 0° **c** −10° **d** −20° **e** −30°

7 a {−3, −1, 1, 3} **b** {−3, 0, 2} **c** {−20, −10, 0} **d** {−20, −10, 10} **e** $\{-1\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\}$ **f** $\{-2, -\frac{1}{2}, 1\}$ **g** {−1·5, 0, 1·5} **h** {−0·5, 0·5, 1·5}

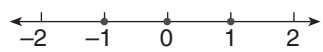
8 a



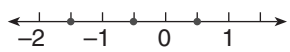
b



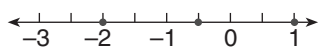
c



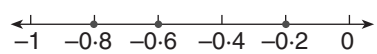
d



e



f



9 a −2° **b** −5° **c** −5° **d** −5° **e** −10° **f** −7°

10 a −2, 1, 4, 5 **b** −4, −1, 0, 1 **c** −4, −3, −2, −1

d −3, −2, 2, 3 **e** −3, −2, 0, 4 **f** −10, −8, 7, 9

Exercise 5:05

1 a

Coordinates chosen	*Result*	*Score*	*Total so far*
(0, 0)	Discovers a lost city	8	8
(0, 4)	Lose your compass	−4	4
(2, 1)	Miss	−1	3
(1, 0)	Bridge destroyed	−3	0
(1, 3)	Photograph a gnu	2	2
(3, 4)	Bitten by a snake	−7	−5

b

Coordinates chosen	*Result*	*Score*	*Total so far*
(3, 1)	Lost in the jungle	−5	−5
(3, 2)	Meet friendly natives	6	1
(0, 2)	Photograph a zebra	1	2
(2, 2)	Shoot yourself in foot	−6	−4
(4, 1)	Miss	−1	−5
(1, 1)	Photograph a tiger	4	−1

2 a

Coordinates chosen	*Score*	*Total so far*
(0, 4)	−4	−4
(1, 4)	−1	−5
(2, 4)	3	−2
(3, 4)	−7	−9
(4, 4)	−8	−17

b

Coordinates chosen	*Score*	*Total so far*
(4, 4)	−8	−8
(3, 3)	−1	−9
(2, 2)	−6	−15
(1, 1)	4	−11
(0, 0)	8	−3

c

Coordinates chosen	*Score*	*Total so far*
(3, 0)	−1	−1
(3, 1)	−5	−6
(3, 2)	6	0
(3, 3)	−1	−1
(3, 4)	−7	−8

3 a

Score	Total so far
−1	−1
4	3
−2	1
1	2
−5	−3

b

Score	Total so far
1	1
−6	−5
−4	−9
−3	−12
5	−7

c

Score	Total so far
−1	−1
−1	−2
−1	−3
−1	−4
−1	−5

d

Score	Total so far
−8	−8
1	−7
2	−5
3	−2
4	2

4 a a loss of 4 **b** a gain of 8 **c** a loss of 1 **d** a loss of 5 **e** a loss of 17
f a loss of 1 **g** −7 **h** 3 **i** −3 **j** 0 **k** −15 **l** −2 **m** −5
n 14 **o** 7 **p** −13 **q** −3 **r** −9 **s** 0 **t** −2 **u** −1 **v** 0

Prep Quiz 5:06

1 1 **2** −3 **3** 0 **4** −1 **5** 5 **6** −2 **7** 0 **8** −5 **9** −10 **10** −2

Exercise 5:06

1 a −4 **b** 2 **c** 0 **d** −7 **e** 11 **f** 2 **g** −3 **h** 4 **i** −5 **j** −7
k −17 **l** −8 **m** −13 **n** −9 **o** −8 **p** −10 **q** 1 **r** −6 **s** 1 **t** 93
u 8 **v** 0 **w** −5 **x** −6

2 a $10 - 30 = -20$ **b** $-10 + 20 = 10$ **c** $10 + 30 = 40$ **d** $-10 - 20 = -30$ **e** $5 - 50 = -45$
f $-20 + 40 = 20$ **g** $-53 + 20 = -33$ **h** $33 - 100 = -67$ **i** $-4 + 33 = 29$ **j** $-6 + 33 = 27$

3 a 2 **b** 1 **c** 0 **d** −1 **e** −2 **f** −3 **g** −2 **h** −3 **i** −7 **j** −3
k −2 **l** −10 **m** −2 **n** −7 **o** −6 **p** −8 **q** −10 **r** −4 **s** −10 **t** −18
u −60 **v** −50 **w** −60 **x** −21

4 a 0 **b** 0 **c** 0 **d** 0 **e** 0 **f** 0 **g** 0 **h** 0 **i** 0 **j** 0
k 0 **l** 0 **m** 0 **n** 0 **p** 0 **q** 0 **r** 0

5 a 3 **b** −2 **c** 7 **d** 3 **e** −2 **f** 7 **g** 2 **h** −4 **i** 5 **j** 2
k −4 **l** 5

6 a −12 **b** 3 **c** −2 **d** −5 **e** 4 **f** −3 **g** −8 **h** −1 **i** −6 **j** −7
k −2 **l** 2 **m** 0 **n** 1 **o** −2 **p** −4

Exercise 5:07

1 a 7 **b** 9 **c** 6 **d** 16 **e** 8 **f** 12 **g** 11 **h** 23 **i** −1 **j** 4
k 7 **l** −1 **m** 3 **n** −5 **o** 5 **p** −3 **q** 10 **r** −6 **s** 6 **t** −56
u 15 **v** −1 **w** 0 **x** 0

2 a 5 **b** 15 **c** 2 **d** 8 **e** 34 **f** 7 **g** 11 **h** 9

3 a 17 **b** −2 **c** 1 **d** 7 **e** −17 **f** 18 **g** −21 **h** −10 **i** 5 **j** 10
k 10 **l** −4 **m** 10 **n** 7 **o** 14 **p** −9 **q** 10 **r** 10 **s** 13 **t** 13
u 1 **v** 8 **w** −14 **x** −4

4 a 0·5 **b** 2·9 **c** −1 **d** −0·54 **e** −0·1 **f** −0·7 **g** $-\frac{6}{10} = -\frac{3}{5}$ **h** $-\frac{1}{2}$ **i** $-\frac{7}{10}$
j $\frac{3}{8}$ **k** $\frac{2}{3}$ **l** −1

ID 5:07

1 $6 - 2 = 4$ **2** $6 + 2 = 8$ **3** $6 \div 2 = 3$ **4** $6 - 2 = 4$ **5** $6 \div 2 = 3$
6 2 **7** 6 **8** $6 \times 2 = 12$ **9** $6 - 2 = 4$ **10** $6 \times 2 = 12$
11 $2 + 6 = 8$ **12** $6 - 2 = 4$ **13** $6^2 = 36$ **14** $\sqrt{36} = 6$ **15** $6 - 2 = 4$
16 $6 \times 2 = 12$ **17** $(6 + 2) \div 2 = 4$ **18** $6 + 2 = 8$ **19** $6^2 = 36$ **20** $6 - 2 = 4$
21 $6 - 2 = 4$ **22** $6 + 2 = 8$ **23** $6 \div 2 = 3$ **24** $6 + 2 = 8$

Prep Quiz 5:08

1 (0, 0) **2** F **3** P **4** K **5** C **6** D **7** M **8** 0 **9** J **10** B

Exercise 5:08

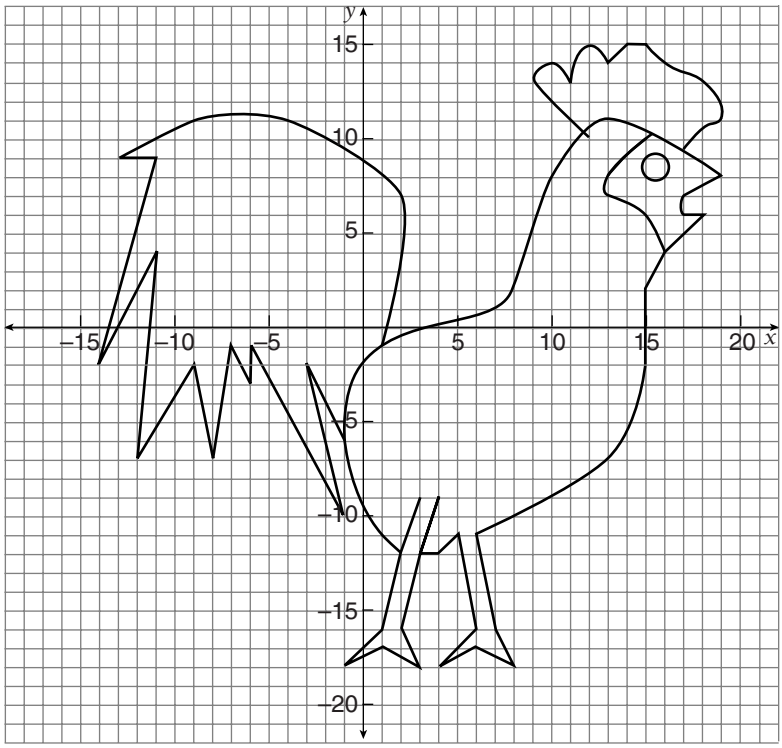
1 a X **b** K **c** O **d** S **e** W **f** I **g** G **h** L **i** U **j** M
k J **l** P **m** Q **n** Z **o** C **p** T **q** A **r** B **s** F **t** D
u H **v** R **w** Y **x** V **y** N **z** E

2 a (2, 1) **b** (3, 0) **c** (2, −1) **d** (3, −3)
e (0, −2) **f** (−1, −1) **g** (−2, 0) **h** (−2, 2)
i (0, 3) **j** (1, 2) **k** (1, −1) **l** (1, −3)
m (−1, −3) **n** (−1, 0) **o** (0, 0) **p** (−2, 1)

q (0, 1) **r** (2, −2) **s** (−2, −2) **t** (−3, −3)
u (−3, 1) **v** (−3, 3) **w** (−1, 2) **x** (3, 2)
y (2, 0) **z** (0, −1)

3 **a** Seek wisdom seek knowledge. **b** Discipline your children while they are young enough to learn.

4



Prep Quiz 5:09

1 4×6 **2** 3×-7 **3** 3×8 **4** 3×-2 **5** $3 \times -10 = -30$
6 $6 \times -3 = -18$ **7** $10 \times -7 = -70$ **8** ..., −2, −4 **9** ..., −3, −6 **10** ..., 3, 6

Exercise 5:09

1 **a** −1 **b** −8 **c** −12 **d** −25 **e** −8 **f** −12 **g** −44 **h** −16 **i** 10 **j** 9
k 8 **l** 1 **m** 0 **n** 0 **o** 0 **p** 0 **q** −21 **r** −6 **s** 12 **t** −12
u −20 **v** 7 **w** −16 **x** 14

2 **a** −35 **b** −28 **c** −27 **d** −45 **e** 16 **f** 32 **g** 42 **h** −48 **i** 24 **j** −81
k −40 **l** −36 **m** −56 **n** −49 **o** 28 **p** −72 **q** 36 **r** −72 **s** 63 **t** −24
u 121 **v** 54 **w** −64 **x** 63

3 **a** 1 **b** 4 **c** 9 **d** 16 **e** 25 **f** 36 **g** 49 **h** 64 **i** 81 **j** 100
k 121 **l** 144

4 **a** −320 **b** 150 **c** −90 **d** −240 **e** −1·8 **f** −4 **g** −0·04 **h** 0·18 **i** −0·07 **j** −0·077
k −0·48 **l** 0·56

5 **a** −27 **b** −8 **c** −125 **d** 50 **e** 18 **f** 64 **g** −285 **h** −40 **i** −36 **j** −12
k 18 **l** −24

Investigation 5:09 Multiplying Directed Numbers

Tape forwards (+) or backwards (−)	*Boy's movement forward (+) or backwards (−)*	*Apparent result on screen*	*Number sentence*
Forward for 3 seconds (+3)	Forward at 4 steps/second (+4)	12 steps forward (+12)	$3 \times 4 = 12$
Forwards for 3 seconds (+3)	Backwards at 4 steps/second (−4)	12 steps backwards (−12)	$3 \times -4 = -12$
Backwards for 3 seconds (−3)	Forward at 4 steps/second (+4)	12 steps backwards (−12)	$-3 \times 4 = -12$
Backwards for 3 seconds (−3)	Backward at 4 steps/second (−4)	12 steps forward (+12)	$-3 \times -4 = 12$

Challenge 5:09 Using Pronumerals

1 a

x	1	2	3	4
y	0	−1	−2	−3

b

x	0	2	4	6
y	0	−8	−16	−24

c

x	−3	−1	1	3
y	9	5	1	−3

2 a 11 **b** 28 **c** 49 **d** 4
3 a $x = -3$ **b** $x = 1$ **c** $x = -14$ **d** $x = 3$ or -3

Prep Quiz 5:10

1 30 **2** −30 **3** 30 **4** −30 **5** $6 \times 5 = 30$ **6** $6 \times -5 = -30$ **7** $-6 \times -5 = 30$ **8** $-6 \times 5 = -30$
9 5 **10** −2

Exercise 5:10

1 a 6 **b** −6 **c** −6 **d** 4 **e** −4 **f** −10 **g** −9 **h** 6 **i** −5 **j** −7
k 8 **l** −10 **m** 5 **n** −4 **o** −3 **p** −8 **q** 9 **r** −5 **s** −1 **t** −1
u 1
2 a −5 **b** −3 **c** 7 **d** −7 **e** 3 **f** −6 **g** −4 **h** 8 **i** −5 **j** −7
k 6 **l** 6 **m** 70 **n** −90 **o** 30

Prep Quiz 5:11

1 −7 **2** −13 **3** −2 **4** 9 **5** −2 **6** 6 **7** −40 **8** 80 **9** −8 **10** 64

Exercise 5:11

1 a −2 **b** −1 **c** −14 **d** 4 **e** −12 **f** −10 **g** 90 **h** 1 **i** −1 **j** 1
k −9 **l** 0 **m** 4 **n** −14
2 a 5·2 **b** −10·1 **c** −1·8 **d** 4·22 **e** −90·44 **f** −5·35 **g** −38·1 **h** 0·25 **i** 0·09 **j** −10·3
k 1·04 **l** −0·01 **m** −1 **n** −1 **o** −1·6
3 a 1, −1, −3 **b** −2, 0, 2 **c** −32, −64, −128 **d** 32, −64, 128 **e** −6, 6, −6
f 4, −2, 1
4 a −2 **b** −12 **c** −5 **d** 5 **e** −2 **f** 5
5 a −11 **b** 8 **c** −4 **d** −18 **e** −15 **f** −6
6 a false **b** true **c** false **d** true **e** true **f** true **g** true **h** true **i** true
7 −2 **8 a** −7, 1, 4 **b** −3, 1, 4 **c** −5, 1, 4
9 a −38 **b** −1·02 **c** −23
10 a −7 **11** −2 **12** −9°C **13** *Southern Cross* won by 7 seconds. **14** Vladivostok is 10 hours ahead.
15 a −4 and −3 **b** 6 and −5 **16** B: a loss of 0·1 kg

Prep Quiz 5:12

1 −2 **2** −10 **3** −8 **4** −9 **5** −7 **6** −5 **7** −24 **9** 12 **9** 16 **10** 4

Exercise 5:12

1

Question	*Without calculator*	*With calculator*
$-3 - 8$	−11	−11
$-9 - 2$	−11	−11
$-7 + 2$	−5	−5
$-15 + 6$	−9	−9
$-7 + 23$	16	16
$-2 + 15$	13	13
$-2 - -3$	1	1
$8 - -2$	10	10

Question	*Without calculator*	*With calculator*
-3×4	−12	−12
-6×3	−18	−18
-7×-2	14	14
-9×-5	45	45
4×-7	−28	−28
10×-6	−60	−60
$20 \div (-5)$	−4	−4
$-18 \div -3$	6	6

2 a −578 **b** −602 **c** 294 **d** −2275 **e** −135 **f** −2508 **g** −48
h −2093 **i** −2121
3 a 165 649 **b** 7·29 **c** 0·0169
4 a 71 **b** 229 **c** 59·24 **d** 1·802 **e** −59 **f** 1080

Fun Spot 5:12 Try this quick quiz!

1

Operation	Start	× (−2)	+ 10	÷ 5	− 3	+ 6
Answer	10	−20	−10	−2	−5	1

2

Operation	Start	− 10	× 3	+ 8	÷ 2	+ 2
Answer	6	− 4	− 12	−4	−2	0

3

Operation	Start	+ 3	× (−5)	− 5	÷ 2	+ 10
Answer	−2	1	−5	−10	−5	5

4

Operation	Start	− 3	÷ 4	+ 4	×(−3)	+ 7
Answer	−5	−8	−2	2	−6	1

5

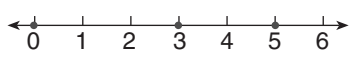
Operation	Start	− 10	× 5	+ 8	÷ 2	− 4
Answer	8	− 2	− 10	− 2	− 1	− 5

6

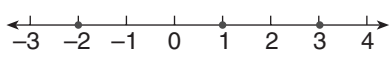
Operation	Start	− 6	÷ 3	+ 5	×(−1)	− 2
Answer	0	− 6	− 2	3	− 3	− 5

Diagnostic Test 5: Directed Numbers and the Number Plane

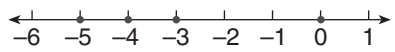
1 a



b



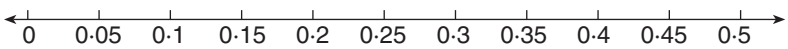
c



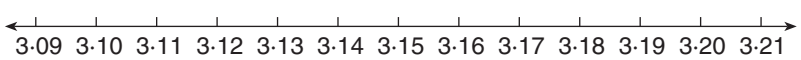
2 a (1, −1) **b** (1, 2) **c** (−1, 0)
3 a G **b** D **c** F **d** H
4 a −2 **b** 4 **c** −15
5 a −9 **b** −10 **c** −1
6 a −8 **b** −5 **c** −7
7 a 13 **b** −2 **c** −1
8 a −56 **b** 40 **c** −24
9 a 36 **b** 1 **c** 100
10 a −10 **b** 7 **c** −7
11 a 5 **b** −6 **c** −8
12 a −4 **b** 0 **c** 7
13 a ..., 0, −3, −6 **b** ..., 0, 10, 20 **c** ..., 81, −243, 729 **d** ..., −4, −2, −1
14 a 1 **b** −4 **c** −1
15 a −3 **b** −11

5A Revision Assignment

1 a 18 **b** 50 **c** 19 **d** 9
2 a 14 **b** 625 **c** 1987 **d** 479
3 a $\frac{5}{100}$ **b** $\frac{5}{10}$ **c** 5 **d** $\frac{5}{1000}$
4 a



b



5 a 7320 **b** 6048 **c** 67 400 **d** 3 050 000

6 a

Operation	Start	− 7	× 3	+ 11	− 35	÷ 6
Answer	5	−2	−6	5	−30	−5

b

Operation	Start	× 3	− 45	÷ 9	+ 5	× 96
Answer	−3	− 9	− 54	−6	−1	−96

7 a $6.00 **b** $5.00 **c** $20.00 **d** $100.00

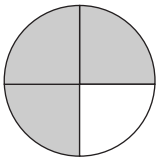
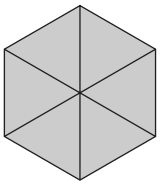
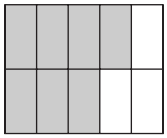
5B Working Mathematically

1 (6) *regular* shapes (7) *net* of a cube (8) *cross-section* (9) *face* (10) *vertex* (11) *edge* (12) axes of *symmetry* (13) *reflection* (or flip) (14) *translation* (or slide) (15) *rotation* (or turn) (16) *tessellation*
2 1 (green) and 6 (red), 6 (green) and 1 (red), 2 (green) and 5 (reds), 5 (green) and 2 (red), 3 (green) and 4 (red), 4 (green) and 3 (red)
3 She made a profit of $30. **4** $216
5 5 games have been played. There have been either 3 draws and 2 wins or 3 wins, 1 draw and a loss in 5 games.
6 The ferry will leave at 9 am, 10:30 am, 12 pm, 2 pm and 3:30 pm. Lunch will be from 1:20 pm to 1:50 pm.
7 a 1997 **b** 1998 **c** 3 **d** 4 **e** $600 000 **f** $800 000
8 a i 2°C **ii** 0°C **iii** −4°C **iv** −3°C **v** 4°C **b** after 50 min **c** after 40 min and 70 min **d** approximately −2°C **e** between 40 min and 50 min after the start of timing; the change was −4°C

Chapter 6: Fractions, Percentages and Probability

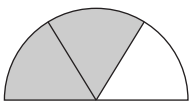
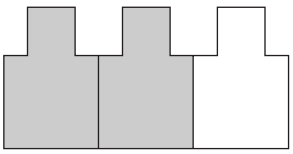
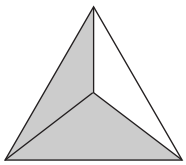
Exercise 6:01

1 **a** $\frac{3}{4}$ **b** $\frac{2}{5}$ **c** $\frac{1}{6}$

2 **a**  **b**  **c** 

3 **a** $\frac{7}{20}$ **b** $\frac{7}{12}$ **c** $\frac{7}{15}$ **4** $\frac{8}{24} = \frac{4}{12} = \frac{2}{6} = \frac{1}{3}$

5 **a** 12 **b** 8 **c** 6 **d** 4 **e** 3 **f** 16 **g** 18 **h** 20 **i** 9

6 **a**  **b**  **c** 

7 **a** 14 **b** 4 **c** 14 **d** 2 **e** $2\frac{1}{2}$ **f** 3 **g** $1\frac{1}{2}$ **h** 4 **i** $3\frac{1}{2}$

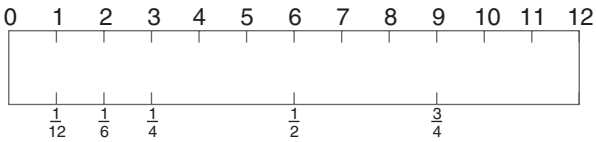
8 **a** \$12.60 **b** 230 **c** 19 500 000

9 **a** 1130 L **b** 4520 L **10** **a** 10 **b** 50

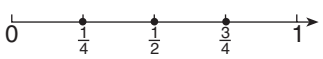
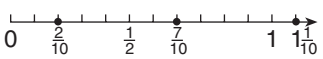
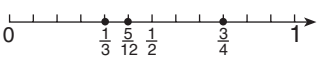
Exercise 6:02

1 **a** $\frac{1}{2}$ **b** $\frac{1}{2}$ **c** $\frac{1}{4}$ **d** $\frac{1}{4}$ **e** decreases

2 **a** $\frac{1}{2}$ **b** $\frac{1}{3}$ **c** $\frac{1}{6}$ **3** **a** 6 **b** 3 **c** 4 **d** 2 **e** 1 **f** 9

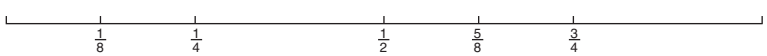
4 

5 $\{\frac{3}{10}, \frac{7}{10}, \frac{9}{10}, 1\frac{1}{10}, 1\frac{3}{10}\}$

6 **a**  **b**  **c** 

7 **a** $\frac{1}{2} > \frac{3}{10}$ **b** $\frac{9}{10} < 1\frac{1}{10}$ **c** $1\frac{1}{2} > \frac{13}{10}$ **8** **a** $\frac{1}{4}, \frac{3}{10}, \frac{1}{2}$ **b** $\frac{1}{5}, \frac{1}{3}, \frac{3}{4}$ **c** $\frac{1}{2}, \frac{7}{10}, \frac{3}{4}$

9 **a** $\frac{2}{2} = \frac{5}{5} = \frac{10}{10}$ **b** $\frac{5}{10}$ **c** $\frac{2}{10}$ **d** $\frac{4}{10}$ **e** $\frac{6}{10}$ **f** $\frac{8}{10}$

10 

11 **a** your estimate, 60 mm **b** your estimate, 40 mm **c** your estimate, 80 mm **d** your estimate, 72 mm

12 **a** D **b** A **c** B **d** C **e** F **f** E **g** G

Exercise 6:03

1 **a** $\frac{1}{2} = \frac{2}{4} = \frac{5}{10}$ **b** $\frac{1}{4} = \frac{2}{8} = \frac{25}{100}$ **c** $\frac{1}{5} = \frac{2}{10} = \frac{20}{100}$ **d** $\frac{3}{2} = \frac{6}{4} = \frac{15}{10}$ **e** $\frac{3}{4} = \frac{6}{8} = \frac{75}{100}$ **f** $\frac{2}{5} = \frac{4}{10} = \frac{40}{100}$

2 **a** 8 **b** 30 **c** 12 **d** 30 **e** 12 **f** 35

3 **a** 12 **b** 9 **c** 16 **d** 32 **e** 2 **f** 36

4 **a** $\frac{3}{5}$ **b** $\frac{3}{4}$ **c** $\frac{1}{3}$ **d** $\frac{5}{8}$ **e** $\frac{7}{20}$ **f** $\frac{1}{3}$ **g** $\frac{1}{3}$ **h** $\frac{5}{7}$ **i** $\frac{3}{5}$

5 **a** $\frac{1}{3}$ **b** $\frac{1}{18}$ **c** $\frac{1}{2}$ **d** $\frac{2}{3}$ **e** $\frac{7}{18}$ **f** $\frac{3}{4}$ **g** $\frac{1}{72}$ **h** $\frac{1}{9}$ **i** $\frac{11}{18}$

6 **a** $\frac{1}{5}, \frac{1}{2}, \frac{7}{10}$ **b** $\frac{1}{2}, \frac{5}{8}, \frac{3}{4}$ **c** $\frac{1}{2}, \frac{11}{24}, \frac{5}{8}$ **d** $\frac{5}{12}, \frac{2}{3}, \frac{3}{4}$ **e** $\frac{73}{100}, \frac{3}{4}, \frac{4}{5}$ **f** $\frac{1}{8}, \frac{11}{40}, \frac{3}{10}$

7 **a** < **b** = **c** > **d** < **e** > **f** <

8 **a** $1\frac{3}{4}$ **b** $1\frac{3}{8}$ **c** $1\frac{1}{2}$ **d** $1\frac{3}{5}$ **e** $1\frac{3}{10}$ **f** $7\frac{23}{100}$

9 **a** $\frac{7}{2}$ **b** $\frac{11}{4}$ **c** $\frac{4}{3}$ **d** $\frac{32}{3}$ **e** $\frac{48}{5}$ **f** $\frac{767}{10}$

10 **a** $\frac{4}{5}$ **b** $\frac{4}{5}$ **c** $\frac{11}{20}$ **d** $1\frac{1}{2}$ **e** 1 **f** $1\frac{1}{20}$ **g** $\frac{3}{4}$ **h** $1\frac{2}{5}$ **i** $1\frac{1}{3}$

11 **a** $\frac{3}{5}$ **b** $\frac{3}{5}$ **c** $\frac{1}{2}$ **d** 1 **e** 1 **f** $\frac{9}{20}$ **g** $\frac{47}{100}$ **h** $\frac{1}{3}$ **i** $\frac{13}{16}$

12 **a** $1\frac{3}{4}$ **b** $4\frac{3}{10}$ **c** $6\frac{2}{3}$ **d** $\frac{1}{4}$ **e** $\frac{7}{10}$ **f** $\frac{1}{3}$

13 **a** $1\frac{1}{2}$ **b** $2\frac{1}{4}$ **c** 3 **d** $2\frac{1}{4}$ **e** $1\frac{1}{2}$ **f** 3

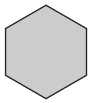
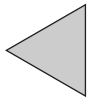
14 **a** $\frac{4}{5}$ **b** 1 **c** $4\frac{2}{3}$ **d** $\frac{9}{10}$ **e** $5\frac{2}{5}$ **f** $3\frac{1}{5}$ **g** $1\frac{17}{100}$ **h** $1\frac{1}{2}$ **i** $9\frac{1}{2}$

15 **a** 5 **b** 20 **c** 3 **d** 15 **e** 80 **f** 21 **g** 105 **h** 300 **i** 990

16 **a** $\frac{3}{4}$ **b** $\frac{5}{8}$ **c** $\frac{7}{10}$ **d** 0 **e** 0 **f** 0 **g** $\frac{9}{10}$ **h** $\frac{7}{8}$ **i** true

j true **k** yes **l** no **m** 6 **n** 6 **o** yes **p** yes

Investigation 6:03 Fractions

What fraction?				
	1	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{4}$
	3	1	$\frac{1}{2}$	$\frac{3}{4}$

ID 6:03

1 *square* **2** *rectangle* **3** *parallelogram* **4** *rhombus*

5 *trapezium* **6** *regular pentagon* **7** *regular hexagon* **8** *regular octagon*

9 *kite* **10** *scalene triangle* **11** *isosceles triangle* **12** *equilateral triangle*

13 *circle* **14** *oval (or ellipse)* **15** *cube* **16** *rectangular prism*

17 *triangular prism* **18** *square pyramid* **19** *rectangular pyramid* **20** *triangular pyramid*

21 *cylinder* **22** *cone* **23** *sphere* **24** *hemisphere*

Prep Quiz 6:04

1 9 **2** 4 **3** $1\frac{1}{12}$ **4** $\frac{5}{12}$ **5** $\frac{12}{15}$ **6** $\frac{10}{15}$ **7** $1\frac{7}{15}$ **8** $\frac{2}{15}$ **9** $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12}$

10 $\frac{1}{5} = \frac{2}{10} = \frac{3}{15} = \frac{4}{20} = \frac{5}{25} = \frac{6}{30}$

Exercise 6:04

1 **a** $\frac{9}{10}$ **b** $\frac{5}{6}$ **c** $\frac{7}{8}$ **d** $\frac{11}{20}$ **e** $\frac{7}{10}$ **f** $\frac{5}{12}$ **g** $\frac{7}{15}$ **h** $\frac{19}{30}$ **i** $\frac{57}{100}$

2 **a** $\frac{7}{10}$ **b** $\frac{7}{15}$ **c** $\frac{1}{8}$ **d** $\frac{23}{30}$ **e** $\frac{1}{4}$ **f** $\frac{7}{20}$ **g** $\frac{9}{20}$ **h** $\frac{53}{100}$ **i** $\frac{3}{10}$

3 **a** $\frac{2}{5}$ **b** $\frac{1}{3}$ **c** $\frac{1}{2}$ **d** $\frac{3}{10}$ **e** 1 **f** $\frac{1}{4}$ **g** $1\frac{1}{5}$ **h** 1 **i** 0

j $\frac{1}{20}$ **k** 0 **l** 2

4 **a** $\frac{9}{10}$ **b** $1\frac{3}{10}$ **c** $\frac{9}{10}$ **d** $\frac{3}{5}$ **e** $\frac{4}{5}$ **f** $1\frac{1}{5}$ **g** $1\frac{1}{12}$ **h** $\frac{11}{12}$ **i** $1\frac{5}{12}$

j $\frac{5}{6}$ **k** $1\frac{1}{6}$ **l** $1\frac{5}{6}$ **m** $\frac{17}{20}$ **n** $1\frac{3}{20}$ **o** $1\frac{11}{20}$ **p** $1\frac{9}{20}$ **q** $1\frac{4}{15}$ **r** $1\frac{1}{4}$

s $1\frac{1}{10}$ **t** $\frac{19}{30}$ **u** $\frac{4}{5}$

5 **a** $\frac{2}{5}$ **b** $\frac{1}{5}$ **c** $\frac{3}{10}$ **d** $\frac{7}{40}$ **e** $\frac{9}{40}$ **f** $\frac{3}{40}$ **g** $\frac{3}{10}$ **h** $\frac{1}{12}$ **i** $\frac{1}{6}$

j $\frac{11}{20}$ **k** $\frac{1}{20}$ **l** $\frac{1}{20}$ **m** $\frac{3}{20}$ **n** $\frac{4}{15}$ **o** $\frac{1}{10}$ **p** $\frac{11}{30}$ **q** $\frac{2}{5}$ **r** $\frac{11}{50}$

s $\frac{5}{24}$ **t** $-\frac{1}{4}$ **u** $\frac{19}{100}$

6 **a** $\frac{11}{12}$ **b** $\frac{2}{9}$ **c** $\frac{19}{24}$ **d** $\frac{11}{50}$ **e** $\frac{37}{100}$ **f** $\frac{11}{24}$ **g** $\frac{7}{24}$ **h** $\frac{61}{100}$ **i** $\frac{2}{3}$

j $\frac{89}{120}$ **k** $\frac{13}{28}$ **l** $\frac{7}{36}$

Prep Quiz 6:05

1 8 **2** 7 **3** 10 **4** 6 **5** 14 **6** 20 **7** 12 **8** 10 **9** 15 **10** 10

Exercise 6:05

1 **a** $1\frac{3}{4}$ **b** $3\frac{1}{5}$ **c** $7\frac{1}{10}$ **d** $13\frac{1}{2}$ **e** $5\frac{1}{3}$ **f** $9\frac{7}{8}$ **g** $10\frac{1}{4}$

2 **a** $5\frac{7}{10}$ **b** $6\frac{7}{8}$ **c** $2\frac{3}{4}$ **d** 6 **e** 10 **f** 4 **g** $8\frac{1}{2}$ **h** $13\frac{4}{5}$ **i** $7\frac{1}{2}$

3 **a** $2\frac{1}{4}$ **b** $4\frac{1}{10}$ **c** $3\frac{1}{2}$ **d** $3\frac{4}{5}$ **e** $13\frac{9}{10}$ **f** $11\frac{1}{4}$ **g** $6\frac{17}{24}$ **h** $7\frac{9}{20}$ **i** $3\frac{33}{40}$
j $4\frac{19}{40}$ **k** $12\frac{1}{6}$ **l** $6\frac{11}{30}$ **m** $9\frac{7}{15}$ **n** $5\frac{9}{10}$ **o** $12\frac{33}{100}$ **p** $7\frac{17}{30}$ **q** $8\frac{7}{12}$ **r** $7\frac{19}{120}$

4 **a** $\frac{9}{10}$ **b** $\frac{3}{10}$ **c** $\frac{1}{4}$ **d** $\frac{3}{8}$ **e** $\frac{7}{10}$ **f** $\frac{3}{20}$ **g** $\frac{7}{100}$ **h** $\frac{3}{4}$ **i** $\frac{11}{15}$

5 **a** $1\frac{9}{10}$ **b** $4\frac{7}{10}$ **c** $6\frac{5}{8}$ **d** $3\frac{5}{12}$ **e** $8\frac{29}{30}$ **f** $2\frac{13}{15}$ **g** $9\frac{3}{10}$ **h** $\frac{19}{24}$ **i** $\frac{69}{100}$

6 **a** $2\frac{3}{4}$ **b** $5\frac{1}{2}$ **c** $5\frac{2}{5}$ **d** $4\frac{1}{10}$ **e** $6\frac{5}{12}$ **f** $1\frac{71}{100}$ **g** $\frac{3}{8}$ **h** 4 **i** $\frac{1}{2}$

7 **a** $4\frac{3}{10}$ **b** $3\frac{5}{8}$ **c** $3\frac{3}{10}$ **d** $2\frac{5}{12}$ **e** $9\frac{1}{30}$ **f** $2\frac{11}{40}$ **g** $\frac{3}{20}$ **h** $\frac{5}{16}$ **i** $\frac{1}{6}$

8 **a** $3\frac{5}{8}$ **b** $4\frac{7}{10}$ **c** $4\frac{2}{3}$ **d** $4\frac{5}{6}$ **e** $7\frac{19}{30}$ **f** $6\frac{23}{40}$ **g** $2\frac{177}{200}$ **h** $\frac{199}{200}$ **i** $6\frac{11}{20}$

Prep Quiz 6:06

1 $2\frac{5}{8}$ **2** $2\frac{4}{5}$ **3** $\frac{2}{5}$ **4** $\frac{2}{5}$ **5** $\frac{9}{4}$ **6** $\frac{11}{3}$ **7** $\frac{9}{5}$ **8** $\frac{1}{3}$ **9** $\frac{1}{4}$ **10** $\frac{1}{12}$

Exercise 6:06

1 **a** $\frac{1}{6}$ **b** $\frac{9}{20}$ **c** $\frac{9}{16}$ **d** $\frac{3}{40}$

2 **a** $\frac{1}{100}$ **b** $\frac{21}{100}$ **c** $\frac{7}{100}$ **d** $\frac{63}{100}$ **e** $\frac{3}{100}$ **f** $\frac{27}{100}$ **g** $\frac{77}{1000}$ **h** $\frac{3}{1000}$ **i** $\frac{63}{1000}$

3 **a** $\frac{1}{4}$ **b** $\frac{1}{8}$ **c** $\frac{1}{6}$ **d** $\frac{3}{20}$ **e** $\frac{9}{40}$ **f** $\frac{3}{16}$ **g** $\frac{3}{25}$ **h** $\frac{35}{48}$ **i** $\frac{63}{80}$

4 **a** $\frac{2}{5}$ **b** $\frac{1}{5}$ **c** $\frac{3}{8}$ **d** $\frac{1}{5}$ **e** $\frac{1}{4}$ **f** $\frac{1}{10}$ **g** $\frac{5}{8}$ **h** $\frac{1}{3}$ **i** $\frac{7}{20}$
j $\frac{3}{5}$ **k** $\frac{27}{40}$ **l** $\frac{1}{6}$ **m** $\frac{9}{16}$ **n** $\frac{135}{600}$ **o** $\frac{27}{50}$ **p** $\frac{9}{32}$ **q** $\frac{1}{2}$ **r** $\frac{3}{8}$

5 **a** 1 **b** 1 **c** 1 **d** 1 **e** 1 **f** 1 **g** 1 **h** 1 **i** 1

6 **a** $1\frac{1}{8}$ **b** $1\frac{2}{3}$ **c** $1\frac{1}{4}$ **d** $1\frac{5}{8}$ **e** $\frac{14}{15}$ **f** $2\frac{1}{10}$ **g** $4\frac{3}{8}$ **h** $4\frac{1}{5}$ **i** $2\frac{4}{5}$
j $2\frac{5}{8}$ **k** $4\frac{1}{20}$ **l** $4\frac{1}{6}$ **m** $18\frac{2}{5}$ **n** $4\frac{1}{5}$ **o** $3\frac{1}{25}$ **p** 8 **q** $14\frac{2}{5}$ **r** $14\frac{1}{4}$
s 1 **t** 1 **u** 1

Prep Quiz 6:07

1 3 **2** 4 **3** 11 **4** 5 **5** 3 **6** yes **7** 4 **8** 10 **9** 9 **10** 11

Exercise 6:07

1 **a** 2 **b** 2 **c** 3 **d** $1\frac{4}{5}$ **e** $2\frac{1}{3}$ **f** 4 **g** $\frac{1}{4}$ **h** $\frac{3}{5}$ **i** $\frac{7}{8}$

2 **a** $\frac{2}{1}$ **b** $\frac{3}{1}$ **c** $\frac{4}{1}$ **d** $\frac{8}{5}$ **e** $\frac{8}{7}$ **f** $\frac{10}{1}$ **g** $\frac{3}{2}$ **h** $\frac{5}{4}$ **i** $\frac{100}{49}$

3 **a** $1\frac{1}{2}$ **b** $1\frac{7}{8}$ **c** $2\frac{4}{5}$ **d** $\frac{16}{25}$ **e** $\frac{12}{35}$ **f** $3\frac{1}{3}$ **g** $\frac{9}{16}$ **h** $\frac{1}{4}$ **i** $\frac{1}{7}$
j $1\frac{1}{4}$ **k** $1\frac{1}{8}$ **l** $1\frac{1}{20}$ **m** $\frac{5}{6}$ **n** 5 **o** $16\frac{2}{3}$ **p** $\frac{14}{25}$ **q** $3\frac{1}{8}$ **r** $1\frac{1}{5}$

4 **a** 20 **b** $3\frac{1}{5}$ **c** $1\frac{1}{2}$ **d** $2\frac{1}{2}$ **e** $3\frac{3}{4}$ **f** $5\frac{1}{3}$ **g** 9 **h** 8 **i** $9\frac{1}{3}$
j $1\frac{5}{6}$ **k** $7\frac{1}{2}$ **l** $6\frac{2}{5}$ **m** $\frac{7}{12}$ **n** $1\frac{11}{32}$ **o** $1\frac{7}{20}$ **p** $1\frac{1}{6}$ **q** $2\frac{2}{3}$ **r** $1\frac{25}{41}$
s $2\frac{7}{10}$ **t** $7\frac{7}{11}$ **u** $1\frac{69}{76}$

Prep Quiz 6:08

1 $2\frac{3}{4}$ **2** $3\frac{1}{7}$ **3** $\frac{7}{12}$ **4** $\frac{73}{10}$ **5** $\frac{3}{4} = \frac{18}{24}$, $\square = 18$ **6** $\frac{9}{10} = \frac{90}{100}$, $\square = 90$ **7** $\frac{3}{8}$ **8** $\frac{2}{3}$ **9** $1\frac{1}{5}$ **10** $\frac{3}{10}$

Exercise 6:08

1 **a** 20 cm **b** 15 min **c** 125 g **d** 30 cm **e** 30°
f 20 min **g** 2 g **h** 10 g **i** 20 mL

2 **a** 80 cm **b** 45 min **c** 625 g **d** 270 cm **e** 60°
f 1 h 40 min **g** 26 g **h** 210 g **i** 1980 mL

3 **a** $\frac{1}{2}$ **b** $\frac{1}{2}$ **c** $\frac{3}{10}$ **d** $\frac{4}{5}$ **e** $\frac{1}{4}$ **f** $\frac{1}{25}$ **g** $\frac{1}{4}$ **h** $\frac{3}{7}$ **i** $\frac{1}{50}$

4 **a** 4500 L **b** $30 000 **c** $480 000 **d** 1·8 kg **e** 6·3 kg **f** 7·875 km
g $4.20 **h** 5670 g **i** 258 cm

5 **a** $\frac{3}{10}$ **b** $\frac{1}{5}$ **c** $\frac{1}{60}$ **d** $\frac{1}{1000}$ **e** $\frac{2}{45}$ **f** $\frac{1}{4}$ **g** $\frac{1}{2}$ **h** $\frac{17}{500}$ **i** $\frac{3}{100}$

Exercise 6:09

1 **a** $\frac{9}{100}$ **b** $\frac{7}{100}$ **c** $\frac{1}{100}$ **d** $\frac{37}{100}$ **e** $\frac{99}{100}$ **f** $\frac{97}{100}$ **g** $\frac{11}{100}$ **h** $\frac{13}{100}$ **i** $\frac{33}{100}$

2 **a** $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{4}{5}$ **d** $\frac{1}{10}$ **e** $\frac{1}{20}$ **f** $\frac{13}{50}$ **g** $\frac{11}{25}$ **h** $\frac{7}{20}$ **i** $\frac{1}{50}$

3 **a** 1 **b** $1\frac{1}{2}$ **c** $2\frac{1}{4}$ **d** $3\frac{1}{10}$ **e** $1\frac{3}{4}$ **f** 4 **g** $2\frac{3}{100}$ **h** $1\frac{3}{25}$ **i** $1\frac{9}{100}$

4 **a** 3% **b** 7% **c** 9% **d** 18% **e** 27% **f** 69% **g** 50% **h** 44% **i** 90%

5 **a** 5% **b** 75% **c** 70% **d** 80% **e** 30% **f** 26% **g** 200% **h** 150% **i** 310%

6 **a** 0·14 **b** 0·37 **c** 0·81 **d** 0·07 **e** 0·02 **f** 0·01 **g** 0·6 **h** 0·3 **i** 0·9
j 1 **k** 1·26 **l** 2·15

7 **a** 35% **b** 76% **c** 88% **d** 4% **e** 6% **f** 5% **g** 40% **h** 60% **i** 50%
j 187% **k** 260% **l** 102%

8 **a** 70% **b** 72% **c** 175% **d** 4% **e** 19% **f** 5% **g** 40% **h** 19% **i** 100%
j 4% **k** 10% **l** 63% **m** 80% **n** 97% **o** 75% **p** 12·5% **q** $6 **r** 7 kg
s **i** $200 **ii** $40 **iii** $100 **iv** $80

Prep Quiz 6:10

1 $\frac{39}{100}$ **2** 3% **3** 35% **4** 0·16 **5** 13% **6** 1 **7** 100% **8** 15 **9** 7 **10** 32·5

Exercise 6:10

1 **a** 60% **b** 90% **c** 14% **d** 85% **e** 25% **f** 73% **g** 50% **h** 32% **i** 15% **j** 75%
k 80% **l** 9% **m** $66\frac{2}{3}$% **n** $16\frac{2}{3}$% **o** $37\frac{1}{2}$% **p** $62\frac{1}{2}$% **q** $66\frac{1}{2}$% **r** $1\frac{4}{5}$% **s** $6\frac{3}{4}$% **t** $20\frac{1}{3}$%
u 150% **v** 225% **w** $333\frac{1}{3}$% **x** $314\frac{2}{7}$%

2 **a** 37% **b** 71% **c** 95% **d** 4% **e** 70% **f** 20% **g** 10% **h** 60% **i** 34·5% **j** 12·5%
k 37·5% **l** 82·5% **m** 30% **n** 4·5% **o** 85% **p** 2% **q** 180% **r** 110% **s** 250% **t** 360%
u 100% **v** 300% **w** 200% **x** 500%

3 **a** 1% **b** $\frac{1}{20}$ **c** $33\frac{1}{3}$% **d** 0·05 **e** 0·1 **f** $\frac{1}{8}$ **g** 0·2 **h** $12\frac{1}{2}$% **i** $\frac{1}{3}$ **j** 0·125

4 **a** 3%, 0·03, $\frac{3}{100}$ **b** 15%, 0·15, $\frac{3}{20}$ **c** 90%, 0·9, $\frac{9}{10}$ **d** 150%, 1·5, $1\frac{1}{2}$ **e** 60%, 0·6, $\frac{3}{5}$
f $66\frac{2}{3}$%, $0.\dot{6}$, $\frac{2}{3}$ **g** 90%, 0·9, $\frac{9}{10}$ **h** 9%, 0·09, $\frac{9}{100}$ **i** 75%, 0·75, $\frac{3}{4}$ **j** $37\frac{1}{2}$%, 0·375, $\frac{3}{8}$

Exercise 6:11

1 **a** $\frac{9}{400}$ **b** $\frac{7}{400}$ **c** $\frac{9}{200}$ **d** $\frac{27}{400}$ **e** $\frac{51}{400}$ **f** $\frac{33}{200}$ **g** $\frac{59}{400}$ **h** $\frac{39}{200}$
i $\frac{49}{250}$ **j** $\frac{143}{1000}$ **k** $\frac{213}{200}$ **l** $\frac{435}{400}$

2 **a** 0·043 **b** 0·186 **c** 0·077 **d** 0·112 **e** 0·215 **f** 0·1725 **g** 0·1375 **h** 0·045
i 0·973 **j** 0·375 **k** 1·125 **l** 2·538

3 **a** $\frac{7}{8}$ **b** $\frac{13}{80}$ **c** $\frac{1}{30}$

Exercise 6:12

1 **a** 88 **b** 99 **c** 60 h **d** 24 mg **e** 8 km **f** 6 g **g** 110 ha **h** 18 min
i 304 m **j** 560 L **k** 112 **l** 33 m

2 **a** $60 **b** $24 **c** $18 **d** $47 **e** $51 **f** $440 **g** $1.20 **h** $0.35
i $1.02 **j** $31.32 **k** $36.63 **l** $21.93

3 **a** 6 people are injured. **b** Ben gave $80. **c** Heather has 18 doves.
d 5100 books are in the fiction section. 900 books are not in this section.
e We had 3880 worms left. **f** 48 ha of parks were made.

Prep Quiz 6:13

1 $\frac{1}{2}$ **2** $\frac{2}{7}$ **3** $\frac{1}{4}$ **4** $\frac{1}{2}$ **5** $\frac{1}{4}$ **6** $\frac{13}{50}$ **7** $\frac{1}{5}$ **8** $\frac{1}{5}$ **9** $\frac{1}{3}$ **10** $\frac{15}{16}$

Exercise 6:13

1 **a** 10% **b** 5% **c** 80% **d** 70% **e** 12%

2 **a** 50% **b** 25% **c** 50% **d** 75% **e** 75% **f** 40% **g** $6\frac{1}{2}$% **h** $5\frac{1}{4}$%

3 **a** 22% **b** 60% are not gouldians. **c** $12\frac{1}{2}$% **d** $42\frac{1}{2}$% **e** $12\frac{1}{2}$%

4 **a** 25% **b** 5% **c** $18\frac{3}{4}$% **d** $51\frac{1}{4}$% **e** 75% **f** $56\frac{1}{4}$%

5 **a** 4% **b** $33\frac{1}{3}$% **c** $47\frac{1}{2}$%. (*Note:* There are 40 years from 1947 to 1986 inclusive.)
d 20% **e** 5% **f** 100% − 16% = 84%

Exercise 6:14

1 100 g **2** 5, 14 **3** **a** 936 **b** 624 **c** 39 **4** **a** 36 **b** 40 **c** 4

5 $\frac{1}{10}$, 10% **6** $5\frac{1}{4}$ h **7** **a** 15 h **b** 4 h **8** $\frac{11}{12}$; $91\frac{2}{3}$%

9 **a** 60% **b** $\frac{2}{5}$ **10** 3750 **11** 24%; 25

12 **a** 80% **b** 20% **c** $\frac{2}{7}$ **d** 71%

13 **a** 520 **b** 52% **c** $\frac{12}{25}$

14 **a** $\frac{1}{5}$ **b** 20% **15** **a** $8\frac{1}{4}$ pages **b** $1\frac{3}{8}$ pages per min **c** $2\frac{3}{4}$ pages per min

16 **a** 10 000 **b** 4 **17** 600 **18** 90% **19** \$43 000 **20** $\frac{1}{6}$ **21** \$2310 **22** $\frac{1}{8}$; $\frac{3}{8}$

23 \$600 **24** 106 **25** $\frac{3}{4}$ **26** $1\frac{3}{4}$ h **27** $\frac{3}{8}$ **28** $\frac{1}{6}$

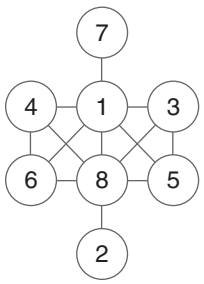
Exercise 6:15

1 25% **2** $37\frac{1}{2}$% **3** $37\frac{1}{2}$% **4** 75% **5** $54\frac{1}{6}$% **6** $65\frac{5}{8}$% **7** $62\frac{1}{2}$%

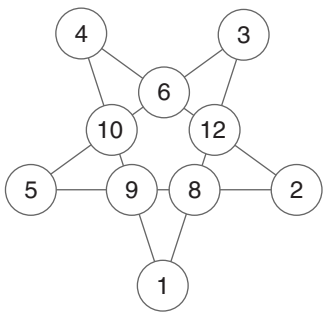
8 $67\frac{68}{96}$% (or $67\frac{17}{24}$% or $\frac{65}{96}$ or $67{\cdot}708\dot{3}$%) **9** 75% **10** $37\frac{1}{2}$% **11** $62\frac{1}{2}$% **12** 59%

Fun Spot 6:15 Where should the numbers go?

1



2



Exercise 6:16

1 **a** to **c** Answers may vary. **d** impossible **e** certain **f** even chance

2 Answers will vary but they will start with **d** and end with **e**.

3 **a** $\frac{1}{6}$ **b** $\frac{0}{6}$ or 0 **c** $\frac{1}{2}$ **d** $\frac{1}{2}$ **e** $\frac{6}{6}$ or 1

4 **a** $\frac{3}{6}$ or $\frac{1}{2}$ **b** $\frac{4}{6}$ or $\frac{2}{3}$ **c** $\frac{2}{6}$ or $\frac{1}{3}$ **d** $\frac{5}{6}$ **5** **a** 50% **b** $66\frac{2}{3}$% **c** $33\frac{1}{3}$% **d** $83\frac{1}{3}$%

6 **a** $\frac{3}{6}$ or $\frac{1}{2}$ **b** $\frac{2}{6}$ or $\frac{1}{3}$ **c** $\frac{4}{6}$ or $\frac{2}{3}$ **d** $\frac{1}{6}$ **7** **a** **i** $\frac{1}{2}$ **ii** $\frac{1}{4}$ **iii** $\frac{1}{4}$ **b** 1

8 **a** $\frac{1}{8}$ **b** $\frac{1}{2}$ **c** $\frac{1}{2}$ **d** $\frac{3}{8}$ **e** 1 **f** $\frac{2}{8}$ or $\frac{1}{4}$ **g** $\frac{1}{2}$ **h** 0
i $\frac{6}{8}$ or $\frac{3}{4}$ **j** $\frac{5}{8}$ **9** **a** $\frac{1}{5}$ **b** $\frac{2}{5}$ **c** $\frac{3}{5}$ **d** $\frac{4}{5}$

10 **a** $\frac{1}{2}$ **b** $\frac{2}{6}$ or $\frac{1}{3}$ **c** $\frac{1}{6}$ (The number 1 is neither prime nor composite.)
d $\frac{4}{6}$ or $\frac{2}{3}$ (1, 2, 3 and 5 are Fibonacci numbers.) **e** $\frac{2}{6}$ or $\frac{1}{3}$ (1 and 4 are square numbers.)
f $\frac{3}{6}$ or $\frac{1}{2}$ (1, 3 and 6 are triangular numbers.)

Diagnostic Test 6: Fractions, Percentages and Probability

1 **a** $\frac{37}{40}$ **b** $1\frac{3}{20}$ **c** $\frac{13}{30}$ **2** **a** $\frac{7}{20}$ **b** $\frac{13}{20}$ **c** $\frac{1}{20}$

3 **a** $5\frac{3}{20}$ **b** $10\frac{7}{10}$ **c** $4\frac{1}{8}$ **4** **a** $2\frac{2}{5}$ **b** $4\frac{13}{20}$ **c** $11\frac{7}{10}$

5 **a** $\frac{63}{80}$ **b** $\frac{2}{5}$ **c** $\frac{27}{32}$ **6** **a** $2\frac{2}{5}$ **b** $5\frac{5}{8}$ **c** 5

7 **a** $1\frac{4}{5}$ **b** $1\frac{1}{5}$ **c** $1\frac{7}{8}$ **8** **a** 1200 km **b** 2100 mL **c** 1 h 20 min

9 **a** $\frac{1}{5}$ **b** $\frac{1}{10}$ **c** $\frac{3}{50}$ **10** **a** $62\frac{1}{2}\%$ **b** 75% **c** 80%

11 **a** 25% **b** 70% **c** $62\frac{1}{2}\%$

12 **a** $\frac{13}{400}$, 0·0325 **b** $\frac{17}{100}$, 0·17 **c** $\frac{93}{500}$, 0·186 **13** **a** 270 mL **b** $1064

14 **a** 20% **b** $18\frac{3}{4}\%$ **15** **a** $\frac{3}{10}$ **b** $\frac{7}{10}$ **16** **a** $\frac{1}{2}$ **b** $\frac{5}{6}$

6A Revision Assignment

1 **a** 1·3 **b** 5·75 **c** 9·75 **d** 15·338 **2** **a** 0·6 **b** 0·8 **c** 5·38 **d** 3·98

3 **a** 0·3 **b** 0·04 **c** 23 **d** 1·24 **4** **a** 63 **b** 3 **c** 125 **d** 0·28

5 **a** 5·5 **b** 0·55 **c** 0·055 **d** 0·55 **6** **a** 1·3 **b** 0·13 **c** 13 **d** 1·3

7 **a** true **b** true **c** true **d** true

8 **a** $y = 15$ **b** $y = 4$ **c** $y = 9$ **d** $y = 8$

9 **a** (3, 2) **b** (−2, −3) **c** (−5, 0) **d** (−2, 3) **e** (0, −5)

10 **a** H **b** T **c** M **d** W **e** Z

11 **a** E **b** E **c** G **d** G

12 **a** 7320 **b** 6048 **c** 67 400 **d** 3 050 000

6B Working Mathematically

1 **a** *hectare* **b** *regular* shapes **c** axes of *symmetry* **d** *coordinates* **e** *tally*
f *picture* graph **g** *column* graph **h** *line* graph **i** *sector (or pie)* graph **j** *bar* graph

2 8 **3** 24 **4** 2 min **5** fifteen each of 1 to 4; six of 5, five each of numbers 6, 7, 8, 9, 0

6 The plumber's name is Farrant, the electrician's is Jones and the carpenter's name is Bagnell.

7 **a** class discussion **b** 51% (plus any company taxes not shown on the graph)
c 37·8c **d** not very accurate

Chapter 7: Calculators

Exercise 7:01

1 **a** 7391 **b** 20 716 **c** 95 064 **d** 5 **e** 200 **f** 0 **g** 160 **h** 30
i 2000 **j** 20 **k** 14 **l** 10 **m** 100 **n** 20 **o** 0 **p** 40
q 0 **r** 2 **s** 3 **t** 5 **u** 10

2 **a** 110 801 **b** 22·05 **c** 41 851 891 **d** 11 869 **e** 1533·186
f 15 002 001 **g** 3 588 288 **h** 6 451 227 **i** 813·5004 **j** 987
k 97·3 **l** 345·6 **m** 7928 **n** 450 **o** 883 736
p 61·77 **q** 500 002 **r** 10 000 **s** 38 **t** 730 **u** 9·8

3 **a** $3871.50 **b** 8034 **c** 165 800 **d** $1250 million **e** 83 916 000
f 9 408 240 million km **g** 236·52 km

Prep Quiz 7:02

1 400 **2** 90 **3** 9 **4** 3000 **5** 0·8 **6** 0·3 **7** 0·9 **8** 0·3 **9** 4 million **10** 0·05

Exercise 7:02

1 **a** 360 **b** 8590 **c** 150 000 **d** 500 000 **e** 800 **f** 210 **g** 24 000 **h** 10
i 180 000 000 **j** 12 000 **k** 3 **l** 16 000

2 **a** $50 \times 20 \times 40$, 40 000, 42 985·53 **b** $90 \div 30 + 9$, 12, 12·042302 **c** 90 000 − 10 000 − 7000, 73 000, 70 217
d $\$500 \times 30 + \$90$, $15 090, $13 504.84 **e** \$7 million $\times 10 \times 40$, 2800 million, 3135·6 million
f $900 - 70 \times 5$, 550, 490·792 **g** $9000 + 3000 \times 8000$, 24 009 000, 28 857 429

3 **a** $513.98 **b** $8806.54 **c** $325.27 **d** $12.50 **e** $101.44

4 **a** 11 814·0 **b** 1048·0 **c** 1 200 220·8 **d** 9087·9 **e** 33·0 **f** 667·3 **g** 5259·0
h 14 746·2 **i** 4·8 **j** 510·0 **k** 95·5 **l** 873·5

5 **a** 32 **b** 89 **c** 156 **d** 13 **e** 18 **f** 37

Prep Quiz 7:03

1 4·8 **2** 498·13 **3** 45 874·386 **4** 0·0042 **5** 78·13 **6** $18·57 **7** $7·46 **8** 56·51 **9** $4.92 **10** 965·12

Exercise 7:03

1 **a** $0.04 **b** $0.11 **c** $1.39 **d** $0.58 **e** $0.02 **f** $0.02 **g** $27.17 **h** $1.91
i $2.57 **j** $0.00

2 Each rubber band costs less than $\frac{1}{2}$ of one cent.

3 **a** 54 **b** 3 **c** 1 **d** 3 **e** 28 **f** 101 **g** 9 **h** 99
4 **a** 2, 3; 2·28 **b** 5, 6; 5·21 **c** 9, 10; 9·70 **d** 7, 8; 7·99
e 1, 2; 1·24 **f** 2, 3; 2·19 **g** 3, 4; 3·11 **h** 0, 1; 0·94
5 **a** 10 **b** 1 **c** 17 **d** 36 **e** 11 **f** 72 **g** 2 **h** 1
i 5 **j** 9 **k** 14 **l** 39
6 **a** 6 **b** 40 320 **c** 120 **d** 479 001 600 **e** 362 880 **f** 39 916 800 **g** 6 227 020 800

Exercise 7:04

1 **a** 33·125 **b** 7703 **c** 123 **d** 13 687·3 **e** 7 kg for $1.70 **f** 17 **g** 88·5 c/L
h 0·4375 **i** 430·142 **j** 1·025
2 **a** 1008·16 **b** 86 589 **c** 458 **d** 5135·2 **e** 14 kg for $4 **f** 20 **g** 92·5 c/L
h 0·875 **i** 313·07 **j** 2·48
3 **a** 22·63 **b** 11 199 **c** 729 **d** 40 228·8 **e** 500 g for $3 **f** 23 **g** 97·5 c/L
h 0·6875 **i** 263·142 **j** 2·3
4 **a** 462·9 **b** 9244 **c** 168 **d** 6417·06 **e** 450 g for $2.85 **f** 9 **g** 94·5 c/L
h 0·625 **i** 515·86 **j** 5·4
5 **a** 453·361 **b** 90 616 **c** 78 **d** 44·4626 **e** 375 g for $2.37 **f** 8 **g** 89·5 c/L
h 0·5625 **i** 733·45 **j** 6·7

Exercise 7:05

1 **a** 12, 24, 36, 48, 60, 72, 84 **b** 37, 74, 111, 148, 185, 222, 259 **c** 56, 112, 168, 224, 280, 336, 392
d 128, 256, 384, 512, 640, 768, 896 **e** 433, 866, 1299, 1732, 2165, 2598, 3031
2 **a** 2, 17, 23, 31 **b** 3, 5, 11, 13, 23 **c** 7, 11, 13, 19, 31 **d** 2, 19, 23, 29, 31 **e** 5, 7, 11, 17, 23, 29
3 **a** 9, 17, 19, 29 **b** 9, 13, 19, 23 **c** 17, 19, 23, 29 **d** 7, 23, 37, 41 **e** 11, 23, 31, 37
f 11, 31, 37, 41 **g** 9, 41, 43, 53 **h** 18, 41, 43, 53 **i** 18, 43, 53, 59 **j** 7, 43, 59, 61
k 7, 59, 61, 64 **l** 7, 49, 61, 64
4 **a** 44, 407, 4400, 48 323 **b** 631, 16 669, 449 695, 12 141 397 **c** 2125, 1900, 1765, 1684
d 356, 2606, 19 481, 146 043·5 **e** 36 864, 37 376, 37 440, 37 448 **f** 2304, 2192, 2185, 2184·5625
5 **a** 1369 **b** 15 625 **c** 287 296 **d** 7921 **e** 244·6096 **f** 0·998001
6 **a** 42 **b** 73 **c** 58 **d** 123 **e** 354 **f** 336
7 **a** 5 **b** 7 **c** 12 **d** 16 **e** 52 **f** 82
8 **a** $910\frac{7}{46}$ **b** $418\frac{73}{93}$ **c** $333\frac{25}{33}$ **d** $3768\frac{28}{107}$ **e** $1103\frac{367}{847}$ **f** $657\frac{450}{761}$

Exercise 7:06

1 **a** 327·6 cm **b** 1·8 cm **c** 198·9 cm **2** **a** 1·36 m **b** 552·16 m **c** 0·56 m
3 **a** 1471·77 m **b** 1·817 m **c** 3·555 m **4** **a** 83·439 m **5** 10·84 m (or 10 m 84 cm), 10·13 min
6 4 00 000 **7** $358·15
8 **a** 600 **b** 941 **c** 1644 **d** 936 **e** 452 **f** 590 **g** 136
9 **a** 0·27 kg **b** 0·213 kg (3 dec. pl.) **c** 0·259 kg (3 dec. pl.)
10 **a** S$13726.70 **b** S$33572.74 **c** S$2258.46
11 **a** US$1.09 **b** US$7.31 **c** US$33.39 **d** US$61.78 **e** US$2431.32
f US$18 940.66 **g** US$265.11 **h** US$13 255.57 **i** US$263 414.63 **j** US$530 222.69
k US$5 705 422.59 **l** US$131 322.80 **12** $98 721.60

13

Rectangle	1	2	3	4	5
Length	45 cm	28·1 cm	17·6 cm	11 cm	6·9 cm
Width	40 cm	25 cm	15·6 cm	9·8 cm	6·1 cm
Area	1800 cm^2	702·5 cm^2	274·6 cm^2	107·8 cm^2	42·1 cm^2
Perimeter	170 cm	106·2 cm	66·4 cm	41·6 cm	26 cm

14 **a** $1949 **b** $205 **c** $24 876 **d** $1395 **e** $1.99 **f** $0.60
15 **a** 3·14085 **b** 3·14152 **c** 3·14054
16 **a** 90 months or $7\frac{1}{2}$ years **b** 98 600 m^3 **c** 73 m^3
17 128, 115, 334, 251, 172
18 **a** 17 and 36 **b** 37 and 63 **c** 42 and 108 **d** 54 and 124 **e** 88 and 88
f 56 and 57 **g** 90 and 92 **h** 123 and 160

Fun Spot 7:06 Make words with your calculator

H	E	L	L	O		H	I	S
I		E			B			O
L	O	G	S		L	E	S	S
L		S	H	O	E		L	
S			E		S		O	H
	B	I	L	L	S		G	O
H	E		L	O		G		S
O			S			E		E
G	I	G		B	I	L	E	S

Challenge 7:06 The Monaco Grand Prix

1 (2) 80·537 s (3) 81·887 s (4) 81·889 s (5) 82·337 s (6) 82·561 s
(7) 82·592 s (8) 82·683 s (9) 82·718 s (10) 83·007 s (11) 83·196 s
(12) 83·426 s (13) 83·597 s (14) 83·758 s

2 0·02219 hours (correct to 5 decimal places), 1 min 19·885 s

3 (2) 168·966 km/h (3) 166·180 km/h (4) 166·176 km/h (5) 165·272 km/h (6) 164·824 km/h
(7) 164·762 km/h (8) 164·580 km/h (9) 164·511 km/h (10) 163·938 km/h (11) 163·566 km/h
(12) 163·115 km/h (13) 162·781 km/h (14) 162·468 km/h

4 154·98 km/h

Diagnostic Test 7: Calculators

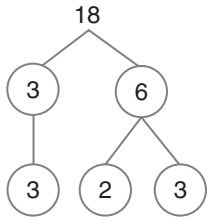
1 **a** 7605 **b** 34·8 **c** 1638

2 **a** 2563·2 **b** 9 **c** 15·16
3 **a** 12 **b** 10 **c** 71 **d** 4 **e** 9 **f** 17
4 **a** 8 **b** 11 **c** 14 **d** 4 **e** 9 **f** 13
5 **a** 63 **b** 6 **c** 10

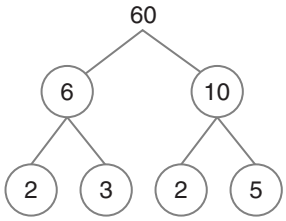
7A Revision Assignment

1 **a** 4561 **b** 3587 **c** 963 **d** 515
2 **a** \$30.50 **b** \$36.10 **c** \$60.50 **d** \$2.10
3 **a** 7 **b** 6 **c** 26 **d** 8
4 **a** 16 **b** 1000 **c** 10 **d** 80 000
5 **a** 4^3 **b** 10^2 **c** 3^4 **d** 7^3
6 **a** 1, 3, 5, 15 **b** 1, 2, 3, 4, 6, 8, 12, 24 **c** 1, 7 **d** 1, 2, 4, 5, 10, 20, 25, 50, 100
7 **a** 10, 20, 30 **b** 11, 13, 17, 19 **c** 10 **d** 24

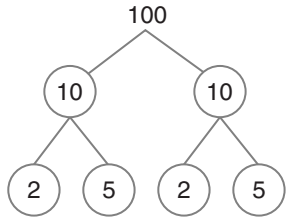
8a



b



c



9 **a** $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{9}{10}$ **d** $\frac{1}{4}$ **10** **a** $\frac{3}{4}$ **b** $\frac{3}{4}$ **c** $\frac{9}{10}$ **d** $\frac{3}{5}$

7B Working Mathematically

1 \$19 900 **2** 35·5 L, 19·5 L **3** 3 cards, 8 cards **4** 7 byes **5** no
6 **a** water heating **b** lighting **c** 126°, $\frac{7}{20}$ **d** 35% **e** \$336

Chapter 8: Patterns and Algebra

Prep Quiz 8:01

1 8 **2** 13 **3** 10 **4** 16 **5** 20 **6** 14 **7** 32 **8** 1 **9** 2 **10** 1·9

Exercise 8:01

1 **a** 9, 13 **b** 18, 29 **c** 21, 30 **d** 17, 8 **e** 24, 11
f 30, 15 **g** 9, 27 **h** 36, 144 **i** 21, 63 **j** 50, 25
k 100, 20 **l** 27, 9 **m** 7, 15 **n** 19, 61 **o** 5, 17
p 41, 21 **q** 35, 17 **r** 9, 81

2 **a** 15, +4 **b** 23, +6 **c** 19, +6 **d** 5, −5 **e** 16, −7
f 11, −2 **g** 16, ×2 **h** 27, ×3 **i** 1000, ×10 **j** 10, ÷2
k 3, ÷3 **l** 1, ÷4

3 **a** 55, 70, 85 **b** 49, 60, 71 **c** 105, 128, 151 **d** 34, 20, 6 **e** 63, 45, 27
f 67, 39, 11 **g** 64, 128, 256 **h** 264, 528, 1056 **i** 270, 810, 2430 **j** 15, 21, 28
k 36, 49, 64 **l** 16, 10, 5

4 **a** 14 **b** 17 **c** 35 **d** 38 **e** 15 **f** 21 **g** 6 **h** 12 **i** 34
j 12 **k** 50 **l** 40

5 **a** 3, 6, 9, 12 **b** 3, 7, 11, 15 **c** 4, 8, 12, 16 **d** 4, 7, 10, 13 **e** 4, 6, 8, 10 **f** 6, 11, 16, 21

6 **a** 1, 3, 6, 10 **b** 1, 4, 9, 16 **7** **a** 14, 30, 55 **b** 9, 36, 100

Exercise 8:02

1 **a**

Figure number	1	2	3	4
Number of coins	1	4	9	16

b number of coins = (number of figure)^2
c **i** 25 **ii** 36 **iii** 100

2 **a** cost = number × 14 cents **b** $1.26

3 **a**

Figure number	1	2	3	4
Number of tiles	2	4	6	8

b number of tiles = (figure number) × 2
c 30

4 **a** number of matches = (number of squares) × 4

b

Number of squares	1	2	3	4	5	6	7	8
Number of matches	4	8	12	16	20	24	28	32

5 **a** 3 **b** number of matches = 1 + (number of squares) × 3

c

Number of squares	1	2	3	4	5	6	7	8
Number of matches	4	7	10	13	16	19	22	25

6 **a** bottom = 5 × top **b** bottom = top + 6 **c** bottom = top + 3
d bottom = 2 × top **e** bottom = 2 × top + 1 **f** bottom = 3 × top −1
g bottom = top −2 **h** bottom = 2 × top + 1

7 **a**

Figure number	1	2	3	4
Fraction coloured	$\frac{2}{8}$	$\frac{4}{8}$	$\frac{6}{8}$	$\frac{8}{8}$

or

Figure number	1	2	3	4
Fraction coloured	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	1

b $F = n \times \frac{2}{8}$ or $F = n \times \frac{1}{4}$ or $F = \frac{1}{4} \times n$

8 **a**

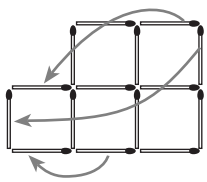
Number of bicycles (B)	1	2	3	4	5	6
Number of wheels (W)	5	7	9	11	13	15

b $W = 2 \times B + 3$
c No. If we double the number of bicycles, the result is even. Adding 3 to this gives an odd answer. 36 is not odd.

9 $N = 2 \times L + 8$ **a** **i** 18 squares **ii** 68 squares **b** yes
10 **a** $C = L^3$ (ie number of cubes = length^3) **b** 125 small cubes

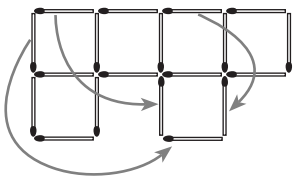
Fun Spot 8:02 Mysteries mixing matches

1



2 IV + I = V

3



4 II + II = IV
or
III + I = IV

Prep Quiz 8:03

1 23, 29 **2** 12, 6 **3** 24, 48 **4** 10, 5 **5** 25, 36 **6** 1, $\frac{1}{4}$ **7** 9 **8** 7 **9** 21
10 bottom = top × 3

Exercise 8:03

1 **a** 3, 4, 5, 6 **b** 3, 6, 9, 12 **c** 8, 10, 12, 14 **d** 10, 20, 30, 40 **e** 3, 4, 5, 6
f 9, 8, 7, 6 **g** 4, 5, 6, 7 **h** 12, 6, 4, 3 **i** 1, 3, 5, 7 **j** 3, 5, 7, 9
k 4, 7, 10, 13 **l** 8, 16, 24, 32

2 **a** 11 **b** 1 **c** 14 **d** 8 **e** 24 **f** 40 **g** 16 **h** 1 **i** 6 **j** 11
k 5 **l** 8

3 **a** 19 **b** 37 **c** 11 **d** 77 **e** 91 **f** 49 **g** 38 **h** 17 **i** 2 **j** 40
k 54 **l** 4

4 **a** $D = S \times T$ **b** $A = L \times B$ **c** $P = 4 \times l$ **d** $T = b + g$ **e** $P = a + p$

f $M = S + 23$ **g** $C = T \div n$ **h** $A = \frac{a+b}{2}$ or $A = (a + b) \div 2$

5 **a** number of matches = 3 × number of triangles

$m = t \times 3$

t	1	2	3	8	12
m	3	6	9	24	36

b number of matches = 2 × number of triangles + 1

$m = t \times 2 + 1$ *or* $m = 1 + t \times 2$

t	1	2	3	10	15
m	3	5	7	21	31

c number of matches = 4 × number of triangles + 2

$m = t \times 4 + 2$ *or* $m = 2 + t \times 4$

t	1	2	3	9	20
m	6	10	14	38	82

6 **a** number of matches = 6 × number of hexagons, $m = 6 \times h$, 120 matches
b number of matches = 5 × number of hexagons + 1, $m = 5 \times h + 1$, 101 matches
c number of matches = 8 × number of matches −2, $m = 8 \times h - 2$, 158 matches

7 **a**

s	1	2	3	4
b	1	4	9	16

, $b = s \times s$ **b**

b	1	2	3	4
f	6	10	14	18

, $f = b \times 4 + 2$

c

b	2	3	4	5	6
n	2	6	12	20	30

, $n = b \times (b - 1)$

8 **a** $b = 4 \times t$ **b** $b = t + 2$ **c** $b = t - 1$ **d** $b = a \times a$ **e** $b = 10 - a$
f $b = 6 - a$ **g** $y = 3 \times x - 1$ **h** $y = 10 \times x$ **i** $y = 10 \times x - 1$ **j** $q = p \div 2$

9 $N = (b^2 + b) \div 2$ or $N = \frac{b \times (b+1)}{2}$

Exercise 8:04

1 **a**

S	1	2	3	4
n	4	8	12	16

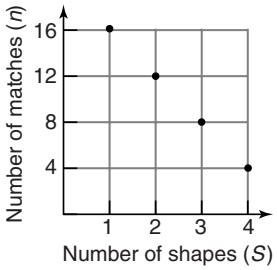
b

S	1	2	3	4
n	6	8	10	12

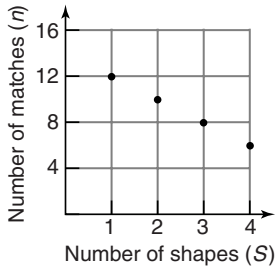
c

N	1	2	3	4	5
n	20	25	30	35	40

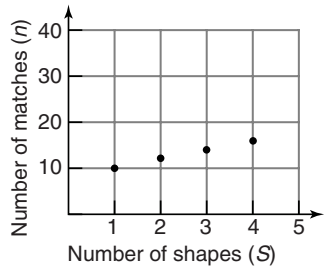
2 **a**



b

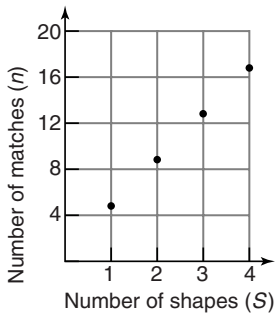


c



3

S	1	2	3	4
n	5	9	13	17



Diagnostic Test 8: Patterns and Algebra

1 **a** 14 **b** 6 **c** 54 **d** 5 **2** **a** add 4 **b** subtract 3 **c** multiply by 3 **d** divide by 2

3 **a**

Triangles	1	2	3	4
Matches	3	6	9	12

b

Squares	1	2	3	4
Matches	4	7	10	13

4 **a**

x	1	2	3	4
y	5	6	7	8

b

t	0	2	4	6
b	0	8	16	24

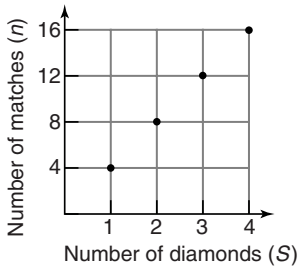
c

m	0	1	2	3
n	3	5	7	9

5 **a** 15 **b** 7 **c** 9 **d** 8

6

Number of diamonds	1	2	3	4
Number of matches	4	8	12	16



8A Revision Assignment

1 **a** 700 **b** 8·5 **c** 96 **2** **a** 0·72 **b** 9·5 **c** 2·4 **d** 2·06

3 **a** (3, 2) **b** (−2, −3) **c** (−5, 0) **d** (−2, 3) **e** (0, −5)

4 **a** H **b** T **c** M **d** W **e** Z

5 **a** $\frac{4}{5}$ **b** $\frac{3}{10}$ **c** $\frac{10}{3}$ or $3\frac{1}{3}$ **d** $\frac{4}{10}$ or $\frac{2}{5}$

6 **a** 40% **b** 30% **c** 80% **d** 0·56 **7** **a** 03:15 **b** 15:15 **c** 23:30 **d** 10:18

8 **a** 5 h **b** 5 h 32 min **c** 3 h 30 min **d** 5 h 6 min

9 **a** 0·72 **b** 0·105 **c** 0·07 **d** 0·045

8B Working Mathematically

1 *(1) add (or plus) (2) subtract (or minus) (3) multiplied by (or times) (4) divided by* (5) *is equal to (6) is approximately equal to (7) is not equal to (8) is less than (9) is less than or equal to (10) is not less than (11) is greater than (12) is greater than or equal to (13) is not greater than (14) is not greater than or equal to (15) 4 squared (16) 4 cubed (17) the square root of 25 (18) the cube root of 27 (19) per cent (20) therefore (21) for example (22) that is (23) numerator (24) denominator*

2 28 800 **3** 361 **4** $1.80 **5** $21 600 **6** Soula has 75c, Kosta has 25c.

7 **a** USA **b** Bulgaria **c** Poland **d** two **e** about 96 million tonnes

Chapter 9: Further Algebra

Prep Quiz 9:01

1 9 **2** 0 **3** 12 **4** 16 **5** 2 **6** 35 **7** 17 **8** 7 **9** 2 **10** 67

Exercise 9:01

1 **a** $3 \times 5 = 15$ **b** $3 \times n$ **c** $3n$ **e** dt **f** $5 \times a \times b$ **g** $\frac{q}{3}$

h $m \div 2$ **i** $34y = 34 \times y$, $3 \times 4 \times y = 12y$

2 **a** $7a$ **b** $10x$ **c** $2p$ **d** $8q$ **e** $9b$ **f** $6m$ **g** $3r$ **h** $12c$

i $9m$ **j** $5n$ **k** $4y$ **l** $12t$ **m** a **n** x **o** n **p** b

q y **r** m **s** $5(a + 2)$ **t** $6(x + 7)$ **u** $2(3x - 2)$ **v** $5(m - n)$ **w** $5 + 2q$ **x** $n - 10$

y $4a + 5b$ **z** $7q - 6p$

3 **a** $\frac{x}{2}$ **b** $\frac{m}{4}$ **c** $\frac{b}{3}$ **d** $\frac{n}{5}$ **e** $\frac{a}{1}$ **f** $\frac{y}{10}$ **g** $\frac{t}{8}$ **h** $\frac{k}{100}$

i $\frac{2b}{3}$ **j** $\frac{3x}{5}$ **k** $\frac{5m}{6}$ **l** $\frac{3a}{4}$ **m** $\frac{a}{b}$ **n** $\frac{6}{n}$ **o** $\frac{2a}{x}$ **p** $\frac{10}{a}$

q $\frac{a+1}{4}$ **r** $\frac{8-x}{5}$ **s** $\frac{7a}{8b}$ **t** $\frac{4x}{3y}$

4 **a** $5ab$ **b** $4mn$ **c** abc **d** $3a^2$ **e** $3xy$ **f** $10ab$
g $6rs$ **h** mn **i** $24a$ **j** $30b$ **k** $28x$ **l** $81p$
m $15a$ **n** $8b$ **o** $30y$ **p** $12m$ **q** $30m$ **r** $6p$
s $28a$ **t** $40b$ **u** $21a$ **v** $80x$ **w** $60b$ **x** $9y$
y $4a + 5b$ **z** $7q - 6p$

5 **a** $2p(q - 1)$ **b** $15(a - 3)$ **c** $ab(c - 7)$ **d** $4(3a + 1)$ **e** $6(1 + 2x)$
f $3(5y - 2z)$ **g** $(x + 2)(x + 3)$ **h** $(a - 3)(a + 3)$ **i** $(4p + 1)(3q - 1)$

6 **a** $5 \times x$ **b** $7 \times a$ **c** $12 \times y$ **d** $14 \times p$ **e** $2 \times p + 7$
f $9 \times x - 5$ **g** $11 - 7 \times m$ **h** $1 + 10 \times a$ **i** $7 \times x + 3 \times y$ **j** $10 \times a - 9 \times b$
k $7 \times a + 3 \times p$ **l** $11 \times m - 9 \times n$ **m** $a \times b + 3$ **n** $k - 3 \times l$ **o** $a \times b \times c$
p $8 \times x \times y$ **q** $6 \times (a + 3)$ **r** $4 \times a \times (6 - a)$ **s** $3 \times (2 \times x + 1)$ **t** $(a + 7) \times (a - 7)$
u $x \times x$ **v** $a \times a + c \times c$ **w** $2 \times y \times y$ **x** $m \times m - 3$

7 **a** $a \div 3$ **b** $5 \div k$ **c** $m \div 7$ **d** $n \div a$ **e** $(2 \times x) \div 5$
f $(3 \times y) \div 2$ **g** $5 \div (8 \times m)$ **h** $6 \div (5 \times p)$ **i** $(5 \times a) \div (6 \times b)$ **j** $(11 \times m) \div (10 \times n)$
k $(a + 7) \div 3$ **l** $(m - 1) \div 2$ **m** $(6 + a) \div (a - 5)$ **n** $(9 + x) \div (7 - x)$ **o** $3 \times (x + 1) \div 5$
p $(a + 6) \div (3 \times a)$

Exercise 9:02

1 **a** $x + 1$ **b** $y + 4$ **c** $2y$ **d** $4x$ **e** $2x + 1$ **f** $2y + 2$ **g** $2x + 4$ **h** $y + 4$
i $3x + 6$ **j** $3x + y$ **k** $x + 2y$ **l** $3y + x$ (or $x + 3y$) **m** $4x + 2$ **n** $2y + 5$ **o** $2x + 2y + 1$
p $3x + 4$ **q** $2y + 4$ **r** $3x + 7$

2 **a** a **b** $a + 1$ **c** $a + 2$ **d** $2a + 1$ **e** $2a + 2$ **f** $2a + 2$ **g** $2a + 3x$ **h** $2a + 3x$
j $2a + 3x$ **j** $3a + 5$ **k** $3a + 5$

3 **a** $2x + 3a + 3$ (or $3a + 2x + 3$) **b** $4y + 2x + 9$ (or $2x + 4y + 9$) **c** $2a + 17$
d $3y + x + 15$ (or $x + 3y + 15$) **e** $3x + 11$ **f** $5a + 10$

Prep Quiz 9:03

1 $4y$ **2** $2m$ **3** $6ab$ **4** $9n + 2$ **5** $9(n + 2)$ **6** $\frac{p}{7}$ **7** $\frac{ab}{c}$ **8** $3 \times p \times q$ **9** $y \times y$ **10** $6 \times a + 4 \times b$

Exercise 9:03

1 **a** 8 **b** 28 **c** 13 **d** 16 **e** 19 **f** 12 **g** 1 **h** 15
i 15 **j** 2 **k** 8 **l** 5

2 **a** 10 **b** 7 **c** 7 **d** 13 **e** 9 **f** 17 **g** 16 **h** 3
i 30 **j** 4 **k** 5 **l** 1

3 **a** 20 **b** 9 **c** 24 **d** 6 **e** 32 **f** 17 **g** 28 **h** 0
i 216 **j** 45 **k** 13 **l** 245 **m** 8 **n** 38 **o** 21 **p** 50
q 0 **r** 12 **s** 1 **t** 2 **u** 2 **v** 1

4 **a** **i** 15 **ii** 0 **iii** 55 **iv** $2\frac{1}{2}$
b **i** 17 **ii** 9 **iii** 37 **iv** 8
c **i** 20 **ii** 2 **iii** 15·5 **iv** 19
d **i** 35 **ii** 105 **iii** 5 **iv** 30

5 **a** 16 **b** 26 **c** 72 **d** 5·6

6 **a** 72 **b** 156 **c** 420 **d** 10 100

7 **a** \$2.50 **b** \$2.75 **c** \$8.75

8 **a** $b = 8, 12, 16, 20$ **b** $y = 0, 3, 6, 9$ **c** $n = 7, 21, 35, 49$ **d** $b = 1, 4, 7, 10$
e $y = 7, 11, 15, 19$ **f** $q = 7, 22, 27, 52$ **g** $b = 3, 8, 13, 18$ **h** $w = 20, 16, 12, 8$
i $g = 2, 5, 8, 11$ **j** $y = 1, 2, 3, 4$ **k** $b = 0, 1, 2, 4$ **l** $n = 0, 1, 4, 9$

9 **a** $y = x + 5$
b $y = x - 23$
c $y = 2 \times x$ or $y = 2x$
d $y = x \div 7$ or $y = \frac{x}{7}$

10 **a** C **b** G **c** A **d** H **e** B **f** F **g** D **h** E
A You add 7 to the first number to get the second number.
B You double the first number and add 1 to get the second number.
C Multiply the first number by 6 to get the second number.
D Double the first number then subtract 4 to get the second number.
E Dividing the first number by 3 produces the second number.
F Multiplying the first number by 3 then adding 2 gives the second number.
G Multiplying the first number by 3 gives the second number.
H If you subtract 4 from the first number, you get the second number.

Prep Quiz 9:04

1 $3a$ **2** mn **3** $12pq$ **4** $3(5 + a)$ **5** $2a + 5b$ **6** 12 **7** 15 **8** 13 **9** 1 **10** 24

Exercise 9:04

1 **a** 2 **b** 1 **c** 2 **d** 1 **e** 3 **f** 1 **g** 3 **h** 4
i 3 **j** 5 **k** 5 **l** 5 **m** 3

2 **a** $2x$, $4x$ **b** $3b$, $7b$ **c** $4m$, $8m$ **d** $2a$, $3a$ **e** a^2, $3a^2$ **f** xy, $3yx$ **g** $9x$, x, $3x$ **h** $2mn$, $6mn$
i a, $2a$, $7a$ **j** ab, ba **k** a^2x, $4a^2x$ **l** $8m^2$, $5m^2$

3 **a** $7a$ **b** $7x$ **c** $13m$ **d** $10y$ **e** $7n$ **f** $8b$ **g** $2a$ **h** $8x$
i $7a$ **j** $3m$ **k** $8k$ **l** $7b$ **m** $7q$ **n** $10x$ **o** 0 **p** m
q $10x$ **r** $13a$ **s** $14m$ **t** $12x$ **u** m **v** $8y$ **w** $3p$ **x** $9a$
y $9ab$ **z** $13xy$

4

	A	B	C		A	B	C
a	$9a + 3b$	$11x + 7y$	$12m + 2n$	**g**	$2p + 6q$	$10k + 3l$	$3x^2 + 4x$
b	$10p - 7q$	$8k + 5l$	$x - 2y$	**h**	$16m^2 + 2m$	$2ab + 4bc$	$2pq + p^2$
c	$11a^2 + 5a$	$7x^2 + 5x$	$9m + m^2$	**i**	$5a^2 + 4a + 7$	$10k + 8l - 2m$	$5x + 2y + 3z$
d	$7ab - 7a$	$11x^2 + 5xy$	$2ab + 3bc$	**j**	$4p - 3q$	$20a + 6b$	$6r + 3t$
e	$7 + a$	$x + 9$	$5a^2 - 7$	**k**	$2x^2 + 5x$	$13p + 4q$	$11a + b$
f	$7a + 5b$	$10x + 3y$	$13m + 12n$				

5 **a** $11n$ chocolates **b** $8b$ bananas **c** $13x$ biscuits

6 **a** $2L + 2B$ **b** **i** 3 L **ii** $4S$ **iii** $2x + y$ **iv** $a + b + 2c$

7 **a** $8s$ **b** $4t$ **c** $6t$ **d** $2s + 3t$ **e** $3s + 3t$ **f** $6s + 12t$
g $10s + 12t$ **h** $ns + mt$

Fun Spot 9:04 Let's improve our algebra skills

1 x, $6x$, $6x + 2$, $6x - 4$, $3x - 2$. This equals 22 when $x = 8$.
2 y, $10y$, $10y + 5$, $20y + 10$, $20y - 10$, $2y - 1$. This equals 9 when $y = 5$.
3 m, $3m$, $3m - 12$, $6m - 24$, $6m - 18$, $2m - 6$. This equals -2 when $m = 2$.
4 $3d$, $8d$, $8d - 7$, $24d - 21$, $24d - 45$. This equals 51 when $d = 4$.
5 $10k$, $9k$, $18kk$ (or $18k^2$), $18kk + 18$ (or $18k^2 + 18$), $9kk + 9$ (or $9k^2 + 9$). This equals 90 when $k = -3$.

Prep Quiz 9:05

1 26 **2** 26 **3** 48 **4** 48 **5** 100 **6** 100 **7** 297 **8** 297 **9** 485 **10** 485

Exercise 9:05

1 **a** $2a + 6$ **b** $3x + 15$ **c** $5y + 15$ **d** $3x - 3$ **e** $7n - 49$
f $4a - 16$ **g** $14 + 7x$ **h** $6 + 6y$ **i** $45 - 9y$ **j** $10 + 2x$
k $3x + 27$ **l** $3a - 6$

2 **a** $6a + 4$ **b** $6x + 3$ **c** $20y + 15$ **d** $30p + 70$ **e** $12y - 6$
f $15q - 20$ **g** $8k - 28$ **h** $36n - 27$ **i** $20 + 10x$ **j** $21 + 28x$
k $5 + 15p$ **l** $24 - 16y$ **m** $8m + 12$ **n** $6y + 12$ **o** $10 + 20x$
p $70 + 30q$ **q** $30n - 18$ **r** $56m - 7$ **s** $14 - 21x$ **t** $55 - 99y$

3 **a** $x^2 + 2x$ **b** $y^2 + 5y$ **c** $m^2 + 7m$ **d** $q^2 - q$ **e** $m^2 - 10m$
f $n^2 - 4n$ **g** $3a + a^2$ **h** $4k + k^2$ **i** $6p - p^2$ **j** $2x^2 + 3x$
k $3y^2 + 4y$ **l** $5n^2 - n$ **m** $2p^2 + 14p$ **n** $3q^2 - 12q$ **o** $2m^2 + 6m$
p $10a^2 + 15a$ **q** $8m^2 - 2m$ **r** $30n - 18n^2$

4 **a** $5x + 5y$ **b** $4a + 4b$ **c** $7p - 7q$ **d** $10m - 10n$ **e** $12a + 6b$
f $21x + 14y$ **g** $36p - 18q$ **h** $10k - 351$ **i** $x^2 + xy$ **j** $p^2 - pq$
k $2y^2 + 2yz$ **l** $15g^2 - 10gh$

5 **a** $2a + 11$ **b** $4m + 29$ **c** $8x + 14$ **d** $5a + 18$ **e** $35 + 8y$
f $13m + 50$ **g** $12n + 20$ **h** $8n + 11$ **i** $12a + 17$ **j** $3m + 20$
k $8a$ **l** $3m + 1$ **m** $6a + 2$ **n** $6m + 6 - 6a$ **o** $30m + 15$
p $11a - 4$ **q** $11x - 20$ **r** $45y - 14$

Prep Quiz 9:06

1 $12a$ **2** $4p$ **3** $12x$ **4** $6m$ **5** $9p + 5q$ **6** $11ab$ **7** $11a + 8$ **8** $5p + 8q$ **9** $3x + 10y$
10 $8t^2 + 3t$

Exercise 9:06

1 **a** a^3 **b** p^5 **c** t^4 **d** y^2 **e** k^5 **f** m^3 **g** n^6 **h** b^4 **i** a^2b^3 **j** x^3y^3
k a^4t **l** mn^3 **m** p^3q^2 **n** a^3b^3 **o** x^3y^2

2 **a** $a \times a \times a \times a \times a$ **b** $q \times q \times q \times q$ **c** $t \times t \times t \times t \times t \times t \times t$ **d** $x \times x \times x$ **e** $m \times m$
f $n \times n \times n \times n \times n$ **g** p **h** $a \times a \times a \times a \times a \times a$ **i** $m \times m \times n \times n \times n$
j $a \times a \times a \times a \times b \times b \times b$ **k** $x \times y \times y \times y \times y \times y$ **l** $k \times k \times k \times k \times l$
m $a \times a \times a \times y$ **n** $h \times h \times n \times n \times n \times n \times n$ **o** $t \times w \times w \times w$
p $x \times x \times x \times x \times x \times y \times y \times y \times y \times y$ **q** $n \times n \times p \times p \times p \times q \times q \times q \times q$ **r** $a \times b \times b \times b \times b \times b \times c \times c$
s $x \times x \times x \times y \times y \times z \times z \times z \times z$ **t** $m \times m \times m \times n \times p \times p \times p \times p$

3 **a** $30a^2$ **b** $12y^3$ **c** $18m^3$ **d** $7t^3$ **e** $28y^4$ **f** $48n^2$ **g** $45m^3$ **h** $72a^3$
i $24x^2y$ **j** $150a^2b^2$ **k** $15k^2l^2$ **l** $15t^3w$

Prep Quiz 9:07

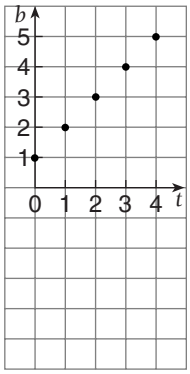
1 15 **2** 16 **3** 31 **4** 4 **5** 64 **6** 8 **7** 12 **8** 9 **9** 4 **10** 4

Exercise 9:07

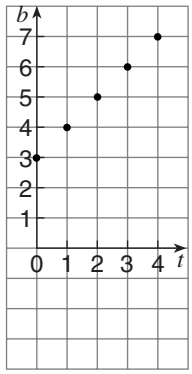
1 **a** L **b** U **c** H **d** V **e** Z **f** D **g** R **h** E **i** J **j** X
k N **l** A **m** P **n** C **o** W

2 **a** (2, 3) **b** (3, 4) **c** (5, 5) **d** (2, 5) **e** (4, 3) **f** (4, 0) **g** (0, 0) **h** (5, 4) **i** (1, 1) **j** (2, 1)
k (3, 2) **l** (4, 1)

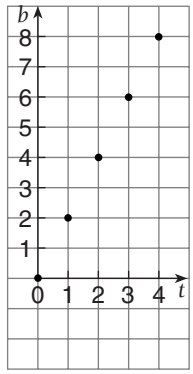
3 **a**



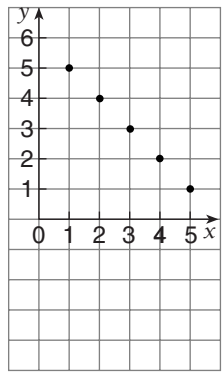
b



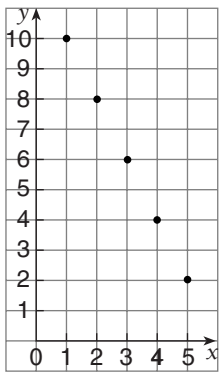
c



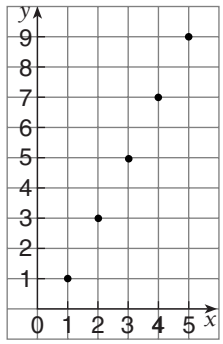
d



e

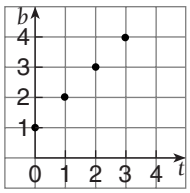


f



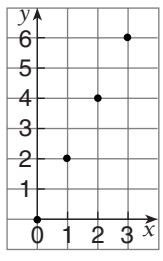
4 **a**

t	0	1	2	3
b	1	2	3	4



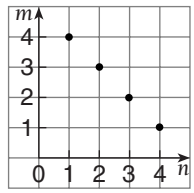
b

x	0	1	2	3
y	0	2	4	6



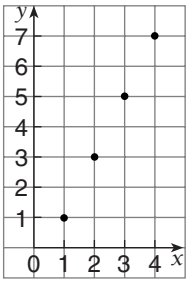
c

n	1	2	3	4
m	4	3	2	1



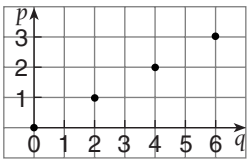
d

x	1	2	3	4
y	1	3	5	7



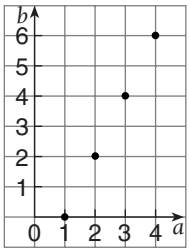
e

q	0	2	4	6
p	0	1	2	3



f

a	1	2	3	4
b	0	2	4	6

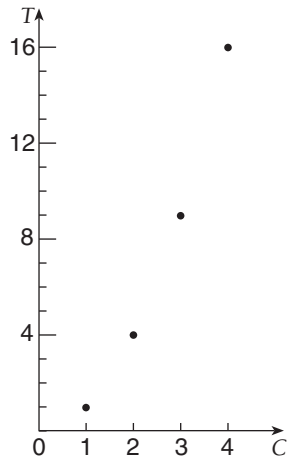


All points lie in straight lines.

5 a

C	1	2	3	4
T	1	4	9	16

b

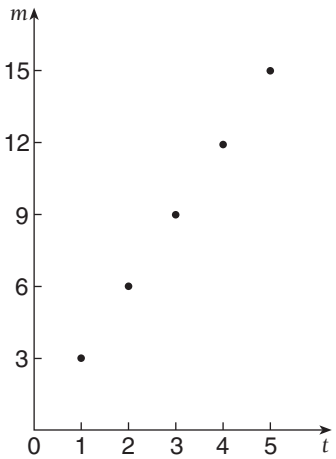


6 a

t	1	2	3	4	5
m	3	6	9	12	15

b $m = 3t$

c



7 a

x	0	1	2	3	4	5
y	2	3	4	5	6	7

$y = x + 2$

b

x	1	2	3	4	5	6
y	0	1	2	3	4	5

$y = x - 1$

c

x	0	1	2	3	4	5
y	6	5	4	3	2	1

$y = 6 - x$

d

a	0	2	4	6	8
b	1	2	3	4	5

$b = \frac{1}{2}a + 1$

e

w	2	3	4	5	6
t	0	2	4	6	8

$t = 2w - 4$

f

m	0	2	4	6	8
n	4	3	2	1	0

$n = 4 - \frac{1}{2}m$

Prep Quiz 9:08

1 −4 **2** 2 **3** −5 **4** 12 **5** −20 **6** 12 **7** −6 **8** 4 **9** 3 **10** 1

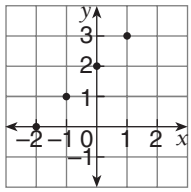
Exercise 9:08

1 **a** −6 **b** −21 **c** 2 **d** −4 **e** 13 **f** 6 **g** −6 **h** 9 **i** −5 **j** 1
k 0 **l** −2

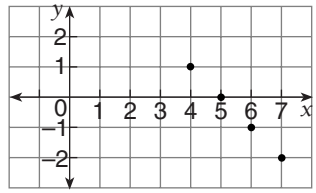
2 **a** −7 **b** −2 **c** −1 **d** −5 **e** 1 **f** −6 **g** −5 **h** −1 **i** 12 **j** −8
k −12 **l** 13 **m** −2 **n** −2 **o** −5 **p** 24

3 **a** **i** −6 **ii** −12 **iii** 15 **iv** 0 **b** **i** −1 **ii** −3 **iii** −7 **iv** −21
c **i** 4 **ii** −2 **iii** 12 **iv** 16 **d** **i** −2 **ii** −8 **iii** −20 **iv** −68

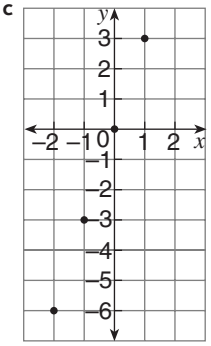
4 **a**



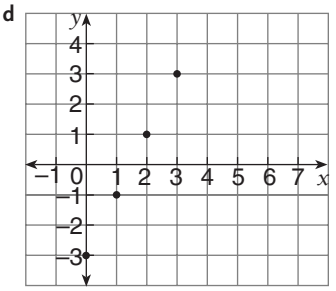
b



c

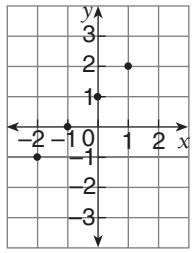


d



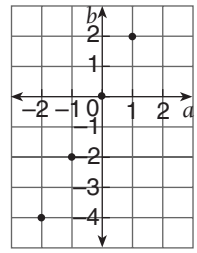
5 **a**

x	-2	-1	0	1
y	-1	0	1	2



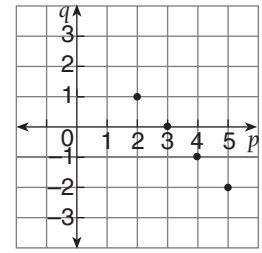
b

a	-2	-1	0	1
b	-4	-2	0	2



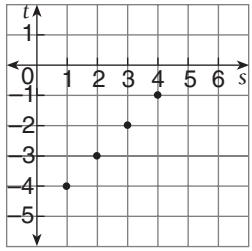
c

p	2	3	4	5
q	1	0	-1	-2



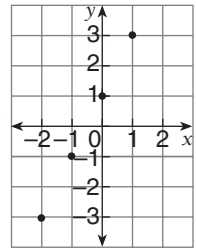
d

s	1	2	3	4
t	-4	-3	-2	-1



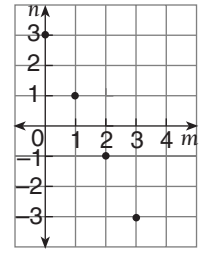
e

x	-2	-1	0	1
y	-3	-1	1	3



f

m	0	1	2	3
n	3	1	-1	-3



6 **a**

x	-2	-1	0	1
y	-2	-1	0	1

, B

b

x	-1	0	1	2
y	2	1	0	-1

, C

c

x	-1	0	1	2
y	-3	-1	1	3

, A

7 **a** **i** 25 **ii** 0 **iii** -5 **iv** -20 **b** 6

Prep Quiz 9:09

1 -10 **2** 4 **3** -3 **4** -15 **5** -3 **6** $3a$ **7** $11m$ **8** $15a$ **9** $4x + 2y$ **10** $2a$

Exercise 9:09

1 **a** $-3a$ **b** $-m$ **c** $-5x$ **d** $-x$ **e** $-n$ **f** $-9p$ **g** $-4a^2$ **h** $-2ab$
i $-10x^2$ **j** $-3a + 3b$ **k** $-5y - 3x$ **l** $-5x + 3y$ **m** $8p - 2q$ **n** $-3a^2 - 5a$ **o** $-t + 8$

2 **a** $-4x$ **b** $-10y$ **c** $-a$ **d** $-b$ **e** $-8m$ **f** $-18a$ **g** $24a$ **h** $-24m$
i $-56n$ **j** $3a$ **k** $-3x$ **l** $2m$ **m** $-3a$ **n** $2m - 2$ **o** $5m + 4$ **p** $3a - 5$
q 4 **r** $30 - 3x$ **s** $12 + 5x$

3 **a** $9a - 11b$ **b** $6x + 5y$ **c** $-m - 3g$ **d** $-3a + 3$ **e** $p + q$ **f** $-2x - 5$ **g** $-3ab + 4bc$
h $t^2 - 5t$ **i** $-z$ **j** $-2k - 2k^2$ **k** $11t - 2w$ **l** $-a - 8b$

4 **a** $2a - 5A + 19a^2 + 2 + 2A^2$ **b** $-3bc + 6ac$

Reading Mathematics 9:09 What's in food?

1 **a** 50% **b** 0·7% **c** 5·0% **2** **a** 32% **b** 11·7% **c** 3·7%
3 **a** 322 kJ/100 g **b** 3318 kJ/100 g **c** 205 kJ/100 g **4** bacon, beef and butter

Prep Quiz 9:10

1 4 **2** 7 **3** 4 **4** 9 **5** 4 **6** F **7** T **8** F **9** T **10** T

Exercise 9:10

1 **a** $x = 9$ **b** $x = 10$ **c** $y = 20$ **d** $y = 11$ **e** $x = 50$ **f** $x = 8$ **g** $y = 12$ **h** $y = 10$
i $y = 16$ **j** $x = 7$

2 **a** What number plus 5 is equal to 7? $x = 2$ **b** What number plus 9 is equal to 21? $a = 12$
c 10 minus what number is equal to 7? $p = 3$ **d** What number minus 5 is equal to 2? $m = 7$
e 4 times what number is equal to 20? $n = 5$ **f** 7 times what number is equal to 56? $m = 8$
g What number divided by 2 is equal to 7? $q = 14$ **h** What number divided by 4 is equal to 4? $n = 16$

3

	A	B	C	D
a	3	1	8	8
b	8	7	5	11
c	3	7	13	15
d	18	4	19	23
e	1	4	9	0
f	6	11	9	12
g	10	10	14	17
h	12	5	25	40
i	2	6	3	5
j	5	4	3	8
k	0	1	4	12
l	6	10	12	16
m	2	3	6	4
n	18	12	15	28
o	8	12	8	12

4 **a** correct **b** incorrect **c** incorrect **d** incorrect **e** correct
f incorrect **g** correct **h** incorrect

5 **a** 3 **b** 2 **c** 3 **d** 3 **e** 4 **f** 5 **g** 3 **h** 1 **i** 5 **j** 2
k 3 **l** 2 **m** 5 **n** 3 **o** 3 **p** 2 **q** 3 **r** 2 **s** 5 **t** 11

6 **a** 1·2 **b** 1·1 **c** 1·0 **d** 3·1 **e** 1·4 **f** 3·1 **g** 7·3 **h** 2·3 **i** 2·4 **j** 1·2
k 2·5 **l** 0·19 **m** 2 **n** 4 **o** 0·3 **p** 3 **q** 3 **r** 10 **s** 0·3 **t** 1·1
u 1·7

7 **a** 4 **b** 10 **c** 6 **d** 4 **e** −3 **f** −3 **g** −2 **h** −5 **i** −6 **j** 3
k 2 **l** −1 **m** −2 **n** −6 **o** 3 **p** −24 **q** −6 **r** −5 **s** 3 **t** −4
u −7 **8** 3 m **9** **a** $5P + A = 86$ **b** 12 cents

Prep Quiz 9:11

1 + **2** × **3** − **4** − **5** + **6** × **7** ÷ **8** $x = 11$ **9** $a = 17$ **10** $m = 7$

Exercise 9:11

1 **a** $x + 4 = 13$; $x = 9$ **b** $9 + x = 17$; $x = 8$ **c** $x - 7 = 9$; $x = 16$ **d** $8x = 56$; $x = 7$
e $x - 7 = 3$; $x = 10$ **f** $x + 6 + 14 = 35$; $x = 15$ **g** $\frac{x}{7} = 12$; $x = 84$ **h** $9x = 117$; $x = 13$
i $\frac{96}{x} = 16$; $x = 6$ **j** $73 - x = 31$; $x = 42$

2 **a** $b + 75 = 220$; book = \$1.45 **b** $t + 17 = 30$; time = 13 min **c** $5{\cdot}25n = 63$; number = 12
d $H + 18 + 15 = 52$; Harry is 19 **e** $\frac{156}{n} = 12$; 13 children

3 **a** $3x + 5 = 17$; $x = 4$ **b** $2x - 10 = 4$; $x = 7$ **c** $\frac{x}{5} + 3 = 7$; $x = 20$ **d** $100 - 8x = 28$; $x = 9$

4 **a** $x + (x + 7) = 13$; numbers are 3 and 10. **b** $x - (20 - x) = 6$; numbers are 7 and 13.
c $x + (x + 19) = 29$; smallest number is 5. **d** $x + (x + 1) = 17$; numbers are 8 and 9.
e $x + (x + 1) + (x + 2) = 18$; smallest number is 5. **f** $x + (x + 5) = 37$; Bill is 21.
g $x + 2x = 36$; Michelle has 12 dolls.

5 **a** 9 **b** 14 **c** 10 **d** 4 **e** 10 **f** 12 **g** 8 **h** 9
6 **a** −7 **b** 0·9 **c** −1 **d** 7 **e** −1 **f** 45 cents **g** 10, 12, 14

Exercise 9:12

1 The distance travelled needs to be known before the cost can be calculated. Lisa pays \$85 rent for the day.
Yasar travelled 172 km. $(111 = 25 + \frac{K}{2})$

2 $2W + 2(W + 12) = 300$. The picture frame has width 69 cm and length 81 cm. (Complications will arise if we consider the width and join of the frame.)

3 $400 = 40 + 15r$, $r = 24$

4 $s = 3L - 20$. If $s = 13$, $L = 11$.

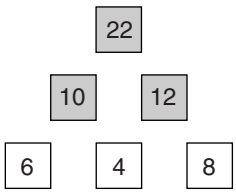
5 $N = 5C - 3$ (or $C = \frac{N+3}{5}$). He will need 147 boards.

6 180 sales → \$60 000 profit, 210 sales → \$120 000 profit, 300 sales → \$300 000 profit, 170 sales → \$40 000 profit, 140 sales → −\$20 000 profit.
In the last year there was a loss of \$20 000. For a profit of \$50 000, 175 sales are needed. The company breaks even with 150 sales.

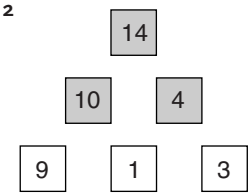
7 $A = 3r^2$. The actual formula is $A = \pi r^2$ (ie $A \doteqdot 3{\cdot}142r^2$). Area of circle $\doteqdot 480\,000\,\text{cm}^2 = 48\,\text{m}^2$.

Fun Spot 9:12 Solving pyramids

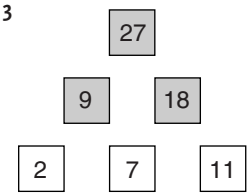
1



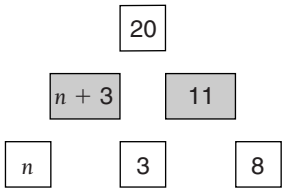
2



3

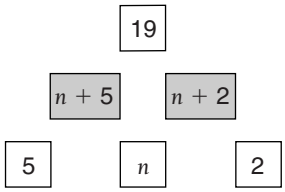


4



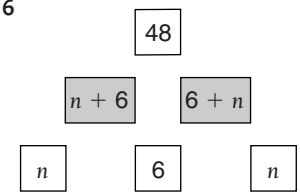
$(n + 3) + 11 = 20$
$\therefore\ n = 6$

5



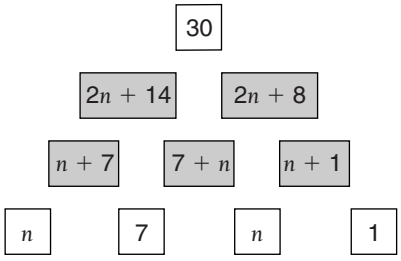
$(n + 5) + (n + 2) = 19$
$2n + 7 = 19$
$\therefore\ n = 6$

6



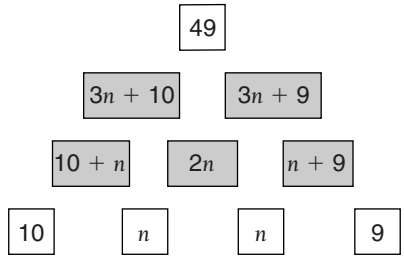
$(n + 6) + (6 + n) = 48$
$2n + 12 = 48$
$\therefore\ n = 18$

7



$(2n + 14) + (2n + 8) = 30$
$4n + 22 = 30$
$\therefore\ n = 2$

8



$(3n + 10) + (3n + 9) = 49$
$6n + 19 = 49$
$\therefore\ n = 5$

Diagnostic Test 9: Further Algebra

1 **a** $5y$ **b** ab **c** $12mn$ **d** $2s + 5t$ **2** **a** a^2 **b** $6p^2$ **c** $6m^2$ **d** wt^2

3 **a** $\frac{y}{5}$ **b** $\frac{10}{a}$ **c** $\frac{a+2}{3}$ **d** $\frac{x}{3+y}$

4 **a** $2y$ **b** $x + 5$ **c** $2x + y$ **d** $x + 2y$ (or $2y + x$) **e** $2x + 8$ **f** $3x + 4$

5 **a** 20 **b** 25 **c** 11 **d** 4 **6** **a** 18 **b** 12 **c** 21 **d** 2

7 **a**

t	1	2	3	4
b	3	6	9	12

b

x	0	2	4	6
y	1	5	9	13

c

n	0	1	2	3
m	0	1	4	9

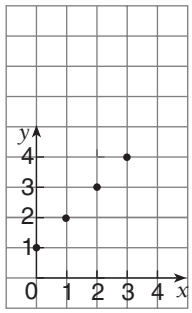
8 **a** 2 **b** 1 **c** 3 **d** 2 **9** **a** $7a$ **b** $6y$ **c** $5ab$ **d** m

10 **a** $3a + 3$ **b** $2x + 2y$ **c** $6m + 8n$ **d** $5p^2 + 2p$

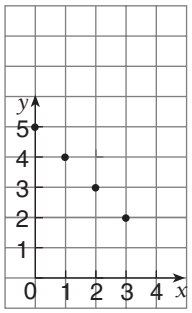
11 **a** $2x + 8$ **b** $5 - 5a$ **c** $6y + 15$ **d** $2x^2 + xy$

12 **a** $4x + 17$ **b** $14a + 3$ **c** $2m - 20$ **13** **a** a^4 **b** y^5 **c** $12a^3$ **d** x^3y^2

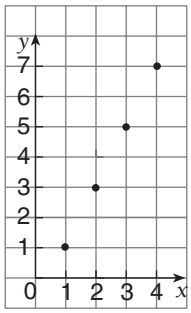
14 **a**



b



c



15 **a** -10 **b** 4 **c** 1 **d** 8

16 **a**

x	-2	-1	0	1
y	0	1	2	3

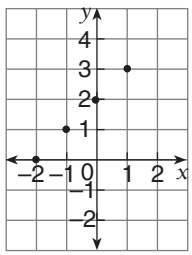
b

t	-2	-1	0	1
b	-3	-1	1	3

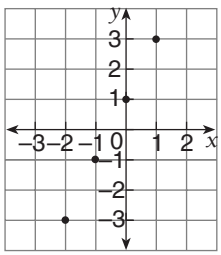
c

m	-2	0	2	4
n	4	2	0	-2

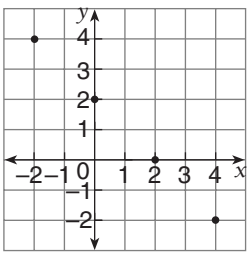
17 **a**



b



c



18 **a** $-2a$ **b** x **c** $-7x^2$ **d** $3a - 3$

19 **a** $-8a$ **b** $-18a$ **c** $3m$ **d** $-a$

20 **a** $a = 6$ **b** $x = 9$ **c** $y = 15$ **d** $m = 6$

21 **a** $x = 8$ **b** $m = 20$ **c** $p = 16$ **d** $n = 36$

22 **a** $a = 3$ **b** $m = 3$ **c** $x = 3$ **d** $m = 3$

23 **a** $n + 7 = 19$, $n = 12$ **b** $3n = 36$, $n = 12$ **c** $n - 13 = 23$, $n = 36$ **d** $72 \div n = 12$, (or $\frac{72}{n} = 12$), $n = 6$

9A Revision Assignment

1 **a** 10 **b** 26 **c** $7\frac{1}{2}$ or 7·5 **d** 5 **2** **a** 500 **b** 5000 **c** $\frac{5}{10}$ or 0·5 **d** $\frac{5}{100}$ or 0·05

3 **a** 7829 **b** 97·38 **c** 430 751

4 **a** $(8 \times 100) + (1 \times 10) + (7 \times 1)$ **b** $(2 \times 1000) + (1 \times 100) + (4 \times 10) + (3 \times 1)$

c $(6 \times 1) + (3 \times \frac{1}{10}) + (5 \times \frac{1}{100})$ **d** $(6 \times 10\,000) + (0 \times 1000) + (0 \times 100) + (8 \times 10) + (1 \times 1)$

5 **a** 4 **b** 75 **c** 1 **d** 1 **6** **a** 25 **b** 36 **c** 49 **d** 64

7 **a** \$7.50 **b** \$1.85 **c** \$5.25 **d** \$8.25

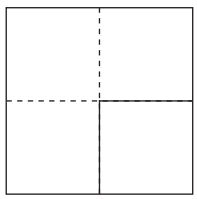
9B Working Mathematically

1 **a** *point A* **b** *interval AB* **c** *line AB* **d** *ray AB*

e *diagonals* **f** *acute-angled triangle* **g** *right-angled triangle*

h *obtuse-angled triangle*

2



3 3 **4** \$5.40 **5** \$4.20, \$1.80 **6** 6

7 **a** July **b** June **c** Tennis Ball Production **d** 50 000 **e** 55 000 **f** 15 000

Chapter 10: Angles

Exercise 10:02

1 **a** A **b** Y **c** P **d** O

2 **a** $\angle CAB$ **b** $\angle XYZ$ **c** $\angle MPQ$ **d** $\angle BOP$

3 **a** $\angle LOT$ or $\angle O$, $\angle OTL$ or $\angle T$, $\angle TLO$ or $\angle L$ **b** $\angle AQP$ or $\angle Q$, $\angle QPB$ or $\angle P$, $\angle PBA$ or $\angle B$, $\angle BAQ$ or $\angle A$
c $\angle PSR$ or $\angle S$, $\angle SRQ$ or $\angle R$, $\angle RQP$ or $\angle Q$, $\angle QPS$ or $\angle P$

4 **a** $\angle DBC$ **b** $\angle ABD$ **c** $\angle ABC$ **d** $\angle DEC$ **e** $\angle AED$
f $\angle ATF$ **g** $\angle DCB$ **h** $\angle ACD$ **i** $\angle ACB$

5 **a** **1** $\angle MTB$ **b** **1** $\angle ADB$ **c** **1** $\angle CFG$
2 $\angle ATN$ **2** $\angle CAB$ **2** $\angle AGF$
3 $\angle BTN$ **3** $\angle DAC$ **3** $\angle HGB$

6 **a** $\angle ACD$ **b** $\angle ABD$ **c** $\angle EBC$ **d** $\angle AEB$ **e** $\angle FCE$

7 **a** 2 **b** 1 **c** 3 **8** **a** Y **b** X **c** Y **d** Y

9 **a** B, A, C, D **b** D, B, A, C **c** B, C, D, A

Prep Quiz 10:03

1 B **2** BA and BC **3** $\angle ABC$ **4** B **5** A **6** B, A, C **7** $\angle ABD$ **8** $\angle D$ **9** $B\hat{C}A$ **10** $\angle BCA$

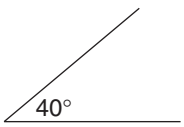
Exercise 10:03 (Practical)

1 **a** 90° **b** 180° **c** 45° **d** 135° **e** 10° **f** 110° **g** 30° **h** 160° **i** 60°

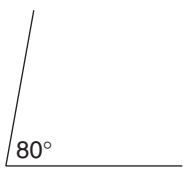
2 **a** 60° **b** 120° **c** 40° **d** 110° **e** 40° **f** 80°

3 **a** 30° **b** 150° **c** 60° **d** 130° **e** 10°

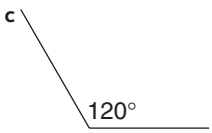
4 **a**



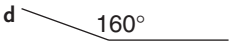
b



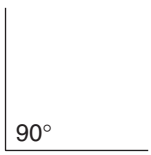
c



d



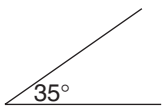
e



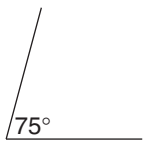
5 Check your efforts with a protractor.

6 **a** 32° **b** 153° **c** 62° **d** 128° **e** 11°

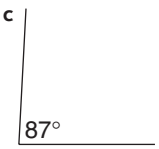
7 **a**



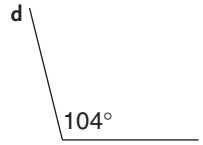
b



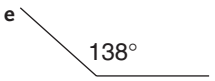
c



d



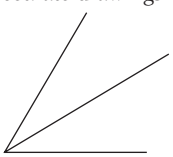
e



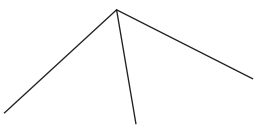
8 **a** 144° **b** 33° **c** 51° **9** **a** $\angle A = 62°$, $\angle C = 107°$ **b** $\angle A = 119°$, $\angle C = 87°$

10 Accurate drawings will be drawn.

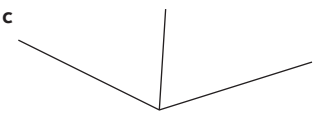
11 **a**



b



c



12 Answers will vary.

13 **a** 334° **b** 224° **c** 270°

Exercise 10:04

1 **a** acute **b** acute **c** acute **d** right **e** right
f right **g** straight **h** straight **i** revolution **j** revolution
k revolution **l** reflex **m** reflex **n** reflex **o** reflex
p obtuse **q** obtuse **r** obtuse **s** obtuse **t** obtuse
u reflex **v** right **w** obtuse **x** acute

2 **a** right **b** straight **c** revolution **d** obtuse **e** acute
f obtuse **g** reflex **h** reflex **i** acute **j** obtuse
k reflex **l** obtuse **m** acute **n** acute **o** obtuse
p revolution **q** right **r** obtuse

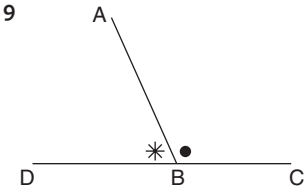
3 **a** acute **b** obtuse **c** straight
4 **a** straight **b** straight **c** acute **d** acute **e** obtuse **f** obtuse
5 **a** right **b** acute **c** acute
6 **a** obtuse **b** acute **c** acute **7** **a** C **b** B
8 **a** all right angles **b** 2 acute and 2 obtuse angles **c** 3 obtuse and 1 acute angle

Reading Mathematics 10:04 Smoking and health facts!

1 At least 154 900 died each year. **2** They were 5 times as great.
3 4 out of 10 or 40% **4** **a** 40% **b** 15% **5** 5·5 minutes **6** $\frac{9}{10}$
7 more than twice as likely. **8** 14 times as great as the non-smoker.

Prep Quiz 10:05

1 right angle **2** acute angle **3** obtuse angle **4** straight angle **5** 90° **6** 180°
7 360° **8** and **9**



10 about 180°

Exercise 10:05

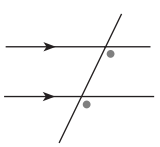
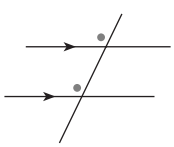
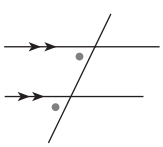
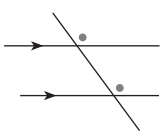
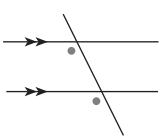
1 **a** $\angle CBD$ and $\angle DBA$ **b** 30° **2** **a** $\angle ABD$ and $\angle DBC$ **b** $\angle ABD$ and $\angle DBC$ **c** 90°
3 **a** $\angle ABD$ and $\angle DBC$ **b** $\angle ABD$ and $\angle DBC$ **c** yes
4 **a** 30° **b** 40° **c** 130° **5** **c** yes, 180° **d** 150° **e** yes **f** yes, yes
6 **b** 180° **e** yes, yes **f** $\angle BAD$ and $\angle CAD$ **7** **b** 180° **c** 50° and 130° **d** yes, yes **e** yes **f** yes
8 **a** Answers will vary. **b** yes, 180° **c** yes, yes **9** **a** Answers will vary. **c** yes, yes
10 $\angle ABD = 150°$ **11** **a** 20° **b** 340° **12** **a** 131° **b** 229°
13 **a** 1 and 3, 2 and 4 **b** They are equal. **c** They match. **d** Vertically opposite angles are equal.
14 **a** 60° because $\angle ABC$ is a straight angle of 180°. **b** 60° because $\angle EBD$ is a straight angle of 180°.
c yes **d** yes
15 **a** $x = 80$ **b** $a = 35$ **c** $p = 60$ **d** $e = 88$ **e** $y = 105$
f $b = 48$ **g** $x = 136$ **h** $a = 90$ **i** $m = 110$ **j** $a = 50$
k $x = 140$ **l** $a = 80$

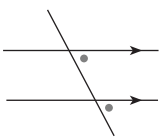
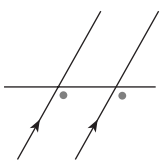
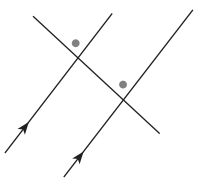
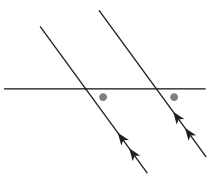
Prep Quiz 10:06

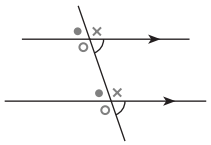
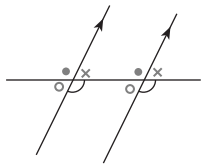
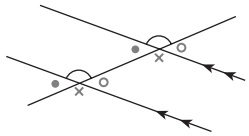
1 3 **2** 4 **3** They add to 180°. **4** They add to 180°. **5** They add to 180°. **6** 360°
7 140° **8** 40° **9** 140° **10** yes

Exercise 10:06

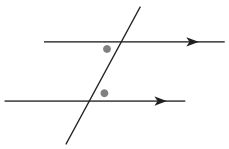
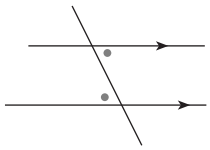
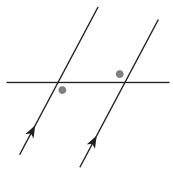
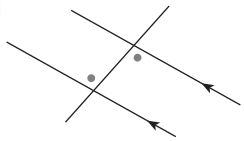
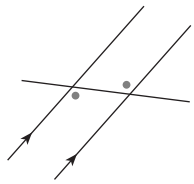
1 **a** 1 and 3, 2 and 4 **b** 5 and 7, 6 and 8
2 **a** 1, 5, 3 and 7 2 6, 4 and 8 **b** 1, 5, 3 and 7 2, 6, 4 and 8
3 **a** equal **b** equal **c** equal **d** equal
4 **a** below a parallel line and to the left of the transversal
b below a parallel line and to the right of the transversal

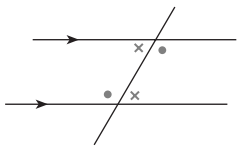
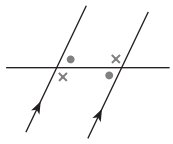
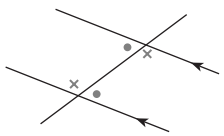
5 **a**  **b**  **c**  **d**  **e** 

f  **g**  **h**  **i** 

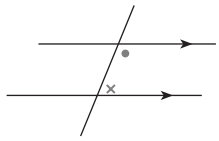
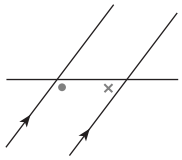
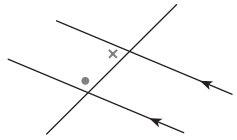
6 **a**  **b**  **c** 

7 **a** equal **b** equal

8 **a**  **b**  **c**  **d**  **e** 

9 **a**  **b**  **c** 

10 **a** They are supplementary. **b** They are supplementary.

11 **a**  **b**  **c** 

12 **a** 5 **b** 3 **c** 5 **d** 8 **e** 5 **f** 3

13 **a** alternate angles, $a = 60$ **b** cointerior angles, $a = 40$
c corresponding angles, $b = 110$ **d** alternate angles, $d = 130$
e alternate angles, $x = 38$ **f** cointerior angles, $x = 110$

14 **a** $a = 140$ **b** $b = 115$ **c** $x = 145$ **d** $y = 75$ **e** $a = 115$ **f** $b = 55$

ID 10:06

1 *parallel* lines **2** *perpendicular* lines **3** *vertical*, *horizontal* **4** *concurrent* lines
5 angle *ABC* or *CBA* **6** *acute* angle **7** *right* angle **8** *obtuse* angle
9 *straight* angle **10** *reflex* angle **11** *revolution* **12** *adjacent* angles
13 *complementary* angles **14** *supplementary* angles **15** *vertically opposite* angles **16** *360°*
17 *transversal* **18** *corresponding* angles **19** *alternate* angles **20** *cointerior* angles

Challenge 10:06 Getting the angle on things: Triangulation

1 **a** symmetry **b** even **c** net **d** odd **e** abacus **f** digits **g** hexagon
h line **i** triangle **j** rectangular **k** circle **l** thermometer
2 attend schools (of fish)

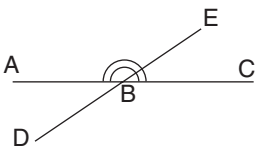
Exercise 10:07

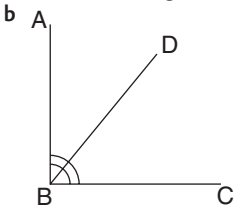
1 **a** Yes. The corresponding angles are equal. **b** Yes. The alternate angles are equal.
c Yes. The cointerior angles are supplementary. **d** No. The cointerior angles are not supplementary.
e Yes. The alternate angles are equal. **f** No. The alternate angles are not equal.
g No. The corresponding angles are not equal.

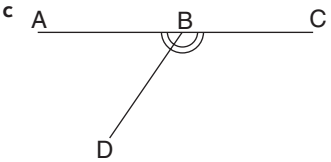
2 **a** yes **b** no **c** yes

Diagnostic Test 10: Angles

1 **a** $\angle CAB$, acute angle **b** reflex $\angle XYZ$, reflex angle **c** $\angle ACB$, obtuse angle

2 **a** 

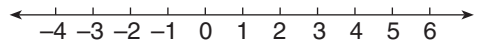
b 

c 

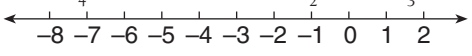
3 **a** $x = 150$ **b** $a = 20$ **c** $b = 105$ **4** **a** $a = 40$ **b** $b = 80$ **c** $a = 65$
5 **a** $a = 120$ **b** $b = 90$ **c** $c = 40$ **6** **a** $a = 150$ **b** $b = 120$ **c** $c = 50$
7 **a** $a = 50$ **b** $b = 110$ **c** $c = 70$

10A Revision Assignment

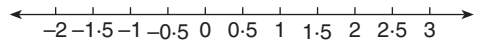
1 **a** 0·7 **b** 0·42 **c** 0·175 **d** 0·05 **e** 0·027
2 **a** 4, 5, 6, 7 **b** 0, 16, 32, 48 **c** 1, 6, 11, 16 **3** 3, 5, 7, 9
4 **a** $6m$ **b** $3mn$ **c** $12y$ **d** $2a + 3b$
5 **a** 18 **b** 81 **c** 45 **d** 4
6 **a** -6 **b** 0 **c** 4 **d** 7 **e** -2
7 **a** 3 h **b** 6 h **c** 4 h **d** 5 h
8 **a** $-5x$ **b** $9x - y$ **c** $-8x$ **d** $3x - 5$
9 **a** 2^3 **b** 3^4 **c** a^2b^3 **d** $2a^3b$
10 **a** 5 **b** 6 **c** 8 **d** 4
11 **a** $\frac{3}{4}$ **b** 18 **c** $3\frac{1}{2}$ **d** $\frac{5}{3}$
12 **a**



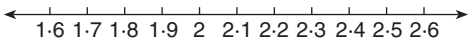
b



c



d



10B Working Mathematically

1 158 km **2** $214.50 **3** $1.60 **4** $271.25 **5** Bob
6 **a** **i** $25 million **ii** $15 million **b** 1997 **c** 2000 **d** $13 million **e** 1997, 2002 and 2003

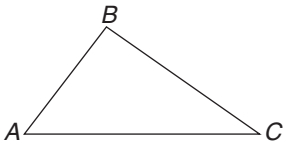
Chapter 11: Shapes

Exercise 11:01

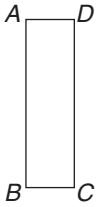
1 **a** It does not have straight sides. **b** It has four sides. **c** It does not have three straight sides.
2 square, rectangle, rhombus, parallelogram, trapezium, kite
3 **a** Two lines are parallel if they lie in the same plane and do not meet.
b square, rhombus, rectangle **c** trapezium **d** trapeze at the circus
4 circle, oval (or ellipse) **5** oval
6

Shape	*Number of sides*	*Number of angles*	*Number of diagonals*
Triangle	3	3	0
Quadrilateral	4	4	2
Pentagon	5	5	5
Hexagon	6	6	9
Octagon	8	8	20

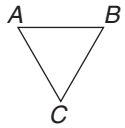
7 diamond **8** **a** I **b** II **c** IV
9



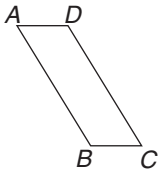
10 **a**



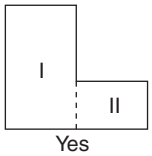
b



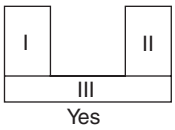
c



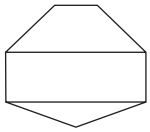
11 **a** rectangle **b** parallelogram **c** kite
12 **a** two triangles **b** triangle and trapezium **c** two trapeziums **d** rectangle and two trapeziums
13 **a** 2 triangles and 2 rhombuses **b** a rectangle and 2 triangles **c** a kite and 2 triangles **d** two trapeziums
14 **a**



b



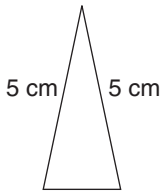
c



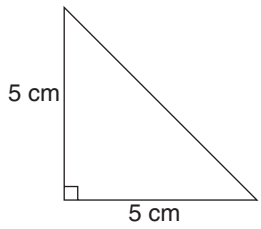
Exercise 11:02

1 **a** isosceles, right-angled **b** equilateral, acute-angled **c** scalene, obtuse-angled
d isosceles, obtuse-angled **e** scalene, right-angled **f** isosceles, acute-angled

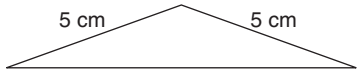
2 a



b



c



These answers have been scaled down to save space.

3 **a** no **b** yes, 2 angles are equal **c** yes, 3 angles are equal
4 In all triangles the angles sum to 180°. **5** **c** The angle sum is 180°.
6 Yes, the equal angles are opposite the equal sides.
7 **a** 80° **b** Yes, the equal sides are opposite the equal angles.
8 **a** They are all 60°. **b** 60°, yes **9** isosceles
10 **a** *AB*, *AC* and *BD*, *DC* **b** $\angle ABC$, $\angle ACB$; $\angle BAD$, $\angle CAD$; $\angle ADB$, $\angle ADC$
11 **a** equilateral, 60° **b** They are all equal in length.
c The largest angle is 90°; the triangle is right angled and scalene. **d** no **e** 45°
12 **a** 40 **b** 90 **c** 30 **d** 40 **e** 65 **f** 6 **g** 60 **h** 6·5
i $b = 70$, $c = 40$ **j** 65 **k** $x = 5{\cdot}8$, $a = 70$ **l** 45

Exercise 11:03

1 **a** square, rectangle, parallelogram, rhombus **b** square, rectangle, parallelogram, rhombus
c square, rhombus **d** square, rectangle **e** Opposite angles are equal.
2 **a** yes **b** yes **c** yes **3** **a** yes **b** no **c** yes
4 **a** no **b** no **c** yes **5** **a** no **b** yes **c** yes
6 **a** square and rhombus **b** square and rhombus **7** The angles add up to 360°. **8** They add up to 360°.

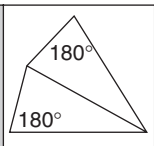
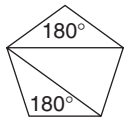
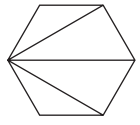
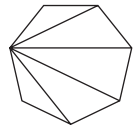
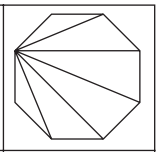
ID 11:03

1 *square* **2** *rectangle* **3** *parallelogram* **4** *rhombus*
5 *trapezium* **6** *regular pentagon* **7** *regular hexagon* **8** *regular octagon*
9 *kite* **10** *scalene triangle* **11** *isosceles triangle* **12** *equilateral triangle*
13 *circle* **14** *oval (or ellipse)*

Exercise 11:04

1 **a** $a = 60$ **b** $a = 50$ **c** $b = 60$ **d** $b = 55$ **e** $b = 70$ **f** $c = 68$ **g** $c = 43$ **h** $c = 45$
i $d = 110$ **j** $d = 80$ **k** $d = 60$ **l** $e = 130$ **m** $e = 65$ **n** $e = 300$
2 **a** $a = 35$ **b** $b = 115$ **c** $c = 40$ **d** $d = 270$ **e** $e = 207$ **f** $f = 67$ **g** $g = 70$ **h** $h = 60$
i $x = 59$ **j** $a = 60$ **k** $b = 35$ **l** $c = 74$

Table 11:05 Angle Sum of a Polygon

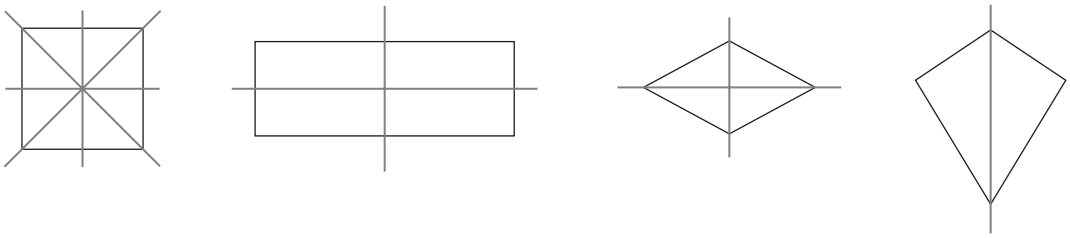
Polygon					
Number of sides	4	5	6	7	8
Number of triangles	2	3	4	5	6
Angle sum of polygon	$2 \times 180°$ $= 360°$	$3 \times 180°$ $= 540°$	$4 \times 180°$ $= 720°$	$5 \times 180°$ $= 900°$	$6 \times 180°$ $= 1080°$

Exercise 11:05

1 **a** 4 **b** 6 **c** 10, 1800° **d** 7, 1260°
2 **a** 100° **b** 70° **c** 55° **d** 60° **e** 74° **f** 113° **g** 94° **h** 40° **i** 140° **j** 90°
k 120° **l** 140° **m** 130° **n** 120°

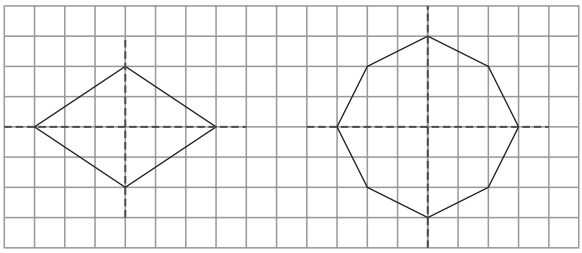
Exercise 11:06

1



2 **a** 4 **b** 2 **c** 0 **d** 0 **e** 2 **f** 1 **3** square, rectangle, parallelogram, rhombus

4 **a** **b**



5 **a** All of the shapes have point symmetry. **b** *B* and *D* **6** Answers will differ. **7** Answers will differ.

Exercise 11:07

1 **a** 1 **b** 5 **c** 9 **d** 4

2 **a** cylinder **b** rectangular prism **c** ball or sphere **d** cone

3 4, 5, 9 **4** 1, 2, 3 **5** 6, 7, 8 **6** 4, 5, 9

7

Solid	*F*	*V*	*E*	*F* + *V*
cube	6	8	12	14
triangular prism	5	6	9	11
rectangular pyramid	5	5	8	10
triangular pyramid	4	4	6	8

8 cylinder, sphere **9** cylinder, sphere, cone

10 **a** triangles and rectangles **b** triangles and square

11 **a** rectangular prism **b** triangular prism **c** cylinder **d** triangular pyramid

12 **a** *AD*, *AE*, *BC*, *BF*
b *DC*, *HG*, *EF*
c *HD*, *GC*, *HE*, *GF*
d *EFGH*

13 **a** cone **b** cylinder **c** square pyramid **d** rectangular prism
e sphere **f** triangular prism **g** triangular pyramid **h** hexagonal prism **i** pentagonal pyramid

14 **a** two square pyramids **b** cone and cylinder **c** two cylinders
d rectangular prism and rectangular pyramid

15 Euler's theorem holds in every case.

16 **a** true **b** true **c** true **d** true **e** true **f** true **g** false **h** true

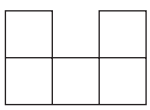
17 **a** B **b** D **c** C **d** A

Exercise 11:08

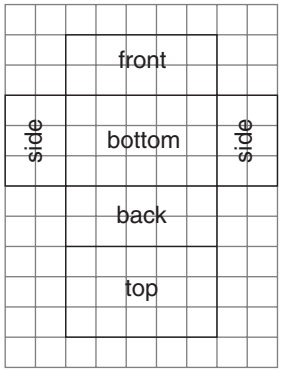
1 **a** C **b** A **c** B

2 **a** B **b** C **c** A

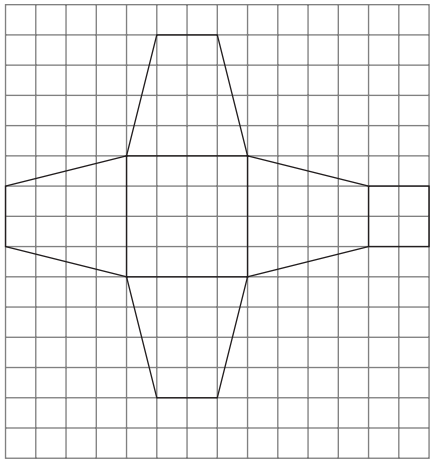
3 **a** B, D, E, F, H, I, J, K **b**



4

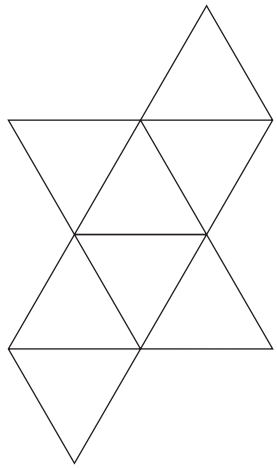


5



Note: This is only one possible answer. Different nets would have different sized trapeziums depending on the height of the pyramid.

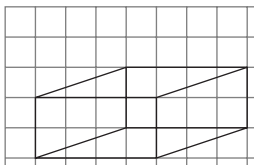
6



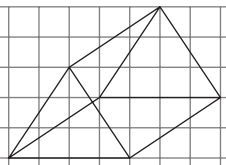
Exercise 11:09

2 square prism

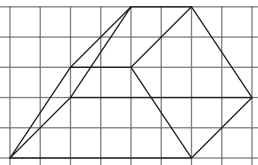
3 **a**



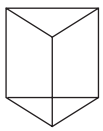
b



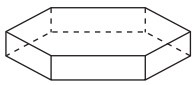
c



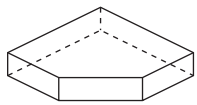
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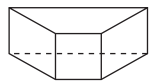
5 **a**



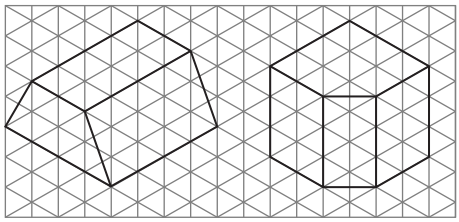
b



c

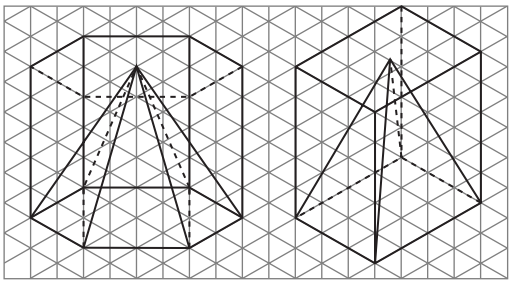


6 **a**



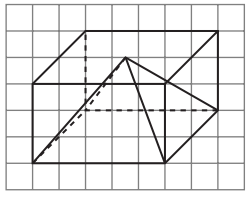
b

7 **a**

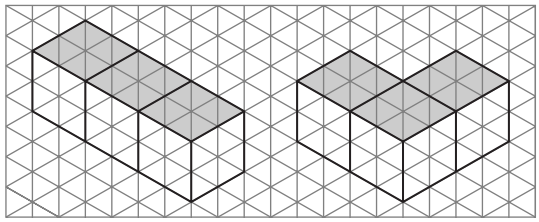


b

c



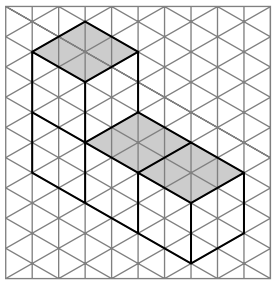
8 The question assumes that the cubes must be joined face to face. If this condition is applied there are only two possible arrangements.



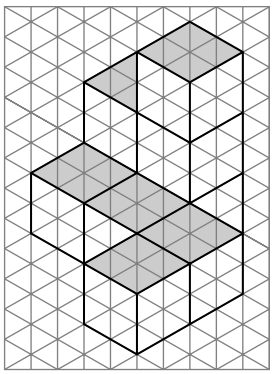
9 Cones and cylinders will have been drawn.

Exercise 11:10

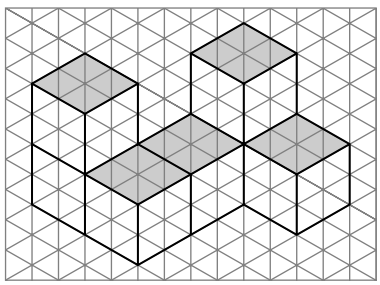
3 a



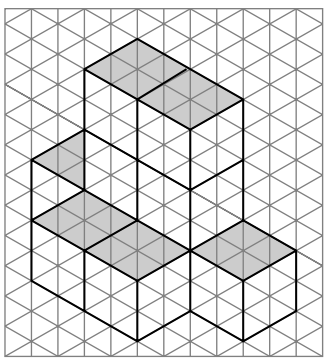
b



4 a

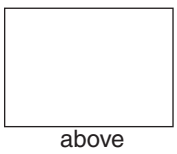


b



Exercise 11:11

1 a





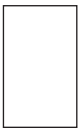
b





c





d

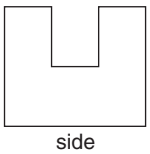




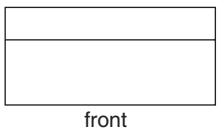
2 A (length) and C (height) **3 a** square pyramid **b** sphere **c** cone **d** triangular prism **e** hexagonal prism **f** octagonal pyramid

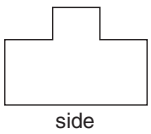
4 a



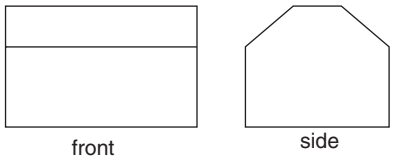


b

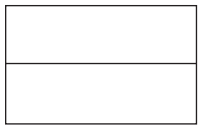




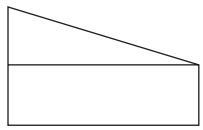
c



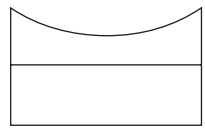
5 a



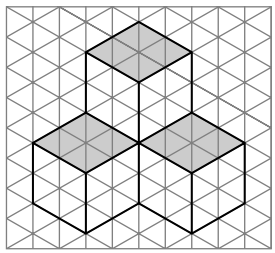
b



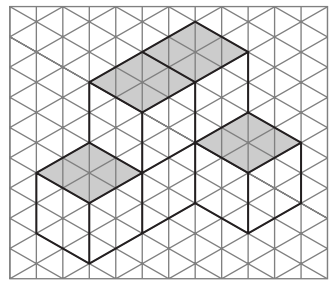
c



6 a



b



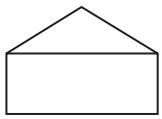
Reading Mathematics 11:11 The Platonic Solids

1 a 6 **b** 8 **c** 4 **d** 12

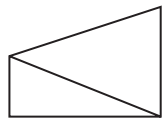
2 Euler's theorem holds for each of the Platonic solids.

Diagnostic Test 11: Shapes

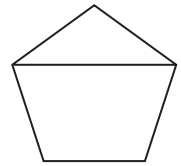
1 a



b



c



2 a right-angled **b** obtuse-angled **c** acute-angled

3 a $a = 80$ **b** $b = 105$ **c** $c = 51$

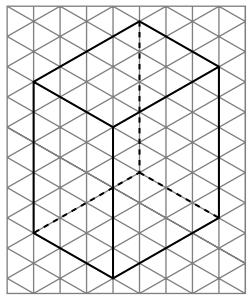
4 a $a = 105$ **b** $b = 75$ **c** $c = 89$

5 a 4 **b** 2 **c** 2

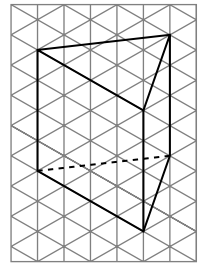
6 a, b and c

7 a cube **b** triangular prism **c** triangular pyramid

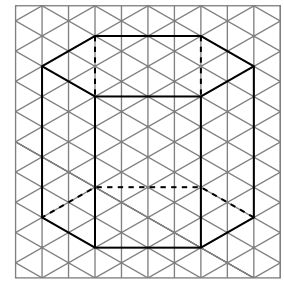
8 a



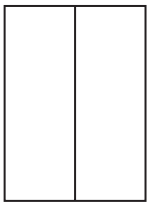
b



c



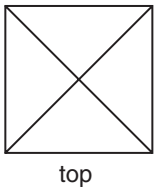
9 a







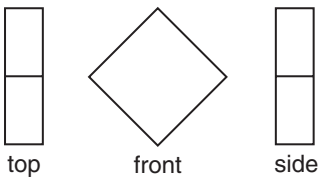
b







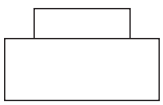
c



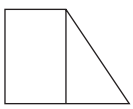
11A Revision Assignment

1 a 6035
b 700 000
c 12 675
d 13 400 089

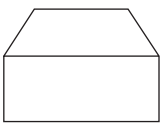
2 a



b



c



3 a 15 **b** 150 **c** 1·56 **d** 0·156
4 a 6 **b** 12 **c** 8 **d** trapezium **e** trapezoidal prism
5 a 4 **b** 13 **c** −5 **d** 16
6 a 40 **b** 3500 **c** 25 **d** 0·42
7 a 800 m **b** 25·2 cm **c** 18 m **d** 4·6 km
8 a 9:05 am **b** 12:45 pm **c** 7:35 am **d** 3:10 pm
9 a 180 km **b** 3 hours **c** 15 km/h
10 a $a = 31$ **b** $b = 38$ **c** $c = 255$

11B Working Mathematically

1 a *cube* **b** *rectangular prism* **c** *triangular prism* **d** *square pyramid* **e** *rectangular pyramid*
f *triangular pyramid* **g** *cylinder* **h** *cone* **i** *sphere* **j** *hemisphere*
2 6:50 am **3** blue **4** 4 **5** 2375 mm
6 a yes **b** about 43 kg **c** about 168 cm and 189 cm **d** about 63 kg and 81 kg **e** about 2·5 kg

Chapter 12: Measurement: Length and Time

ID 12:01

1 *metres* **2** *decimetres* **3** *centimetres* **4** *millimetres* **5** *kilometres*
6 *square metres* **7** *square centimetres* **8** *square kilometres* **9** *hectares* **10** *cubic metres*
11 *cubic centimetres* **12** *seconds* **13** *minutes* **14** *hours* **15** *metres per second*
16 *kilometres per hour* **17** *grams* **18** *milligrams* **19** *kilograms* **20** *tonnes*
21 *litres* **22** *millilitres* **23** *kilolitres* **24** *degrees Celsius*

Exercise 12:01

1 4 cm, 40 mm **2 a** 2·5 m **b** 2 m
3 a a quarter past eight, before noon **b** a quarter to eight, after noon **c** seven minutes to nine, before noon
d one minute to four, after noon **e** a quarter past eight, after noon **f** a quarter to eight, before noon
g one minute to four, before noon **h** seven minutes to nine, after noon
4 a 30°C **b** 40°C **c** 44°C **d** 14°C **e** 0°C
5 a 60° **b** 120° **c** 10 mL **d** 40 km **e** 35 km
6 a 1·5 L **b** 3 cups **c** 0·25 L **d** $1\frac{1}{4}$ L
7 a 81 057·6 km **b** 100 913·0 km **8 a** 30 km/h **b** 140 km/h
9 a almost full **b** $\frac{1}{4}$ full **c** warm **d** very hot **10 a** 1802·711 **b** 9447

Prep Quiz 12:02

1 620 **2** 2340 **3** 6·7 **4** 5·25 **5** 3·54 **6** 5·49 **7** 4·7 **8** 5·37 **9** 25 cm **10** 83 mm or 84 mm

Exercise 12:02

1 **a** 3 cm **b** 5 cm **c** 7 cm **d** 9 cm **e** 11 or 12 cm
2 **a** 32 mm **b** 50 mm **c** 69 mm **d** 94 mm **e** 115 mm
3 **a** 8 mm **b** 22 mm **c** 45 mm **d** 80 mm **e** 118 mm
4 **a** 0·8 cm **b** 2·2 cm **c** 4·5 cm **d** 8·0 cm **e** 11·8 cm
5 **a** **i** 12·0 cm **ii** 14·5 cm **iii** 16·7 cm **iv** 17·9 cm **v** 19·3 cm
b **i** 12·0 cm **ii** 14·5 cm **iii** 16·7 cm **iv** 17·9 cm **v** 19·3 cm
6 **a** 3200 mm **b** 2120 mm **c** 4670 mm **d** 1485 mm **e** 3844 mm **f** 5438 mm
7 **a** 50 **b** 170 **c** 62 **d** 256 **e** 600 **f** 2300 **g** 160 **h** 235
i 5000 **j** 67 000 **k** 7300 **l** 9320 **m** 9000 **n** 3500 **o** 2900 **p** 8471
q 300 000 **r** 436 000 **s** 6 000 000 **t** 9 217 000
8 **a** 3 **b** 12 **c** 0·6 **d** 5·37 **e** 5 **f** 25 **g** 2·5 **h** 0·63
i 2 **j** 17 **k** 6·7 **l** 0·58 **m** 4 **n** 26 **o** 6·5 **p** 0·7
q 0·1 **r** 0·27 **s** 0·09 **t** 0·2
9 **a** 2 **b** 3 **c** 6000 **d** 5 **e** 25 000 **f** 1500 **g** 230 **h** 5·2
i 6·7 **j** 1700 **k** 2600 **l** 75 **m** 6·35 **n** 195 **o** 1·96 **p** 93·6
q 7630 **r** 93·5 **s** 0·75 **t** 0·87 **u** 0·62 **v** 320 000 **w** 0·73 **x** 170
10 **a** 1 km **b** 24·4 km **c** 180 cm **d** 8 pieces, 1·2 m long **e** $4.50 **f** 13
11 **a** 150 **b** 310 **c** 525 **d** 3500 **e** 250 **f** 2200 **g** 5500 **h** 2250
i 1300
12 **a** 63 km **b** 7 L
13 **a** 300 **b** 70 **c** 5 **d** 90 **e** 6 **f** 7 **g** 40 **h** 5
14 **a** 1000 **b** 10 000 **c** 6 Mm **d** 20 μm

Prep Quiz 12:03

1 1000 **2** 300 **3** 5 **4** 7 **5** 60 **6** 70 **7** 5000 **8** 7·5 **9** 6350 **10** 7·65

Exercise 12:03

1 **a** cm **b** km **c** m **d** mm **e** cm **f** m **g** cm **h** mm
2 **a** ruler or tape **b** odometer **c** trundle wheel **d** ruler **e** ruler
f tape or ruler **g** tape **h** ruler
3 **a** 5 cm **b** 7 cm **c** 4 cm **d** 9 cm **e** 11 cm **f** 5 cm **g** 2 cm **h** 4 cm
4 **a** 50 mm **b** 71 mm **c** 39 mm **d** 94 mm **e** 111 mm **f** 54 mm **g** 22 mm **h** 39 mm
5 EF, CD, GH, AB **6** RS, WZ, XY, TU, KL, MN
7 About: **a** 7 cm **b** 8 cm **c** 13 cm **d** 20 cm
8 **a** 53 096 km **b** 3·6 km **c** 53 121·9 **d** 46 903·6 km **e** 40 km/h

Prep Quiz 12:04

1 5·3 cm **2** 53 cm **3** 5·3 m **4** 53 000 mm **5** 0·53 km **6** 12 cm **7** 8 cm **8** 5 cm **9** 3 cm **10** 9 cm

Exercise 12:04

1 **a** 5 cm **b** 3 cm **c** 10 cm **d** 2 cm **e** 8 cm **f** 7 cm **g** 4 cm
2 C **3** **a** 4 cm **b** 6 cm **c** 3 cm **d** 5 cm **e** 4 cm **f** 6 cm **g** 2 cm **h** 5 cm
4 $AB = 40$ mm, $CD = 50$ mm, $EF = 16$ mm, $GH = 55$ mm, $IJ = 62$ mm, $KL = 37$ mm, $MN = 59$ mm, $OP = 27$ mm, $QR = 70$ mm, $ST = 45$ mm

Prep Quiz 12:05

1 12 m **2** 8·4 m **3** 7·1 m **4** 2·33 m **5** 86 cm **6** 4·25 km **7** 10 m **8** 1·05 m **9** 4 m **10** 4 m

Exercise 12:05

1 **a** 15 cm **b** 12 cm **c** 39 cm **d** 80 cm **e** 36 cm **f** 13·8 cm **g** 10·6 cm **k** 4 cm
l 37·6 cm **j** 15·6 cm
2 **a** 12·8 cm **b** 224 mm **c** 8 cm **d** 17·6 cm
3 **a** Perimeter = 16 cm **b** Perimeter = 12 cm **c** Perimeter = 16 cm **d** Perimeter = 16 cm
4 **a** 1·5 cm **b** 2 cm **c** 2·5 cm **d** 3 cm
5 **a** Perimeter = 170 mm = 17 cm **b** Perimeter = 108 mm $\div$ 11 cm
c Perimeter = 163 mm $\div$ 16 cm **d** Perimeter = 156 mm $\div$ 16 cm No. If we measure the perimeter in mm and then round off to cm our answer is usually much more accurate.
6 **a** 360 m **b** $2700 **7** **a** 1080 m **b** $918 **8** 5·8 km
9 **a** 3·7 cm **b** 1·6 cm **10** **a** 64 m **b** 50 m **11** 102 m
12 **a** 42 cm **b** 32 cm **c** 22·4 cm **d** 36·6 cm

Exercise 12:06

1 **a** 365 **b** 366 **c** 12 **d** 52 **e** 7 **f** 2 **g** 14 **h** 26 **i** 10 **j** 100

2

Mth	*Days*	*Mth*	*Days*	*Mth*	*Days*
J	31	M	31	S	30
F	28	J	30	O	31
M	31	J	31	N	30
A	30	A	31	D	31

a 7 **b** 4 **c** 29 February

3 SUMMER: Dec, Jan, Feb; AUTUMN, Mar, Apr, May;
WINTER: June, July, Aug; SPRING: Sep, Oct, Nov

4 **a** 2000, 2004, 2008, 2012, 2016 **b** Olympic Games

5 **a** 19th **b** 16th **c** 20th **d** 6th **e** 22nd

6 **a** 3 January 1956 **b** 15 April 1964
c 31 July 1980 **d** 27 November 1984
e 10 April 1902 or 10 April 2002 **f** 13 May 1984
g 3 December 1947 **h** 30 January 1951
i 15 September 1980 **j** 5 August 1903 or 5 August 2003

7 **a** **i** 10 October **ii** 24 October **iii** 7 November
b **i** 19 **ii** 31 **c** 6 **d** 65 **e** 12 March 2020
f **i** Thursday **ii** Sunday **iii** Thursday

8 **a** Wednesday **b** Sunday **c** Tuesday if my birthday is in January or February; Wednesday if after February.

Exercise 12:07

1 **a** 120 min **b** 180 s **c** 3 h **d** 4 min **e** 15 min **f** 30 s **g** $\frac{1}{2}$ h **h** $\frac{1}{4}$ min
i 2 days

2 **a** **i** 3600 s **ii** 1440 min **iii** 168 h **b** 132 h

3 **a** 2:25 or 25 past 2 **b** 9:30 or half past 9 **c** 6:50 or 10 to 7 **d** 4:45 or a quarter to 5
e 10:15 or a quarter past 10 **f** 8:36 or 24 to 9 **g** 11:20 or 20 past 11 **h** 7:55 to 5 to 8

4 **a** 5:20 am **b** 10:30 am **c** 1:10 pm **d** 4:00 pm **e** 10:40 pm **f** 9:15 am **g** 12:05 pm **h** 8:55 am
i 12:20 am **j** 11:59 pm **k** 2:43 pm **l** 11:01 am

5 **a** 0520 **b** 1050 **c** 1515 **d** 2120 **e** 0230 **f** 1710 **g** 0245 **h** 1200

6

Program	*Timer Settings*
10:30 pm to 11:30 pm	22:30 to 23:30
9:15 am to 10:45 am	09:15 to 10:45
7:45 pm to 9:10 pm	19:45 to 21:10
5:30 am to 6:40 am	05:30 to 06:40
12 noon to 1:30 pm	12:00 to 13:30
5:40 pm to 7:20 pm	17:40 to 19:20

Exercise 12:08

1 **a** 3 h 50 min **b** 8 h 56 min **c** 4 h 10 min **d** 13 h 21 min

2 **a** 3 h 20 min **b** 2 h 23 min **c** 1 h 30 min **d** 3 h 30 min **e** 2 h 38 min
f 0 h 28 min

3 **a** 50 min **b** 1 h 50 min **c** 6 h 15 min **d** 6 h 10 min **e** 3 h
f 5 h 20 min **g** 10 h 55 min **h** 19 h 10 min

4 **a** 10 h **b** 9 h 15 min **c** 22 h 40 min **d** 26 h 40 min **e** 38 h 45 min

5 **a** 9 pm **b** 1:30 pm **c** 8:05 am **d** 2:20 am **e** 1:45 am **f** 11:15 am **g** 9 am **h** 6:05 am

6 **a** **i** 7 h **ii** 6 h 30 min
b **i** 4 h **ii** 3 h 45 min
c **i** 4 h **ii** 4 h 15 min
d **i** 7 h **ii** 7 h 20 min
e **i** 9 h **ii** 80 h 40 min
f **i** 2 h **ii** 2 h 24 min
g **i** 2 h **ii** 1 h 54 min
h **i** 7 h **ii** 7 h 8 min
i **i** 13 h **ii** 12 h 46 min
j **i** 12 h **ii** 11 h 50 min

7 **a** 6:30 am **b** 12:15 pm

8 **a** 8:00 pm **b** 5:00 am **c** 6:15 am **d** 1:10 am **e** 10:20 am **f** 7:45 am **g** 3:30 pm **h** 11:30 pm

9 **a** 2:00 pm **b** 1:00 pm **c** 9:00 am **d** 1:00 am **e** 4:00 pm the previous day

10 **a** 10:00 pm **b** 7:30 am

Exercise 12:09

1 **a** *B* is ahead by 2 h. **b** *A* is ahead by 7 h. **c** *A* is ahead by 5 h. **d** *B* is ahead by 5 h.
e *A* is ahead by 4 h. **f** *B* is ahead by 12 h. **g** *A* is ahead by 20 h. **h** *B* is ahead by 8 h.

2 **a**

Longitude	105°W	90°W	75°W	60°W	45°W	30°W	15°W	0°	15°E	30°E	45°E	60°E	75°E	90°E	105°E
Time	5 am	6 am	7 am	8 am	9 am	10 am	11 am	noon	1 pm	2 pm	3 pm	4 pm	5 pm	6 pm	7 pm

b

Longitude	150°W	120°W	90°W	60°W	45°W	30°W	15°W	0°	15°E	30°E	45°E	60°E	90°E	120°E	150°E
Time	5 am	7 am	9 am	11 am	noon	1 pm	2 pm	3 pm	4 pm	5 pm	6 pm	7 pm	9 pm	11 pm	1 am

c

Longitude	135°W	120°W	105°W	90°W	60°W	45°W	15°W	0°	30°E	45°E	75°E	120°E	135°E	150°E	165°E
Time	1 am	2 am	3 am	4 am	6 am	7 am	9 am	10 am	noon	1 pm	3 pm	6 pm	7 pm	8 pm	9 pm

3 **a** 11 am **b** 1 pm **c** 8 am **d** 1 am **e** 11 pm (the previous day) **f** 2 am **g** 7 pm (the previous day)

Exercise 12:10

1 **a** **i** 3:30 am **ii** 10:00 pm **iii** 4:00 am
b **i** 14.5 h (assuming upto midnight) **ii** 1 h **iii** 2 min **c** 2 h 35 min
d Tennis – possibly a big tournament like Wimbledon.
2 **a** 7 times **b** 1 h 50 min **c** About 11:45 pm **d** 2 min **e** 2.5 h
3 **a** **i** 2:47 **ii** 2:52 **iii** 3:04
b **i** 23 min **ii** 12 min **iii** 11 min **c** 3:00 pm **d** 7:05 pm **e** 4:14 pm
4 **a** 6 **b** **i** 7:44 am **ii** 11:06 am **iii** 9:01 am
c **i** 4 min **ii** 19 min **iii** 1 h 14 min
5 **a** **i** 30 min **ii** 12 min **iii** 6 min **iv** 8 min **b** 24 min
c **i** 42 min **ii** $61\frac{1}{2}$ min
6 **a** 0·3 m, 4:14 am (0414) on day 1 **b** 12 h 15 min **c** 11 h 39 min **d** 5 h 51 min
e 1·3 m **f** 10:27 pm (2227) on day 1 and again at 11:04 pm (2304) on day 2
g 12:05 pm (1205) [and 12:13 am (0013) day 4, but this is not in day 3]

ID 12:10

1 *metres* **2** *decimetres* **3** *centimetres* **4** *millimetres* **5** *kilometres*
6 *square metres* **7** *square centimetres* **8** *square kilometres* **9** *hectares* **10** *cubic metres*
11 *cubic centimetres* **12** *seconds* **13** *minutes* **14** *hours* **15** *metres per second*
16 *kilometres per hour* **17** *grams* **18** *milligrams* **19** *kilograms* **20** *tonnes*
21 *litres* **22** *millilitres* **23** *kilolitres* **24** *degrees Celsius*

Fun Spot 12:10 And now for something light

1 **a** 18 000 000 km **b** 1 080 000 000 km **c** 25 920 000 000 km **d** 9 460 800 000 000 km
2 63 000
3 Mercury = 58 500 000 km, Venus = 108 000 000 km, Mars = 228 000 000 km, Saturn = 1 431 000 000 km, Pluto = 5 910 000 000 km
4 **a** 41 154 480 000 000 km **b** approx 274 000
5 **a** 31 000 000 000 000 km approx. **b** 3·27 light years approx.

Diagnostic Test 12: Measurement: Length and Time

1 **a** 1·5 cm **b** 3·8 cm **c** 6·1 cm **d** 8·4 cm
2 **a** 30 mm **b** 7000 m **c** 250 cm
3 **a** 5 cm **b** 6·5 m **c** 7·15 km
4 **a** 45 mm **b** 18 mm **c** 32 mm **d** 41 mm
5 **a** 20 cm **b** 6·4 cm **c** 10·0 cm **d** 11·4 cm
6 **a** 0530 (or 05:30) **b** 1320 **c** 1957 **d** 0020
7 **a** 13 h 50 min **b** 5 h 12 min **c** 2 h 53 min **d** 5 h 18 min
8 **a** 11:30 am **b** 1 pm **c** 1:30 pm **d** 1:30 pm
9 **a** 6 h 10 min **b** 4 h 35 min
10 **a** 4 h 55 min **b** 45 min

12A Revision Assignment

1 **a** reflex angle **b** acute angle **c** obtuse angle
2 **a** $a = 130$ **b** $b = 20$ **c** $c = 120$
3 **a** $m = 150$ **b** $x = 125$ **c** $y = 62$

4 **a** $a = 45$ **b** $b = 88$ **c** $c = 135$ **d** $a = 52$, $x = 102$ **e** $b = 77$, $y = 77$ **f** $c = 273$, $d = 73$

5 **a**

M	1	2	5	10
L	5	6	9	14

b

B	0	1	2	4
P	4	6	8	12

c

m	2	3	4	10
n	0	6	14	104

d

x	1	2	3	4
y	3	4	5	6

e

m	0	1	2	3
n	6	5	4	3

f

M	2	3	4	5
P	5	7	9	11

6 **a** 21 **b** 18 **c** 12 **d** 32

7 **a** 35 **b** 38 **c** 31 **d** 28

8 **a** 9 **b** 6 **c** 1

12B Working Mathematically

1 **a** *anno Domini* **b** *before Christ* **c** *ante meridiem* **d** *post meridiem* **e** *hectare*
f *regular* shapes **g** *net* of a cube **h** *cross-section* **i** *coordinates* **j** *tally*

2 \$700 and \$960 **3** \$12.46 **4** 160 m and 20 m **5** \$3.40 + \$1.60; \$2.20 + \$1.20 + \$1.60

6 5·2 t

7 **a** **i** 5·3 m **ii** 59·4 m **iii** 48·1 m **iv** 0·6 m
b **i** 60 km/h **ii** 90 km/h **iii** 120 km/h **iv** 80 km/h
c **i** 5·6 m/s **ii** 13·9 m/s **iii** 11·1 m/s **iv** 44·4 m/s
d 16·7 m/s **e** 80 km/h

Chapter 13: Area and Volume

Exercise 13:01

1 **a** 5 units^2 **b** 18 units^2 **c** 15 units^2 **d** 28 units^2 **e** 60 units^2 **f** 56 units^2 **g** 10 units^2
h 19 units^2 **i** 15 units^2

2 **a** 6 cm^2 **b** 6 cm^2 **c** 15 cm^2 **d** 10 cm^2 **e** 12 cm^2 **f** 23 cm^2

3 **a** 12 cm^2, 12 cm^2 **b** 14 cm, 16 cm **c** no

4 **a** 10 cm^2 **b** 9 cm^2 **c** 8 cm^2 **d** 11 cm^2 **e** 9 cm^2

5 **a** $\frac{1}{2}$ cm^2 **b** $\frac{1}{2}$ cm^2 **c** $\frac{1}{4}$ cm^2 **d** $\frac{3}{4}$ cm^2

6 **a** 9 cm^2 **b** 6 cm^2 **c** 6 cm^2 **d** $6\frac{3}{4}$ cm^2 **e** 4 cm^2 **f** 9 cm^2 **g** $11\frac{1}{2}$ cm^2 **h** $9\frac{1}{2}$ cm^2

7 **a** 100 **b** 10 000

Investigation 13:01 Finding area

1 **a** 12 cm^2 **b** 20 cm^2 **c** 16 cm^2 **2** to **5** Answers will vary.

Prep Quiz 13:02

1 5 **2** 12 **3** 8 **4** 12 **5** 12 **6** 16 **7** 12 **8** 14 **9** 24 **10** 20

Exercise 13:02

1 **a** 16 units^2 **b** 21 units^2

2 **a** 50 cm^2 **b** 18 cm^2 **c** 56 cm^2 **d** 120 mm^2 **e** 200 mm^2
f 153 m^2

3 **a** 64 cm^2 **b** 400 cm^2 **c** 49 mm^2

4 **a** 25 cm^2 **b** 16 cm^2

5 **a** 180 mm^2 **b** 1·5 m^2 **c** 10 500 cm^2 **d** 10 cm^2 **e** 2·92 m^2

6 **a** 100 cm^2 **b** 70 cm^2 **c** 52 cm^2 **d** 73 cm^2 **e** 36 cm^2
f 69 cm^2

7 **a** 64 cm^2 **b** 58 cm^2 **c** 276 cm^2

8 **a** 10 ha **b** 18 ha **c** 800 ha

9 **a** 90 cm^2 **b** 8 ha **c** 1 m^2 **d** 800 m^2 **e** 6 cm^2
f 2 ha **g** 10 ha

10 **a** 9 m^2 **b** 7·5 m^2 **c** 20·25 m^2 **d** 27 m^2 **e** 63·75 m^2

11 **a** 13·12 m^2 **b** 14·24 m^2 **c** 40·48 m^2

Prep Quiz 13:03

1 10 cm^2 **2** 15 cm^2 **3** 108 cm^2 **4** 15 cm^2 **5** 40 mm^2 **6** 300 cm^2 **7** 49 cm^2 **8** 100 **9** 10 000 **10** 10 000

Exercise 13:03

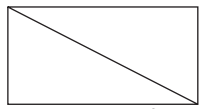
1 **a** 30 cm^2 **b** 27 cm^2 **c** 56 cm^2
2 **a** 25 cm^2 **b** 42 cm^2 **c** 90 cm^2 **d** 28 cm^2 **e** $49\frac{1}{2}\text{ cm}^2$ **f** 21 cm^2
3 **a** 16 cm^2 **b** 40 cm^2 **c** $17\frac{1}{2}\text{ cm}^2$
4 **a** 120 cm^2 **b** 8 mm **c** 24 cm
5 **a** 65 cm^2 **b** 28 cm^2 **c** 30 cm^2 **d** 65 cm^2 **e** 50 cm^2 **f** 136 cm^2
6 **a** 16 cm^2 **b** 7 cm^2 **c** $5\frac{1}{2}\text{ cm}^2$ **d** 14 cm^2

Prep Quiz 13:04

1 24 cm^2 **2** 36 cm^2 **3** 10 cm^2 **4** 12 cm^2 **5** 14 cm^2 **6** 8 **7** 20 **8** 25 **9** 250 **10** 22 cm

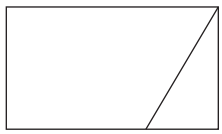
Exercise 13:04

1 **a**



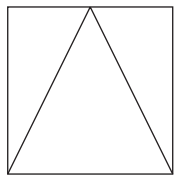
$A = 8\text{ cm}^2$

b



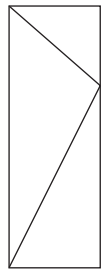
$A = 15\text{ cm}^2$

c



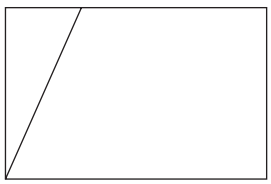
$A = 12\text{ cm}^2$

d



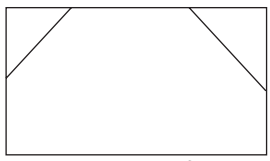
$A = 12\text{ cm}^2$

e



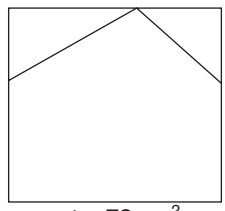
$A = 24{\cdot}5\text{ cm}^2$

f



$A = 66\text{ cm}^2$

g



$A = 72\text{ cm}^2$

2 \$804 **3** 10 L **4** 225 cm^2 **5** **a** 15 cm^2 **b** $24{\cdot}5\text{ cm}^2$ **c** $9{\cdot}5\text{ cm}^2$
6 1254 cm^2 **7** 5·4 kg **8** **a** 3·2 ha **b** yes **9** 100 **10** 15 m^2 **11** 63 m
12 96 **13** 75 m^2 **14** **a** 840 **b** \$3528 **c** \$2751 could have been saved. **15** **a** 726 **b** 117

ID 13:04

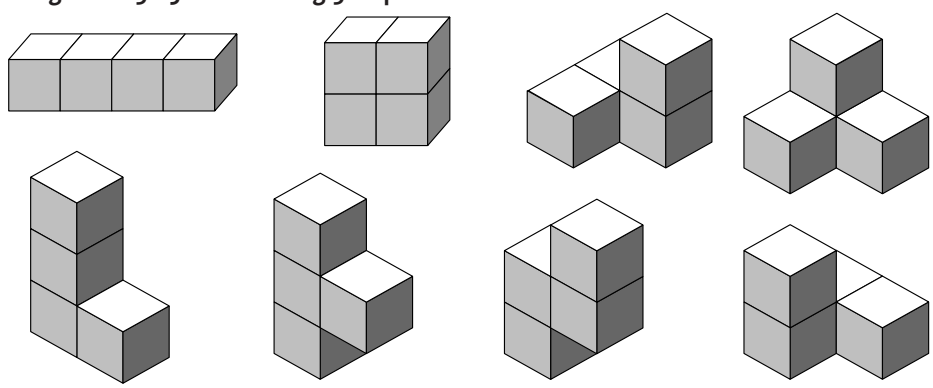
1 *square* **2** *rectangle* **3** *parallelogram* **4** *rhombus* **5** *trapezium*
6 *regular pentagon* **7** *regular hexagon* **8** *regular octagon* **9** *kite* **10** *scalene triangle*
11 *isosceles triangle* **12** *equilateral triangle* **13** *circle* **14** *oval (or ellipse)* **15** *cube*
16 *rectangular prism* **17** *triangular prism* **18** *square pyramid* **19** *rectangular pyramid*
20 *triangular pyramid* **21** *cylinder* **22** *cone* **23** *sphere* **24** *hemisphere*

Exercise 13:05

1 **a** 5 units^3 **b** 12 units^3 **c** 10 units^3 **d** 14 units^3 **e** 29 units^3 **f** 9 units^3 **g** 30 units^3 **h** 13 units^3
2 **a** 8 units^3 **b** 27 units^3 **c** 64 units^3 **d** 20 units^3 **e** 48 units^3 **f** 48 units^3
3 **a** 8 cm^3 **b** 12 cm^3 **c** 18 cm^3 **d** 15 cm^3 **e** 14 cm^3
4 **a** 10 **b** 100 **c** 1000

Investigation 13:05 Measuring 3D space

1



2 **a** 3
b 4
c 6

Prep Quiz 13:06

1 8 units^3 **2** 8 units^3 **3** 16 units^3 **4** 18 units^3 **5** 10 units^3 **6** 40 cm^2 **7** 36 cm^2 **8** 36 mm^2
9 36 mm^2 **10** 25 cm^2

Exercise 13:06

1 a 42 cm^3 **b** 30 cm^3 **c** 42 cm^3 **d** 48 cm^3 **e** 28 cm^3 **f** 36 cm^3 **g** 72 cm^3 **h** 72 cm^3
i 90 cm^3 **j** 96 cm^3
2 a 6 cm^3 **b** 60 cm^3 **c** 12 cm^3 **d** 54 cm^3 **e** 125 cm^3 **f** 320 cm^3
3 a 1 000 000 cm^3 **b** 1000 mm^3 **4 a** 27 cm^3 **b** 125 cm^3 **c** 729 cm^3 **d** 64 mm^3
5 a 210 m^3 **b** 84 m^3 **c** 45 m^3 **d** 1980 m^3 **e** 2520 m^3 **6 a** 48 cm^3 **b** 36 cm^3
7 a $A = 4$ units^3, $B = 32$ units^3 **b** 8 **8 a** 320 cm^3 **b** 8 **9 a** 96 cm^3 **b** 180 cm^3

Exercise 13:07

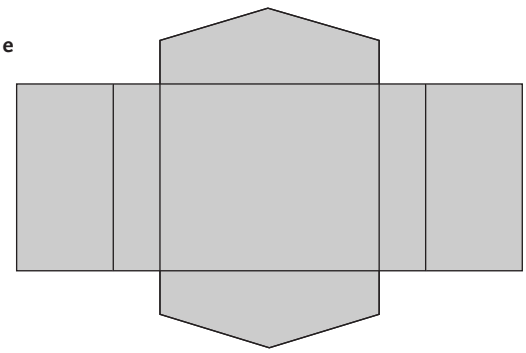
1 a 2000 **b** 5000 **c** 500 **d** 400 **e** 3500 **f** 100 000 **g** 50 **h** 1 000 000
2 a 3 **b** 9 **c** 7·5 **d** 8·3 **e** 0·5 **f** 0·3 **g** 20 **h** 0·05
3 a 500 mL **b** 50 L **c** 50 mL **4** 250 mL **5** 6 **6** 14 **7** 20
8 10 **9** 600 mL bottle, by 10c per L
10 a 60 mL **b** 360 mL **c** 8·64 L **d** 60·48 L **e** 3144·96 L
11 a 15 cm^3 **b** 8 mL **c** 183 cm^3 **d** 5 L **e** 35 000 cm^3 **f** 2500 cm^3
g 15 kL **h** 4·5 m^3 **i** 8·2 kL
12 a 60 mL **b** 200 mL **c** 5000 mL **13** 500 cm^3 **14 a** 1 000 000 cm^3 **b** 1000 L
15 a 1 000 000 cm^3 **b i** 1 000 000 mL **ii** 1000 L **iii** 1 kL
16 7·5 L **17 a** 2400 cm^3 **b** 1·6 L

Diagnostic Test 13: Area and Volume

1 a 12 units^2 **b** 16 units^2 **c** 14 units^2 **d** 12 units^2
2 a 7 cm^2 **b** 2 cm^2 **c** 4 cm^2
3 a 100 mm^2 **b** 10 000 cm^2 **c** 10 000 m^2 **d** 5 m^2
4 a 36 cm^2 **b** 150 mm^2 **c** 21 m^2
5 a 22 cm^2 **b** 38 cm^2 **c** 54 cm^2
6 a 20 cm^2 **b** 10 cm^2 **c** 17·5 cm^2
7 a 20 units^3 **b** 15 units^3 **c** 33 units^3
8 a 48 cm^3 **b** 105 cm^3
9 a 64 cm^3 **b** 84 cm^3 **c** 30 m^3
10 a 1000 mL **b** 2 kL **c** 3·5 L **d** 1500 L
11 a 3 cm^3 **b** 50 mL **c** 1000 cm^3 **d** 1 kL
12 a 48 mL **b** 105 mL

13A Revision Assignment

1 a 5 **b** 2 **c** pentagon **d** pentagonal prism **e**



2 a rhombus **b** no **c** no **d** yes **e** heptagon
3 a 89 **b** 70 **c** 89 **d** 99 **4** 21 Jan, 4 Feb, 18 Feb, 4 Mar
5 a 3:30 pm **b** 7:20 am **c** 12:30 am **d** 11:50 pm **e** 1:40 pm **f** 6:20 am **g** 5:15 pm **h** 8:00 pm

6

Name	*No. of faces*	*No. of vertices*	*No. of edges*
triangular prism	5	6	9
triangular pyramid	4	4	6
cube	6	8	12
rectangular prism	6	8	12

7 **a** 1 **b** 2 **c** $1\frac{1}{4}$ **d** $5\frac{1}{4}$ **8** **a** 0·81 **b** 8·7 **c** 11·1 **d** 19·96
9 **a** 0·24 **b** 0·24 **c** 0·024 **d** 24 **10** **a** 4·2 **b** 0·35 **c** 0·35 **d** 3·95

13B Working Mathematically

1 215 km **2** 1840 **3** 20 m **4** 201 **5** **a** 6 **b** 12 **c** 8
6 **a** **i** 180 cm **ii** 70 kg **iii** 1·84 m^2
b **i** 150 cm **ii** 50 kg **iii** 1·41 m^2
c **i** 160 cm **ii** 100 kg **iii** 1·93 m^2

Chapter 14: Using Geometrical Instruments

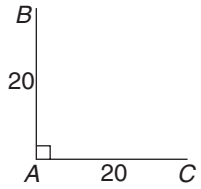
Exercise 14:01

4 **a** 5 cm **b** 8·6 cm **c** 14·1 cm **d** 3·6 cm **e** 9·1 cm **f** 5·5 cm
5 **a** yes, yes **b** yes, yes **c** yes, yes
7 **d** yes **8** **c** parallelogram **9** **b** rectangle **12** **d** yes **e** yes

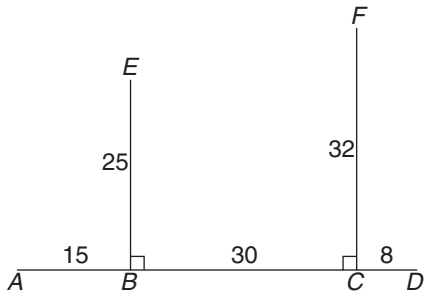
Exercise 14:02

2 **a** between 31 mm and 32 mm **b** 50 mm **3** *BC* should be 5 cm long.
4 *AD* and *BD* are between 3·3 cm and 3·4 cm.
5 **a** 7·1 cm (to nearest mm) **b** Both diagonals are 5·8 cm long to the nearest mm.
6 **a** **i** yes **ii** no **iii** yes **iv** yes **b** *AB*, *DC*, *AE* and *DH* **c** *AE*, *DH*
8 **c** yes **9** **b** yes **10** **c** They are the same length. **12** **d** parallelogram
13 **d** rhombus **14** **c** kite or rhombus **15** **c** square

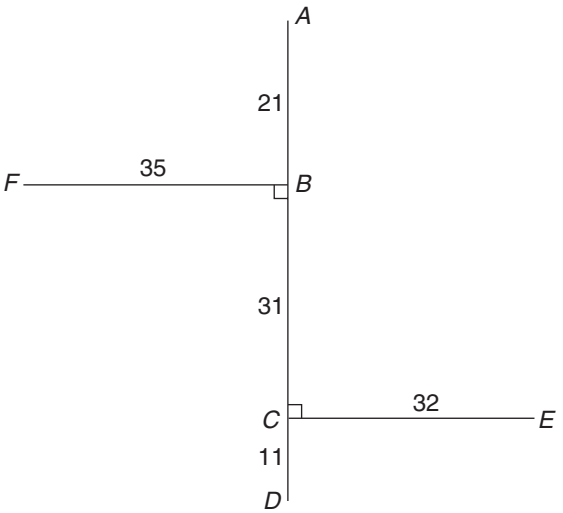
16 **a**



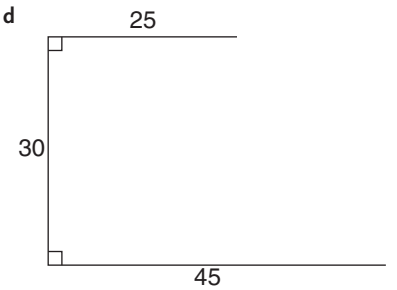
b



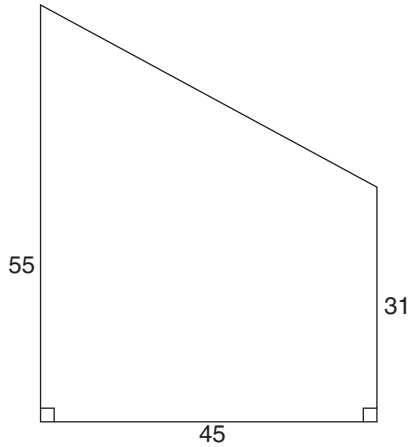
c



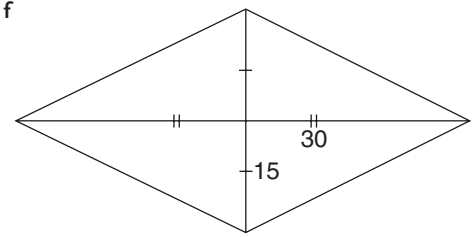
d



e



f

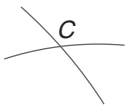


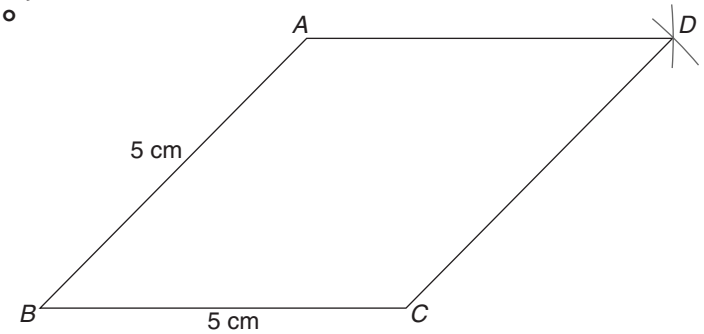
Exercise 14:03

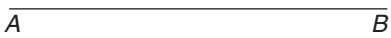
1 **b** yes, yes **4** **d** rhombus **7** **c** yes **8** **d** 5 cm, 4 cm **e** 5 cm, 4 cm

9 **a** and **b**. There are two points *C* and *C'*.

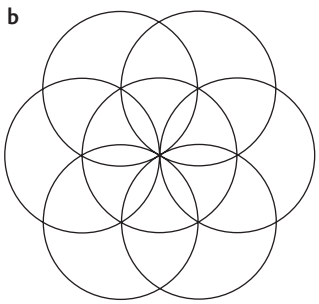
10



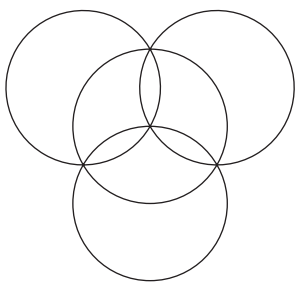


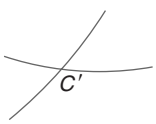


11 **b**



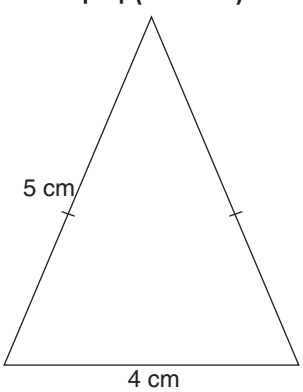
c



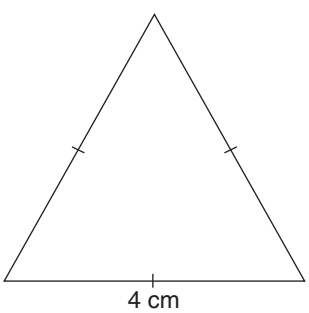


Exercise 14:04 (Practical)

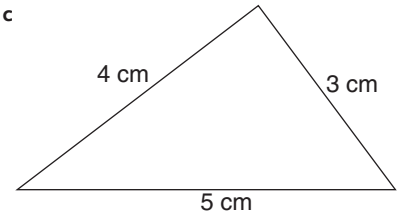
1 **a**



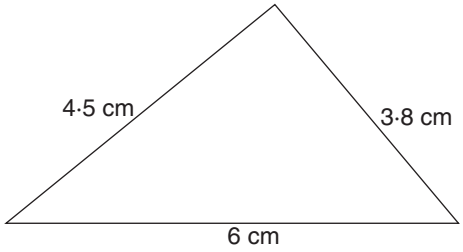
b



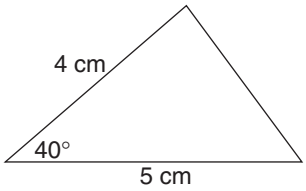
c

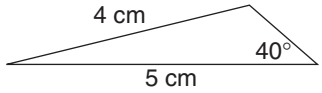


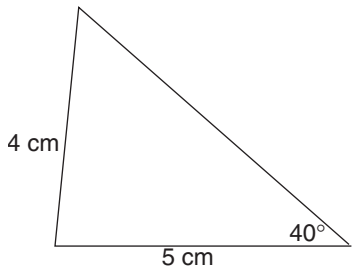
d

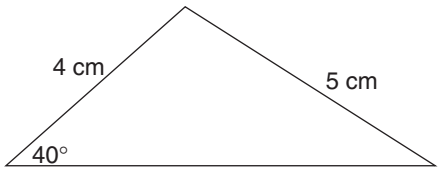


2 **a** no **b** no **3** **c** yes **d** no **4** **b** 83° **c** 6 cm
5 **b** The largest side is always opposite the largest angle.
6 **a** 5 cm **b** 27 mm **c** 7·6 or 7·7 cm
7 **a** 70° **b** 40° **c** 105°
8 **a** There are four different triangles:

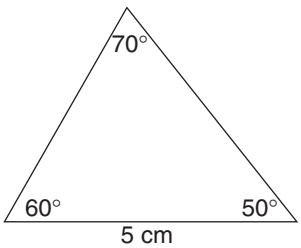


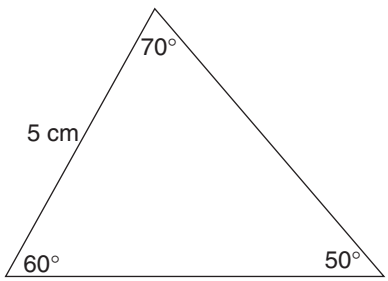


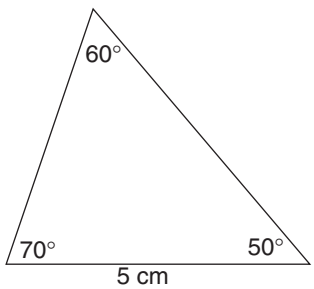




b There are 3 different triangles:



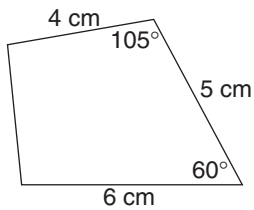


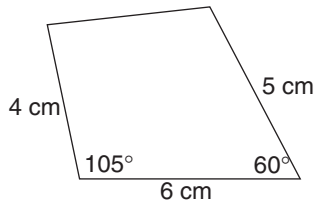


Exercise 14:05 (Practical)

1 **a** yes, yes **b** Diagonals should be equal.

7 There are many answers. Two are given below.

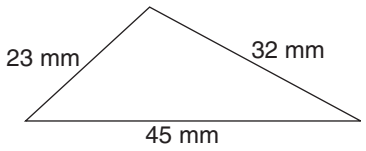




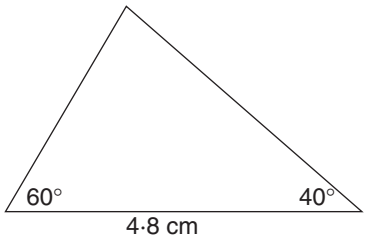
8 **a** You can draw an infinite number of such quadrilaterals.
b You would need to know the length of a diagonal or the size of one angle.

Diagnostic Test 14: Using Geometrical Instruments

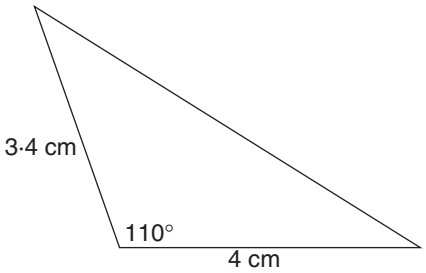
1 a



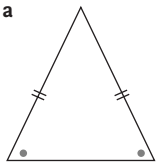
b



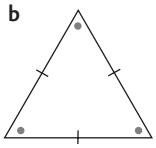
c



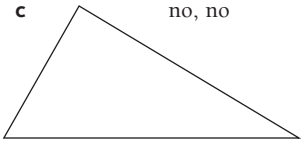
2 a



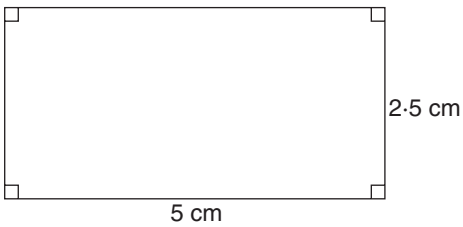
b



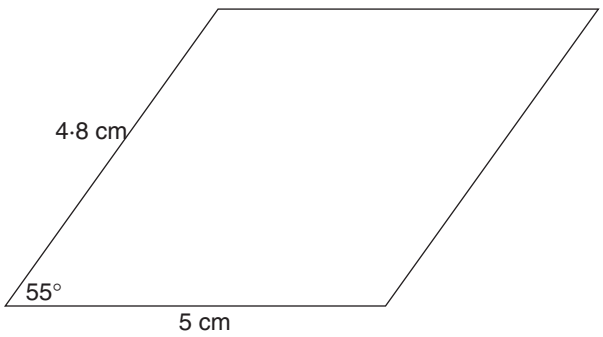
c no, no



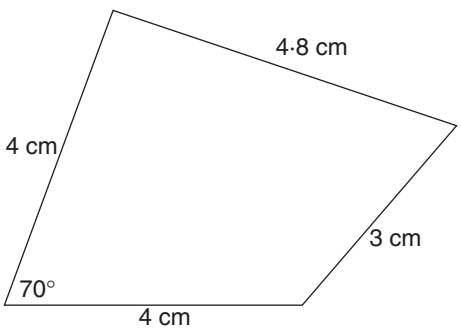
3 a



b



c



14A Revision Assignment

1 **a** $15y$ **b** $6xy$ **c** $4m - 7n$ **d** $6 + 12x$

2 **a** 5 **b** 1 **c** 13 **d** 5

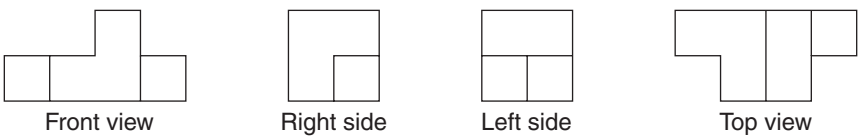
3 **a** 5 **b** 36 **c** $3\frac{3}{4}$ **d** 24 **e** -14 **f** 12 **g** 9 **h** 97
i -1 **j** $5x + 3$ **k** $2x - 2y$ **l** $23 - x$ or $-x + 23$ **m** x **n** x **o** $2x^2 - x$

4 **a** $7{\cdot}6\,\text{cm}^2$ **b** $6{\cdot}44\,\text{cm}^2$ **c** $18{\cdot}5\,\text{cm}^2$

5

Operation	Start	− 7	× 3	+ 11	− 35	÷ 6
Answer	5	−2	−6	5	−30	−5

6



7 **a** 03:15 (or 0315) **b** 15:15 **c** 23:30 **d** 10:18

8 **a** 5 h **b** 5 h 32 min **c** 3 h 30 min **d** 5 h 6 min

9 **a** 915 mm **b** 1190 mm **c** 559 mm (or 620 mm)

10 **a** 30° **b** 22° **c** 65°

14B Working Mathematically

1 **a** *parallel* lines **b** *perpendicular* lines **c** *vertical*, *horizontal* **d** *concurrent* lines
e *bisecting* an interval **f** *bisecting* an angle **g** $\angle CAB = 60°$ **h** *CD* is *perpendicular* to *AB*.
2 333 **3** 20 **4** 3 **5** 77 km, 13 weeks
6 **a** Australian rules **b** soccer **c** 49% **d** 5% **e** Some students played more than one sport.

Chapter 15: Sets

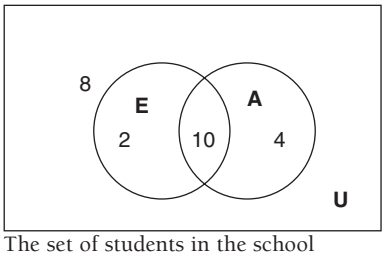
Exercise 15:01

1 **a** {3, 6, 9, 12, 15, 18} **b** {1, 2, 3, 4, 6, 8, 12, 24}
c {February, April, June, September, November} **d** {black, yellow, blue, green, red}
e {Africa, Antarctica, Asia, Australia, Europe, North America, South America}
2 **a** False **b** True **c** False **d** False **e** True
f True **g** True **h** False **i** False **j** True

3 *These answers are examples only.*
a {rivers in North America} **b** {modes of transport} **c** {parts of a computer}
d {footwear} **e** {oceans of the world}
4 **a** **i** *E* and *F* **ii** *D* and *H* **iii** *C* and *I* **iv** *C* **v** *A* and *J*
vi *B* and *G* **vii** *I* **viii** *D* **ix** *G* **x** *F*
b Indonesia **c** any capital city not in Africa
d any two-legged mammal **e** any four-legged animal that does not only live on land, eg a crocodile
5 The set of capital cities in Europe or the set of cities of Europe
6 1, 2, 3, 4, 5, 8 ∈ *A*
1, 6, 9 ∈ *B*
1, 2, 3, 4, 5, 6, 7, 8, 9, 10 ∈ *C*
2, 3, 7, 8, 10 ∈ *D*

Exercise 15:02

1 **a** {1, 5} **b** {Bangkok} **c** {equilateral triangle}
d {10, 20, 30, 40, 50, 60, 70, 80, 90} **e** *depends on your class*
2 **i** **a** 8 **b** 11 **c** 7 **d** 3 **e** 5
ii They might speak another language other than Korean or Japanese.
3 **i** 23
ii **a** 17 **b** 13 **c** 10 **d** 3 **e** 3
iii Maybe they just didn't watch those programmes on that particular night.
4 **i** All the people on the bus **ii** 4 **iii** **a** 19 **b** 17 **c** 8 **d** 13
5 **i** 8 **ii** All the students in Marc's class.
iii



iv **a** 2 **b** 10

6 **a** The set of students in the school
b None of the boys in Carol's class wear glasses.
c The students in Carol's school who are not in her class
d All the girls in Carol's school and all the boys who do not wear glasses
e no
f ∅
7 **a** Girls in the class who wear glasses **b** Students in the class who are girls or wear glasses
c Boys in the class **d** Students in the class who do not wear glasses
e Boys in the class who do not wear glasses **f** Boys in the class who wear glasses
8 There are no houses made of brick and wood.

Exercise 15:03

1 **a** False **b** False **c** True **d** True **e** True

2 **a** False **b** False **c** True **d** True **e** False **f** True

3 **a** $L = \{56, 64, 72, 80, 88, 96\}$
$M = \{55, 60, 65, 70, 75, 80, 85, 90, 95\}$

b the numbers from 50 to 100 that are multiples of 5 or 8

c **i** True **ii** False **iii** False **iv** True **v** True

4 **a** **i** $\not\subset$ **ii** $\not\subset$ **iii** $\subset$ **iv** $\subset$ **v** $\subset$

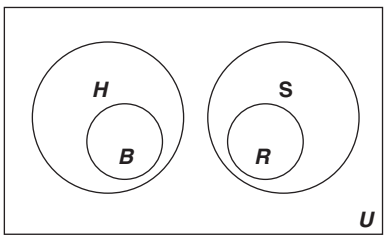
b They are all blue. **c** They are all parked in garages. **d** yes **e** no

5 **a** Candy tastes sweet. **b** Things that are sweet taste good.

c Things that taste sour, most people don't like. **d** Candy does not taste sour.

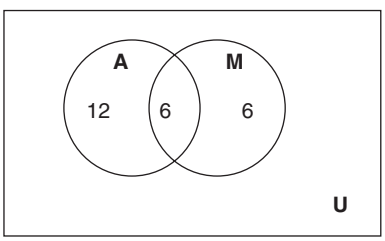
e Things most people don't like do not taste good.

6



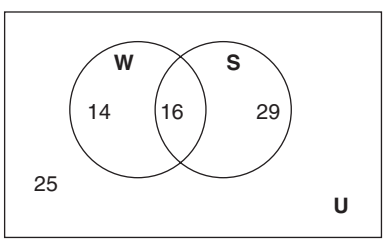
Exercise 15:04

1 **a**



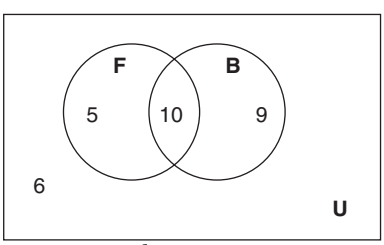
b 6 **c** 12

2 **a** All of the cars in the dealership **b**



c 16 **d** 14

3

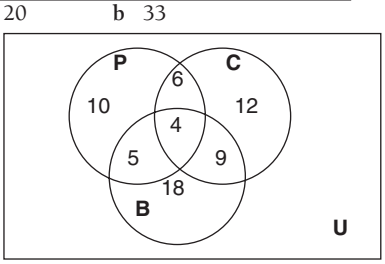


a 10 **b** 9 **c** 14

4 **a** 20 **b** 33

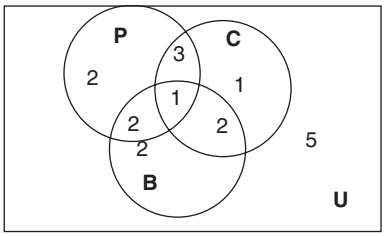
5 **a** 3 **b** 45 **c** 58

6 **a**



b 10 **c** 40 **d** 20

7 **a**



b 5 **c** 13 **d** 7 **e** 5

Diagnostic Test 15: Sets

1 i a {1, 2, 3, 6, 9, 18} **b** {1, 2, 3, 4, 6, 9, 12, 18, 36} **c** 6 **d** 9

ii yes

2 a {2} **b** 14 **c** 24 **d** 37 **e** 13

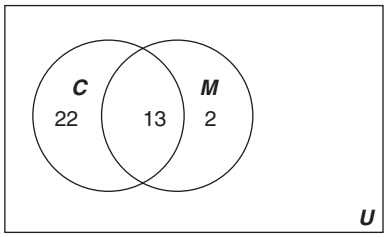
3 a Group D teams in the second round

b Teams in the second round that were not in group D

c Teams that were either in group D or in the second round or both

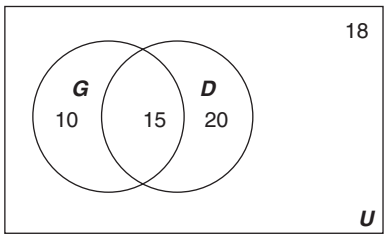
d Teams that were neither in group D nor the second round

4 a



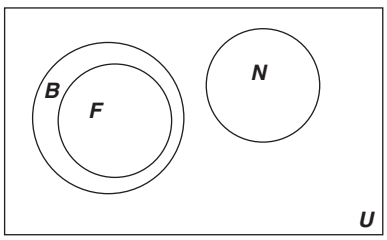
b 13

5 a

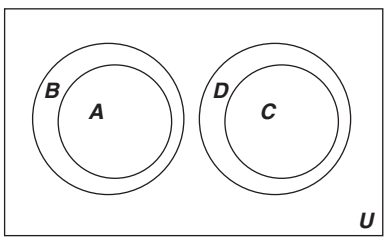


b 63 **c** 30

6



7



Answers to ID Cards

ID Card 1 (Metric Units) page xiv

1 *metres*	**2** *decimetres*	**3** *centimetres*	**4** *millimetres*
5 *kilometres*	**6** *square metres*	**7** *square centimetres*	**8** *square kilometres*
9 *hectares*	**10** *cubic metres*	**11** *cubic centimetres*	**12** *seconds*
13 *minutes*	**14** *hours*	**15** *metres per second*	**16** *kilometres per hour*
17 *grams*	**18** *milligrams*	**19** *kilograms*	**20** *tonnes*
21 *litres*	**22** *millilitres*	**23** *kilolitres*	**24** *degrees Celsius*

ID Card 2 (Symbols) page xiv

1 *add (or plus)*	**2** *subtract (or minus)*	**3** *multiplied by (or times)*	**4** *divided by*
5 *is equal to*	**6** *is approximately equal to*	**7** *is not equal to*	**8** *is less than*
9 *is less than or equal to*	**10** *is not less than*	**11** *is greater than*	**12** *is greater than or equal to*
13 *is not greater than*	**14** *is not greater than or equal to*	**15** *4 squared*	**16** *4 cubed*
17 *the square root of 25*	**18** *the cube root of 27*	**19** *per cent*	**20** *therefore*
21 *for example*	**22** *that is*	**23** *numerator*	**24** *denominator*

ID Card 3 (Language) page xv

1 $6 - 2 = 4$	**2** $6 + 2 = 8$	**3** $6 \div 2 = 3$	**4** $6 - 2 = 4$	**5** $6 \div 2 = 3$
6 2	**7** 6	**8** $6 \times 2 = 12$	**9** $6 - 2 = 4$	**10** $6 \times 2 = 12$
11 $2 + 6 = 8$	**12** $6 - 2 = 4$	**13** $6^2 = 36$	**14** $\sqrt{36} = 6$	**15** $6 - 2 = 4$
16 $6 \times 2 = 12$	**17** $(6 + 2) \div 2 = 4$	**18** $6 + 2 = 8$	**19** $6^2 = 36$	**20** $6 - 2 = 4$
21 $6 - 2 = 4$	**22** $6 + 2 = 8$	**23** $6 \div 2 = 3$	**24** $6 + 2 = 8$	

ID Card 4 (Language) page xvi

1 *square*	**2** *rectangle*	**3** *parallelogram*	**4** *rhombus*
5 *trapezium*	**6** *regular pentagon*	**7** *regular hexagon*	**8** *regular octagon*
9 *kite*	**10** *scalene triangle*	**11** *isosceles triangle*	**12** *equilateral triangle*
13 *circle*	**14** *oval (or ellipse)*	**15** *cube*	**16** *rectangular prism*
17 *triangular prism*	**18** *square pyramid*	**19** *rectangular pyramid*	**20** *triangular pyramid*
21 *cylinder*	**22** *cone*	**23** *sphere*	**24** *hemisphere*

ID Card 5 (Language) page xvii

1 *point A*	**2** *interval AB*	**3** *line AB*	**4** *ray AB*
5 *collinear* points	**6** *midpoint*	**7** *number line*	**8** *diagonals*
9 *acute-angled triangle*	**10** *right-angled triangle*	**11** *obtuse-angled triangle*	**12** *vertices*
13 ΔABC	**14** *hypotenuse*	**15** $180°$	**16** $(a + b)°$
17 $360°$	**18** $a° = b°$	**19** $a° = 60°$	**20** $3 \times 180° = 540°$
21 *AB is a diameter. OC is a radius.*		**22** *circumference*	**23** *semicircle*
24 *AB* is a *tangent*. *CD* is an *arc*. *EF* is a *chord*.			

ID Card 6 (Language) page viii

1 *parallel* lines	**2** *perpendicular* lines	**3** *vertical, horizontal*	**4** *concurrent* lines
5 angle *ABC* or *CBA*	**6** *acute* angle	**7** *right* angle	**8** *obtuse* angle
9 *straight* angle	**10** *reflex* angle	**11** *revolution*	**12** *adjacent* angles
13 *complementary* angles	**14** *supplementary* angles	**15** *vertically opposite* angles	**16** $360°$
17 *transversal*	**18** *corresponding* angles	**19** *alternate* angles	**20** *cointerior* angles
21 *bisecting* an interval	**22** *bisecting* an angle	**23** $\angle CAD = 60°$	**24** *CD* is *perpendicular* to *AB*

ID Card 7 (Language) page xix

1 *anno Domini*	**2** *before Christ*	**3** *ante meridiem*	**4** *post meridiem*
5 *hectare*	**6** *regular* shapes	**7** *net* of a cube	**8** *cross-section*
9 *face*	**10** *vertex*	**11** *edge*	**12** *axes of symmetry*
13 *reflection (or flip)*	**14** *translation (or slide)*	**15** *rotation (or turn)*	**16** *tessellation*
17 *coordinates*	**18** *tally*	**19** *picture* graph	**20** *column* graph
21 *line* graph	**22** *sector (or pie)* graph	**23** *bar* graph	**24** *scatter diagram*

Index

Most references are to chapter topics, eg 1:02 refers to Chapter 1: Topic 2 (Place value).
A:01 … to I:01 refers to the appendix section of the CD.
The following abbreviations refer to:

D & D	Drag and Drops
FS	Fun Spot
Inv	Investigation
MT	Mathematical Terms
RM	Reading Mathematics
Ch	Challenge Worksheet

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Navigating International Maths for the Middle Years 1

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Rotate view 90° clockwise

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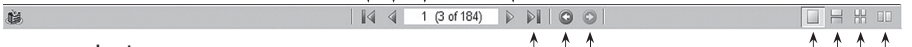


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Next view

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APPENDIX A

A:01 | The Four Operations

The combinations in the following exercises need to be practised until known.

Exercise A:01

1 Have someone time you, as you do each line four times. Record the times in a table like the one on the right, and graph the results.

a

2 +	3	1	0	4	7	2	8	5	9	6

b

10 +	0	2	1	9	3	5	7	8	6	4

c

3 +	3	4	0	2	8	1	7	5	9	6

d

4 +	1	4	2	6	0	3	7	8	5	9

e

5 +	5	2	0	6	4	1	5	3	9	8

f

6 +	2	4	0	1	6	3	7	9	5	8

g

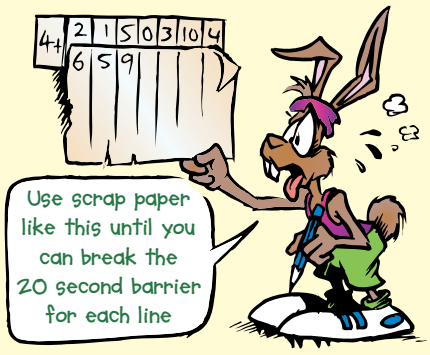
7 +	0	1	4	3	7	2	8	6	5	9

h

8 +	2	3	8	1	0	7	5	9	4	6

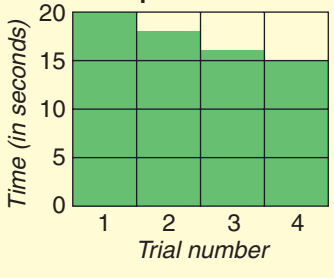
i

9 +	4	0	2	6	5	8	3	1	9	7



	Time for each trial			
Table	1	2	3	4
2 +	20 s	18 s	16 s	15 s
10 +				
3 +				
4 +				
5 +				
6 +				
7 +				
8 +				
9 +				

Graph for table 2+



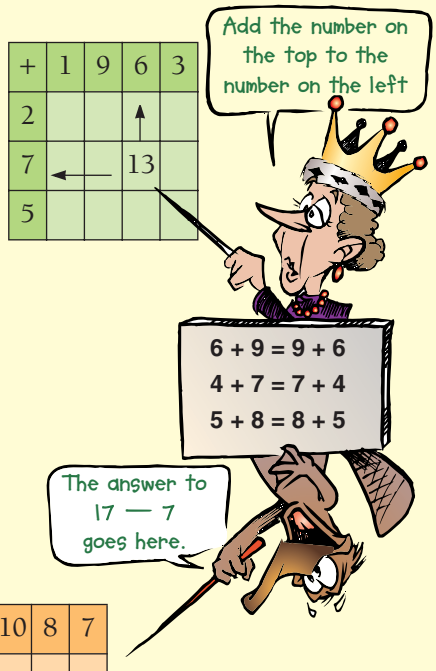
2 Try to do each of these in 2 minutes.

a

+	1	9	0	5	8	2	6	4	7	3
1										
9										
0										
5										
8										

b

+	1	9	0	5	8	2	6	4	7	3
2										
6										
4										
7										
3										



3 Try to do each line in 20 seconds.

a

18 –	10	8	9

b

17 –	9	10	8	7

c

16 –	7	9	6	10	8

d

15 –	8	5	10	6	9	7

e

14 –	10	6	8	4	5	9	7

f

13 –	5	10	4	7	9	3	6	8

g

12 –	5	2	9	4	8	3	6	10	7

h

11 –	2	8	5	1	9	6	3	10	7	4

i

10 –	1	10	2	5	7	3	9	4	0	8	6

■ If 10 + 9 = 19,
then
19 – 10 = 9
and
19 – 9 = 10

4 Each box should be completed in 10 seconds.

a

5 – 3	
8 – 2	
6 – 3	
9 – 2	
8 – 7	

b

6 – 6	
3 – 1	
9 – 7	
8 – 5	
7 – 6	

c

6 – 5	
7 – 4	
4 – 2	
8 – 3	
9 – 3	

d

9 – 9	
6 – 4	
7 – 3	
9 – 4	
8 – 6	

e

9 – 8	
7 – 5	
9 – 6	
8 – 4	
9 – 5	

5 Have someone time you as you do each part four times. Record the times in a table like the one on the right, and graph the results.

a

2 ×	2	1	5	0	3	10	4	9	6	8	7	11

b

3 ×	1	0	3	2	5	10	6	4	9	8	7	11

c

10 ×	4	2	0	3	1	7	6	8	10	9	5	11

d

5 ×	0	8	2	5	10	4	3	1	6	7	9	11

e

4 ×	2	1	5	0	3	10	4	9	6	8	7	11

f

6 ×	1	0	3	2	5	10	6	4	9	8	7	11

g

7 ×	0	2	1	5	10	3	7	4	8	6	9	11

h

8 ×	2	1	5	0	3	10	4	9	6	8	7	11

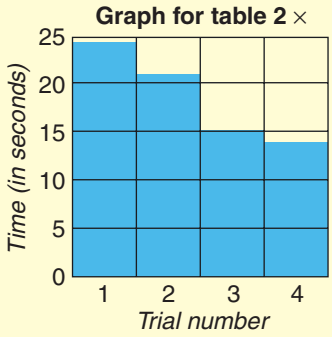
i

9 ×	1	0	3	2	5	10	6	4	9	8	7	11

j

11 ×	0	2	1	5	10	3	7	4	8	6	9	11

	Time for each trial			
Table	1	2	3	4
2 ×	24 s	21 s	15 s	13 s
3 ×				
10 ×				
5 ×				
4 ×				
6 ×				
7 ×				
8 ×				
9 ×				
11 ×				



■ 7 × 4 = 4 × 7

6 Try to do this in 3 minutes.

×	0	2	1	5	10	3	7	4	8	6	11	9
3												
7												
5												
9												
8												
6												

APPENDIX B

The Diagnostic Checkups in Section 1:08 will show you which number skills you need to revise.

Before each practice exercise you will find:

- explanations
- rules
- worked examples

Read these carefully before attempting the exercise.

Using the Four Operations

B:01 | Addition

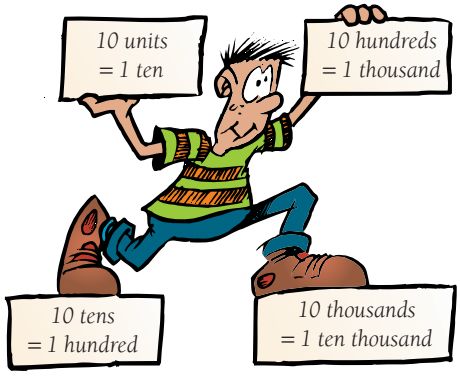
worked examples

1

$$\begin{array}{r} 5\,1\,5 \\ 3\,0\,7 \\ 0\,9\,6 \\ {}_{2}9{}_{2}8{}_{2}3 \\ \hline 1\,9\,0\,1 \end{array}$$

2

$$\begin{array}{r} 1\,9\,2\,0\,8 \\ 7\,5\,3\,7 \\ 3{}_{2}8{}_{1}6{}_{1}9{}_{1}0 \\ \hline 6\,5\,4\,3\,5 \end{array}$$



1 ten-thousand
9 thousands
2 hundreds
0 tens
8 units
1 9 2 0 9

Exercise B:01

1 a

$$\begin{array}{r} 63 \\ 50 \\ 82 \\ 8 \\ \hline \end{array}$$

b

$$\begin{array}{r} 93 \\ 88 \\ 7 \\ 46 \\ \hline \end{array}$$

c

$$\begin{array}{r} 75 \\ 69 \\ 18 \\ 34 \\ \hline \end{array}$$

d

$$\begin{array}{r} 92 \\ 48 \\ 67 \\ 90 \\ \hline \end{array}$$

e

$$\begin{array}{r} 735 \\ 41 \\ 686 \\ 317 \\ \hline \end{array}$$

f

$$\begin{array}{r} 806 \\ 40 \\ 71 \\ 93 \\ \hline \end{array}$$

g

$$\begin{array}{r} 472 \\ 88 \\ 693 \\ 486 \\ \hline \end{array}$$

h

$$\begin{array}{r} 655 \\ 788 \\ 391 \\ 849 \\ \hline \end{array}$$

2 a

$$\begin{array}{r} 18\,550 \\ 9\,275 \\ 43\,827 \\ \hline \end{array}$$

b

$$\begin{array}{r} 48\,657 \\ 66\,982 \\ 14\,361 \\ \hline \end{array}$$

c

$$\begin{array}{r} 69\,435 \\ 88\,647 \\ 93\,156 \\ \hline \end{array}$$

d

$$\begin{array}{r} 5\,843 \\ 92\,549 \\ 87\,104 \\ \hline \end{array}$$

e

$$\begin{array}{r} 73\,186 \\ 264\,092 \\ 181\,925 \\ 9\,135 \\ \hline \end{array}$$

f

$$\begin{array}{r} 168\,936 \\ 39\,214 \\ 610\,293 \\ 88\,150 \\ \hline \end{array}$$

g

$$\begin{array}{r} 845\,329 \\ 693\,246 \\ 816\,094 \\ 135\,298 \\ \hline \end{array}$$

h

$$\begin{array}{r} 99\,999 \\ 156\,085 \\ 565\,117 \\ 318\,628 \\ \hline \end{array}$$

3 **a** In 1984 the Australian Commonwealth parliament had 224 members while the state parliaments' memberships were: Victoria 125, New South Wales 144, Queensland 82, Western Australia 91, South Australia 69 and Tasmania 54. How many members of parliament were there altogether in 1984?

b The Australian Armed Forces in 1984 had 10 482 officers, 57 961 other ranks, 1306 cadets, 1817 apprentices and 76 junior recruits. How many people were in the armed forces in 1984?

c In 1980, 94 500 settlers arrived in Australia. There were 118 740 in 1981, 107 170 in 1982 and 78 400 in 1983. How many settlers arrived in Australia in those four years?

d In 1980, 11 450 former settlers permanently left Australia. There were 11 280 in 1981, 13 350 in 1982 and 16 920 in 1983. How many former settlers permanently left Australia in those four years?

B:02 | Subtraction

worked examples

1
$$\begin{array}{r} 6_{1}2\ 8_{1}5 \\ -\ 1_{1}9\ 1_{1}7 \\ \hline 4\ 3\ 6\ 8 \\ \hline \end{array}$$

Method 1
Equal additions

Method 2
Decomposition

2
$$\begin{array}{r} {}^{5}\not{6}_{1}2^{2}\not{8}_{1}5 \\ -\ 1\ 9\ 1\ 7 \\ \hline 4\ 3\ 6\ 8 \\ \hline \end{array}$$



Exercise B:02

1 **a**
$$\begin{array}{r} 839 \\ -\ 256 \\ \hline \end{array}$$

b
$$\begin{array}{r} 574 \\ -\ 139 \\ \hline \end{array}$$

c
$$\begin{array}{r} 750 \\ -\ 506 \\ \hline \end{array}$$

d
$$\begin{array}{r} 617 \\ -\ 583 \\ \hline \end{array}$$

e
$$\begin{array}{r} 4605 \\ -\ 1821 \\ \hline \end{array}$$

f
$$\begin{array}{r} 3511 \\ -\ 1076 \\ \hline \end{array}$$

g
$$\begin{array}{r} 8000 \\ -\ 1960 \\ \hline \end{array}$$

h
$$\begin{array}{r} 8436 \\ -\ 7737 \\ \hline \end{array}$$

2 **a**
$$\begin{array}{r} 56\,453 \\ -\ 19\,216 \\ \hline \end{array}$$

b
$$\begin{array}{r} 80\,473 \\ -\ 1\,984 \\ \hline \end{array}$$

c
$$\begin{array}{r} 375\,314 \\ -\ 218\,109 \\ \hline \end{array}$$

d
$$\begin{array}{r} 680\,935 \\ -\ 48\,566 \\ \hline \end{array}$$

e
$$\begin{array}{r} 1\,650\,000 \\ -\ 940\,000 \\ \hline \end{array}$$

f
$$\begin{array}{r} 8\,654\,000 \\ -\ 1\,672\,000 \\ \hline \end{array}$$

g
$$\begin{array}{r} 5\,000\,000 \\ -\ 1\,456\,000 \\ \hline \end{array}$$

h
$$\begin{array}{r} 9\,500\,000 \\ -\ 2\,754\,000 \\ \hline \end{array}$$

3 **a** Of the permanent defence force of 71 642 in 1984, 4673 were female. How many were male?

b The population of Australia in 1983 was 15 378 600, to the nearest hundred. If the population was 15 543 600 twelve months later, what was the approximate increase in population during those twelve months?

c The area of Australia is 7 682 300 square kilometres. The area of the United States of America is 9 372 614 square kilometres. By how much is the area of the United States of America greater than the area of Australia?

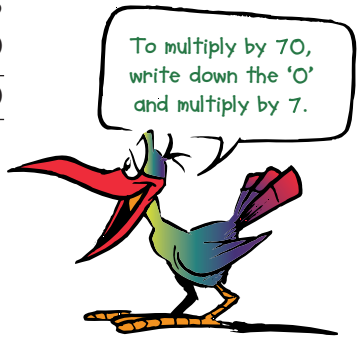
d Spiros was surprised to discover that of the 111 280 tonnes of butter produced in Australia last year, 93 884 tonnes were produced in Victoria. How many tonnes were produced altogether in the other states last year?

B:03 | Multiplication

worked examples

1

$$\begin{array}{r} {}_{1}1_{2}3\ 0_{3}5 \\ \times\ 7\ 0 \\ \hline 9\ 1\ 3\ 5\ 0 \end{array}$$



2

$$\begin{array}{rl} 8\,653 & \\ \times\ 45 & \\ \hline 43\,265 & (8653 \times 5) \\ 346\,120 & (8653 \times 40) \\ \hline 389\,385 & \end{array}$$

'Long' multiplication

$8653 \times 45 = (8653 \times 5) + (8653 \times 40)$

Exercise B:03

1

a	$\begin{array}{r} 872 \\ \times\ 6 \\ \hline \end{array}$	b	$\begin{array}{r} 8046 \\ \times\ 7 \\ \hline \end{array}$	c	$\begin{array}{r} 2559 \\ \times\ 8 \\ \hline \end{array}$	d	$\begin{array}{r} 1746 \\ \times\ 9 \\ \hline \end{array}$
e	$\begin{array}{r} 495 \\ \times\ 20 \\ \hline \end{array}$	f	$\begin{array}{r} 6140 \\ \times\ 60 \\ \hline \end{array}$	g	$\begin{array}{r} 7086 \\ \times\ 50 \\ \hline \end{array}$	h	$\begin{array}{r} 7823 \\ \times\ 90 \\ \hline \end{array}$

2

a	$\begin{array}{r} 172 \\ \times\ 26 \\ \hline \end{array}$	b	$\begin{array}{r} 793 \\ \times\ 57 \\ \hline \end{array}$	c	$\begin{array}{r} 4658 \\ \times\ 41 \\ \hline \end{array}$	d	$\begin{array}{r} 2009 \\ \times\ 63 \\ \hline \end{array}$
e	$\begin{array}{r} 5186 \\ \times\ 45 \\ \hline \end{array}$	f	$\begin{array}{r} 13\,814 \\ \times\ 78 \\ \hline \end{array}$	g	$\begin{array}{r} 65\,046 \\ \times\ 93 \\ \hline \end{array}$	h	$\begin{array}{r} 16\,287 \\ \times\ 59 \\ \hline \end{array}$

3 a Bulli football club paid each of its first grade team members $2937 in fees. How much money did the club pay altogether if there were 24 members of the team?

b If 65 046 kilograms of metal are required to make one kilometre of cable, how many kilograms of metal are required to make 25 kilometres of cable?

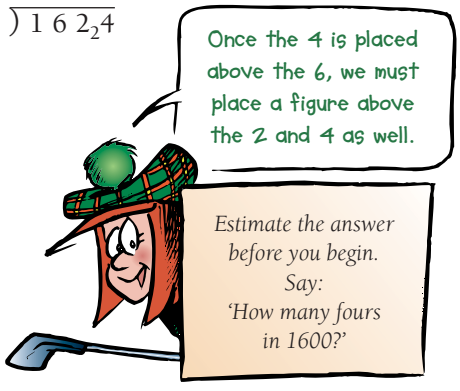
c In 1999 the average monthly production of crude oil was 1913 megalitres. What was the total production of crude oil in 1999?

d From 1 July 1999 to 30 June 2000, average weekly milk production was 73 million litres. How much milk was produced during that financial year (52 weeks)?

B:04 | Division

worked examples

1
$$4\overline{)1624}$$
$$406$$



2 $91\overline{)9686}$

Method 1 Contracted form

$$\begin{array}{r} 106 \text{ r } 40 \\ 91\overline{)9686} \\ -\,91 \\ \hline 586 \\ -\,546 \\ \hline 40 \end{array}$$

Method 2 Preferred multiples

$$\begin{array}{r|l} 106 \text{ r } 40 & \\ 91\overline{)9686} & \\ -\,9100 & 100 \\ \hline 586 & \\ -\,546 & 6 \\ \hline 40 & 106 \end{array}$$

Answer: 106 remainder 40 or $106\frac{40}{91}$

Exercise B:04

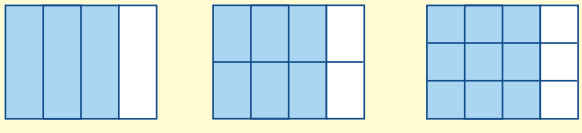
1 **a** $8\overline{)880}$ **b** $6\overline{)397}$ **c** $7\overline{)843}$ **d** $4\overline{)1111}$
e $3\overline{)7418}$ **f** $8\overline{)38\,168}$ **g** $9\overline{)1469}$ **h** $7\overline{)68\,142}$

2 **a** $23\overline{)8104}$ **b** $59\overline{)6183}$ **c** $32\overline{)7894}$ **d** $19\overline{)3856}$
e $500\overline{)7265}$ **f** $36\overline{)60\,660}$ **g** $42\overline{)96\,143}$ **h** $91\overline{)14\,965}$

3 **a** 23 009 books are to be packed into 19 crates so that each crate contains the same number of books. How many books will be placed in each crate?
b How many trips must a truck make to carry 1246 sheep to the stock yards if the truck can carry 54 sheep on each trip?
c 36 450 mL of milk must be placed into bottles so that each bottle contains 600 mL of milk. How many bottles can be filled?
d A tank that holds 10 000 litres is filled by means of a hose in 24 hours. Find the amount added each hour if in each hour the same amount is added.

Fractions

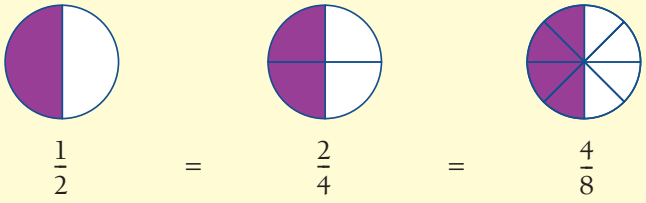
B:05 | Equivalent Fractions



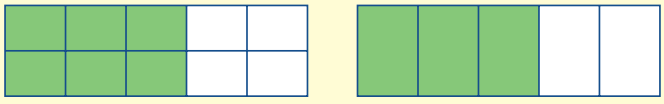
$\frac{3}{4} = \frac{6}{8} = \frac{9}{12}$

$\frac{3}{4} \times \frac{2}{2} = \frac{6}{8}$

$\frac{3}{4} \times \frac{3}{3} = \frac{9}{12}$



$\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$



$\frac{6}{10} = \frac{3}{5}$

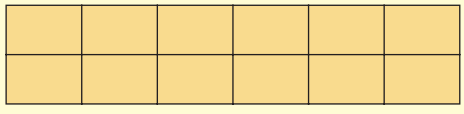
$\frac{1}{2} \times \frac{2}{2} = \frac{2}{4}$

$\frac{1}{2} \times \frac{4}{4} = \frac{4}{8}$

$\frac{6}{10} \div \frac{2}{2} = \frac{3}{5}$

The size of a fraction is unchanged if both the numerator and the denominator are multiplied or divided by the same number.

$1 = \frac{2}{2} = \frac{4}{4} = \frac{5}{5} = \frac{8}{8} = \frac{10}{10} = \frac{12}{12}$



There are many different ways to write the number one.

worked examples

1 $\frac{3}{10} = \frac{\ldots}{100}$

Multiply top and bottom by 10.

$\therefore \frac{3}{10} \times \frac{10}{10} = \frac{30}{100}$

2 $\frac{6}{8} = \frac{\ldots}{4}$

Divide top and bottom by 2.

$\therefore \frac{6}{8} \div \frac{2}{2} = \frac{3}{4}$

3 Simplify $\frac{20}{50}$

$\frac{20}{50} = \frac{20}{50} \div \frac{10}{10}$

$= \frac{2}{5}$



Exercise B:05

1 Complete the following to make equivalent fractions:

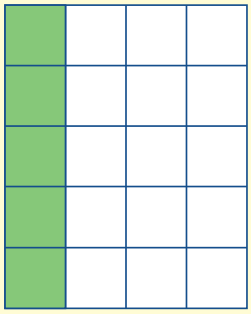
a $\frac{1}{10} = \frac{\square}{100}$ **b** $\frac{1}{2} = \frac{\square}{4}$ **c** $\frac{1}{4} = \frac{\square}{20}$ **d** $\frac{3}{4} = \frac{\square}{8}$ **e** $\frac{7}{10} = \frac{\square}{100}$

f $\frac{5}{10} = \frac{\square}{50}$ **g** $\frac{1}{1} = \frac{\square}{8}$ **h** $\frac{3}{4} = \frac{\square}{20}$ **i** $\frac{3}{5} = \frac{\square}{10}$ **j** $\frac{2}{10} = \frac{\square}{100}$

k $\frac{1}{5} = \frac{\square}{10}$ **l** $\frac{3}{10} = \frac{\square}{100}$ **m** $\frac{1}{3} = \frac{\square}{12}$ **n** $\frac{5}{8} = \frac{\square}{16}$ **o** $\frac{2}{5} = \frac{\square}{10}$

p $\frac{4}{10} = \frac{\square}{100}$ **q** $\frac{2}{5} = \frac{\square}{100}$ **r** $\frac{7}{20} = \frac{\square}{40}$ **s** $\frac{9}{10} = \frac{\square}{100}$ **t** $\frac{4}{5} = \frac{\square}{10}$

2 Use the diagram on the right to complete the following:



a $\frac{5}{20} = \frac{\square}{4}$	**b** $\frac{10}{20} = \frac{\square}{4}$	**c** $\frac{15}{20} = \frac{\square}{4}$	**d** $\frac{20}{20} = \frac{\square}{4}$	
e $\frac{4}{20} = \frac{\square}{5}$	**f** $\frac{8}{20} = \frac{\square}{5}$	**g** $\frac{12}{20} = \frac{\square}{5}$	**h** $\frac{16}{20} = \frac{\square}{5}$	
i $\frac{2}{20} = \frac{\square}{10}$	**j** $\frac{6}{20} = \frac{\square}{10}$	**k** $\frac{10}{20} = \frac{\square}{10}$	**l** $\frac{14}{20} = \frac{\square}{10}$	

3 Write each fraction in its simplest form:

a $\frac{4}{10}$	**b** $\frac{4}{8}$	**c** $\frac{6}{12}$	**d** $\frac{2}{10}$	**e** $\frac{8}{20}$
f $\frac{18}{20}$	**g** $\frac{5}{10}$	**h** $\frac{70}{100}$	**i** $\frac{6}{10}$	**j** $\frac{10}{100}$
k $\frac{2}{12}$	**l** $\frac{15}{20}$	**m** $\frac{10}{16}$	**n** $\frac{3}{9}$	**o** $\frac{8}{10}$
p $\frac{10}{12}$	**q** $\frac{14}{16}$	**r** $\frac{10}{10}$	**s** $\frac{20}{25}$	**t** $\frac{12}{60}$
u $\frac{33}{44}$	**v** $\frac{100}{800}$	**w** $\frac{20}{100}$	**x** $\frac{240}{360}$	**y** $\frac{625}{1000}$

B:06 | Comparing Fractions

Use equivalent fractions to make the denominators the same.
Then compare the numerators.

worked examples

Place in order, from smallest to largest:

1 $\{\frac{3}{10}, 1, \frac{1}{10}\}$

2 $\{\frac{1}{2}, \frac{2}{5}, \frac{6}{10}\}$

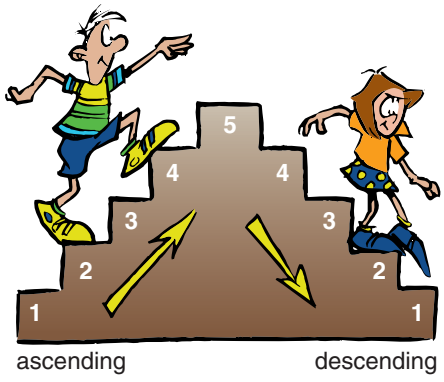
Give each fraction the same denominator.

■ $1 = \frac{10}{10}$, $\frac{1}{2} = \frac{5}{10}$, $\frac{2}{5} = \frac{4}{10}$

$\{\frac{3}{10}, \frac{10}{10}, \frac{1}{10}\}$ $\{\frac{5}{10}, \frac{4}{10}, \frac{6}{10}\}$

In ascending order these become:

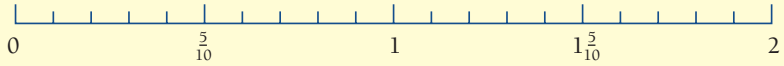
$\{\frac{1}{10}, \frac{3}{10}, 1\}$ $\{\frac{2}{5}, \frac{1}{2}, \frac{6}{10}\}$



ascending descending

Exercise B:06

1 Use this number line to find which of the two numbers is smaller.



a $\frac{3}{10}, \frac{7}{10}$ **b** $\frac{1}{10}, \frac{1}{2}$ **c** $\frac{1}{10}, \frac{9}{10}$ **d** $1, \frac{7}{10}$ **e** $1\frac{3}{10}, 1\frac{1}{2}$

f $\frac{4}{10}, \frac{9}{10}$ **g** $\frac{2}{5}, \frac{1}{5}$ **h** $\frac{2}{5}, \frac{1}{10}$ **i** $\frac{4}{5}, \frac{7}{10}$ **j** $\frac{1}{2}, \frac{2}{5}$

2 Which of the three fractions is smallest?

a $\frac{4}{10}, \frac{3}{10}, \frac{7}{10}$ **b** $\frac{4}{5}, \frac{2}{5}, \frac{3}{5}$ **c** $\frac{2}{10}, \frac{1}{2}, \frac{3}{10}$ **d** $\frac{3}{5}, \frac{1}{2}, \frac{2}{5}$

e $\frac{1}{2}, \frac{1}{4}, \frac{3}{4}$ **f** $\frac{7}{10}, \frac{1}{2}, \frac{1}{5}$ **g** $\frac{3}{4}, \frac{6}{10}, \frac{1}{2}$ **h** $\frac{2}{5}, \frac{1}{2}, \frac{7}{10}$

3 Arrange in order, from smallest to largest:

a $\{\frac{2}{10}, \frac{1}{10}, \frac{7}{10}\}$ **b** $\{\frac{1}{2}, \frac{1}{4}, \frac{3}{4}\}$ **c** $\{1, \frac{4}{5}, \frac{3}{5}\}$ **d** $\{\frac{9}{10}, \frac{7}{10}, \frac{1}{2}\}$

e $\{\frac{7}{10}, 1\frac{1}{10}, \frac{1}{5}\}$ **f** $\{\frac{1}{2}, \frac{1}{4}, \frac{1}{10}\}$ **g** $\{\frac{2}{5}, \frac{3}{4}, \frac{1}{2}\}$ **h** $\{2\frac{1}{10}, 1\frac{1}{2}, \frac{9}{10}\}$

i $\{\frac{3}{4}, \frac{1}{2}, \frac{3}{5}\}$ **j** $\{\frac{3}{4}, \frac{9}{10}, \frac{3}{5}\}$ **k** $\{\frac{7}{40}, \frac{2}{10}, \frac{1}{4}\}$ **l** $\{\frac{23}{100}, \frac{1}{5}, \frac{3}{10}\}$

B:07 | Improper Fractions and Mixed Numbers

An improper fraction has a numerator that is greater than its denominator.
eg $\frac{5}{2}, \frac{13}{10}$.

A mixed number is one that has a whole number part and a fraction part, **eg** $2\frac{1}{2}, 1\frac{3}{10}$.

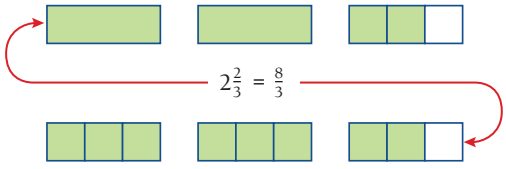
worked examples

1 Write 5 as an improper fraction in quarters.

Now $1 = 4$ quarters

$\therefore \quad 5 = 20$ quarters

$= \frac{20}{4}$





2 Write $2\frac{2}{3}$ as an improper fraction.

$2\frac{2}{3} = 2 + \frac{2}{3}$

$= \frac{6}{3} + \frac{2}{3}$

$= \frac{8}{3}$

3 Change $\frac{8}{3}$ to a mixed number.

$\frac{8}{3} = \frac{6}{3} + \frac{2}{3}$ OR $\frac{8}{3} = 8 \div 3$

$= 2 + \frac{2}{3}$ $= 2\frac{2}{3}$

$= 2\frac{2}{3}$

■ $\frac{25}{4}$ means $25 \div 4$.

Exercise B:07

1 Write each mixed number as an improper fraction.

a $1\frac{1}{2}$	**b** $1\frac{1}{4}$	**c** $1\frac{1}{10}$	**d** $1\frac{1}{3}$	**e** $1\frac{1}{5}$	**f** $1\frac{1}{8}$
g $2\frac{3}{10}$	**h** $1\frac{3}{4}$	**i** $2\frac{1}{2}$	**j** $3\frac{2}{3}$	**k** $4\frac{3}{5}$	**l** $1\frac{9}{10}$
m $5\frac{4}{5}$	**n** $2\frac{3}{4}$	**o** $2\frac{7}{10}$	**p** $5\frac{7}{10}$	**q** $4\frac{3}{8}$	**r** $3\frac{3}{5}$
s $1\frac{5}{8}$	**t** $7\frac{17}{100}$	**u** $4\frac{13}{20}$	**v** $2\frac{1}{100}$	**w** $8\frac{7}{8}$	**x** $6\frac{3}{4}$

2 Write each improper fraction as a mixed or whole number.

a $\frac{5}{4}$	**b** $\frac{13}{10}$	**c** $\frac{7}{4}$	**d** $\frac{57}{10}$	**e** $\frac{20}{5}$	**f** $\frac{44}{5}$
g $\frac{21}{8}$	**h** $\frac{9}{4}$	**i** $\frac{61}{10}$	**j** $\frac{11}{4}$	**k** $\frac{7}{5}$	**l** $\frac{11}{8}$
m $\frac{15}{2}$	**n** $\frac{9}{5}$	**o** $\frac{9}{8}$	**p** $\frac{89}{10}$	**q** $\frac{24}{3}$	**r** $\frac{15}{4}$
s $\frac{36}{4}$	**t** $\frac{13}{4}$	**u** $\frac{13}{5}$	**v** $\frac{15}{8}$	**w** $\frac{17}{4}$	**x** $\frac{19}{8}$

3 Simplify by writing each as a mixed or whole number.

a $\frac{120}{4}$	**b** $\frac{73}{2}$	**c** $\frac{49}{3}$	**d** $\frac{65}{5}$	**e** $\frac{127}{10}$	**f** $\frac{81}{8}$
g $\frac{29}{24}$	**h** $\frac{17}{10}$	**i** $\frac{100}{3}$	**j** $\frac{100}{5}$	**k** $\frac{46}{22}$	**l** $\frac{140}{7}$
m $\frac{18}{10}$	**n** $\frac{36}{8}$	**o** $\frac{122}{4}$	**p** $\frac{185}{10}$	**q** $\frac{110}{20}$	**r** $\frac{274}{100}$

4 **a** Thirty-one children were each given one-quarter of an apple. How many apples were given away.

b When Karen was injured working in the factory, each of her friends donated $\frac{1}{5}$ of a day's pay to help her. How many days' pay were given if 33 friends contributed money?

c Ray used $\frac{1}{4}$ of a page for each graph he drew. How many pages would he use to draw 15 graphs?

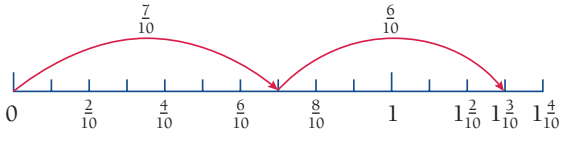
d Ruby used $6\frac{3}{10}$ pages to do many similar division questions. If each question took $\frac{1}{10}$ of a page, how many questions did she do?

B:08 | Adding and Subtracting Fractions

When fractions have the same denominator, we can add them by adding numerators; we can subtract them by subtracting numerators.

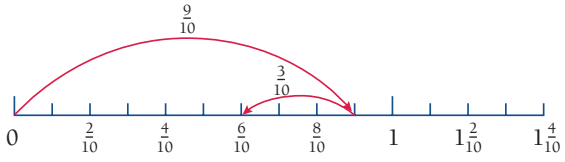
worked examples

1 $\frac{7}{10} + \frac{6}{10} = \frac{7+6}{10} = \frac{13}{10} = 1\frac{3}{10}$





2 $\frac{9}{10} - \frac{3}{10} = \frac{9-3}{10} = \frac{6}{10} = \frac{3}{5}$



Exercise B:08

1 **a** $\frac{5}{10} + \frac{4}{10}$ $= \frac{\ldots + \ldots}{10}$ $= \frac{\ldots}{10}$

b $\frac{8}{10} - \frac{5}{10}$ $= \frac{\ldots - \ldots}{10}$ $= \frac{\ldots}{10}$

c $\frac{9}{10} - \frac{8}{10}$ $= \frac{\ldots - \ldots}{10}$ $= \frac{\ldots}{10}$

d $\frac{1}{10} + \frac{6}{10}$ $= \frac{\ldots + \ldots}{10}$ $= \frac{\ldots}{10}$

e $\frac{2}{8} + \frac{3}{8}$ $= \frac{\ldots + \ldots}{8}$ $= \frac{\ldots}{8}$

f $\frac{7}{10} + \frac{2}{10}$ **g** $\frac{3}{10} + \frac{6}{10}$ **h** $\frac{4}{10} + \frac{3}{10}$ **i** $\frac{5}{8} + \frac{2}{8}$ **j** $\frac{2}{5} + \frac{1}{5}$

k $\frac{7}{10} - \frac{4}{10}$ **l** $\frac{3}{10} - \frac{2}{10}$ **m** $\frac{4}{10} - \frac{3}{10}$ **n** $\frac{5}{8} - \frac{4}{8}$ **o** $\frac{2}{5} - \frac{1}{5}$

p $\frac{3}{100} + \frac{8}{100}$ **q** $\frac{17}{100} + \frac{2}{100}$ **r** $\frac{35}{100} + \frac{26}{100}$ **s** $\frac{51}{100} + \frac{16}{100}$ **t** $\frac{82}{100} + \frac{17}{100}$

u $\frac{18}{100} - \frac{15}{100}$ **v** $\frac{41}{100} - \frac{18}{100}$ **w** $\frac{55}{100} - \frac{34}{100}$ **x** $\frac{98}{100} - \frac{89}{100}$ **y** $\frac{31}{100} - \frac{18}{100}$

2 Give answers as mixed numbers or in simplest form.

a $\frac{3}{10} + \frac{2}{10}$ **b** $\frac{3}{8} + \frac{3}{8}$ **c** $\frac{6}{10} + \frac{5}{10}$ **d** $\frac{8}{10} + \frac{9}{10}$ **e** $\frac{3}{12} + \frac{7}{12}$

f $\frac{9}{16} - \frac{3}{16}$ **g** $\frac{8}{10} - \frac{4}{10}$ **h** $\frac{7}{8} - \frac{5}{8}$ **i** $\frac{7}{10} - \frac{1}{10}$ **j** $\frac{11}{10} - \frac{3}{10}$

k $\frac{6}{10} + \frac{6}{10}$ **l** $\frac{7}{100} + \frac{7}{100}$ **m** $\frac{9}{10} + \frac{9}{10}$ **n** $\frac{3}{20} + \frac{9}{20}$ **o** $\frac{95}{100} + \frac{9}{100}$

p $\frac{5}{4} - \frac{1}{4}$ **q** $\frac{13}{10} - \frac{4}{10}$ **r** $\frac{5}{8} - \frac{1}{8}$ **s** $\frac{15}{12} - \frac{3}{12}$ **t** $\frac{10}{10} - \frac{3}{10}$

u $\frac{5}{12} + \frac{9}{12}$ **v** $\frac{4}{5} + \frac{3}{5}$ **w** $\frac{3}{4} + \frac{1}{4}$ **x** $\frac{95}{100} + \frac{95}{100}$ **y** $\frac{61}{100} + \frac{89}{100}$

3 Where there are only '+' and '–', work from left to right.

a $\frac{2}{10}+\frac{1}{10}+\frac{3}{10}$ b $\frac{7}{10}+\frac{2}{10}+\frac{1}{10}$ c $\frac{8}{10}+\frac{5}{10}+\frac{1}{10}$ d $\frac{6}{10}+\frac{3}{10}+\frac{1}{10}$

e $\frac{9}{10}-\frac{2}{10}-\frac{1}{10}$ f $\frac{8}{12}-\frac{3}{12}-\frac{4}{12}$ g $\frac{17}{100}-\frac{8}{100}-\frac{3}{100}$ h $\frac{7}{8}-\frac{1}{8}-\frac{3}{8}$

i $\frac{11}{15}+\frac{6}{15}-\frac{9}{15}$ j $\frac{9}{10}+\frac{4}{10}-\frac{7}{10}$ k $\frac{7}{8}-\frac{5}{8}+\frac{3}{8}$ l $\frac{4}{5}-\frac{3}{5}+\frac{2}{5}$

B:09 | Multiplying a Whole Number by a Fraction

$5\times\frac{3}{4}$ means '5 lots of $\frac{3}{4}$', *or* $\frac{3+3+3+3+3}{4}$ *or* $\frac{5\times3}{4}$

worked examples

1 $8\times\frac{3}{10}=\frac{8\times3}{10}$

(eight lots of $\frac{3}{10}$)

$=\frac{24}{10}$

$=2\frac{4}{10}$

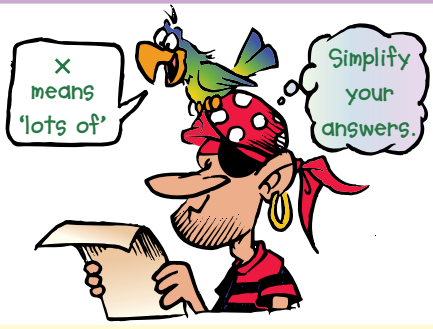
$=2\frac{2}{5}$

2 $10\times\frac{2}{5}=\frac{10\times2}{5}$

(ten lots of $\frac{2}{5}$)

$=\frac{20}{5}$

$=4$



Exercise B:09

1 Give the simplest answer for:

a $2\times\frac{3}{10}$ b $3\times\frac{1}{4}$ c $7\times\frac{3}{100}$ d $7\times\frac{2}{15}$ e $3\times\frac{3}{10}$

f $3\times\frac{1}{12}$ g $6\times\frac{1}{10}$ h $8\times\frac{9}{100}$ i $5\times\frac{1}{8}$ j $11\times\frac{7}{100}$

k $3\times\frac{7}{10}$ l $7\times\frac{3}{4}$ m $2\times\frac{3}{5}$ n $5\times\frac{3}{4}$ o $7\times\frac{31}{100}$

p $8\times\frac{7}{10}$ q $9\times\frac{5}{8}$ r $6\times\frac{9}{10}$ s $7\times\frac{4}{5}$ t $10\times\frac{3}{8}$

u $11\times\frac{7}{8}$ v $7\times\frac{21}{100}$ w $8\times\frac{3}{4}$ x $100\times\frac{3}{4}$ y $9\times\frac{8}{15}$

B:10 | Finding a Fraction of a Number

To find $\frac{7}{8}$ of a number, find $\frac{1}{8}$ of the number and then multiply by 7.
(To find $\frac{1}{8}$ of a number, divide it by 8.)

worked examples

1 $\frac{3}{5}$ of 100 = □

$\frac{1}{5}$ of 100 = 100 ÷ 5

= 20

$\frac{3}{5}$ of 100 = 3 × 20

= 60

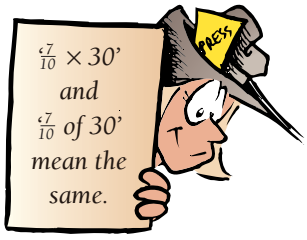
2 $\frac{7}{10} \times 30 = \square$

$\frac{1}{10}$ of 30 = 30 ÷ 10

= 3

$\frac{7}{10}$ of 30 = 7 × 3

= 21



Exercise B:10

1 **a** $\frac{1}{4}$ of 20 **b** $\frac{1}{6}$ of 66 **c** $\frac{1}{5}$ of 30 **d** $\frac{1}{8}$ of 24 **e** $\frac{1}{10}$ of 90

f $\frac{3}{4}$ of 20 **g** $\frac{5}{6}$ of 66 **h** $\frac{4}{5}$ of 30 **i** $\frac{3}{8}$ of 24 **j** $\frac{7}{10}$ of 90

k $\frac{3}{10}$ of 200 **l** $\frac{3}{4}$ of 40 **m** $\frac{5}{8}$ of 16 **n** $\frac{3}{10}$ of 100 **o** $\frac{2}{3}$ of 15

p $\frac{1}{10}$ of 80 **q** $\frac{1}{5}$ of 60 **r** $\frac{1}{3}$ of 66 **s** $\frac{3}{100}$ of 600 **t** $\frac{7}{20}$ of 100

u $\frac{1}{8} \times 40$ **v** $\frac{1}{4} \times 24$ **w** $\frac{3}{10} \times 30$ **x** $\frac{2}{3} \times 60$ **y** $\frac{7}{8} \times 8$

Decimals and Money

B:11 | Conversions: Fractions and Decimals

The examples below show the connection between fractions and decimals.

worked examples

1 $\frac{7}{10}$ = 0·7

$\frac{7}{10}$: 1 zero

0·7: 1 figure after the point

2 $2\frac{13}{100}$ = 2·13

$2\frac{13}{100}$: 2 zeros

2·13: 2 figures after the point

3 $\frac{9}{1000}$ = 0·009

$\frac{9}{1000}$: 3 zeros

0·009: 3 figures after the point

■ If a fraction has a denominator of 10, 100, 1000, etc, it can easily by changed to a decimal. If the fraction has some other denominator, it may be possible to change the denominator using equivalent fractions, eg $\frac{3}{4} = \frac{3 \times 25}{4 \times 25} = \frac{75}{100} = 0{\cdot}75$.



Exercise B:11

1 Write these fractions as decimals.

a $\frac{1}{10}$ **b** $\frac{23}{100}$ **c** $\frac{9}{10}$ **d** $\frac{9}{100}$ **e** $\frac{8}{10}$ **f** $\frac{25}{100}$

g $\frac{1}{1000}$ h $\frac{3}{10}$ i $\frac{67}{1000}$ j $\frac{2}{10}$ k $\frac{135}{1000}$ l $\frac{95}{100}$

m $\frac{63}{100}$ n $\frac{6}{1000}$ o $\frac{4}{10}$ p $\frac{31}{100}$ q $\frac{11}{1000}$ r $\frac{5}{10}$

s $\frac{999}{1000}$ t $\frac{48}{100}$ u $\frac{375}{1000}$ v $\frac{6}{10}$ w $\frac{68}{100}$ x $\frac{84}{100}$

2 Write these mixed numbers as decimals.

a $1\frac{7}{10}$ b $3\frac{43}{100}$ c $6\frac{3}{100}$ d $4\frac{4}{10}$ e $9\frac{4}{100}$ f $7\frac{3}{10}$

g $2\frac{412}{1000}$ h $5\frac{1}{100}$ i $8\frac{96}{1000}$ j $10\frac{7}{100}$ k $12\frac{71}{100}$ l $20\frac{2}{100}$

m $79\frac{875}{1000}$ n $14\frac{8}{100}$ o $15\frac{73}{100}$ p $9\frac{22}{1000}$ q $5\frac{5}{100}$ r $41\frac{147}{1000}$

3 Write these decimals as fractions or mixed numbers in simplest form.

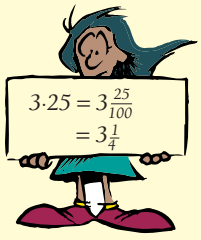
a 0·1 b 0·57 c 0·3 d 0·03 e 0·9

f 0·2 g 0·42 h 0·81 i 0·8 j 0·729

k 9·61 l 11·03 m 8·107 n 26·5 o 73·6

p 0·07 q 5·04 r 7·9 s 0·632 t 16·25

u 0·805 v 1·125 w 0·346 x 75·2 y 94·625



B:12 | Comparing Decimals

When comparing two decimals, you can place zeros at the end of one so that the two decimals have the same number of decimal places. This makes them easier to compare.

worked examples

1 Which is smaller?

a 0·11, 0·2 **b** 0·5, 0·49

2 Put in order, smallest to largest:
0·12, 0·8, 0·5

Solutions

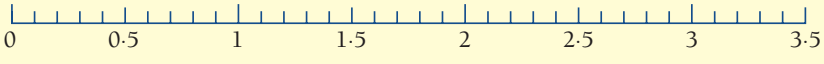
1 a 0·2 = 0·20
∴ 0·11 is smaller than 0·2 (since 11 is less than 20).

b 0·5 = 0·50
∴ 0·49 is smaller than 0·5 (since 49 is less than 50).

2 0·8 = 0·80, 0·5 = 0·50
∴ in order we have
0·12, 0·5, 0·8 (since 12 is less than 50, which is less than 80).

Exercise B:12

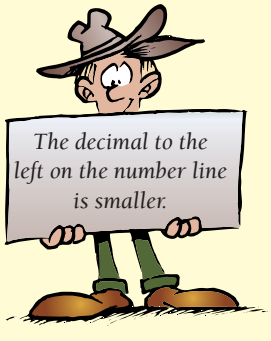
1 By referring to the number line below, say which is smaller.



a 0·6, 0·9 **b** 0·5, 0·1 **c** 1, 0·9 **d** 2·5, 1·8

e 1·1, 1 **f** 3·4, 2·8 **g** 0·9, 2·2 **h** 2·4, 2·1

i 0·2, 0·1 **j** 2, 0·3 **k** 3·5, 2·9 **l** 0·8, 1·6



2 Which is smaller?

a 0·8, 0·3	**b** 0·93, 0·15	**c** 0·61, 0·51	**d** 0·09, 0·15
e 0·8, 0·14	**f** 0·22, 0·3	**g** 0·9, 0·33	**h** 0·09, 0·6
i 0·03, 0·2	**j** 0·4, 0·39	**k** 0·5, 0·88	**l** 0·2, 0·19

3 Put in order, smallest to largest:

a 0·2, 0·5, 0·3	**b** 0·8, 0·1, 0·3	**c** 0·9, 0·5, 0·2
d 0·3, 0·41, 0·21	**e** 0·33, 0·5, 0·4	**f** 0·1, 0·09, 0·08
g 0·6, 0·55, 0·3	**h** 0·5, 1, 0·7	**i** 0·6, 0·3, 1
j 0·31, 0·4, 1	**k** 1, 0·39, 0·4	**l** 1·84, 1·9, 1·79

■ 0·1 = 0·10
0·2 = 0·20
0·3 = 0·30
0·4 = 0·40
0·5 = 0·50
0·6 = 0·60
0·7 = 0·70
0·8 = 0·80
0·9 = 0·90

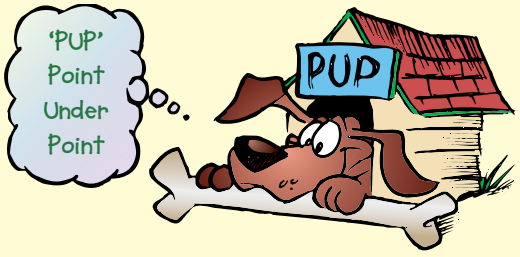
B:13 | Adding Decimals

The column in which a figure is placed determines its size. Therefore in adding (or subtracting) decimals, we must be sure to put figures in the same column, under one another.

To add (or subtract) decimals use the PUP rule: Place Points Under Points. An empty space can be filled by a zero.

5·85
0·60
7·00
·

■ 0·6 = 0·60
7 = 7·00



worked examples

1 2·74 + 0·4
= 2 · 7 4
+ 0_1· 4 0
3 · 1 4

2 5·18 + 3
= 5·18
+ 3·00
8·18

3 7·2 + 16 + 4·1
= 7·2
16·0
$_1$4·1
27·3

Exercise B:13

1

a 3·8 + 6·3	**b** 9·15 + 8·31	**c** 16·03 + 9·97	**d** 9·7 + 18·6
e 8·2, 0·6, 1·8	**f** 18·31, 9·45, 17·88	**g** 19·9, 8·6, 3·8	**h** 47·8, 11·3, 9·5
i 8·65, 1·6, 9·3	**j** 16·3, 9·15, 10·9	**k** 8·83, 7·5, 12·08	**l** 3·4, 9·18, 16·02

m 8·06
0·9
0·04

n 4·9
5
6·15

o 10
9·04
7·95

p 23·8
4·99
6

2 **a** 0·15 + 0·58 **b** 3·5 + 9·6 **c** 23·1 + 9·8
d 6·4 + 10·6 **e** 0·8 + 2·5 + 1·8 **f** 3·8 + 10·6 + 9·1
g 0·85 + 1·15 + 0·65 **h** 0·95 + 8·75 + 1·11 **i** 2·4 + 12
j 6 + 0·6 **k** 5 + 0·5 **l** 8·2 + 8·33
m 8·15 + 6 + 1·8 **n** 6·7 + 7·15 + 4 **o** 8·15 + 0·8 + 0·08
p 0·9 + 9 + 0·09 **q** 60 + 9·3 + 0·7 **r** 18·4 + 3·21 + 10
s 0·06 + 6 + 6·66 **t** 8·3 + 9·81 + 2·4 **u** 3·85 + 1·4 + 1·85
v 6·21 + 9·9 + 1·86 **w** 5·1 + 6 + 2·85 **x** 9·35 + 8·9 + 6·95

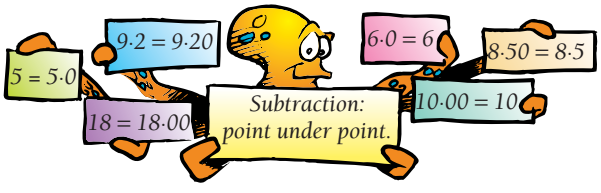
B:14 | Subtracting Decimals

To subtract (or add) decimals use the PUP rule: Place Point Under Points.
An empty space can be filled by a zero.

worked examples

1 18 − 9·18
= 18·00
− 9·18
8·82

2 9·2 − 1·25
= 9·20
− 1·25
7·95



Exercise B:14

1 **a** 14·95 − 9·21 **b** 90·8 − 49·3 **c** 48·5 − 28·9



d 9·65 − 3·4 **e** 19·85 − 1·6 **f** 23·4 − 1·5

g 12·6 − 4·24 **h** 8·5 − 3·15 **i** 19·8 − 3·85

j 15 − 3·15 **k** 10 − 4·8 **l** 6 − 3·9 **m** 5 − 4·1

n 16·4 − 3·89 **o** 8·04 − 6·4 **p** 63·18 − 9·5 **q** 160·05 − 45

2 **a** 18·6 − 3·4 **b** 23·41 − 8·62 **c** 4·95 − 0·85 **d** 6·92 − 1·08
e 45·83 − 9·1 **f** 6·85 − 5·8 **g** 43·6 − 13 **h** 91·45 − 6·7
i 8·4 − 0·35 **j** 10·4 − 3·85 **k** 8·9 − 4·15 **l** 6·8 − 0·37
m 9 − 0·3 **n** 6 − 1·25 **o** 9 − 0·08 **p** 15 − 1·8
q 20·9 − 1·35 **r** 6·08 − 1·6 **s** 19·45 − 6·3 **t** 23·8 − 0·09

u 76 – 19·8 v 154 – 3·72 w 9·08 – 3 x 17·85 – 1·9

3 a Mrs Leong bought 9·5 metres of ribbon. This was cut from a roll that had 32·8 metres of ribbon on it. How much was left?
b Mrs Hooker owned 80 hectares of land in Orange. She sold 8·7 hectares as a primary school site. How many hectares did she have left?
c Brian began to train in order to lose weight. He reduced his original weight of 86 kilograms by 9·2 kilograms. What did he weigh in the end?
d I am 37 km from Parkes post office. If I live 3·2 km this side of the post office, how far am I from home?

B:15 | Multiplying a Decimal by a Whole Number

worked examples

1 $1{\cdot}6 \times 5$

Solutions

1
$$\begin{array}{r} 1{\cdot}6 \\ \times\ 5 \\ \hline 8{\cdot}0 \\ \hline \end{array}$$

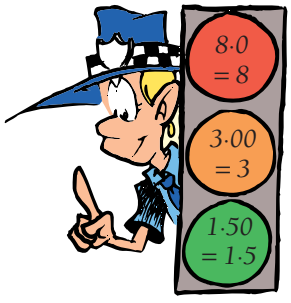


One figure after the point

Three figures after the point

2 $0{\cdot}181 \times 3$

2
$$\begin{array}{r} 0{\cdot}181 \\ \times\ 3 \\ \hline 0{\cdot}543 \\ \hline \end{array}$$



Exercise B:15

1 a
$$\begin{array}{r} 1{\cdot}8 \\ \times\ 3 \\ \hline \end{array}$$
b
$$\begin{array}{r} 3{\cdot}7 \\ \times\ 5 \\ \hline \end{array}$$
c
$$\begin{array}{r} 2{\cdot}4 \\ \times\ 6 \\ \hline \end{array}$$
d
$$\begin{array}{r} 5{\cdot}9 \\ \times\ 4 \\ \hline \end{array}$$

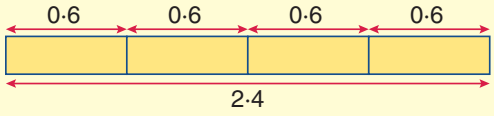
e
$$\begin{array}{r} 0{\cdot}6 \\ \times\ 7 \\ \hline \end{array}$$
f
$$\begin{array}{r} 1{\cdot}31 \\ \times\ 4 \\ \hline \end{array}$$
g
$$\begin{array}{r} 0{\cdot}9 \\ \times\ 5 \\ \hline \end{array}$$
h
$$\begin{array}{r} 2{\cdot}14 \\ \times\ 3 \\ \hline \end{array}$$

i
$$\begin{array}{r} 0{\cdot}152 \\ \times\ 3 \\ \hline \end{array}$$
j
$$\begin{array}{r} 1{\cdot}82 \\ \times\ 4 \\ \hline \end{array}$$
k
$$\begin{array}{r} 9{\cdot}31 \\ \times\ 2 \\ \hline \end{array}$$
l
$$\begin{array}{r} 1{\cdot}811 \\ \times\ 6 \\ \hline \end{array}$$

2 a $0{\cdot}5 \times 9$ b $0{\cdot}6 \times 8$ c $0{\cdot}7 \times 6$ d $0{\cdot}4 \times 9$
e $0{\cdot}8 \times 8$ f $0{\cdot}6 \times 9$ g $0{\cdot}7 \times 9$ h $0{\cdot}9 \times 9$
i $0{\cdot}11 \times 6$ j $0{\cdot}97 \times 3$ k $0{\cdot}85 \times 5$ l $0{\cdot}91 \times 3$
m $7{\cdot}2 \times 4$ n $3{\cdot}6 \times 5$ o $8{\cdot}5 \times 8$ p $1{\cdot}4 \times 9$
q $6{\cdot}04 \times 7$ r $9{\cdot}09 \times 2$ s $3{\cdot}04 \times 6$ t $8{\cdot}05 \times 8$

3 a For one dollar Luke can buy 3·2 metres of hose. How many metres of hose can he buy for five dollars?
b In one second Alana can walk 1·4 metres. How far would she walk in 8 seconds?
c Her younger sister, Naomi, can walk 1·2 metres in one second. How far would she walk in 8 seconds?
d If Alana and Naomi started at the same place and walked in opposite directions for 8 seconds, how far apart would they be?

B:16 | Dividing a Decimal by a Whole Number



$0{\cdot}6 \times 4 = 2{\cdot}4$ so $4\overline{)2{\cdot}4}$ $0{\cdot}6$

When dividing a decimal by a whole number, place the point in the answer above the point in the question.

worked examples

1 $3\overline{)4_{1}8{\cdot}6}$ $1\,6{\cdot}2$

2 $6\overline{)1{\cdot}_{1}8}$ $0{\cdot}\,3$

3 $3\overline{)0{\cdot}1_{1}2\,3}$ $0{\cdot}0\,4\,1$

In each case we have kept Point Under Point.

Exercise B:16

1 a $5\overline{)15{\cdot}5}$ b $4\overline{)8{\cdot}52}$ c $3\overline{)7{\cdot}11}$ d $6\overline{)37{\cdot}2}$

e $8\overline{)0{\cdot}96}$ f $5\overline{)5{\cdot}05}$ g $4\overline{)38{\cdot}4}$ h $7\overline{)15{\cdot}4}$

i $2\overline{)2{\cdot}16}$ j $9\overline{)2{\cdot}79}$ k $5\overline{)5{\cdot}45}$ l $6\overline{)6{\cdot}24}$

2 a $0{\cdot}72 \div 3$ b $0{\cdot}303 \div 3$ c $0{\cdot}726 \div 2$ d $0{\cdot}145 \div 5$

e $4{\cdot}88 \div 8$ f $12{\cdot}04 \div 4$ g $45{\cdot}6 \div 6$ h $6{\cdot}72 \div 6$

i $4{\cdot}56 \div 8$ j $1{\cdot}82 \div 7$ k $9{\cdot}27 \div 9$ l $74{\cdot}9 \div 7$

m $41{\cdot}64 \div 4$ n $10{\cdot}12 \div 2$ o $15{\cdot}05 \div 7$ p $56{\cdot}79 \div 9$

3 a Rachel walked 10·5 metres in 7 seconds. If she walked at a constant speed, how far did she travel in one second?

b Three brothers inherited 54·6 hectares of land. If the land was divided equally, how many hectares did each inherit?

c Six tug-of-war ropes of equal length are to be made from a length of rope 57·6 metres long. How long will each rope be?

d Each member of a four-girl relay team completed her lap in the same time. If the four laps took 137·2 seconds, how long did each swimmer take?

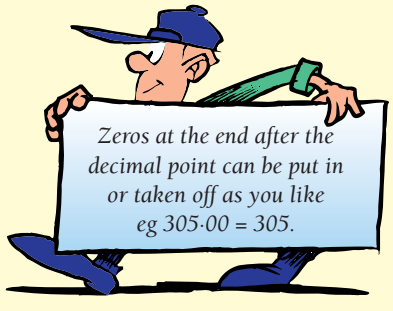
B:17 | Multiplying and Dividing Decimals by 10

Investigation

1 $0{\cdot}8 \times 10 = 8{\cdot}0$
$= 8$

2 $3{\cdot}05 \times 10 = 30{\cdot}50$
$= 30{\cdot}5$

3 $0{\cdot}00125 \times 10 = 0{\cdot}01250$
$0{\cdot}00125 \times 10 = 0{\cdot}0125$



4 $10 \overline{)\,0{\cdot}80}$ $0{\cdot}08$

5 $10 \overline{)\,1{\cdot}80}$ $0{\cdot}18$

6 $10 \overline{)\,56{\cdot}80}$ $5{\cdot}68$

From the examples above we see that:
- **when we multiply a decimal by 10, we move the decimal point 1 place to the right**
- **when we divide a decimal by 10, we move the decimal point 1 place to the left**

worked examples

1 $0{\cdot}6 \times 10 = 6$

2 $34{\cdot}180 \times 10 = 341{\cdot}8$

3 $0{\cdot}013 \times 10 = 0{\cdot}13$

4 $0{\cdot}035 \times 10 = 0{\cdot}35$

5 $30{\cdot}7 \div 10 = 3{\cdot}07$

6 $56{\cdot} \div 10 = 5{\cdot}6$

7 $0{\cdot}045 \div 10 = 0{\cdot}0045$

8 $845{\cdot}6 \div 10 = 84{\cdot}56$

Exercise B:17

1 a $0{\cdot}5 \times 10$ b $0{\cdot}37 \times 10$ c $0{\cdot}9 \times 10$ d $0{\cdot}08 \times 10$
e $3{\cdot}85 \times 10$ f $0{\cdot}815 \times 10$ g $17{\cdot}125 \times 10$ h $0{\cdot}071 \times 10$
i $0{\cdot}91 \times 10$ j $8{\cdot}5 \times 10$ k $0{\cdot}01 \times 10$ l $2{\cdot}375 \times 10$
m $0{\cdot}005 \times 10$ n $15{\cdot}8 \times 10$ o $1{\cdot}382 \times 10$ p $7{\cdot}8 \times 10$

2 a $3{\cdot}8 \div 10$ b $52 \div 10$ c $78{\cdot}6 \div 10$ d $84{\cdot}15 \div 10$
e $0{\cdot}04 \div 10$ f $108{\cdot}5 \div 10$ g $0{\cdot}74 \div 10$ h $15{\cdot}7 \div 10$
i $7{\cdot}2 \div 10$ j $8 \div 10$ k $0{\cdot}7 \div 10$ l $6{\cdot}7 \div 10$
m $0{\cdot}125 \div 10$ n $635 \div 10$ o $3{\cdot}6 \div 10$ p $845{\cdot}1 \div 10$

B:18 | Decimal Currency

The rules for decimals also apply to money, eg 5 dollars 63 cents = $5.63.

worked examples

1 \$201 + \$1.20

2 \$50 − \$3.50

3 \$1.85 × 6

4 \$70 ÷ 8

Solutions

1
$$\begin{array}{r} \$201.00 \\ +\quad 1.20 \\ \hline \$202.20 \end{array}$$

2
$$\begin{array}{r} \$50.00 \\ -\quad 3.50 \\ \hline \$46.50 \end{array}$$

3
$$\begin{array}{r} \$1.85 \\ \times\quad 6 \\ \hline \$11.10 \end{array}$$

4 $8 \overline{)\,\$7\,0.{}_{6}0{}_{4}0}$ $\$\ 8.\ 7\ 5$

Exercise B:18

1 Write each amount as dollars using a decimal.

a 8 dollars 15 cents	**b** 16 dollars 50 cents	**c** 146 dollars 95 cents
d 37 dollars 15 cents	**e** 93 dollars 5 cents	**f** 2 dollars 35 cents
g 100 dollars 50 cents	**h** 40 dollars 60 cents	**i** 18 dollars 65 cents

2

a \$1.45 + \$6.50	**b** \$6.80 + \$9.20	**c** \$3.45 + \$6.15	**d** \$2.90 + \$9.55
e \$20 + \$1.65	**f** \$3.85 + \$25	**g** \$14.30 + \$8	**h** \$165 + 19.35
i \$36.40 8.35 11.35 0.60	**j** \$84.00 9.15 63.40 6.90	**k** \$145.30 9.45 16.80 101.75	**l** \$29.40 56.85 17.00 67.95

3

a \$8.70 − 3.45	**b** \$10.00 − 7.65	**c** \$7.60 − 5.45	**d** \$76.00 − 9.40
e \$8.60 − \$3.15	**f** \$9.85 − \$3.15	**g** \$6.50 − \$6.35	**h** \$8.75 − 4.80
i \$20 − \$1.35	**j** \$40 − \$23.40	**k** \$50 − \$7.30	**l** \$100 − \$40.70
m \$465 − \$71.15	**n** \$96.10 − \$9.95	**o** \$30 − \$27.95	**p** \$63.80 − \$36.45

4

a \$4.05 × 3	**b** \$3.15 × 4	**c** \$6.05 × 2	**d** \$7.25 × 5
e \$7.45 × 7	**f** \$8.60 × 9	**g** \$3.85 × 6	**h** \$9.35 × 8
i \$17.00 × 6	**j** \$18.00 × 5	**k** \$95.00 × 4	**l** \$103.00 × 6
m \$16.70 × 8	**n** \$78.40 × 9	**o** \$37.45 × 7	**p** \$103.00 × 11

5

a \$8.25 ÷ 3	**b** \$9.30 ÷ 2	**c** \$8.00 ÷ 5	**d** \$3.80 ÷ 4
e \$18.45 ÷ 9	**f** \$36.00 ÷ 6	**g** \$40.40 ÷ 8	**h** \$63.70 ÷ 7
i \$20 ÷ 8	**j** \$21 ÷ 6	**k** \$52 ÷ 4	**l** \$81 ÷ 4
m \$136.20 ÷ 6	**n** \$43.40 ÷ 7	**o** \$736.00 ÷ 80	**p** \$30 ÷ 8

Percentages

B:19 | Changing Percentages to Fractions

worked examples

1 $7\% = \frac{7}{100}$
(7 for every 100)

2 $53\% = \frac{53}{100}$
(53 for every 100)

3 $90\% = \frac{90}{100} \begin{matrix}\div 10\\ \div 10\end{matrix}$
$= \frac{9}{10}$

4 $64\% = \frac{64}{100} \begin{matrix}\div 4\\ \div 4\end{matrix}$
$= \frac{16}{25}$



% has a 'one' and two 'zeros'.

Exercise B:19

1 Write these as hundredths:

a 13%	b 1%	c 5%	d 3%	e 11%
f 9%	g 15%	h 2%	i 10%	j 77%
k 97%	l 4%	m 58%	n 14%	o 30%
p 63%	q 35%	r 8%	s 50%	t 81%
u 90%	v 12%	w 43%	x 6%	y 29%

2 Write each percentage as a fraction in its simplest form.

a 10%	b 50%	c 2%	d 4%	e 7%
f 8%	g 25%	h 18%	i 64%	j 5%
k 85%	l 52%	m 60%	n 75%	o 76%
p 24%	q 15%	r 28%	s 30%	t 80%
u 46%	v 66%	w 92%	x 55%	y 16%

B:20 | Changing Percentages to Whole and Mixed Numbers

To write a percentage as a fraction or mixed number, write it as a fraction and simplify.

worked examples

1 $100\% = \frac{100}{100}$

$= 1$

2 $400\% = \frac{400}{100}$

$= 4$

3 $125\% = \frac{125}{100}$

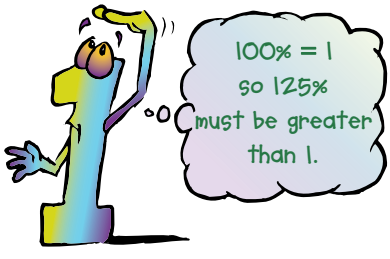
$= 1\frac{125}{100}$ $\div 25$, $\div 25$

$= 1\frac{1}{4}$

4 $306\% = \frac{306}{100}$

$= 3\frac{6}{100}$ $\div 2$, $\div 2$

$= 3\frac{3}{50}$



Exercise B:20

1 Change these percentages to whole or mixed numbers in simplest form.

a 200%	b 500%	c 100%	d 300%	e 600%
f 150%	g 170%	h 250%	i 220%	j 350%
k 106%	l 138%	m 175%	n 144%	o 185%
p 410%	q 635%	r 365%	s 880%	t 478%

2 Write each as a fraction, whole number or mixed number in simplest form.

a 173%	b 84%	c 375%	d 450%	e 66%
f 800%	g 550%	h 425%	i 302%	j 900%
k 205%	l 700%	m 181%	n 980%	o 4%
p 623%	q 475%	r 16%	s 111%	t 1000%

B:21 | Changing Fractions and Mixed Numbers to Percentages

To change fractions to percentages, first change the denominator of the fraction to 100.

worked examples

1 $\frac{37}{100} = 37\%$

(% means 'per 100')

2 $\frac{3}{20} = \frac{3}{20} \times \frac{5}{5}$

$= \frac{15}{100}$

$= 15\%$

1 = 100%
∴ 7 = 700%

3 $4 = \frac{4}{1} \times \frac{100}{100}$

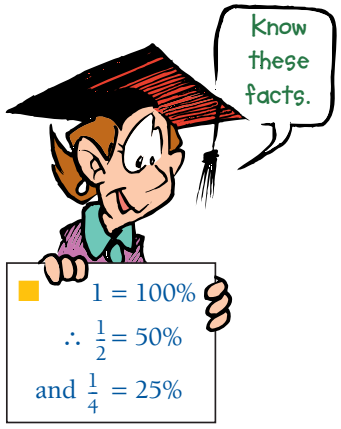
$= \frac{400}{100}$

$= 400\%$

4 $7\frac{3}{10} = 7 + \frac{3}{10} \times \frac{10}{10}$

$= \frac{700}{100} + \frac{30}{100}$

$= 730\%$



Exercise B:21

1 Write each as a percentage.

a $\frac{7}{100}$	b $\frac{50}{100}$	c $\frac{18}{100}$	d $\frac{73}{100}$	e $\frac{8}{100}$
f $\frac{90}{100}$	g $\frac{77}{100}$	h $\frac{20}{100}$	i $\frac{99}{100}$	j $\frac{40}{100}$
k $\frac{67}{100}$	l $\frac{80}{100}$	m $\frac{1}{100}$	n $\frac{19}{100}$	o $\frac{22}{100}$
p $\frac{87}{100}$	q $\frac{75}{100}$	r $\frac{30}{100}$	s $\frac{25}{100}$	t $\frac{3}{100}$

2 Write each as a percentage.

a $\frac{1}{2}$	b $\frac{3}{10}$	c $\frac{9}{10}$	d $\frac{1}{10}$	e $\frac{7}{10}$
f $\frac{2}{5}$	g $\frac{3}{20}$	h $\frac{1}{5}$	i $\frac{11}{20}$	j $\frac{4}{5}$
k $\frac{3}{50}$	l $\frac{1}{4}$	m $\frac{8}{25}$	n $\frac{3}{4}$	o $\frac{24}{25}$
p $\frac{2}{25}$	q $\frac{3}{5}$	r $\frac{17}{50}$	s $\frac{8}{10}$	t $\frac{21}{50}$

3 Change each to a percentage.

a 1	b 3	c 5	d 2	e 8
f $1\frac{1}{2}$	g $3\frac{1}{2}$	h $4\frac{1}{2}$	i $6\frac{1}{2}$	j $7\frac{1}{2}$
k $2\frac{3}{10}$	l $5\frac{7}{10}$	m $6\frac{1}{10}$	n $1\frac{3}{10}$	o $2\frac{9}{10}$
p $1\frac{13}{50}$	q $3\frac{1}{4}$	r $5\frac{7}{20}$	s $2\frac{3}{5}$	t $6\frac{3}{25}$

B:22 | Changing Percentages to Decimals

To change a percentage to a decimal, we can write it first as hundredths and then write it as a decimal.

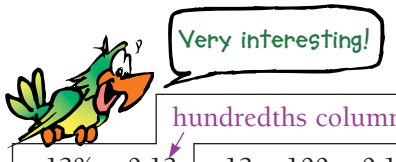
worked examples

1 $7\% = \frac{7}{100}$
$= 0{\cdot}07$

2 $93\% = \frac{93}{100}$
$= 0{\cdot}93$

3 $10\% = \frac{10}{100}$
$= 0{\cdot}10$
$= 0{\cdot}1$

4 $425\% = \frac{425}{100}$
$= 4\frac{25}{100}$
$= 4{\cdot}25$



hundredths column	
13% = 0·13	13 ÷ 100 = 0·13
150% = 1·50	150 ÷ 100 = 1·50
4% = 0·04	4 = 100 = 0·04

Exercise B:22

1 Write each percentage as a decimal.

a 6%	b 5%	c 2%	d 9%	e 1%
f 17%	g 24%	h 83%	i 65%	j 88%
k 20%	l 50%	m 90%	n 60%	o 40%
p 43%	q 3%	r 70%	s 11%	t 95%

2 Express each percentage as a decimal.

a 112%	b 97%	c 204%	d 86%	e 427%
f 114%	g 293%	h 147%	i 342%	j 886%
k 250%	l 160%	m 420%	n 390%	o 180%
p 5%	q 200%	r 193%	s 100%	t 106%

B:23 | Changing Decimals to Percentages

To change a decimal to percentage, we can write it first as a fraction, then change it to a decimal, or we can multiply the decimal by 100%.

worked examples

1 $0{\cdot}08 = \frac{8}{100}$
$= 8\%$

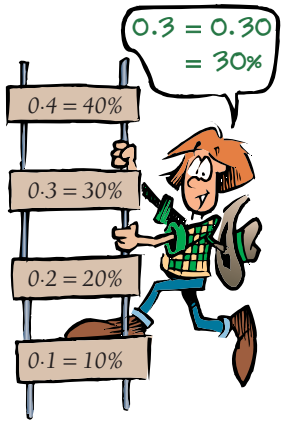
2 $5{\cdot}9 = 5{\cdot}90$
$= \frac{590}{100}$
$= 590\%$

3 $0{\cdot}93 = \frac{93}{100}$
$= 93\%$

OR

4 $0{\cdot}93 = 0{\cdot}93 \times 100\%$
$= 93\%$

■ When multiplying by 100, move the decimal point 2 places right.



Exercise B:23

1 Write each decimal as a percentage.

a 0·15	**b** 0·33	**c** 0·92	**d** 0·52	**e** 0·38
f 0·83	**g** 0·55	**h** 0·19	**i** 0·25	**j** 0·75
k 0·03	**l** 0·05	**m** 0·08	**n** 0·09	**o** 0·01
p 0·67	**q** 0·86	**r** 0·99	**s** 0·42	**t** 0·21

2 Change to percentages.

a 0·4	**b** 0·6	**c** 0·9	**d** 0·8	**e** 0·2
f 1·77	**g** 1·06	**h** 2·64	**i** 2·95	**j** 3·24
k 1·02	**l** 4·34	**m** 2·61	**n** 1·93	**o** 3·88
p 1·7	**q** 2·5	**r** 3·1	**s** 1·3	**t** 2·6

3 Express each as a percentage.

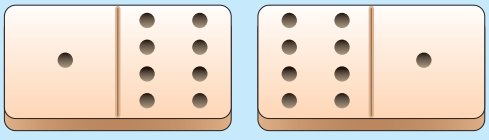
a 0·4	**b** 1·53	**c** 0·09	**d** 0·28	**e** 1·62
f 1·11	**g** 0·6	**h** 0·32	**i** 0·84	**j** 1·97
k 0·07	**l** 1·46	**m** 0·7	**n** 0·54	**o** 0·98
p 2·69	**q** 0·72	**r** 0·03	**s** 0·2	**t** 0·78

Appendix B | Working Mathematically

assignment B

1 A boy is playing with a pile of matches. He notices that when he divides them into piles of 2, 3 or 5 there is always one left over. What is the smallest number of matches that he could have?

2 A domino is made by joining two squares and placing a number of dots from 0 to 9 in each square so that each domino is different. How many dominoes are there in a complete set?





3 A girl has a box containing a total of 8 spiders and beetles. She counted the legs and found that there were 54 altogether. Find how many of each were in the box. (You will need to know that spiders have 8 legs and beetles have 6 legs.)

4 What is the largest area that could be covered by a rectangle with a perimeter of 40 cm?

5 Jason cut an apple into 20 equal parts. If he ate $\frac{1}{4}$ of the apple, how many parts remained?

6



a What amount of coal was exported in 2000?
b What amount of steel was exported in 2003?
c In which year was the smallest amount of steel exported?
d In which year was the largest amount of coal exported?
e What was the average amount of steel exported for the four-year period? (Answer to the nearest million tonnes.)

APPENDIX C

C:01 | HCF and LCM by Prime Factors (Extension)

prep quiz C:01

1 List all factors of 30.
2 List all factors of 54.
3 What is the highest common factor of 30 and 54?
4 List all multiples of 4 that are less than 40.
5 List all multiples of 3 that are less than 40.
6 What is the lowest common multiple of 4 and 3?
7 Use a factor tree to write 48 as a product of its prime factors.

Write the following products of prime numbers in index notation.

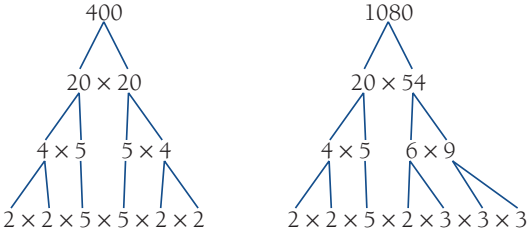
8 $2 \times 2 \times 2 \times 2 \times 2$
9 $2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$
10 $3 \times 3 \times 5 \times 5 \times 5 \times 7$

When we express numbers as products of their prime factors, it is sometimes easier to find their highest common factor (HCF) and their lowest common multiple (LCM).

worked example 1

Find the highest common factor of 400 and 1080.

Solution



From the factor trees:

Number	*Product of prime factors*
400	(2) × (2) × (2) × 2 × (5) × 5 or $2^4 \times 5^2$
1080	(2) × (2) × (2) × 3 × 3 × 3 × (5) or $2^3 \times 3^3 \times 5$

The two numbers have $2 \times 2 \times 2 \times 5$ in common.
$2 \times 2 \times 2 \times 5 = 2^3 \times 5 = 40$, so the highest common factor of 400 and 1080 is 40.

worked example 2

Find the lowest common multiple of 400 and 1080.

Solution

We write each number as a product of its primes:

400	$2 \times 2 \times 2 \times 2 \times 5 \times 5$ *or* $2^4 \times 5^2$

1080	$2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5$ *or* $2^3 \times 3^3 \times 5$

LCM = $(2 \times 2 \times 2 \times 2 \times 5 \times 5) \times (3 \times 3 \times 3)$



LCM = $2^4 \times 3^3 \times 5^2 = 10\,800$, so the lowest common multiple of 400 and 1080 is 10 800.

worked example 3

Find the highest common factor and the lowest common multiple of 12 150 and 39 375, if $12\,150 = 2 \times 3^5 \times 5^2$ and $39\,375 = 3^2 \times 5^4 \times 7$.

Solution

$12\,150 = 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5$
$39\,375 = 3 \times 3 \times 5 \times 5 \times 5 \times 5 \times 7$

$3 \times 3 \times 5 \times 5$ is contained in both,
$\therefore$ the HCF is $3^2 \times 5^2$ or 225.

$2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5 \times 5 \times 7$ contains both numbers,
$\therefore$ the LCM is $2 \times 3^5 \times 5^4 \times 7$ or 2 126 250.

The highest common factor of 12 150 and 39 375 is 225 and the lowest common multiple of these two numbers is 2 126 250.

Exercise C:01

1 Use this table to answer the questions.

Number	*Products of prime factors*	
144	$2 \times 2 \times 2 \times 2 \times 3 \times 3$	$2^4 \times 3^2$
324	$2 \times 2 \times 3 \times 3 \times 3 \times 3$	$2^2 \times 3^4$
1890	$2 \times 3 \times 3 \times 3 \times 5 \times 7$	$2 \times 3^3 \times 5 \times 7$
4900	$2 \times 2 \times 5 \times 5 \times 7 \times 7$	$2^2 \times 5^2 \times 7^2$
1960	$2 \times 2 \times 2 \times 5 \times 7 \times 7$	$2^3 \times 5 \times 7^2$
3375	$3 \times 3 \times 3 \times 5 \times 5 \times 5$	$3^3 \times 5^3$
8232	$2 \times 2 \times 2 \times 3 \times 7 \times 7 \times 7$	$2^3 \times 3 \times 7^3$
1568	$2 \times 2 \times 2 \times 2 \times 2 \times 7 \times 7$	$2^5 \times 7^2$

a Find the highest common factor of:

- **i** 144 and 324
- **ii** 324 and 1890
- **iii** 1890 and 4900
- **iv** 4900 and 1960
- **v** 1960 and 3375
- **vi** 3375 and 8232
- **vii** 8232 and 1568
- **viii** 1960 and 8232
- **ix** 1890 and 3375
- **x** 1568 and 3375

b Find the lowest common multiple of:

i 144 and 324	**ii** 324 and 1890	**iii** 1890 and 4900	**iv** 4900 and 1960
v 1960 and 3375	**vi** 3375 and 8232	**vii** 8232 and 1568	**viii** 1960 and 8232
ix 1899 and 3375	**x** 1568 and 3375		

2 Complete the table below and then answer the questions.

Number	*Products of prime factors*	
18		
36		
24		
300		
1050		
1250		
2475		
2310		

a Find the highest common factor of:

i 18 and 36
ii 36 and 24
iii 24 and 300
iv 300 and 1050
v 1250 and 1050
vi 2475 and 1250
vii 2475 and 2310
viii 1050 and 2475
ix 1250 and 2310
x 24 and 2475

b Find the lowest common multiple of:

i 18 and 36	**ii** 36 and 24	**iii** 24 and 300	**iv** 300 and 1050
v 1250 and 1050	**vi** 2475 and 1250	**vii** 2475 and 2310	**viii** 1050 and 2475
ix 1250 and 2310	**x** 24 and 2475		

3 **a** Find the highest common factor of 1155 and 2079.
b Find the lowest common multiple of 1155 and 2079.
c What is the highest common factor of 264 and 1386?
d What is the lowest common multiple of 264 and 1386?

challenge
C:01

4 **a** Two people are jogging around an oval. They start together and one takes 168 seconds to complete exactly one lap while the other takes 189 seconds. How long after they start will it take before they again meet at the starting point?

b Two cannons are fired together, then one is fired every 72 minutes while the other is fired every 108 minutes. How long after the first shot will they again be fired together?

c Judy is told that she may purchase chairs for $44 each and tables for $231 each, as long as she pays exactly the same amount for chairs as for tables. What is the least amount she needs to spend to take advantage of these prices?



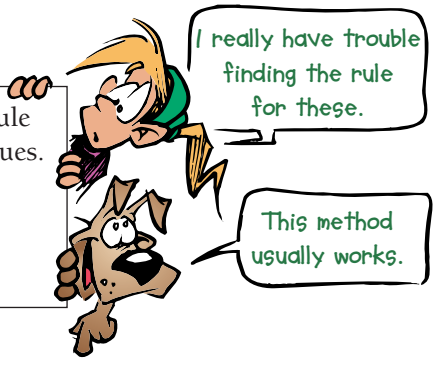
APPENDIX D

D:01 | Finite Differences (Finding the Rule)

worked example

Using the pronumerals shown, find the rule that would give the following table of values.

x	1	2	3	4	5
y	9	16	23	30	37



Solution

x	y	
1	9	
2	16	7
3	23	7
4	30	7
5	37	7

■ The difference between the two numbers

Step 1 Make sure the values of x increase by just one at a time.

Step 2 Find the difference between the values for y.
$(16 - 9 = 7, \quad 23 - 16 = 7, \quad 30 - 23 = 7, \quad 37 - 30 = 7)$

Step 3 The difference is always 7, so the rule will take the form
$y = 7 \times x + \Delta$ or $y = 7 \times x - \Delta$.

Step 4 Since $y = 9$ when $x = 1$, $\quad 9 = 7 \times 1 + \Delta$.
So $\Delta = 2$.
$\therefore$ The rule is $y = 7 \times x + 2$.

Discussion

If we are given a rule, we can test it to see if the difference between y values remains the same.

1 $y = 3 \times x + 4$

x	y	difference
1	7	
2	10	3
3	13	3
4	16	3

2 $y = 6 \times x + 8$

x	y	difference
1	14	
2	20	6
3	26	6
4	32	6

3 $y = 2 \times x - 1$

x	y	difference
1	1	
2	3	2
3	5	2
4	7	2

Conclusion: Rules like $y = 3x + 4$ have equal differences between successive y values.

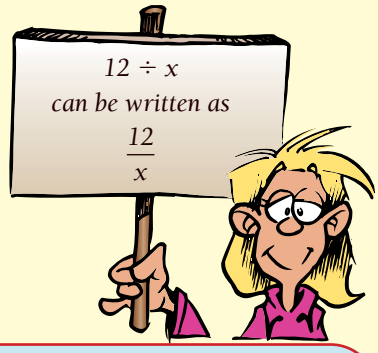
Now let's test other rules.

4 $y = x^2$

x	y	difference
1	1	
		3
2	4	
		5
3	9	
		7
4	16	

5 $y = 12 \div x$

x	y	difference
1	12	
		6
2	6	
		2
3	4	
		1
4	3	



Conclusion: Be careful! Some rules do not have equal differences between successive y values.

Exercise D:01

1 For each of the rules below, find the differences between successive y values.

a $y = 4 \times x - 4$

x	y	difference
1	0	
		□
2	4	
		□
3	8	
		□
4	12	

b $y = 9 \times x + 5$

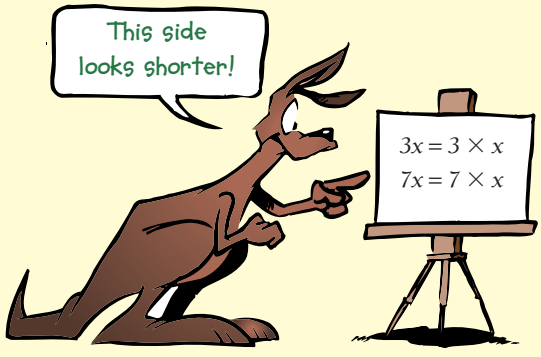
x	y	difference
0	5	
		□
1	14	
		□
2	23	
		□
3	32	

c $y = 11 \times x - 10$

x	y	difference
1	1	
		□
2	12	
		□
3	23	
		□
4	34	

d $y = 3x + 2$

x	y	difference
1	5	
		□
2	8	
		□
3	11	
		□
4	14	



e $y = 7x - 12$

x	y	difference
2	2	
		□
3	9	
		□
4	16	
		□
5	23	

f $y = 2x - 5$

x	3	4	5	6
y	1	3	5	7
difference		□	□	□

g $y = 5x + 9$

x	1	2	3	4
y	14	19	24	29
difference		□	□	□

2 Use the rule given to find the y values and the differences.

a $y = 2x + 1$

x	y	difference
1	□	
		□
2	□	
		□
3	□	
		□
4	□	

b $y = 5x - 5$

x	y	difference
1	□	
		□
2	□	
		□
3	□	
		□
4	□	

c $y = 3x - 2$

x	y	difference
1	□	
		□
2	□	
		□
3	□	
		□
4	□	

d $y = 6x + 3$

x	0	1	2	3
y				

difference □ □ □

e $y = 10x - 3$

x	1	2	3	4
y				

difference □ □ □

3 Find the differences between successive y values and use the differences to discover the rule.

a

x	1	2	3	4
y	11	15	19	23

difference □ □ □

Rule: $y = \square\, x + 7$

b

x	2	3	4	5
y	6	13	20	27

difference □ □ □

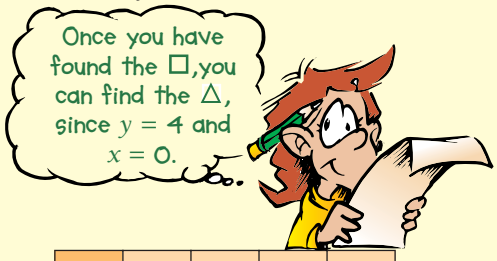
Rule: $y = \square\, x - 8$

c

x	0	1	2	3
y	4	6	8	10

difference □ □ □

Rule: $y = \square\, x + \Delta$



d

x	1	2	3	4
y	6	10	14	18

difference □ □ □

Rule: $y = \square\, x + \Delta$

e

x	0	1	2	3
y	5	6	7	8

difference □ □ □

Rule: $y = \square\, x + \Delta$

4 Use the method of finding differences to discover the rule for each table below.

a

x	1	2	3	4
y	3	6	9	12

b

x	2	3	4	5
y	7	8	9	10

c

x	1	2	3	4
y	3	5	7	9

d

a	0	1	2	3
b	7	9	11	13

e

a	3	4	5	6
b	5	10	15	20

f

a	1	2	3	4
b	0	4	8	12

g

x	1	2	3	4
y	10	17	24	31

h

x	2	3	4	5
y	1	7	13	19

i

x	0	1	2	3
y	8	18	28	38

j

s	10	11	12	13
m	0	3	6	9

k

s	5	6	7	8
m	13	15	17	19

l

s	4	5	6	7
m	26	31	36	41

5 Farmer Powell's water was getting low because of the drought, so he had water delivered in a truck. As the water was being pumped from the truck into the tank, he measured the height of water in the tank. After 1 minute the height of water was 23 cm. After 2 minutes it was 38 cm. After 3 minutes it was 53 cm. After 4 minutes it was 68 cm. Draw a table of values showing the connection between minutes passed (m) and height of water (H). Find the rule that gives the height (H) of the water in the tank as the minutes (m) pass.



6 Let's consider what happens when the values of x do not increase by one at a time.

a $y = 2x + 5$

x	1	3	5	7
y				

difference □ □ □

b $y = 3x - 4$

x	2	4	6	8
y				

difference □ □ □

Can you explain why the difference in each does not equal the number multiplying the x?

APPENDIX E

E:01 | Changing Repeating Decimals to Fractions

Here is a clever shortcut method for writing a repeating decimal as a fraction. Follow the steps carefully.

worked example

1 $0.\dot{2}\dot{6} = \frac{26 - 0}{99}$

$= \frac{26}{99}$

Step 1 (numerator)
Subtract the digits before the repeating digits from all the digits.
Step 2 (denominator)
Write down a 9 for each repeating digit and then a zero for each non-repeating digit in the decimal.
Step 3
Simplify the fraction if possible.

2 $0.32\dot{7} = \frac{327 - 32}{900}$ (one 9, two 0s)

$= \frac{295}{900}$

$= \frac{59}{180}$



3 $8.60\dot{7}\dot{7} = 8\frac{607 - 6}{990}$ (two 9s, one 0)

$= 8\frac{601}{990}$

Exercise E:01

1 Convert these repeating decimals to fractions.

a $0.\dot{7}$	**b** $0.\dot{1}$	**c** $0.\dot{3}$	**d** $0.\dot{8}$
e $0.\dot{2}$	**f** $0.\dot{6}$	**g** $0.\dot{5}$	**h** $0.\dot{9}$
i $0.\dot{1}\dot{5}$	**j** $0.\dot{8}\dot{4}$	**k** $0.\dot{1}\dot{2}$	**l** $0.\dot{8}\dot{2}$
m $0.\dot{7}\dot{3}$	**n** $0.\dot{5}\dot{8}$	**o** $0.\dot{0}\dot{7}$	**p** $0.\dot{3}\dot{0}$
q $0.1\dot{4}\dot{8}$	**r** $0.7\dot{3}\dot{2}$	**s** $0.1\dot{8}\dot{6}$	**t** $0.0\dot{4}\dot{2}$
u $0.0\dot{6}\dot{3}$	**v** $0.9\dot{4}\dot{7}$	**w** $0.2\dot{5}\dot{4}$	**x** $0.9\dot{1}\dot{0}$

2 Convert these repeating decimals to fractions.

a $0.32\dot{1}\dot{7}$	**b** $0.45\dot{0}\dot{8}$	**c** $0.05\dot{2}\dot{8}$	**d** $0.1256\dot{}$
e $1.\dot{6}$	**f** $7.\dot{0}\dot{4}$	**g** $9.\dot{8}\dot{6}$	**h** $4.\dot{9}$
i $8.3\dot{0}\dot{4}$	**j** $7.6\dot{1}\dot{5}$	**k** $3.8\dot{7}\dot{5}$	**l** $9.05\dot{2}$

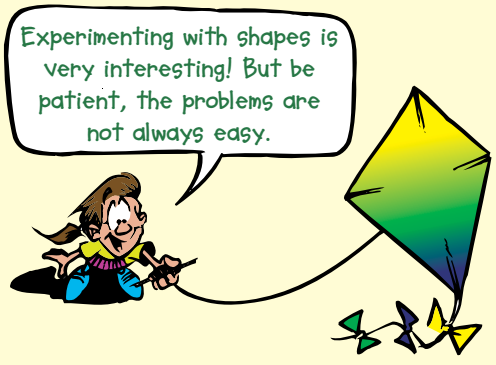
3 Why is $0.\dot{9}$ equal to 1?

APPENDIX F

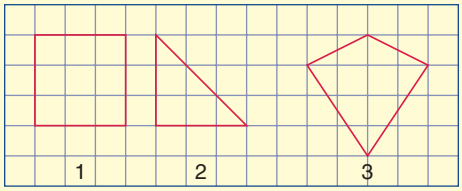
F:01 | Investigation of Plane Shapes

Exercise F:01

Equipment needed: square grid paper, scissors

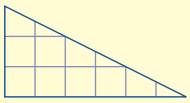


1 Copy the following figures onto a piece of square grid paper.



- **a** Name the shapes.
- **b** Cut the shapes out and, by moving them about, trace them in different orientations.
- **c** Make two of shape 2. What shapes can be made by joining the two together along an edge?
- **d** Cut shape 3 along its longest diagonal and use the two triangular pieces to make a parallelogram. Can more than one parallelogram be made?

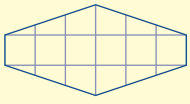
2



On the remaining grid paper from question 1, draw two triangles like the one shown. What shapes can be made if the two triangles are joined along:

- **a** the shortest edge?
- **b** the middle-sized edge?
- **c** the longest edge?

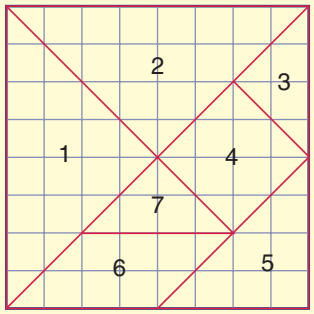
3 On a piece of square grid paper, copy the figure shown and cut it out.

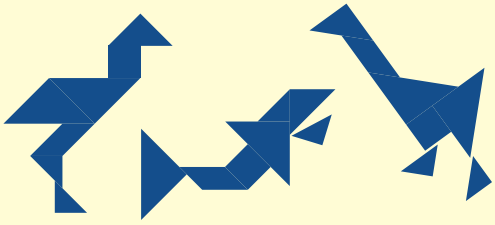


- **a** Name the figure.
- **b** Cut the figure into a rectangle and two identical triangles.
- **c** Use the two triangular pieces to make a rhombus.
- **d** Use the two triangular pieces to make a parallelogram.
- **e** Cut each of the triangular pieces in half and use the four triangular pieces to make the rectangle.

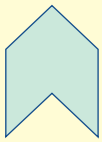
4 Mark out a 8 cm square on a piece of square grid paper and carefully copy the shapes shown on the right. Cut them out. These seven shapes form the ancient Chinese puzzle called the **tangram**.

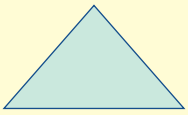
- **a** Name the two shapes that are not triangles.
- **b** Which two shapes make number 5?
- **c** Which two shapes make number 6?
- **d** Use three smaller pieces to make shape 1. Can this be done in more than one way?
- **e** Use pieces 3, 4 and 7 to make a parallelogram.
- **f** Use pieces 3, 4 and 7 to make a trapezium.
- **g** Use pieces 3, 4 and 7 to make a rectangle.
- **h** Make a square with the five smallest pieces.
- **i** Use the seven pieces of your tangram to make each of the pictures on the right.

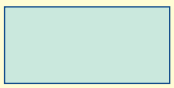




5 Draw a square of side 4 cm on a piece of square grid paper. Mark in the diagonals, then cut out the square and cut along the diagonals. Use the 4 triangular pieces obtained to make the following shapes.

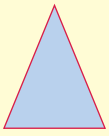
a 

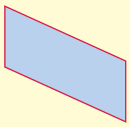
b 

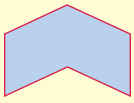
c 

d 

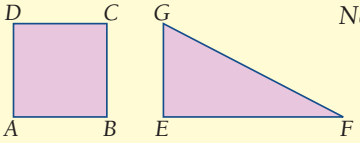
6 On a piece of square grid paper, draw a rectangle 6 cm long and 3 cm wide. Mark in the diagonals, then cut out the rectangle and cut along the diagonals. Use the 4 pieces obtained to make the following shapes.

a 

b 

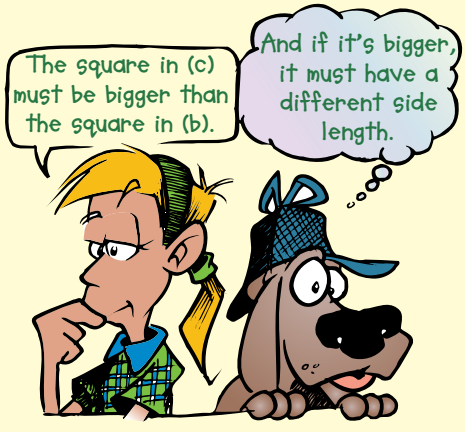
c 

7 Make one square and four triangles like these drawn below.

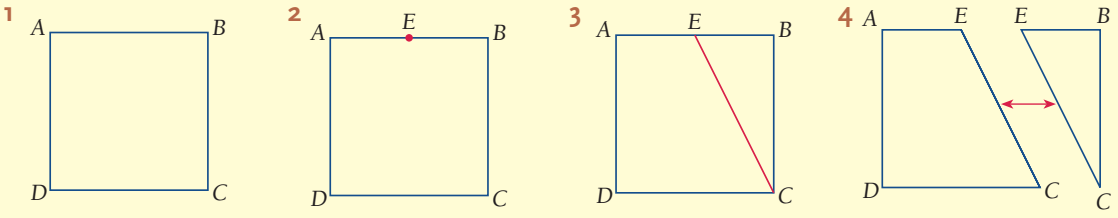


Note: $GE = BC$
$EF = 2 \times AB$

- **a** Use two of the triangles to make a rectangle.
- **b** Use four triangles to make a square.
- **c** Use the five pieces to make a square.



8 The diagram shows how two pieces are made from a square *ABCD*. *E* is the midpoint of the side *AB*. Starting with any square *ABCD*, follow the steps shown to make the pieces shown in step 4 of the diagram.



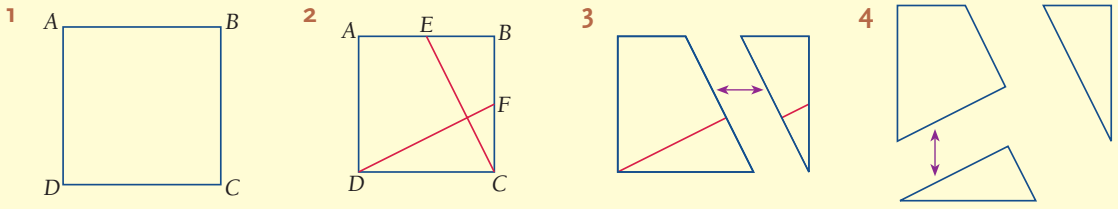
Using the two pieces make:

a a triangle **b** a trapezium **c** a parallelogram

d a quadrilateral that has all its sides different

e a pentagon that has a pair of parallel sides

9 The diagram shows how a square is dissected into three pieces. *E* and *F* are the midpoints of the sides *AB* and *BC* respectively. Follow the steps in the diagram to make the three pieces shown in step 4 of the diagram.

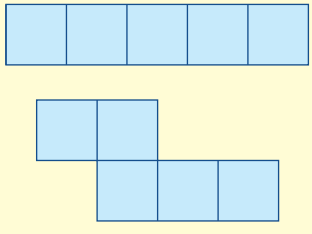


a Use the two larger pieces to make a triangle.

b Use the three pieces to make a triangle.

c Make a rectangle, parallelogram and trapezium from the three pieces.

10 A **pentomino** is formed by joining 5 identical squares together so that each square is joined to the one alongside it by an edge. Two of the 12 possible pentominoes are shown.



a Find the other ten pentominoes.

b Use three of the pieces to form a 5×3 rectangle.

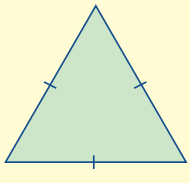
c Use four of the pieces to form a 5×4 rectangle.

11 The word 'congruent' means 'identical'. Divide the shape given into:

a 2 congruent shapes

b 3 congruent shapes

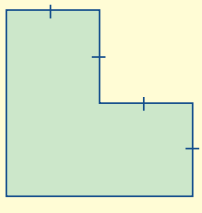
c 4 congruent shapes



challenge

F:01

12 Repeat question 11 for this shape:



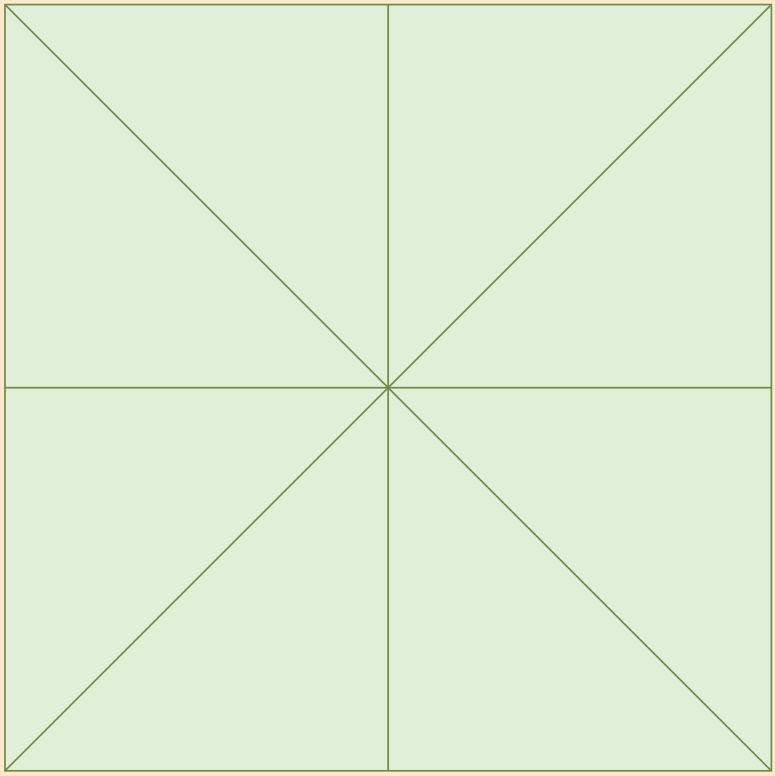
fun spot

2:04

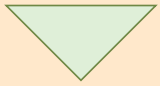
Fun Spot F:01 | Hidden shapes

Many different shapes can be hidden in one design.

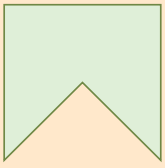
A shape may appear in any position, even flipped over.

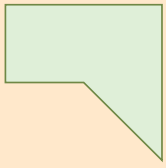


How many times does each of the following shapes appear in the design above?

a 

b 

c 

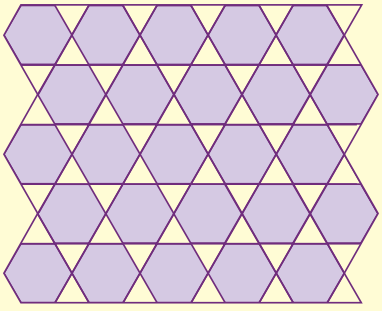
d 

F:02 | Plane Shapes and Patterns

Plane shapes are often used to make patterns. When a single shape is repeated to make a pattern, leaving no gaps, the pattern is called a **tessellation**.

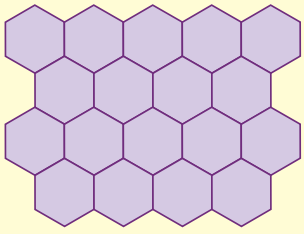
Examples

1



This pattern is not a tessellation as two different shapes are used.

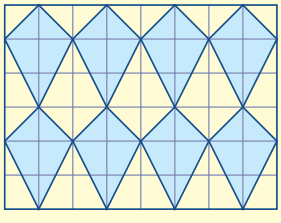
2



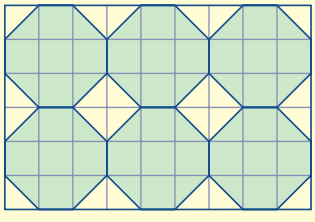
This pattern is a tessellation as one shape is used to cover the entire surface.

Exercise F:02 (Practical)

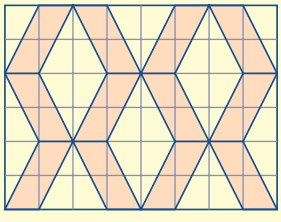
Equipment needed: square grid paper, scissors, pencils and erasers



Pattern 1



Pattern 2



Pattern 3

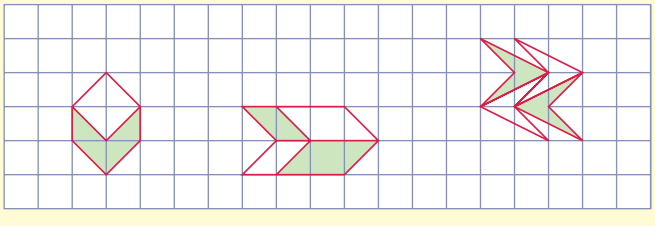
1 Which of these patterns above is a tessellation?

2 a Name the shape used in pattern 1.
b Copy this pattern (8 units by 6 units) onto a piece of grid paper and extend it until it is 16 units by 12 units.

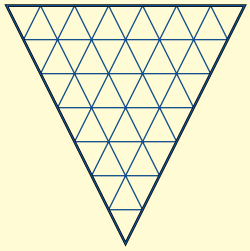
3 a Name the shapes used in pattern 2.
b Copy this pattern (9 units by 6 units) onto a piece of grid paper and extend it until it is 18 units by 12 units.

4 a Name the shapes used in pattern 3.
b Copy this pattern (8 units by 6 units) onto a piece of grid paper and extend it until it is 16 units by 12 units.

5 On a piece of square grid paper, use the shapes given below to make a tile pattern.



6



In the pattern, find the following shapes:

- **a** hexagon
- **b** trapezium
- **c** parallelogram
- **d** rhombus



7

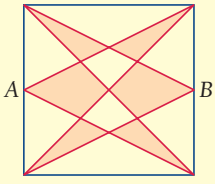


Figure 1

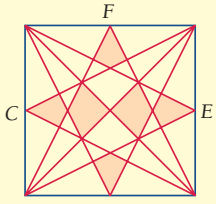
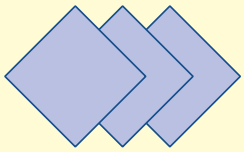


Figure 2

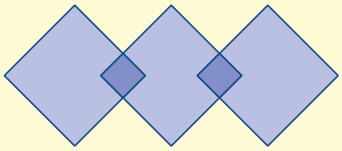
Draw two squares and copy the figures shown. The points A, B, C, D, E and F are midpoints of the sides of the squares.

- **a** In figure 1, find the following shapes:
  - **i** kite **ii** rhombus
- **b** In figure 2, find and sketch the following shapes:
  - **i** kite **ii** rhombus **iii** square
  - **iv** parallelogram **v** regular octagon

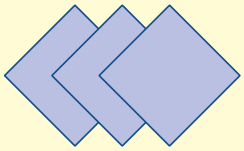
8 Sometimes patterns are formed by shapes overlapping. In the following example, some patterns involving overlapping squares are given.



A



B

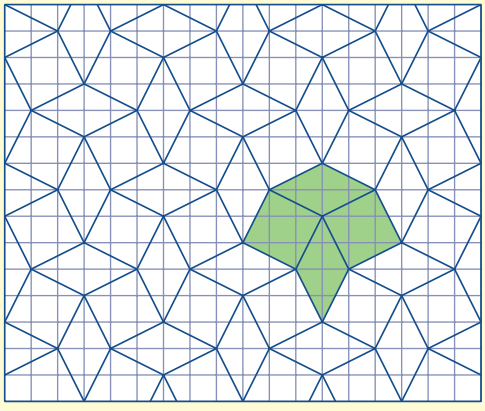


C

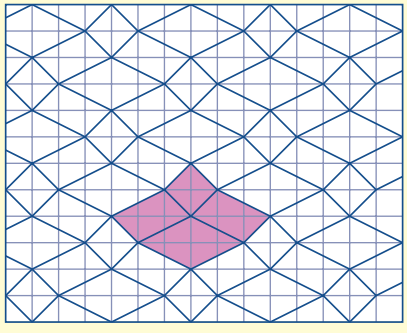
- **a** In which pattern does it appear that the squares are transparent?
- **b** In which pattern does the square to the right appear to be on top?
- **c** Repeat the three types of patterns shown above using circles.
- **d** Use square grid paper to make a diamond and repeat the three types of patterns shown above using the diamond.

9 When we look at tile patterns we can often see other shapes within the pattern. Copy the tile pattern given, and then colour it in using the shape given as the basic tile shape.

a



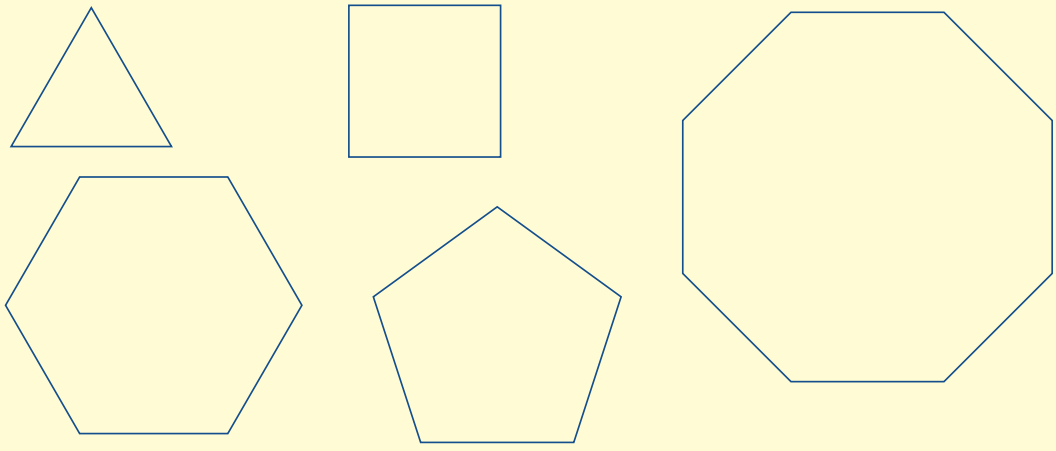
b



10 Make accurate templates of the following shapes and find:

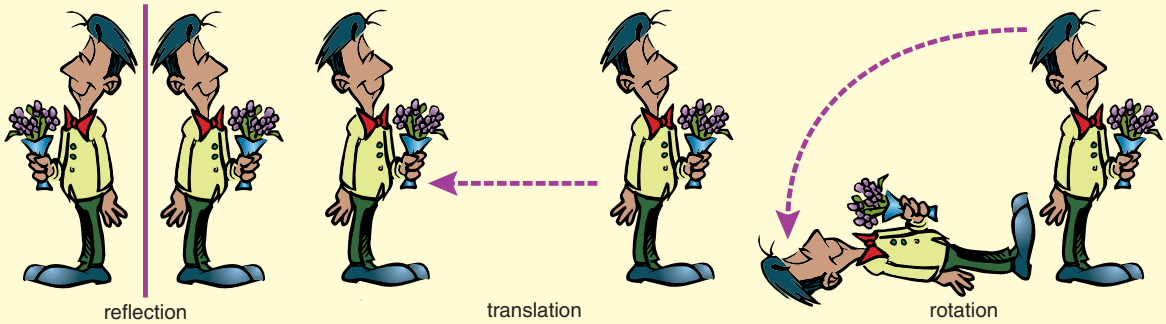
a which shapes will form a tile pattern on their own

b which of these shapes can combine to form a tile pattern



F:03 | Transformations: Reflections, Translations and Rotations

In primary school you used the **flip**, the **slide** and the **turn** to make patterns. These transformations will now be called the **reflection**, the **translation** and the **rotation**.

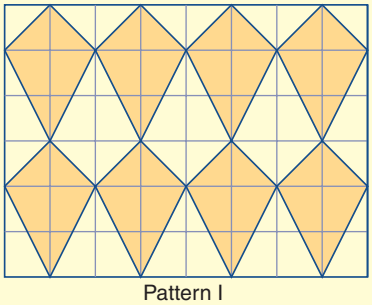


reflection (or flip)

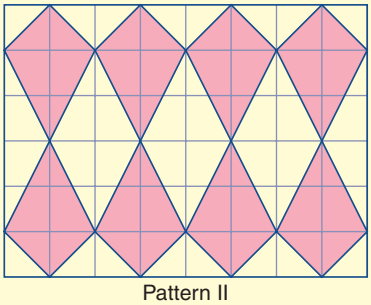
translation (or slide)

rotation (or turn)

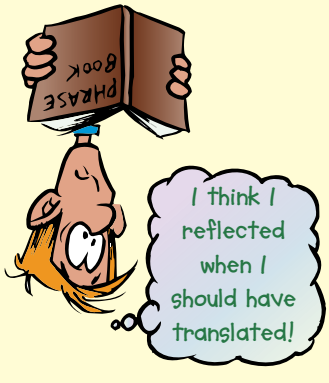
Examples

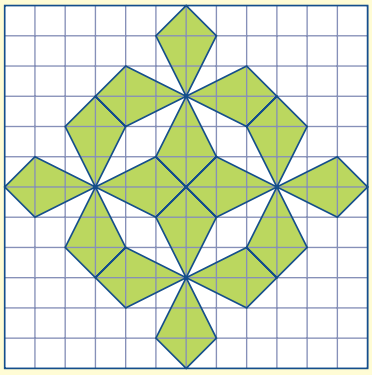


Pattern I



Pattern II





Pattern III

Pattern I has been made by translating (or sliding) the kites horizontally and vertically.

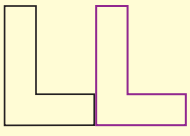
Pattern II has been made by reflecting (or flipping) the kites horizontally and vertically. It could also be made by a combination of translations and reflections.

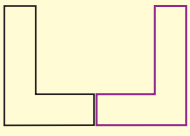
Pattern III has been made by rotating the kites about different points. It could also have been made by a combination of rotation and reflection or rotation and translation.

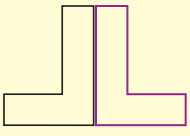
Exercise F:03 (Practical)

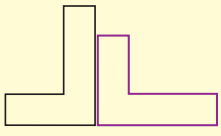
Equipment needed: square grid paper, a pencil and an eraser

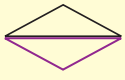
1 In each of the following, say whether the coloured figure has been produced from the other figure by a reflection (flip), a translation (slide) or a rotation (turn).

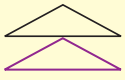
a 

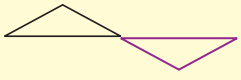
b 

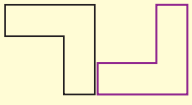
c 

d 

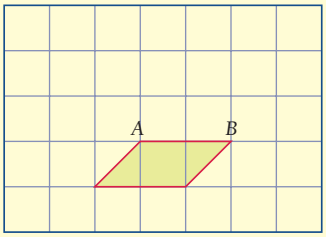
e 

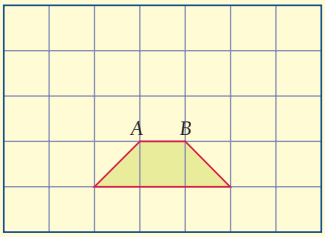
f 

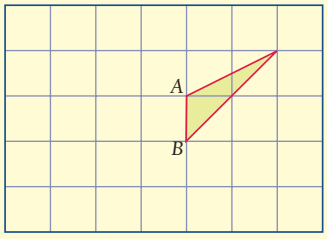
g 

h 

2 Copy the following onto square grid paper and sketch the shape when it is reflected (flipped) on side *AB*.

a 

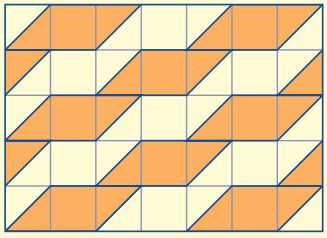
b 

c 



If you want a model, cut the shape out and sticky-tape down the edge *AB*. You can then use the sticky tape as a hinge and it's easy to flip the shape over.

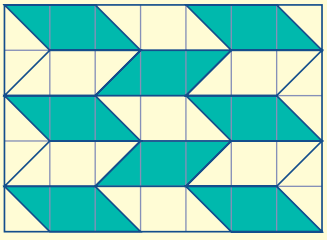
3



This pattern was formed by taking the parallelogram in question **2a** and sliding it into new positions. We used *translation* to make the pattern.

By sliding the shapes in **2b** and **2c**, make patterns in a similar way.

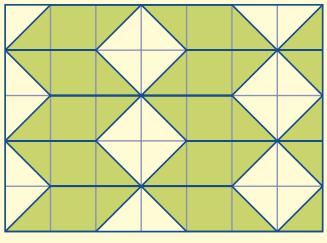
4



This pattern was made by taking the parallelogram in question **2a** and doing a combination of slides and flips (ie translations and reflections).

Repeat this procedure using the shapes in **2b** and **2c**.

5

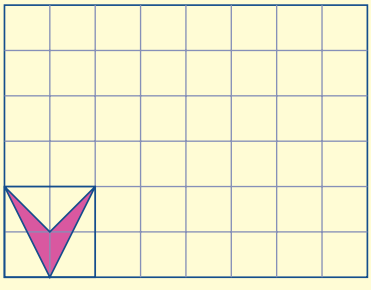


This pattern has been formed by taking a parallelogram and flipping it vertically and horizontally.

Repeat this procedure using the shapes in **2b** and **2c**.

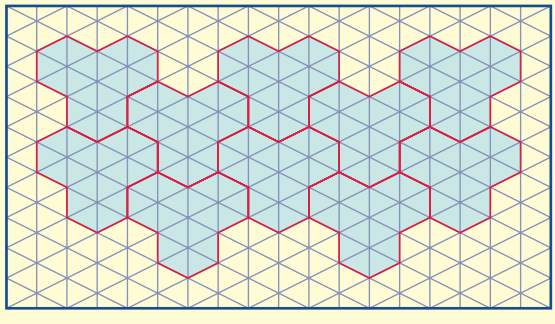
6 Complete a 12-tile pattern using the tile given and:

- **a** translation (slide)
- **b** reflection (flip)
- **c** rotation (turn)
- **d** a combination of the transformations above

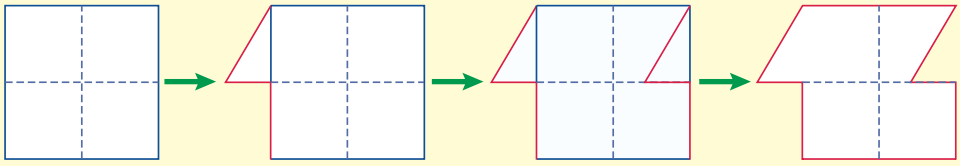


7 This pattern was drawn on triangular grid paper.

- **a** Could this pattern have been made by translating the tile shape?
- **b** Could this pattern have been made by reflecting the tile shape?
- **c** Is this pattern a tessellation?



8 The diagrams below show you how to make a tile. On a piece of squared grid paper, draw a square with sides of 2 cm. By altering one side and then sliding this side across to the opposite side, design a tile. Use this tile to form a tile pattern.



Start with a square. | Alter a side. | Slide the altered side to the side opposite it. | The finished piece.

9 **a** Repeat question **8**, but this time alter both pairs of opposite sides.
b Repeat question **8**, but start with a parallelogram instead of a square.

10 Reflections are not only found in patterns. On the front of an ambulance the word is often painted like this:

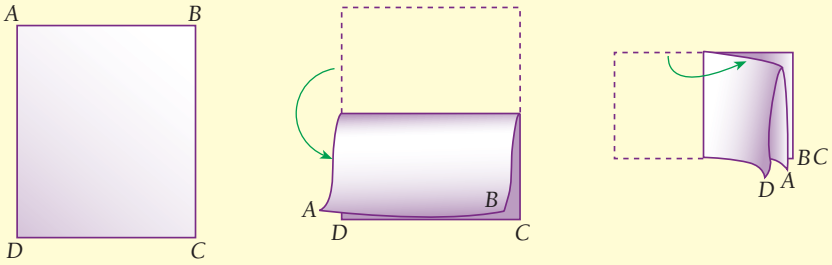
challenge F:03

ƎɔNΛꓶUꓶMA



Why is this so? Can you write the word POLICE in the same way?

11 An eight-page newsletter is produced on a single piece of paper. It is printed on both sides and folded as shown in the diagram. Find how the pages 1 to 8 should be printed on the sheet *ABCD*.

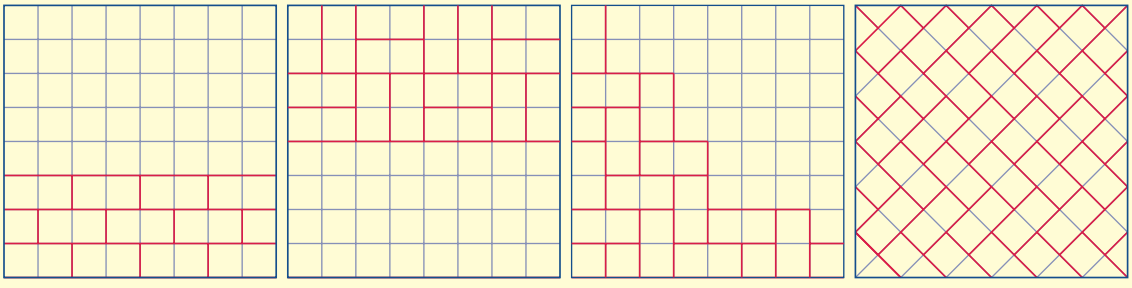


F:04 | Using Plane Shapes

Plane shapes are used in many applications in the real world. The following exercise is intended to show you some of these applications. It is hoped that some of these applications could be expanded in projects. The list is by no means exhaustive.

Exercise F:04 (Practical research)

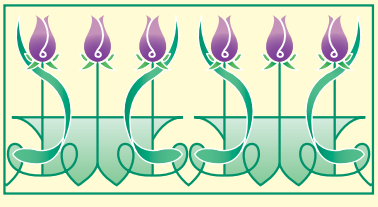
1 A paving brick is twice as long as it is wide. Several different patterns are shown below. Copy the pattern onto square grid paper and complete the *longitudinal*, *basket* and *header and stretcher* patterns.



Longitudinal | Basket | Header and stretcher | Herringbone

2 Many tiles have symmetrical properties. List the types of symmetry found in each of these tiles.

a



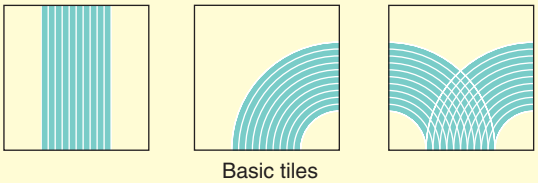
b



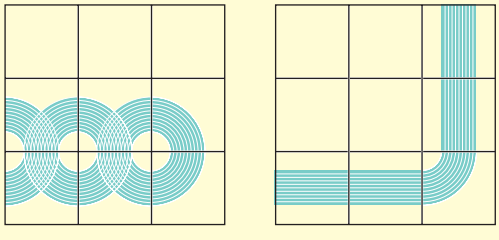
c



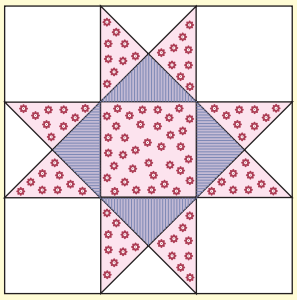
3 Notice how the patterns illustrated below have been made from the basic tiles. Sketch other patterns that could be made from these basic tiles.



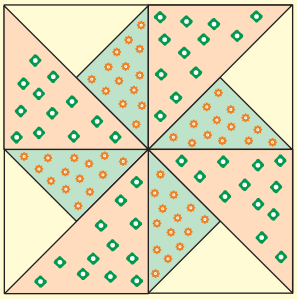
Basic tiles



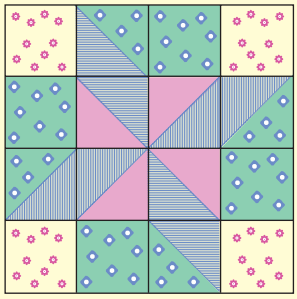
4 A knowledge of shapes is useful in learning to make patchwork quilts. Some common patches are given below.



Ohio Star



Windmill



Clay's Choice

Copy each of these onto a square grid to make your own patchwork design.

a What shapes are present in 'Ohio Star'?
b What shapes are present in the 'Windmill'?
c What shapes are present in 'Clay's Choice'?
d Which patterns have point symmetry?
e Which patch has line symmetry?

5 The symmetry of shapes is often of practical use. Use your knowledge of symmetry to explain why such things as saucepan lids and manhole covers are circular rather than some other shape.

6 Many car parts come in left- and right-hand forms (or near-side and off-side). What does this mean? How are the shapes of the parts related?

7 Many wallpapers and curtain materials are patterned. Find out the meaning of the term 'pattern matching' and why it is important in calculating the amount of wallpaper or curtain material that is needed to do a job.

8 A knowledge of geometric shapes is used by graphic designers to design insignias and logos. What shapes can you see in the following logos?

a



Monier

b



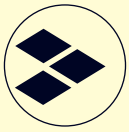
State Rail

c



Mitsubishi

d



Readymix

e



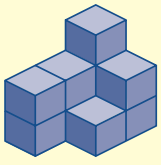
Renault

F:05 | Building Solids Using Blocks

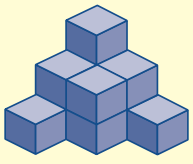
Exercise F:05

1 Construct each of the following solids from cubes.

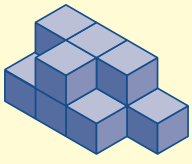
a



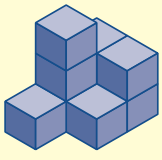
b



c

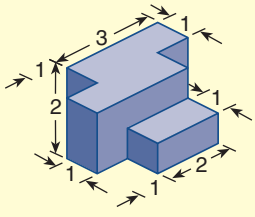


d

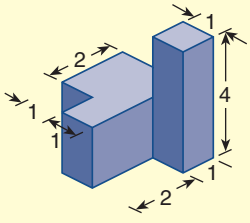


2 Build the solids shown. (The numbers indicate how many cubes are needed for that length.)

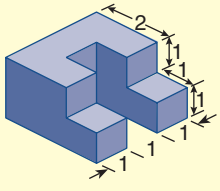
a



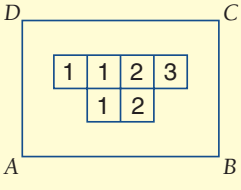
b

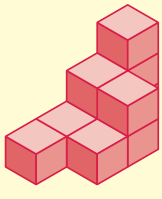


c



3 The diagram below is a way of showing the solid on the right of it. The numbers indicate how high each stack of cubes should be. The picture of the solid shown is drawn looking from corner *A*.

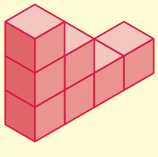




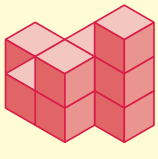


From which corner would you get the following views?

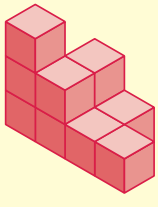
a



b

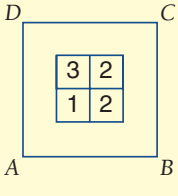


c

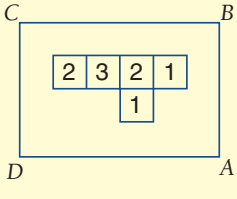


4 Build each of the following solids from cubes, and draw on isometric grid paper the view from corner *A* and from corner *C*.

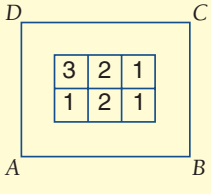
a



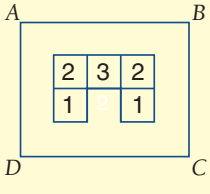
b



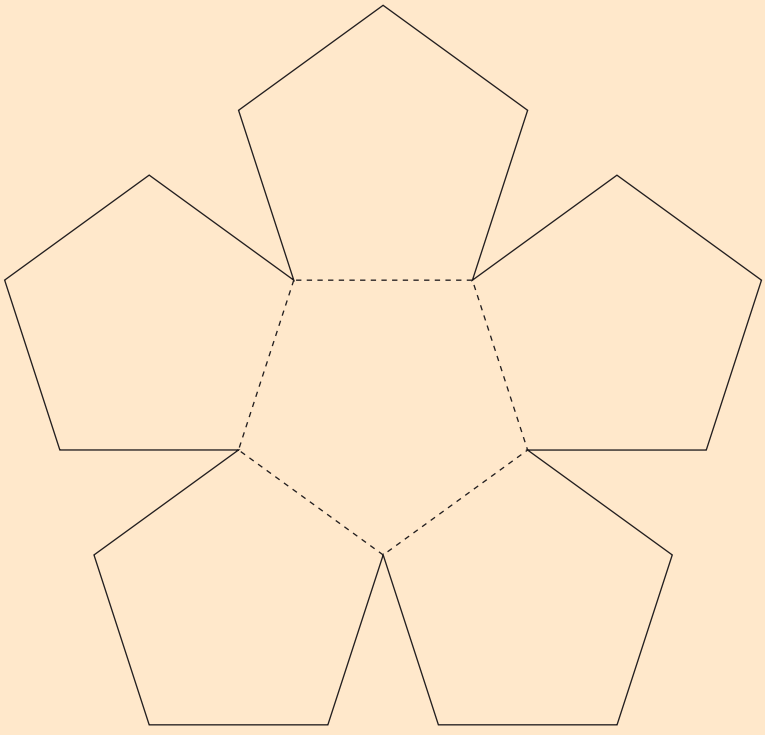
c



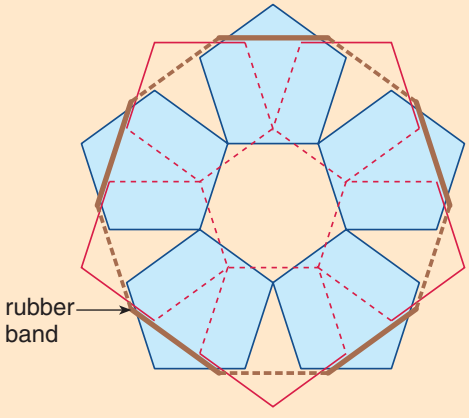
d



Fun Spot F:05 | Making a pop-up dodecahedron



Trace two of these nets involving pentagons on fairly stiff cardboard. Then score (crease) along the dotted lines and place one net on top of the other as shown in the diagram. Carefully thread a rubber band around the corners as shown, holding the two nets of pentagons flat. When the rubber band is in place, let go and the pentagons should pop into place to form a dodecahedron.

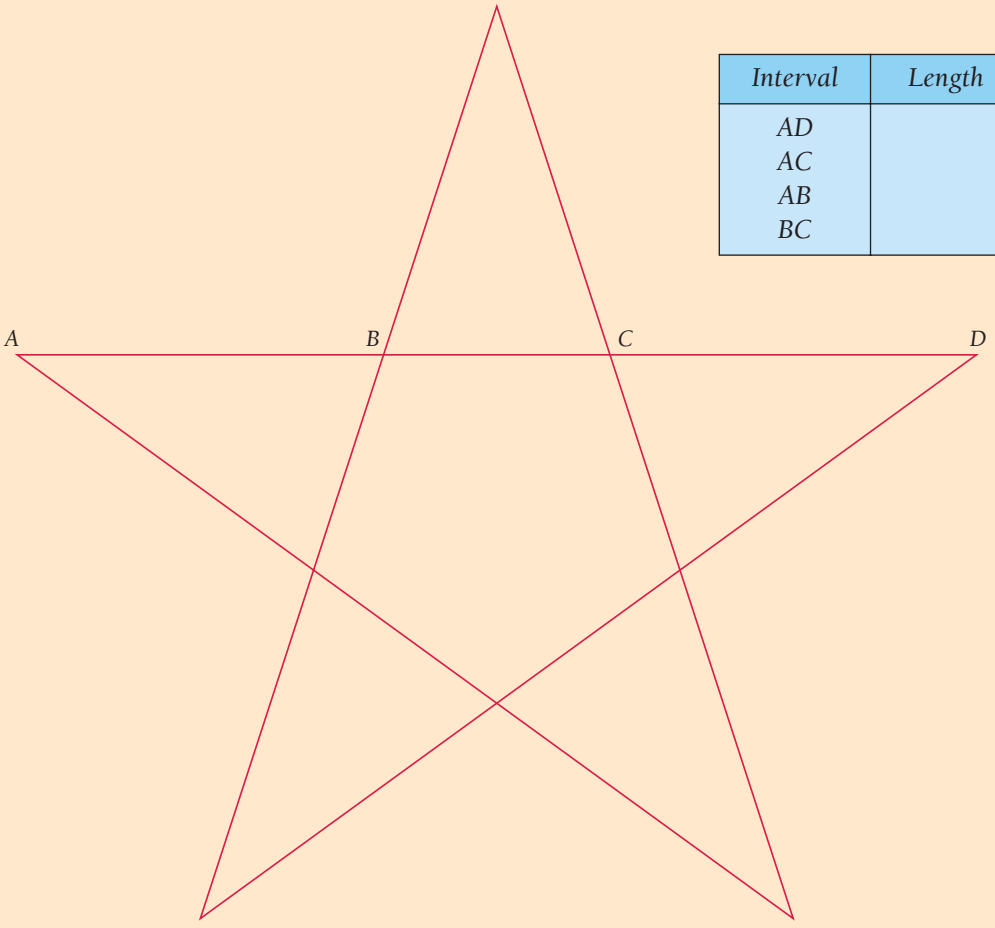


APPENDIX G

Investigation G:01 | Measurement extension

investigation G:01

The different lengths in this star are very interesting. First complete the table below, measuring each section or interval to the nearest millimetre.

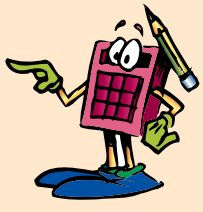


Interval	*Length*
AD	
AC	
AB	
BC	

Now complete these divisions and write your answers correct to 1 decimal place.

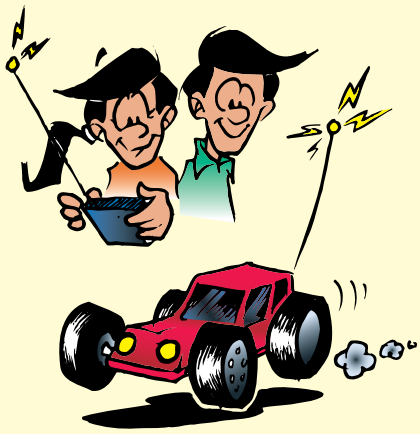
$AD \div AC =$ $AC \div AB =$ $AB \div BC =$

What do you notice about your answers?



G:02 | Distance, Speed and Time

Rajiv and Taya used their Christmas present, a model car that had four speed settings, to carry out an experiment. They marked out 20 metres of straight track, set the car at a given speed and measured how long it took the car to travel the 20 metres. The results are listed in the table below.



Distance	20 m	20 m	20 m	20 m
Speed	1 m/s	2 m/s	4 m/s	5 m/s
Time	20 s	10 s	5 s	4 s

From their results they discovered some important formulas.

Distance = Speed × Time Speed = Distance ÷ Time Time = Distance ÷ Speed
We could write these formulas using letters instead of words:
$D = S \times T$ $S = D \div T$ $T = D \div S$

worked examples

1 Rajiv ran at a speed of 5 metres per second for 20 seconds. How far did he run?

2 Taya walked at a constant speed for 50 seconds. During this time she travelled 150 metres. What was her speed?

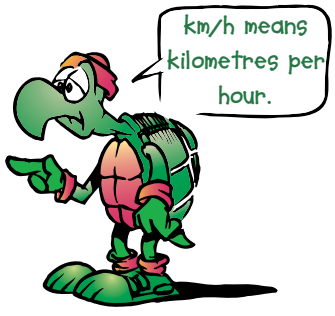
3 A train travelling with a speed of 30 kilometres per hour travelled a distance of 120 kilometres. How long did it take?

Solutions

1 Speed = 5 m/s, Time = 20 s.
Find the distance.
Distance = Speed × Time
$D = S \times T$
$\therefore D = 5 \times 20$ m
$= 100$ m
$\therefore$ Rajiv ran 100 metres.

2 Time = 50 s, Distance = 150 m.
Find the speed.
Speed = Distance ÷ Time
$S = D \div T$
$\therefore S = 150 \div 50$ m/s
$= 3$ m/s
$\therefore$ Taya's speed was 3 m/s.

3 Speed = 30 km/h, Distance = 120 km.
Find the time.
Time = Distance ÷ Speed
$T = D \div S$
$= 120 \div 30$ h
$= 4$ h
$\therefore$ The train took 4 hours.



Exercise G:02

1 Use the formula $D = S \times T$ to find the distance travelled if:

a speed = 10 m/s, time = 12 s

b speed = 8 km/h, time = 2 h

c speed = 3 km/h, time = 9 h

d time = 4 h, speed = 60 km/h

e time = 10 s, speed = 50 m/s

f time = 8 min, speed = 2 km/min

2 Use the formula $S = D \div T$ to find the speed if:

a distance = 100 m, time = 10 s

b distance = 15 km, time = 5 h

c distance = 56 m, time = 2 min

d time = 3 s, distance = 24 m

e time = 4 h, distance = 48 km

f time = 40 min, distance = 80 km

3 Use the formula $T = D \div S$ to find the time taken if:

a distance = 80 m, speed = 40 m/s

b distance = 60 km, speed = 10 km/h

c distance = 72 m, speed = 4 m/s

d speed = 100 km/h, distance = 800 km

e speed = 5 m/s, distance = 105 m

f speed = 60 km/h, distance = 420 km

4 If Zac took 12 seconds to run 60 metres, at what speed was he travelling?

5 Su-Lin ran at 6 metres per second for 8 seconds. How far did she run?

6 How long would it take for a car travelling at 20 km/h to go 100 km?

7 Mark's beetle runs at a speed of 10 cm/s.

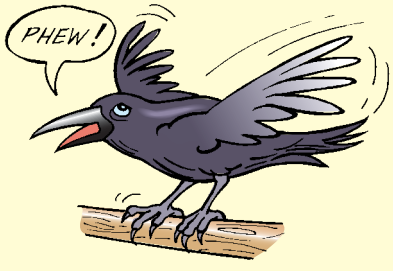
a How long would it take the beetle to run 60 cm?

b How far would it run in 8 seconds?

8 A crow flew from one bridge to another. The bridges are 200 metres apart.

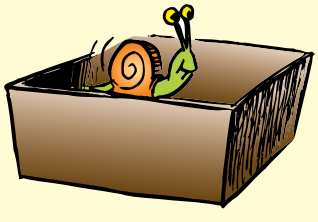
a Find the speed in metres per second if it took the crow 10 seconds.

b Find the time taken by the crow if its speed was 25 m/s.



9 When returning to Nelson's Bay from her holiday, Mary's average speed was 71·86 km/h. If the trip took three and a half hours, how far had she travelled?

10 How long would it take a snail travelling with an average speed of 5 cm/min to crawl around the top edge of an open wooden box that has length 30 cm, breadth 20 cm and height 8 cm.



APPENDIX H

H:01 | Operations with Indices (Extension)

prep quiz H:01

Rewrite in index notation:

1 $a \times a \times a$ **2** $m \times m \times m \times m \times m$ **3** $p \times p$ **4** $t \times t \times t \times t$

5 $n \times n \times n \times n \times n \times n \times n \times n$

Rewrite in expanded form: **6** x^4 **7** y^3 **8** a^2b^3

Simplify: **9** $2 \times x \times x \times 4 \times x$ **10** $5 \times y \times 3 \times y \times 2 \times y$

Multiplication Using Indices

1 $a^3 \times a^2 = (a \times a \times a) \times (a \times a)$
$= a^5$

2 $6y^4 \times 3y^3 = 6 \times y \times y \times y \times y \times 3 \times y \times y \times y$
$= 18y^7$

You should note the rule stated below:

When multiplying terms together, add the indices of the like pronumerals.
$a^m \times a^n = a^{m+n}$

So $3a^4b^2 \times 5a^3b$ would simplify to $15a^{4+3}b^{2+1}$, ie $15a^7b^3$.

Remember: $b = b^1$

Division Using Indices

3 $a^5 \div a^2 = \dfrac{a \times a \times a \times {}^1\!\cancel{a} \times {}^1\!\cancel{a}}{{}_1\!\cancel{a} \times {}_1\!\cancel{a}}$
$= a^3$

4 $10m^3n^5 \div 2mn^2 = \dfrac{{}^5\!\cancel{10} \times {}^1\!\cancel{m} \times m \times m \times n \times n \times n \times {}^1\!\cancel{n} \times {}^1\!\cancel{n}}{{}_1\!\cancel{2} \times {}_1\!\cancel{m} \times {}_1\!\cancel{n} \times {}_1\!\cancel{n}}$
$= 5m^2n^3$

You should see the following rule for divisions:

When dividing terms, subtract the indices of the like pronumerals.
$a^m \div a^n = a^{m-n}$

So $20m^7n^5 \div 10m^4n$ would simplify to $2m^{7-4}n^{5-1}$, ie $2m^3n^4$.

Powers of Indices

5 $(x^3)^2 = x^3 \times x^3$
$= x^{3+3}$
$= x^6$

6 $(2a^5)^3 = 2a^5 \times 2a^5 \times 2a^5$
$= 8a^{5+5+5}$
$= 8a^{15}$

For expressions like $(a^5)^4$, multiply the indices. $(a^m)^n = a^{m \times n}$

So $(5m^3n^4)^2$ would simplify to
$5^2m^{3\times2}n^{4\times2}$,
ie $25m^6n^8$

Index Laws
$a^m \times a^n = a^{m+n}$ (add)
$a^m \div a^n = a^{m-n}$ (subtract)
$(a^m)^n = a^{mn}$ (multiply)



Exercise H:01

1 Simplify these products, writing your answers in index form.

a $x^5 \times x^2$	**b** $a^3 \times a^2$	**c** $m^4 \times m^5$	**d** $p^2 \times p^7$
e $n^7 \times n^3$	**f** $x^7 \times x$	**g** $h^3 \times h^4$	**h** $m^6 \times m^6$
i $a \times a^4$	**j** $p \times p$	**k** $s^5 \times s^6$	**l** $t^8 \times t^{10}$
m $w^{11} \times w^9$	**n** $q^6 \times q$	**o** $x^{10} \times x^{10}$	**p** $y^7 \times y^{17}$
q $a^2 \times a^3 \times a^4$	**r** $x^3 \times x^2 \times x^3$	**s** $m^7 \times m^5 \times m^3$	**t** $n \times n^3 \times n^5$
u $y^7 \times y^7 \times y^7$	**v** $a \times a \times a$	**w** $h^4 \times h^6 \times h^2$	**x** $t^3 \times t^6 \times t^9$

2 Simplify these divisions.

a $x^6 \div x^4$	**b** $a^5 \div a^4$	**c** $q^6 \div q^5$	**d** $m^5 \div m$
e $t^4 \div t^2$	**f** $h^7 \div h^5$	**g** $n^2 \div n$	**h** $a^7 \div a^6$
i $m^{10} \div m^9$	**j** $l^{12} \div l^9$	**k** $y^{20} \div y^{19}$	**l** $k^{15} \div k$
m $\frac{x^6}{x^4}$	**n** $\frac{y^7}{y^3}$	**o** $\frac{a^{10}}{a}$	**p** $\frac{m^{10}}{m^9}$

3 Simplify:

a $(x^3)^2$	**b** $(a^3)^4$	**c** $(m^2)^3$	**d** $(n^6)^2$
e $(y^2)^5$	**f** $(m^7)^3$	**g** $(n^2)^6$	**h** $(x^5)^5$
i $(2x^3)^2$	**j** $(3a^4)^2$	**k** $(2m^3)^3$	**l** $(5m^7)^2$
m $(2y^3)^4$	**n** $(10n^3)^2$	**o** $(5t^3)^3$	**p** $(10y^4)^3$

4 Simplify each expression.

a $2x^2 \times x^5$	**b** $3y^4 \times y^2$	**c** $a^7 \times 5a^2$	**d** $m^6 \times 4m^2$
e $3n^5 \times 5n^2$	**f** $4a^2 \times 2a^4$	**g** $7k^3 \times 3k^4$	**h** $5t^4 \times 4t^4$
i $x^2y^2 \times x^4y^3$	**j** $a^4b^3 \times ab^2$	**k** $m^3n^2 \times m^7n^3$	**l** $p^7q^3 \times pq$
m $2a^2b \times 4ab^2$	**n** $5x^3y^4 \times 4x^4y^6$	**o** $10k^7l^2 \times 7k^2l^4$	**p** $6m^2n^4 \times 4m^5n^5$

5 Simplify:

a $12x^5 \div 3$	**b** $15a^3 \div 3$	**c** $14m^2 \div 2$	**d** $60m^6 \div 10$
e $10a^4 \div a^2$	**f** $15x^2 \div x$	**g** $20y^7 \div y^5$	**h** $7m^4 \div m^3$
i $20m^7 \div 5m^3$	**j** $12a^7 \div 6a^5$	**k** $27y^4 \div 9y^3$	**l** $40y^7 \div 10y^5$
m $12a^4b^3 \div a^2b^2$	**n** $18x^4y^6 \div 6y^4$	**o** $20m^3n^6 \div mn$	**p** $16p^4q^7 \div 8p^4q^3$

6 Simplify:

a $(x^2y^4)^5$	**b** $(m^7n)^6$	**c** $(2a^4b^6)^2$	**d** $(3p^4q^2)^3$

7 Simplify, using the index laws.

a $\frac{x^4 \times x^7}{x^6}$	**b** $\frac{a^6 \times a^7}{a^4 \times a^8}$	**c** $\frac{2a^4 \times 6a^3}{4a^5}$	**d** $\left(\frac{5m^2 \times 4m^3}{2m \times 2m^2}\right)^2$

challenge

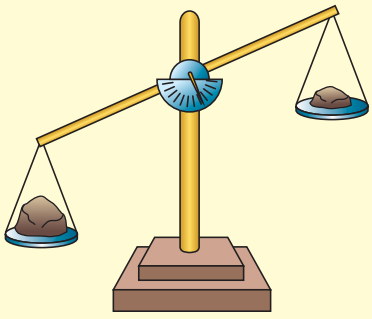
H:01

APPENDIX I

I:01 | Mass

Mass is a measure of the amount of matter that makes up an object. The **weight** of an object is related to its mass, and the pull of gravity on that object. Because this pull is virtually constant on the Earth, we can measure the mass of an object by measuring its weight.

We can compare the masses of two objects by placing them on a set of scales.



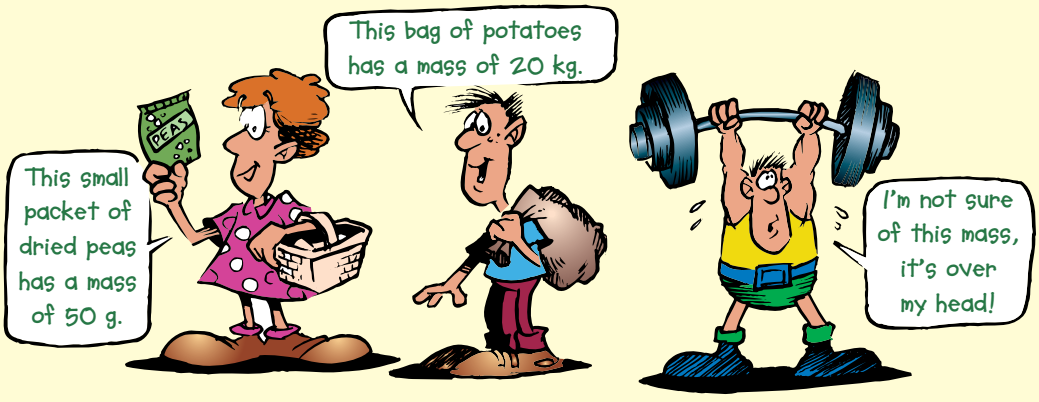


Interesting Note!
Because the pull of gravity on the moon is less than on the Earth, an object's weight on the moon would be less. However, its mass (the amount of matter in it) would not change.

To get a measure of an object's mass, we need some standard units.
The metric units for mass are:

1000 milligrams (mg) = 1 gram (g)
1000 grams (g) = 1 kilogram (kg)
1000 kilograms (kg) = 1 tonne (t)





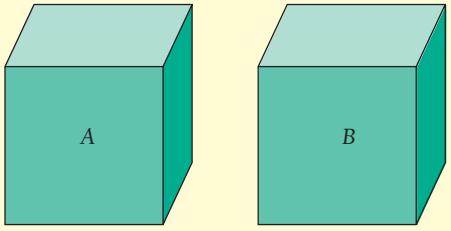
Exercise 1:01

1 Convert each measurement to the units given.

a	2000 g	= . . . . . kg	**b**	5000 g	= . . . . . kg
c	12 000 g	= . . . . . kg	**d**	15 000 g	= . . . . . kg
e	7400 g	= . . . . . kg	**f**	250 g	= . . . . . kg
g	3000 kg	= . . . . . t	**h**	10 000 kg	= . . . . . t
i	5500 kg	= . . . . . t	**j**	7 kg	= . . . . . g
k	4·5 kg	= . . . . . g	**l**	2·7 kg	= . . . . . g
m	2 t	= . . . . . kg	**n**	8·1 t	= . . . . . kg
o	1·45 t	= . . . . . kg	**p**	4000 mg	= . . . . . g
q	3 g	= . . . . . mg	**r**	2·4 g	= . . . . . mg
s	1 t	= . . . . . g	**t**	5 000 000 g	= . . . . . t

Investigation
Investigate these non-metric units of mass.
- pound
- stone
- hundredweight
- ton

2



Cubes *A* and *B* have the same volume, but *A* is made of wood and *B* is made of steel.

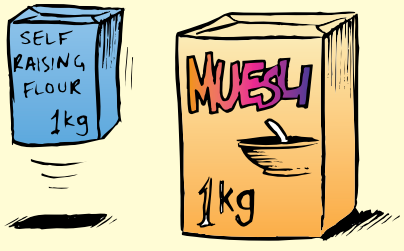
a Which cube would have the greater mass? (Which is heavier?)

b True or false? 'Objects with the same volume must have the same mass.'

3 A packet of flour and a packet of muesli were both marked as having a mass of 1 kg.

a Which do you think had the greater volume? (Which packet was bigger?)

b True or false? 'Objects with the same mass must have the same volume.'



4 Arrange these items in order of their mass, lightest to heaviest:
car, matchbox, hammer, bicycle, cup of water, bag of cement

5 Five containers of food were labelled with the following masses:
tea 250 g; curry powder 100 g; peaches 800 g; condensed milk 400 g; cocoa 375 g

a What is the total mass of these items?

b What is the average mass of these 5 items?

6 A carton contains 24 cans of spaghetti, each having a mass of 500 g. What is the mass of the carton and its contents in kilograms?

7 A filled carton has a mass of 10·8 kg. If it contains 12 cans of peaches, what is the mass of each can?

8 A worker lifts 3·5 kg of gravel on his shovel and places it in a wheelbarrow.
- **a** If the wheelbarrow will hold 70 kg of gravel, how many shovelfuls is this?
- **b** If the worker shifts 40 barrow loads of gravel, how many tonnes is this?

9 The mass given on the outside of a can of food is the mass of the contents only. A can of fruit had 800 g on the label, but when it was weighed its mass was found to be 910 g. What must have been the mass of the empty can?

- **The combined mass of a container and its contents is its gross mass.**
- **The mass of the contents only is the net mass.**

10 What would be the mass of a container if:
- **a** gross mass = 125 g, net mass = 110 g?
- **b** gross mass = 1·12 kg, net mass = 1 kg?
- **c** gross mass = 480 g, net mass = 400 g?
- **d** gross mass = 1·1 kg, net mass = 950 g?

11 What would be the mass of the contents (net mass) if:
- **a** gross mass = 500 g, container's mass = 100 g?
- **b** gross mass = 150 g, container's mass = 15 g?
- **c** gross mass = 1 kg, container's mass = 150 g?
- **d** gross mass = 510 g, container's mass = 70 g?

12 The mass of a loaded truck is 23·8 t. If the mass of the truck is 9·6 t, what must be the mass of the load?

13

Airmail		
Mass	*Asia*	*Europe*
up to 20 g	$0.95	$1.20
over 20 g up to 50 g	$1.30	$1.70
over 50 g up to 125 g	$2.30	$3.00
over 125 g up to 250 g	$4.00	$5.30
over 250 g up to 500 g	$7.50	$10.00

This table shows the cost of mailing letters overseas by airmail. What is the cost of sending the following?
- **a** 25 g letter to Thailand
- **b** 150 g letter to Ireland
- **c** 50 g letter to Vietnam
- **d** 15 g letter to England
- **e** 420 g letter to Germany
- **f** 130 g letter to Taiwan, 200 g letter to France and a 90 g letter to China

14 The cost of sending a parcel to Watsonville by rail is 82 cents per kg plus an initial charge of $2.80. How much will it cost to send a parcel with a mass of 2·5 kg?

15 On a chocolate milk powder label, it is stated that there are 15 mg of vitamin C in 10 g of the powder. What mass of vitamin C would be present in:
- **a** 50 g of the powder?
- **b** a 350 g tin of this powder?

Investigation I:01 | Mass

1. Use a set of kitchen scales to measure the masses of various objects, such as a book, ball, apple or a shoe.
2. Estimate the masses of a set of objects and then measure them to test your accuracy.
3. Find your own mass in kg using a set of bathroom scales.
4. Find the mass of a sheet of paper by weighing 50 or 100 sheets and then calculating the mass of a single sheet by division.
5. Measure out 1 litre of water into a container, and then find the mass. Then weigh the container by itself and determine the mass of 1 litre of water. What would be the mass of water that a 10-litre bucket would hold?



Answers

Appendix A

Exercise A:01

1 a

2 +	3	1	0	4	7	2	8	5	9	6
	5	3	2	6	9	4	10	7	11	8

b

10 +	0	2	1	9	3	5	7	8	6	4
	10	12	11	19	13	15	17	18	16	14

c

3 +	3	4	0	2	8	1	7	5	9	6
	6	7	3	5	11	4	10	8	12	9

d

4 +	1	4	2	6	0	3	7	8	5	9
	5	8	6	10	4	7	11	12	9	13

e

5 +	5	2	0	6	4	1	5	3	9	8
	10	7	5	11	9	6	10	8	14	13

f

6 +	2	4	0	1	6	3	7	9	5	8
	8	10	6	7	12	9	13	15	11	14

g

7 +	0	1	4	3	7	2	8	6	5	9
	7	8	11	10	14	9	15	13	12	16

h

8 +	2	3	8	1	0	7	5	9	4	6
	10	11	16	9	8	15	13	17	12	14

i

9 +	4	0	2	6	5	8	3	1	9	7
	13	9	11	15	14	17	12	10	18	16

2 a

+	1	9	0	5	8	2	6	4	7	3
1	2	10	1	6	9	3	7	5	8	4
9	10	18	9	14	17	11	15	13	16	12
0	1	9	0	5	8	2	6	4	7	3
5	6	14	5	10	13	7	11	9	12	8
8	9	17	8	13	16	10	14	12	15	11

b

+	1	9	0	5	8	2	6	4	7	3
2	3	11	2	7	10	4	8	6	9	5
6	7	15	6	11	14	8	12	10	13	9
4	5	13	4	9	12	6	10	8	11	7
7	8	16	7	12	15	9	13	11	14	10
3	4	12	3	8	11	5	9	7	10	6

3 a

18 –	10	8	9
	8	10	9

b

17 –	9	10	8	7
	8	7	9	10

c

16 –	7	9	6	10	8
	9	7	10	6	8

d

15 –	8	5	10	6	9	7
	7	10	5	9	6	8

e

14 –	10	6	8	4	5	9	7
	4	8	6	10	9	5	7

f

13 –	5	10	4	7	9	3	6	8
	8	3	9	6	4	10	7	5

g

12 –	5	2	9	4	8	3	6	10	7
	7	10	3	8	4	9	6	2	5

h

11 –	2	8	5	1	9	6	3	10	7	4
	9	3	6	10	2	5	8	1	4	7

i

10 –	1	10	2	5	7	3	9	4	0	8	6
	9	0	8	5	3	7	1	6	10	2	4

4 a

2
6
3
7
1

b

0
2
2
3
1

c

1
3
2
5
6

d

0
2
4
5
2

e

1
2
3
4
4

5 a 4, 2, 10, 0, 6, 20, 8, 18, 12, 16, 14, 22, 24
b 3, 0, 9, 6, 15, 30, 18, 12, 27, 24, 21, 33, 36
c 40, 20, 0, 30, 10, 70, 60, 80, 100, 90, 50, 110, 120
d 0, 40, 10, 25, 50, 20, 15, 5, 30, 35, 45, 55, 60
e 8, 4, 20, 0, 12, 40, 16, 36, 24, 32, 28, 44, 48
f 6, 0, 18, 12, 30, 60, 36, 24, 54, 48, 42, 66, 72
g 0, 14, 7, 35, 70, 21, 49, 28, 56, 42, 63, 77, 84
h 16, 8, 40, 0, 24, 80, 32, 72, 48, 64, 56, 88, 96
i 9, 0, 27, 18, 45, 90, 54, 36, 81, 72, 63, 99, 108
j 0, 22, 11, 55, 110, 33, 77, 44, 88, 66, 99, 121, 132

6 a

×	0	2	1	5	10	3	7	4	8	6	11	9
3	0	6	3	15	30	9	21	12	24	18	33	27
7	0	14	7	35	70	21	49	28	56	42	77	63
5	0	10	5	25	50	15	35	20	40	30	55	45
9	0	18	9	45	90	27	63	36	72	54	99	81
8	0	16	8	40	80	24	56	32	64	48	88	72
6	0	12	6	30	60	18	42	24	48	36	66	54

Appendix B

Exercise B:01

1 a 203 **b** 234 **c** 196 **d** 297 **e** 1779 **f** 1010 **g** 1739 **h** 2683
2 a 71 652 **b** 130 000 **c** 251 238 **d** 185 496 **e** 528 338 **f** 906 593 **g** 2 489 967 **h** 1 139 829
3 a 789 **b** 71 642 **c** 398 810 **d** 53 000

Exercise B:02

1 a 583 **b** 435 **c** 244 **d** 34 **e** 2784 **f** 2435 **g** 6040 **h** 699
2 a 37 237 **b** 78 489 **c** 157 205 **d** 632 369 **e** 710 000 **f** 6 982 000 **g** 3 544 000 **h** 6 746 000
3 a 66 969 **b** 165 000 **c** 1 690 314 km^2 **d** 17 396 tonnes

Exercise B:03

1 a 5232 **b** 56 322 **c** 20 472 **d** 15 714 **e** 9900 **f** 368 400 **g** 354 300 **h** 704 070
2 a 4472 **b** 45 201 **c** 190 978 **d** 126 567 **e** 233 370 **f** 1 077 492 **g** 6 049 278 **h** 960 933
3 a $70 488 **b** 1 626 150 kg **c** 22 956 megalitres **d** 3796 million litres

Exercise B:04

1 a 110 **b** $66\frac{1}{6}$ **c** $120\frac{3}{7}$ **d** $277\frac{3}{4}$ **e** $2472\frac{2}{3}$ **f** 4771 **g** $163\frac{2}{9}$ **h** $9734\frac{4}{7}$
2 a $352\frac{8}{23}$ **b** $104\frac{47}{59}$ **c** $246\frac{11}{16}$ **d** $202\frac{18}{19}$ **e** $14\frac{53}{100}$ **f** 1685 **g** $2289\frac{5}{42}$ **h** $164\frac{41}{91}$
3 a 1211 **b** 24 **c** 60 **d** $416\frac{2}{3}$

Exercise B:05

1 a 10 **b** 2 **c** 5 **d** 6 **e** 70 **f** 25 **g** 8 **h** 15 **i** 6 **j** 20
k 2 **l** 30 **m** 4 **n** 10 **o** 4 **p** 40 **q** 40 **r** 14 **s** 90 **t** 8
2 a 1 **b** 2 **c** 3 **d** 4 **e** 1 **f** 2 **g** 3 **h** 4 **i** 1 **j** 3
k 5 **l** 7
3 a $\frac{2}{5}$ **b** $\frac{1}{2}$ **c** $\frac{1}{2}$ **d** $\frac{1}{5}$ **e** $\frac{2}{5}$ **f** $\frac{9}{10}$ **g** $\frac{1}{2}$ **h** $\frac{7}{10}$ **i** $\frac{3}{5}$ **j** $\frac{1}{10}$
k $\frac{1}{6}$ **l** $\frac{3}{4}$ **m** $\frac{5}{8}$ **n** $\frac{1}{3}$ **o** $\frac{4}{5}$ **p** $\frac{5}{6}$ **q** $\frac{7}{8}$ **r** 1 **s** $\frac{4}{5}$ **t** $\frac{1}{5}$
u $\frac{3}{4}$ **v** $\frac{1}{8}$ **w** $\frac{1}{5}$ **x** $\frac{2}{3}$ **y** $\frac{5}{8}$

Exercise B:06

1 a $\frac{3}{10}$ **b** $\frac{1}{10}$ **c** $\frac{1}{10}$ **d** $\frac{7}{10}$ **e** $1\frac{3}{10}$ **f** $\frac{4}{10}$ **g** $\frac{1}{5}$ **h** $\frac{1}{10}$ **i** $\frac{7}{10}$ **j** $\frac{2}{5}$
2 a $\frac{3}{10}$ **b** $\frac{2}{5}$ **c** $\frac{2}{10}$ **d** $\frac{2}{5}$ **e** $\frac{1}{4}$ **f** $\frac{1}{5}$ **g** $\frac{1}{2}$ **h** $\frac{2}{5}$
3 a $\frac{1}{10}, \frac{2}{10}, \frac{7}{10}$ **b** $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}$ **c** $\frac{3}{5}, \frac{4}{5}, 1$ **d** $\frac{1}{2}, \frac{7}{10}, \frac{9}{10}$ **e** $\frac{1}{5}, \frac{7}{10}, 1\frac{1}{10}$
f $\frac{1}{10}, \frac{1}{4}, \frac{1}{2}$ **g** $\frac{2}{5}, \frac{1}{2}, \frac{3}{4}$ **h** $\frac{9}{10}, 1\frac{1}{2}, 2\frac{1}{10}$ **i** $\frac{1}{2}, \frac{3}{5}, \frac{3}{4}$ **j** $\frac{3}{5}, \frac{3}{4}, \frac{9}{10}$
k $\frac{7}{40}, \frac{2}{10}, \frac{1}{4}$ **l** $\frac{1}{5}, \frac{23}{100}, \frac{3}{10}$

Exercise B:07

1 **a** $\frac{3}{2}$ **b** $\frac{5}{4}$ **c** $\frac{11}{10}$ **d** $\frac{4}{3}$ **e** $\frac{6}{5}$ **f** $\frac{9}{8}$ **g** $\frac{23}{10}$ **h** $\frac{7}{4}$ **i** $\frac{5}{2}$ **j** $\frac{11}{3}$
k $\frac{23}{5}$ **l** $\frac{19}{10}$ **m** $\frac{29}{5}$ **n** $\frac{11}{4}$ **o** $\frac{27}{10}$ **p** $\frac{57}{10}$ **q** $\frac{35}{8}$ **r** $\frac{18}{5}$ **s** $\frac{13}{8}$ **t** $\frac{717}{100}$
u $\frac{93}{20}$ **v** $\frac{201}{100}$ **w** $\frac{71}{8}$ **x** $\frac{27}{4}$

2 **a** $1\frac{1}{4}$ **b** $1\frac{3}{10}$ **c** $1\frac{3}{4}$ **d** $5\frac{7}{10}$ **e** 4 **f** $8\frac{4}{5}$ **g** $2\frac{5}{8}$ **h** $2\frac{1}{4}$ **i** $6\frac{1}{10}$ **j** $2\frac{3}{4}$
k $1\frac{2}{5}$ **l** $1\frac{3}{8}$ **m** $7\frac{1}{2}$ **n** $1\frac{4}{5}$ **o** $1\frac{1}{8}$ **p** $8\frac{9}{10}$ **q** 8 **r** $3\frac{3}{4}$ **s** 9 **t** $3\frac{1}{4}$
u $2\frac{3}{5}$ **v** $1\frac{7}{8}$ **w** $4\frac{1}{4}$ **x** $2\frac{3}{8}$

3 **a** 30 **b** $36\frac{1}{2}$ **c** $16\frac{1}{3}$ **d** 13 **e** $12\frac{7}{10}$ **f** $10\frac{1}{8}$ **g** $1\frac{5}{24}$ **h** $1\frac{7}{10}$ **i** $33\frac{1}{3}$ **j** 20
k $2\frac{1}{11}$ **l** 20 **m** $1\frac{4}{5}$ **n** $4\frac{1}{2}$ **o** $30\frac{1}{2}$ **p** $18\frac{1}{2}$ **q** $5\frac{1}{2}$ **r** $2\frac{37}{50}$

4 **a** $7\frac{3}{4}$ **b** $6\frac{3}{5}$ **c** $3\frac{3}{4}$ **d** 63

Exercise B:08

1 **a** 5 + 4, 9 **b** 8 − 5, 3 **c** 9 − 8, 1 **d** 1 + 6, 7 **e** 2 + 3, 5 **f** $\frac{9}{10}$ **g** $\frac{9}{10}$ **h** $\frac{7}{10}$
i $\frac{7}{8}$ **j** $\frac{3}{5}$ **k** $\frac{3}{10}$ **l** $\frac{1}{10}$ **m** $\frac{1}{10}$ **n** $\frac{1}{8}$ **o** $\frac{1}{5}$ **p** $\frac{11}{100}$ **q** $\frac{19}{100}$ **r** $\frac{61}{100}$
s $\frac{67}{100}$ **t** $\frac{99}{100}$ **u** $\frac{3}{100}$ **v** $\frac{23}{100}$ **w** $\frac{21}{100}$ **x** $\frac{9}{100}$ **y** $\frac{13}{100}$

2 **a** $\frac{1}{2}$ **b** $\frac{3}{4}$ **c** $1\frac{1}{10}$ **d** $1\frac{7}{10}$ **e** $\frac{5}{6}$ **f** $\frac{3}{8}$ **g** $\frac{2}{5}$ **h** $\frac{1}{4}$ **i** $\frac{3}{5}$ **j** $\frac{4}{5}$
k $1\frac{1}{5}$ **l** $\frac{7}{50}$ **m** $1\frac{4}{5}$ **n** $\frac{3}{5}$ **o** $1\frac{1}{25}$ **p** 1 **q** $\frac{9}{10}$ **r** $\frac{1}{2}$ **s** 1 **t** $\frac{7}{10}$
u $1\frac{1}{6}$ **v** $1\frac{2}{5}$ **w** 1 **x** $1\frac{9}{20}$ **y** $1\frac{1}{2}$

3 **a** $\frac{3}{5}$ **b** 1 **c** $1\frac{2}{5}$ **d** 1 **e** $\frac{3}{5}$ **f** $\frac{1}{12}$ **g** $\frac{3}{50}$ **h** $\frac{3}{8}$ **i** $\frac{8}{15}$ **j** $\frac{3}{5}$
k $\frac{5}{8}$ **l** $\frac{3}{5}$

Exercise B:09

1 **a** $\frac{3}{5}$ **b** $\frac{3}{4}$ **c** $\frac{21}{100}$ **d** $\frac{14}{15}$ **e** $\frac{9}{10}$ **f** $\frac{1}{4}$ **g** $\frac{3}{5}$ **h** $\frac{18}{25}$ **i** $\frac{5}{8}$ **j** $\frac{77}{100}$
k $2\frac{1}{10}$ **l** $5\frac{1}{4}$ **m** $1\frac{1}{5}$ **n** $3\frac{3}{4}$ **o** $2\frac{17}{100}$ **p** $5\frac{3}{5}$ **q** $5\frac{5}{8}$ **r** $5\frac{2}{5}$ **s** $5\frac{3}{5}$ **t** $3\frac{3}{4}$
u $9\frac{5}{8}$ **v** $1\frac{47}{100}$ **w** 6 **x** 75 **y** $4\frac{4}{5}$

Exercise B:10

1 **a** 5 **b** 11 **c** 6 **d** 3 **e** 9 **f** 15 **g** 55 **h** 24 **i** 9 **j** 63
k 60 **l** 30 **m** 10 **n** 30 **o** 10 **p** 8 **q** 12 **r** 22 **s** 18 **t** 35
u 5 **v** 6 **w** 9 **x** 40 **y** 7

Exercise B:11

1 **a** 0·1 **b** 0·23 **c** 0·9 **d** 0·09 **e** 0·8 **f** 0·25 **g** 0·001 **h** 0·3 **i** 0·067 **j** 0·2
k 0·135 **l** 0·95 **m** 0·63 **n** 0·006 **o** 0·4 **p** 0·31 **q** 0·011 **r** 0·5 **s** 0·999 **t** 0·48
u 0·375 **v** 0·6 **w** 0·68 **x** 0·84

2 **a** 1·7 **b** 3·43 **c** 6·03 **d** 4·4 **e** 9·04 **f** 7·3 **g** 2·412 **h** 5·01 **i** 8·096 **j** 10·07
k 12·71 **l** 20·02 **m** 79·875 **n** 14·08 **o** 15·73 **p** 9·022 **q** 5·05 **r** 41·147

3 **a** $\frac{1}{10}$ **b** $\frac{57}{100}$ **c** $\frac{3}{10}$ **d** $\frac{3}{100}$ **e** $\frac{9}{10}$ **f** $\frac{1}{5}$ **g** $\frac{21}{50}$ **h** $\frac{81}{100}$ **i** $\frac{4}{5}$ **j** $\frac{729}{1000}$
k $9\frac{61}{100}$ **l** $11\frac{3}{100}$ **m** $8\frac{107}{1000}$ **n** $26\frac{1}{2}$ **o** $73\frac{3}{5}$ **p** $\frac{7}{100}$ **q** $5\frac{1}{25}$ **r** $7\frac{9}{10}$ **s** $\frac{79}{125}$ **t** $16\frac{1}{4}$
u $\frac{161}{200}$ **v** $1\frac{1}{8}$ **w** $\frac{173}{500}$ **x** $75\frac{1}{5}$ **y** $94\frac{5}{8}$

Exercise B:12

1 **a** 0·6 **b** 0·1 **c** 0·9 **d** 1·8 **e** 1 **f** 2·8 **g** 0·9 **h** 2·1 **i** 0·1 **j** 0·3
k 2·9 **l** 0·8

2 **a** 0·3 **b** 0·15 **c** 0·51 **d** 0·09 **e** 0·14 **f** 0·22 **g** 0·33 **h** 0·09 **i** 0·03 **j** 0·39
k 0·5 **l** 0·19

3 **a** 0·2, 0·3, 0·5 **b** 0·1, 0·3, 0·8 **c** 0·2, 0·5, 0·9 **d** 0·21, 0·3, 0·41 **e** 0·33, 0·4, 0·5
f 0·08, 0·09, 0·1 **g** 0·3, 0·55, 0·6 **h** 0·5, 0·7, 1 **i** 0·3, 0·6, 1 **j** 0·31, 0·4, 1
k 0·39, 0·4, 1 **l** 1·79, 1·84, 1·9

Exercise B:13

1 **a** 10·1 **b** 17·46 **c** 26 **d** 28·3 **e** 10·6 **f** 45·64 **g** 32·3 **h** 68·6 **i** 19·55 **j** 36·35
k 28·41 **l** 28·6 **m** 9 **n** 16·05 **o** 26·99 **p** 34·79
2 **a** 0·73 **b** 13·1 **c** 32·9 **d** 17 **e** 5·1 **f** 23·5 **g** 2·65 **h** 10·81 **i** 14·4 **j** 6·6
k 5·5 **l** 16·53 **m** 15·95 **n** 17·85 **o** 9·03 **p** 9·99 **q** 70 **r** 31·61 **s** 12·72 **t** 20·51
u 7·1 **v** 17·97 **w** 13·95 **x** 25·2

Exercise B:14

1 **a** 5·64 **b** 41·5 **c** 19·6 **d** 6·25 **e** 18·25 **f** 21·9 **g** 8·36 **h** 5·35 **i** 15·95 **j** 11·85
k 5·2 **l** 2·1 **m** 0·9 **n** 12·51 **o** 1·64 **p** 53·68 **q** 115·05
2 **a** 15·2 **b** 14·79 **c** 4·1 **d** 8·84 **e** 36·73 **f** 1·05 **g** 30·6 **h** 84·75 **i** 8·05 **j** 6·55
k 4·75 **l** 6·43 **m** 8·7 **n** 4·75 **o** 8·92 **p** 13·2 **q** 19·55 **r** 4·48 **s** 13·15 **t** 23·71
u 56·2 **v** 150·28 **w** 6·08 **x** 15·95
3 **a** 23·3 m **b** 71·3 ha **c** 76·8 kg **d** 33·8 km

Exercise B:15

1 **a** 5·4 **b** 18·5 **c** 14·4 **d** 23·6 **e** 4·2 **f** 5·24 **g** 4·5 **h** 6·42 **i** 0·456 **j** 7·28
k 18·62 **l** 10·866
2 **a** 4·5 **b** 4·8 **c** 4·2 **d** 3·6 **e** 6·4 **f** 5·4 **g** 6·3 **h** 8·1 **i** 0·66 **j** 2·37
k 4·25 **l** 2·73 **m** 28·8 **n** 18 **o** 68 **p** 12·6 **q** 42·28 **r** 18·18 **s** 18·24 **t** 64·4
3 **a** 16 m **b** 11·2 m **c** 9·6 m **d** 20·8 m

Exercise B:16

1 **a** 3·1 **b** 2·13 **c** 2·37 **d** 6·2 **e** 0·12 **f** 1·01 **g** 9·6 **h** 2·2 **i** 1·08 **j** 0·31
k 1·09 **l** 1·04
2 **a** 0·24 **b** 0·101 **c** 0·363 **d** 0·029 **e** 0·61 **f** 3·01 **g** 7·6 **h** 1·12 **i** 0·57 **j** 0·26
k 1·03 **l** 10·7 **m** 10·41 **n** 5·06 **o** 2·15 **p** 6·31
3 **a** 1·5 m **b** 18·2 ha **c** 9·6 m **d** 34·3 s

Exercise B:17

1 **a** 5 **b** 3·7 **c** 9 **d** 0·8 **e** 38·5 **f** 8·15 **g** 171·25 **h** 0·71 **i** 9·1 **j** 85
k 0·1 **l** 23·75 **m** 0·05 **n** 158 **o** 13·82 **p** 78
2 **a** 0·38 **b** 5·2 **c** 7·86 **d** 8·415 **e** 0·004 **f** 10·85 **g** 0·074 **h** 1·57 **i** 0·72 **j** 0·8
k 0·07 **l** 0·67 **m** 0·0125 **n** 63·5 **o** 0·36 **p** 84·51

Exercise B:18

1 **a** \$8.15 **b** \$16.50 **c** \$146.95 **d** \$37.15 **e** \$93.05 **f** \$2.35 **g** \$100.50 **h** \$40.60
i \$18.65
2 **a** \$7.95 **b** \$16 **c** \$9.60 **d** \$12.45 **e** \$21.65 **f** \$28.85 **g** \$22.30 **h** \$184.35
i \$56.70 **j** \$163.45 **k** \$273.30 **l** \$171.20
3 **a** \$5.25 **b** \$2.35 **c** \$2.15 **d** \$66.60 **e** \$5.45 **f** \$6.70 **g** \$0.15 **h** \$3.95
i \$18.65 **j** \$16.60 **k** \$42.70 **l** \$59.30 **m** \$393.85 **n** \$86.15 **o** \$2.05 **p** \$27.35
4 **a** \$12.15 **b** \$12.60 **c** \$12.10 **d** \$36.25 **e** \$52.15 **f** \$77.40 **g** \$23.10 **h** \$74.80
i \$102 **j** \$90 **k** \$380 **l** \$618 **m** \$133.60 **n** \$705.60 **o** \$262.15 **p** \$1133.00
5 **a** \$2.75 **b** \$4.65 **c** \$1.60 **d** \$0.95 **e** \$2.05 **f** \$6.00 **g** \$5.05 **h** \$9.10
i \$2.50 **j** \$3.50 **k** \$13 **l** \$20.25 **m** \$22.70 **n** \$6.20 **o** \$9.20 **p** \$3.75

Exercise B:19

1 **a** $\frac{13}{100}$ **b** $\frac{1}{100}$ **c** $\frac{5}{100}$ **d** $\frac{3}{100}$ **e** $\frac{11}{100}$ **f** $\frac{9}{100}$ **g** $\frac{15}{100}$ **h** $\frac{2}{100}$ **i** $\frac{10}{100}$ **j** $\frac{77}{100}$
k $\frac{97}{100}$ **l** $\frac{4}{100}$ **m** $\frac{58}{100}$ **n** $\frac{14}{100}$ **o** $\frac{30}{100}$ **p** $\frac{63}{100}$ **q** $\frac{35}{100}$ **r** $\frac{8}{100}$ **s** $\frac{50}{100}$ **t** $\frac{81}{100}$
u $\frac{90}{100}$ **v** $\frac{12}{100}$ **w** $\frac{43}{100}$ **x** $\frac{6}{100}$ **y** $\frac{29}{100}$
2 **a** $\frac{1}{10}$ **b** $\frac{1}{2}$ **c** $\frac{1}{50}$ **d** $\frac{1}{25}$ **e** $\frac{7}{100}$ **f** $\frac{2}{25}$ **g** $\frac{1}{4}$ **h** $\frac{9}{50}$ **i** $\frac{16}{23}$ **j** $\frac{1}{20}$
k $\frac{17}{20}$ **l** $\frac{13}{25}$ **m** $\frac{3}{5}$ **n** $\frac{3}{4}$ **o** $\frac{19}{25}$ **p** $\frac{6}{25}$ **q** $\frac{3}{20}$ **r** $\frac{7}{25}$ **s** $\frac{3}{10}$ **t** $\frac{4}{5}$
u $\frac{23}{50}$ **v** $\frac{33}{50}$ **w** $\frac{23}{25}$ **x** $\frac{11}{20}$ **y** $\frac{4}{25}$

Exercise B:20

1 **a** 2 **b** 5 **c** 1 **d** 3 **e** 6 **f** $1\frac{1}{2}$ **g** $1\frac{7}{10}$ **h** $2\frac{1}{2}$ **i** $2\frac{1}{5}$ **j** $3\frac{1}{2}$
k $1\frac{3}{50}$ **l** $1\frac{19}{50}$ **m** $1\frac{3}{4}$ **n** $1\frac{11}{25}$ **o** $1\frac{17}{20}$ **p** $4\frac{1}{10}$ **q** $6\frac{7}{20}$ **r** $3\frac{13}{20}$ **s** $8\frac{4}{5}$ **t** $4\frac{39}{50}$

2 **a** $1\frac{73}{100}$ **b** $\frac{11}{25}$ **c** $3\frac{3}{4}$ **d** $4\frac{1}{2}$ **e** $\frac{33}{50}$ **f** 8 **g** $5\frac{1}{2}$ **h** $4\frac{1}{4}$ **i** $3\frac{1}{50}$ **j** 9
k $2\frac{1}{20}$ **l** 7 **m** $1\frac{81}{100}$ **n** $9\frac{4}{5}$ **o** $\frac{1}{25}$ **p** $6\frac{23}{100}$ **q** $4\frac{3}{4}$ **r** $\frac{4}{25}$ **s** $1\frac{11}{100}$ **t** 10

Exercise B:21

1 **a** 7% **b** 50% **c** 18% **d** 73% **e** 8% **f** 90% **g** 77% **h** 20% **i** 99% **j** 40%
k 67% **l** 80% **m** 1% **n** 19% **o** 22% **p** 87% **q** 75% **r** 30% **s** 25% **t** 3%

2 **a** 50% **b** 30% **c** 90% **d** 10% **e** 70% **f** 40% **g** 15% **h** 20% **i** 55% **j** 80%
k 6% **l** 25% **m** 32% **n** 75% **o** 96% **p** 8% **q** 60% **r** 34% **s** 80% **t** 42%

3 **a** 100% **b** 300% **c** 500% **d** 200% **e** 800% **f** 150% **g** 350% **h** 450% **i** 650% **j** 750%
k 230% **l** 570% **m** 610% **n** 130% **o** 290% **p** 126% **q** 325% **r** 535% **s** 260% **t** 612%

Exercise B:22

1 **a** 0·06 **b** 0·05 **c** 0·02 **d** 0·09 **e** 0·01 **f** 0·17 **g** 0·24 **h** 0·83 **i** 0·65 **j** 0·88
k 0·2 **l** 0·5 **m** 0·9 **n** 0·6 **o** 0·4 **p** 0·43 **q** 0·03 **r** 0·7 **s** 0·11 **t** 0·95

2 **a** 1·12 **b** 0·97 **c** 2·04 **d** 0·86 **e** 4·27 **f** 1·14 **g** 2·93 **h** 1·47 **i** 3·42 **j** 8·86
k 2·5 **l** 1·6 **m** 4·2 **n** 3·9 **o** 1·8 **p** 0·05 **q** 2 **r** 1·93 **s** 1 **t** 1·06

Exercise B:23

1 **a** 15% **b** 33% **c** 92% **d** 52% **e** 38% **f** 83% **g** 55% **h** 19% **i** 25% **j** 75%
k 3% **l** 5% **m** 8% **n** 9% **o** 1% **p** 67% **q** 86% **r** 99% **s** 42% **t** 21%

2 **a** 40% **b** 60% **c** 90% **d** 80% **e** 20% **f** 177% **g** 106% **h** 264% **i** 295% **j** 324%
k 102% **l** 434% **m** 261% **n** 193% **o** 388% **p** 170% **q** 250% **r** 310% **s** 130% **t** 260%

3 **a** 40% **b** 153% **c** 9% **d** 28% **e** 162% **f** 11% **g** 60% **h** 32% **i** 84% **j** 197%
k 7% **l** 146% **m** 70% **n** 54% **o** 98% **p** 269% **q** 72% **r** 3% **s** 20% **t** 78%

Working Mathematically

1 31 matches **2** 45 **3** 3 spiders and 5 beetles **4** 100 cm^2 **5** 15 **6** **a** 11 million tonnes
b 6 million tonnes **c** 2002 **d** 2000 **e** 7 million tonnes

Appendix C

Prep Quiz C:01

1 1, 2, 3, 5, 6, 10, 15, 30 **2** 1, 2, 3, 6, 9, 18, 27, 54 **3** 6 **4** 4, 8, 12, 16, 20, 24, 28, 32, 36
5 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39 **6** 12 **7** $2 \times 2 \times 2 \times 2 \times 3$ **8** 2^5 **9** $2^3 \times 3^4$ **10** $3^2 \times 5^3 \times 7$

Exercise C:01

1 **a** **i** 36 **ii** 54 **iii** 70 **iv** 980 **v** 5 **vi** 3 **vii** 392 **viii** 392
ix 135 **x** 1
b **i** 1296 **ii** 11 340 **iii** 132 300 **iv** 9800 **v** 1 323 000 **vi** 9 261 000 **vii** 32 928 **viii** 41 160
ix 47 250 **x** 5 292 000

2

Number	*Product of prime factors*
18	2×3^2
36	$2^2 \times 3^2$
24	$2^3 \times 3$
300	$2^2 \times 3 \times 5^2$
1050	$2 \times 3 \times 5^2 \times 7$
1250	2×5^4
2475	$3^2 \times 5^2 \times 11$
2310	$2 \times 3 \times 5 \times 7 \times 11$

3 **a** **i** 18 **ii** 12 **iii** 12 **iv** 150 **v** 50
vi 25 **vii** 165 **viii** 75 **ix** 10 **x** 3
b **i** 36 **ii** 72 **iii** 600 **iv** 2100 **v** 26 250
vi 123 750 **vii** 34 650 **viii** 34 650 **ix** 288 750 **x** 19 800

4 **a** 231 **b** 10 395 **c** 66 **d** 5544

5 **a** 1512 s or 25 min 12 s **b** 216 min or 3 h 36 min
c $1848 ($924 for chairs and $924 for tables)

Appendix D

Exercise D:01

1 **a** 4, 4, 4 **b** 9, 9, 9 **c** 11, 11, 11 **d** 3, 3, 3 **e** 7, 7, 7 **f** 2, 2, 2 **g** 5, 5, 5

2 **a** 3, 5, 7, 9 difference 2 **b** 0, 5, 10, 15 difference 5 **c** 1, 4, 7, 10 difference 3 **d** 3, 9, 15, 21 difference 6
e 7, 17, 27, 37 difference 10

3 **a** 4, 4, 4, $y = 4x + 7$ **b** 7, 7, 7, $y = 7x - 8$ **c** 2, 2, 2, $y = 2x + 4$ **d** 4, 4, 4, $y = 4x + 2$
e 1, 1, 1, $y = 1x + 5$ or $y = x + 5$

4 **a** $y = 3x$ **b** $y = x + 5$ **c** $y = 2x + 1$ **d** $b = 2a + 7$
e $b = 5a - 10$ **f** $b = 4a - 4$ **g** $y = 7x + 3$ **h** $y = 6x - 11$
i $y = 10x + 8$ **j** $m = 3s - 30$ **k** $m = 2s + 3$ **l** $m = 5s + 6$

5 $H = 15T + 8$ **6** **a** 4, 4, 4, **b** 6, 6, 6,

Appendix E

Exercise E:01

1 **a** $\frac{7}{9}$ **b** $\frac{1}{9}$ **c** $\frac{3}{9}$ or $\frac{1}{3}$ **d** $\frac{8}{9}$ **e** $\frac{2}{9}$ **f** $\frac{6}{9}$ or $\frac{2}{3}$ **g** $\frac{5}{9}$ **h** $\frac{9}{9}$ or 1
i $\frac{15}{99}$ or $\frac{5}{33}$ **j** $\frac{84}{99}$ or $\frac{28}{33}$ **k** $\frac{12}{99}$ or $\frac{4}{33}$ **l** $\frac{82}{99}$ **m** $\frac{73}{99}$ **n** $\frac{58}{99}$
o $\frac{7}{99}$ **p** $\frac{30}{99}$ or $\frac{10}{33}$ **q** $\frac{147}{990}$ or $\frac{49}{330}$ **r** $\frac{725}{990}$ or $\frac{145}{198}$ **s** $\frac{185}{990}$ or $\frac{37}{198}$ **t** $\frac{42}{990}$ or $\frac{7}{165}$
u $\frac{63}{990}$ or $\frac{21}{330}$ or $\frac{7}{110}$ **v** $\frac{938}{990}$ or $\frac{469}{495}$ **w** $\frac{252}{990}$ or $\frac{126}{495}$ **x** $\frac{901}{990}$

2 **a** $\frac{3185}{9900}$ or $\frac{637}{1980}$ **b** $\frac{4463}{9900}$ **c** $\frac{523}{9900}$ **d** $\frac{1131}{9000}$ or $\frac{377}{3000}$ **e** $1\frac{6}{9}$ or $1\frac{2}{3}$ **f** $7\frac{4}{99}$ **g** $9\frac{86}{99}$
h $4\frac{9}{9}$ or 5 **i** $8\frac{301}{990}$ **j** $7\frac{609}{990}$ or $7\frac{203}{330}$ **k** $3\frac{867}{990}$ or $3\frac{289}{330}$ **l** $9\frac{52}{990}$ or $9\frac{26}{495}$

3 $0\cdot\dot{9}$ and 1 are two different ways of writing the number 1. Both are equal to $\frac{9}{9}$.
$0\cdot\dot{9}$ is short for $0 + 0{\cdot}9 + 0{\cdot}09 + 0{\cdot}009 + 0{\cdot}0009 + 0{\cdot}000\,09 + \ldots$ and mathematicians have shown that this is equal to 1.

Also, ten lots of $0\cdot\dot{9} = 9\cdot\dot{9}$,
$-$ one lot of $0\cdot\dot{9} = 0\cdot\dot{9}$.

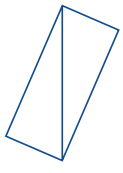
nine lots of $0\cdot\dot{9} = 9\cdot\dot{9} - 0\cdot\dot{9}$
$= 9$

If nine lots of $0\cdot\dot{9}$ is 9, then one lot of $0\cdot\dot{9}$ is 1.

Appendix F

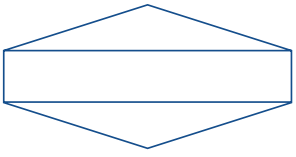
Exercise F:01

1 **a** 1 square, 2 triangles, 3 kites **b** There are an infinite number of answers.
c square, triangle, parallelogram **d** Yes, three can be made.

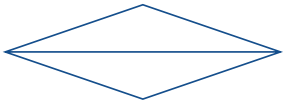


2 **a** a triangle or a parallelogram
b a triangle or a parallelogram
c a rectangle or a kite

3 **a** hexagon **b**



c



d

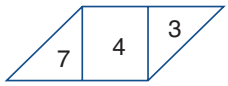


e

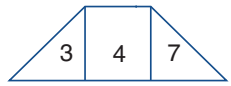


4 **a** 4 is a square; 6 is a parallelogram. **b** 3 and 7 **c** 3 and 7 **d** 3, 5 and 7; yes

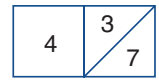
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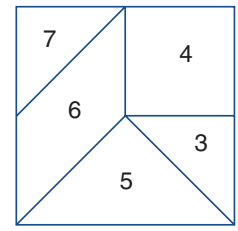
f



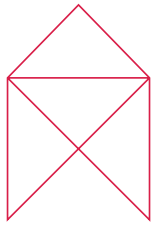
g



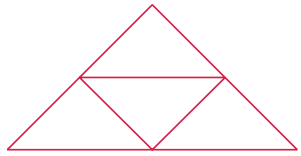
h



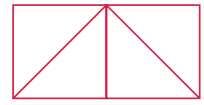
5 **a**



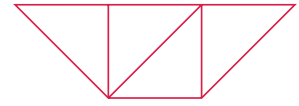
b



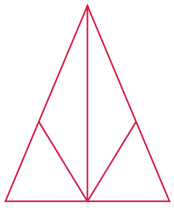
c



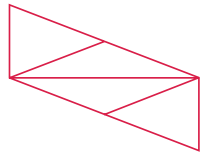
d



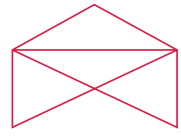
6 **a**



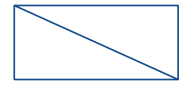
b



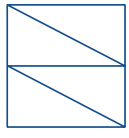
c



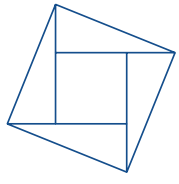
7 **a**



b



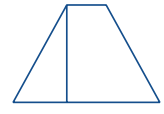
c



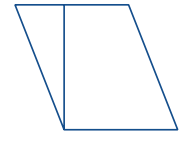
8 **a**



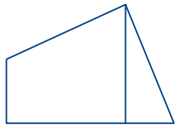
b



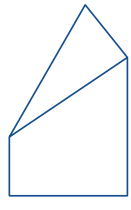
c



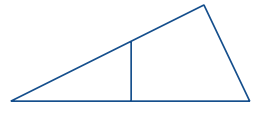
d



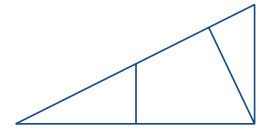
e



9 **a**



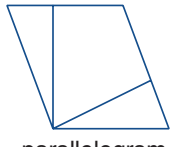
b



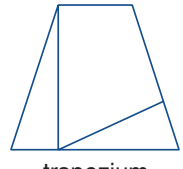
c



rectangle

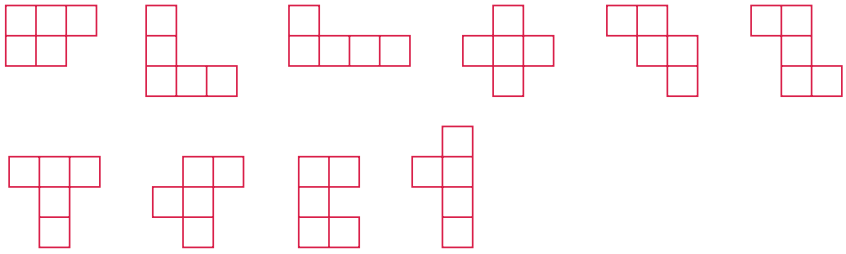


parallelogram

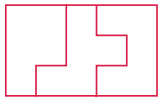


trapezium

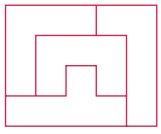
10 a



b

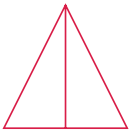


c

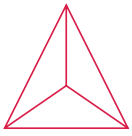


Note: There are many different answers to **b** and **c**.

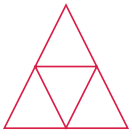
11 a



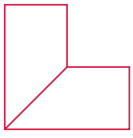
b



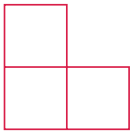
c



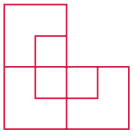
12 a



b



c



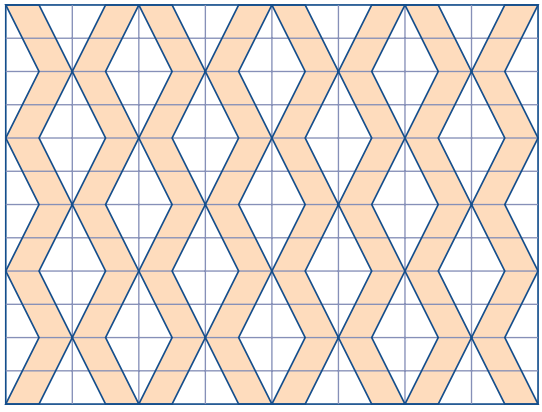
Fun Spot F:01

a 16 **b** 8 **c** 4 **d** 8

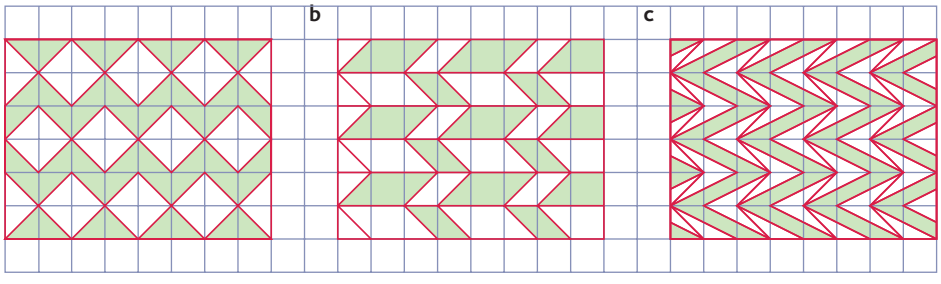
Exercise F:02

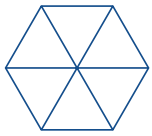
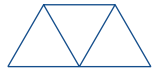
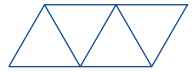
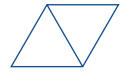
1 a Pattern 1 only **2 a** kite **b** The pattern will consist of four rows of 8 coloured kites. **c** triangles **3 a** octagons and squares **b** The pattern will be four times larger. It will consist of four rows of 6 coloured octagons divided by three rows of 5 squares. **4 a** rhombus and parallelogram

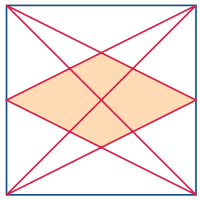
b



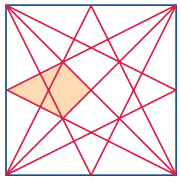
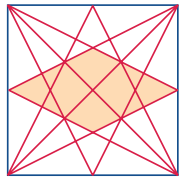
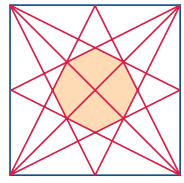
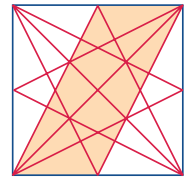
5 a

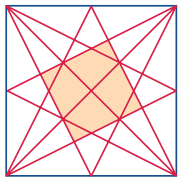


6 **a**  **b**  **c**  **d** 

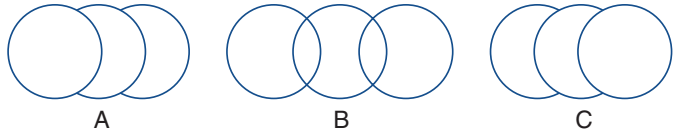
7 **a** 

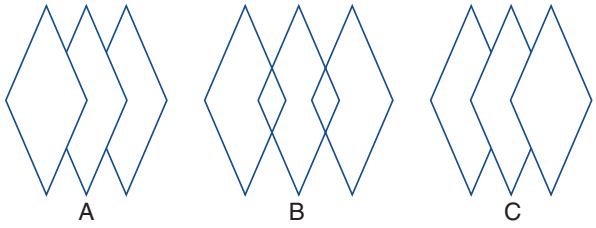
The rhombus is shown in colour. It is made from four kites.

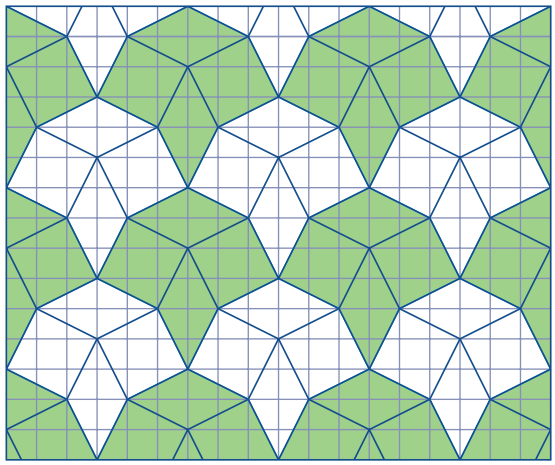
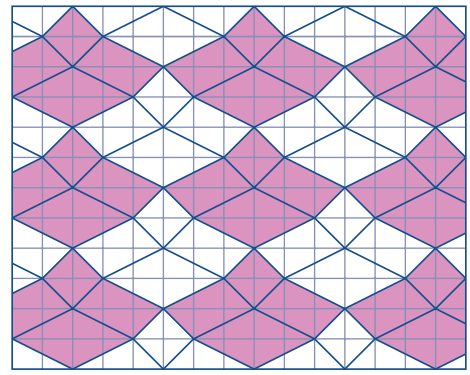
b **i**  **ii**  **iii**  **iv** 

v 

Please note that there is more than one answer to parts **i**, **iv**.

8 **a** B **b** C **c** 

d 

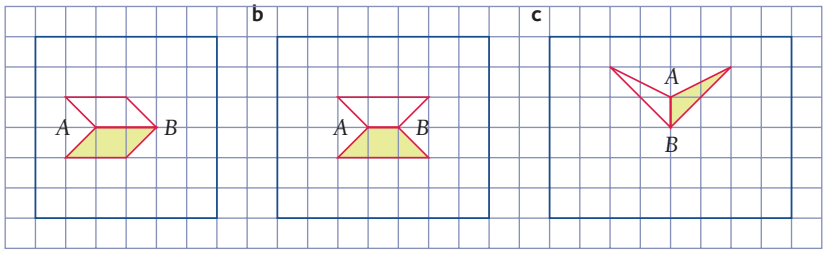
9 **a**  **b** 

10 **a** the triangle, square and hexagon
b square and triangle; hexagon and triangle; octagon and square; hexagon, square and triangle

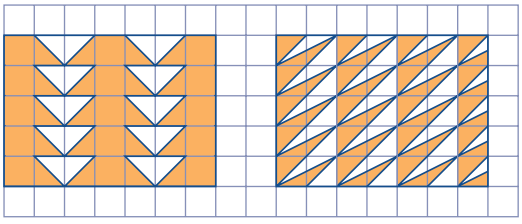
Exercise F:03 (Practical)

1 **a** slide **b** flip **c** flip **d** turn **e** flip or turn **f** slide **g** turn **h** turn

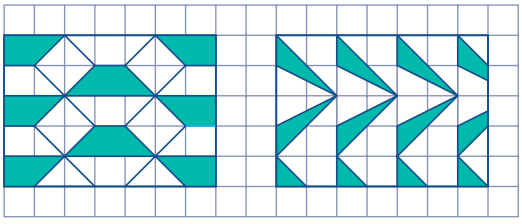
2 **a** **b** **c**



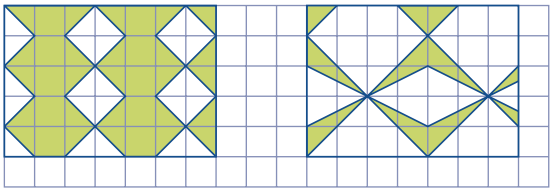
3 **a** **b**



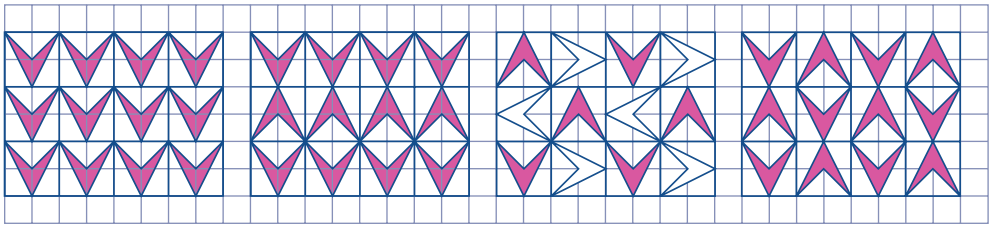
4 **a** **b**



5 **a** **b**

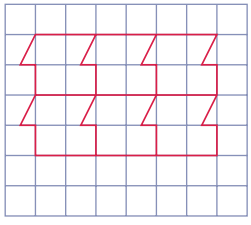


6 **a** **b** **c** **d**

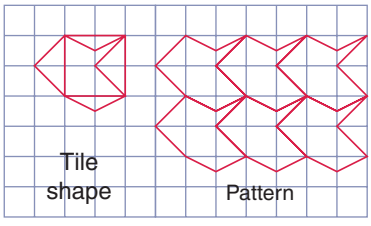


7 **a** yes **b** no **c** There are several different patterns possible.

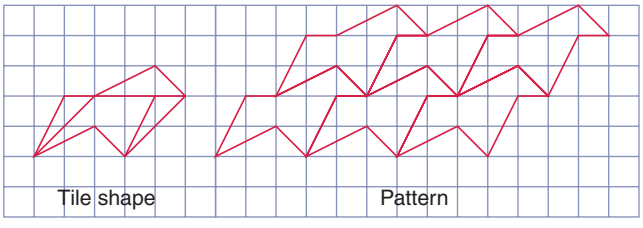
8 This is one example only.



9 **a**



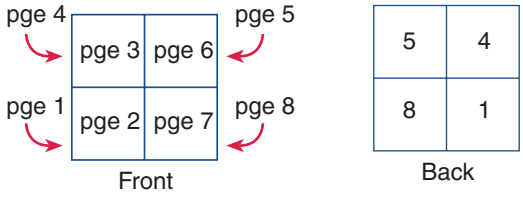
b This is one example only.



10 The word is written like this so that when you look in your rear vision mirror while driving your car the word will appear in the correct way.

POLICE

11



Exercise F:04 (Practical research)

2 **a** line symmetry **b** line and point symmetry **c** line symmetry

3 There are many different patterns possible.

4 **a** The basic shapes are triangles and squares. These have been combined to form a larger square.
b The basic shapes are two different sized triangles. These can be used to make a larger triangle, a square or trapezium.
c The basic shapes are triangles and squares. These have been used to make triangles, squares, parallelograms and trapeziums.
d All of them **e** Ohio star

5 The circle has an infinite number of axes of symmetry. Hence saucepan lids that are circular can fit in an infinite number of ways whereas a square lid can only fit in four ways.

6 The left-hand or nearside is the side of the car to the driver's left. The right-hand side is the side to the driver's right. Right-hand and left-hand are reflections of each other.

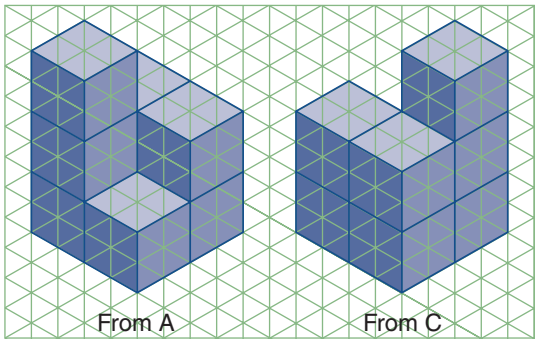
7 'Pattern matching' simply refers to matching the pattern in patterned wallpapers or curtains. When buying patterned wallpapers, you have to allow for an amount of wastage because of the need to match the pattern. This means you will require more material than if you were using unpatterned material.

8 **a** Two trapeziums joined on their shorter side inside a circle.
b The top and bottom halves are made from a trapezium and a parallelogram.
c The logo is made from three rhombuses.
d The logo consists of three rhombuses arranged in an arrowhead pattern inside a circle.
e The central shape is a rhombus. On opposite sides of the rhombus are two parallelograms and two rhombuses.

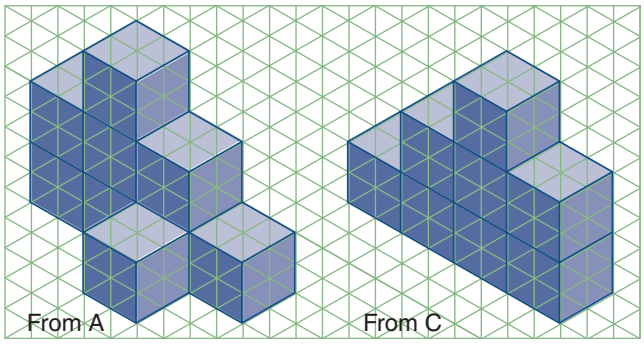
Exercise F:05

3 **a** C **b** B **c** D

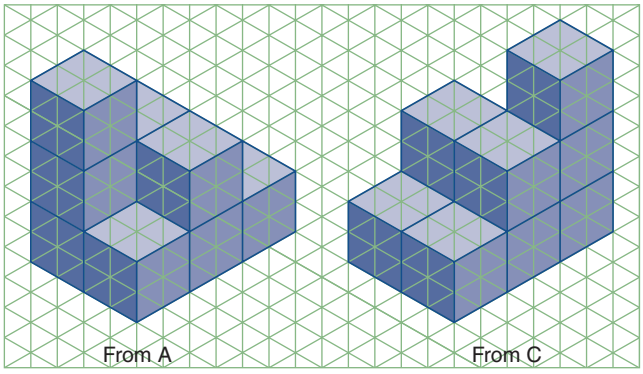
4 **a**



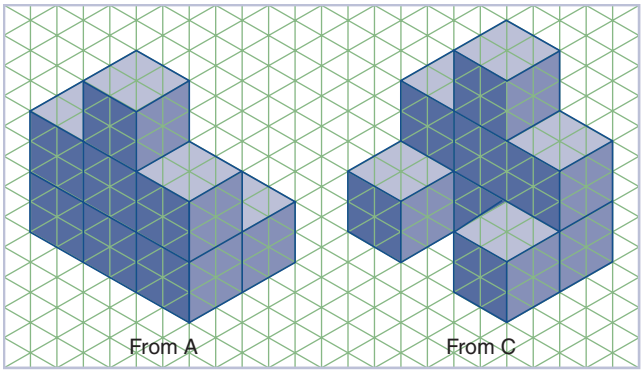
b



c



d



Appendix G

Investigation G:01

Interval	*Length*
AD	12·6
AC	7·8
AB	4·8
BC	3·0

All divisions equal 1·6 to 1 dec. pl.

Exercise G:02

1 **a** 120 m **b** 16 km **c** 27 km **d** 240 km **e** 500 m **f** 16 km
2 **a** 10 m/s **b** 3 km/h **c** 28 m/min **d** 8 m/s **e** 12 km/h **f** 2 km/min
3 **a** 2 s **b** 6 h **c** 18 s **d** 8 h **e** 21 s **f** 7 h
4 5 m/s **5** 48 m **6** 5 hours **7** **a** 6 s **b** 80 cm
8 **a** 20 m/s **b** 8 s **9** 251·51 km **10** 20 min

Appendix H

Prep Quiz H:01

1 a^3 **2** m^5 **3** p^2 **4** t^4 **5** n^8 **6** $x \times x \times x \times x \times x$ **7** $y \times y \times y$ **8** $a \times a \times b \times b \times b$ **9** $8x^3$ **10** $30y^3$

Exercise H:01

1 **a** x^7 **b** a^5 **c** m^9 **d** p^9 **e** n^{10} **f** x^8 **g** h^7 **h** m^{12}
i a^5 **j** p^2 **k** s^{11} **l** t^{18} **m** w^{20} **n** q^7 **o** x^{20} **p** y^{24}
q a^9 **r** x^8 **s** m^{15} **t** n^9 **u** y^{21} **v** a^3 **w** h^{12} **x** t^{18}

2 **a** x^2 **b** a **c** q **d** m^4 **e** t^2 **f** h^2 **g** n **h** a
i m **j** l^3 **k** y **l** k^{14} **m** x^2 **n** y^4 **o** a^9 **p** m

3 **a** x^6 **b** a^{12} **c** m^6 **d** n^{12} **e** y^{10} **f** m^{21} **g** n^{12} **h** x^{25}
i $4x^6$ **j** $9a^8$ **k** $8m^9$ **l** $25m^{14}$ **m** $16y^{12}$ **n** $100n^6$ **o** $125t^9$ **p** $1000y^{12}$

4 **a** $2x^7$ **b** $3y^6$ **c** $5a^9$ **d** $4m^8$ **e** $15n^7$ **f** $8a^6$ **g** $21k^7$ **h** $20t^8$
i x^6y^5 **j** a^5b^5 **k** $m^{10}n^5$ **l** p^8q^4 **m** $8a^3b^3$ **n** $20x^7y^{10}$ **o** $70k^9l^6$ **p** $24m^7n^9$

5 **a** $4x^5$ **b** $5a^3$ **c** $7m^2$ **d** $6m^6$ **e** $10a^2$ **f** $15x$ **g** $20y^2$ **h** $7m$
i $4m^4$ **j** $2a^2$ **k** $3y$ **l** $4y^2$ **m** $12a^2b$ **n** $3x^4y^2$ **o** $20m^2n^5$ **p** $2q^4$

6 **a** $x^{10}y^{20}$ **b** $m^{42}n^6$ **c** $4a^8b^{12}$ **d** $27p^{12}q^6$

7 **a** x^5 **b** a **c** $3a^2$ **d** $25m^4$

Appendix I

Exercise I:01

1 **a** 2 kg **b** 5 kg **c** 12 kg **d** 15 kg **e** 7·4 kg **f** 0·250 kg **g** 3 t **h** 10 t
i 5·5 t **j** 7000 g **k** 4500 g **l** 2700 g **m** 2000 kg **n** 8100 kg **o** 1450 kg **p** 4 g
q 3000 mg **r** 2400 mg **s** 1 000 000 g **t** 5 t

2 **a** B **b** false **3** **a** muesli **b** false

4 matchbox, cup of water, hammer, chair, bicycle, bag of cement **5** **a** 1925 g **b** 385 g

6 12 kg **7** 900 g **8** **a** 20 **b** 2·8 t **9** 110 g

10 **a** 15 g **b** 120 g **c** 80 g **d** 150 g

11 **a** 400 g **b** 135 g **c** 850 g **d** 440 g

12 14·2 t **13** **a** $1.30 **b** $5.30 **c** $1.30 **d** $1.20 **e** $10.00 **f** $11.60

14 $4.85 **15** **a** 75 mg **b** 525 mg

Foundation Worksheet

1:03 | Know Your Tables

Name: _____ **Class:** _____

Exercise

Complete each table using the operation shown.

1	7 +	0	1	4	3	7	2	8	6	5	9		+
2	11 −	2	8	5	1	9	6	3	10	7	4		−
3	7 ×	0	2	1	5	10	3	7	4	8	6	9	11	×
4	4 ×	3	6	2	7	8	1	4	9	5	10	0	11	×
5	5 +	4	7	3	6	0	1	5	9	8	10		+
6	9 −	3	8	7	4	1	6	9	2	0	5		−
7	3 ×	8	4	6	9	7	5	2	1	0	11	3	10	×
8	8 +	5	6	2	4	8	0	7	3	9	1		+
9	6 ×	7	2	4	10	6	8	9	3	11	1	0	5	×
10	3 +	6	5	9	2	4	7	0	10	3	8		+
11	9 ×	1	3	8	11	2	9	10	0	4	7	6	5	×
12	10 −	9	0	6	8	3	4	1	7	10	5		−

You can practise more by choosing new numbers for the end box and repeating the sheet.

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Foundation Worksheet

1:06 | Powers of Numbers

Name: _____ **Class:** _____

Examples

1 Write the answer.

$7 \times 7 \times 7$
$= 49 \times 7$
$= 343$

2 Write the answer.

$5^3 = 5 \times 5 \times 5$
$= 25 \times 5$
$= 125$

3 Write as a power.

$6 \times 6 \times 6 \times 6$
$= 6^4$



The 6 is used 4 times in the product. This is *6 to the power of 4*.

Exercise

1 Write the simplest answer for each.

a 4×4	**b** 8×8	**c** 3×3	**d** 1×1	**e** 9×9
f 2×2	**g** 5×5	**h** 10×10	**i** 7×7	**j** 6×6
k $2 \times 2 \times 2$	**l** $1 \times 1 \times 1$	**m** $5 \times 5 \times 5$	**n** $3 \times 3 \times 3$	**o** $10 \times 10 \times 10$

2 Write the simplest answer for each.

a 7^2	**b** 10^2	**c** 5^2	**d** 8^2	**e** 2^2
f 1^2	**g** 6^2	**h** 4^2	**i** 3^2	**j** 9^2
k 3^3	**l** 2^3	**m** 4^3	**n** 10^3	**o** 6^3

3 Write each as a power.

a $10 \times 10 \times 10$	**b** 6×6	**c** $2 \times 2 \times 2 \times 2 \times 2 \times 2$
d 7×7	**e** $5 \times 5 \times 5 \times 5$	**f** $9 \times 9 \times 9 \times 9 \times 9$
g $8 \times 8 \times 8 \times 8$	**h** $4 \times 4 \times 4$	**i** $10 \times 10 \times 10 \times 10 \times 10$

Fun Spot 1:06 | Why did the chicken cross over the road?



Work out the answer to each part and put the letter for that part in the box above the correct answer.

A 3×4	**B** 7×2	**C** 9×3	**D** 5×6	**E** 4×6
H 5×7	**I** 8×4	**L** 2×10	**N** 3×7	**O** 9×2
R 1^2	**S** 2^2	**T** 3^2	**U** 4^2	

14	24	27	12	16	4	24		35	24		27	18	16	20	30				

21	18	9		27	1	18	4	4		16	21	30	24	1		32	9		

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Foundation Worksheet

1:08 | Rounding Numbers

Name: _____ **Class:** _____

Each digit has a place value:

	Thousands	*Hundreds*	*Tens*	*Units*
6542	6	5	4	2

When rounding, look at the next digit.
If it is 5 or more, round up.
If it is less than 5, leave the digit.

Examples

1 Round off to the nearest ten.

a 532 **b** 1678

For tens, look at the units.

a 53②: 2 is less than 5, so leave the tens digit.
Answer is 530.

b 167⑧: 8 is more than 5, so round up the tens digit.
Answer is 1680.

2 Round off to the nearest hundred.

a 658 **b** 9240

For hundreds, look at the tens.

a 6⑤8: 5 is 5 or more, so round up the hundreds.
Answer is 700.

b 92④0: 4 is less than 5, so leave the hundreds digit.
Answer is 9200.

Exercise

1 Round off to the nearest ten.

a 368	**b** 79	**c** 35	**d** 136
e 574	**f** 6077	**g** 591	**h** 8018
i 43	**j** 882	**k** 62	**l** 207

2 Round off to the nearest hundred.

a 8731	**b** 1234	**c** 309	**d** 484
e 6550	**f** 938	**g** 2789	**h** 161
i 528	**j** 9876	**k** 7007	**l** 299

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Foundation Worksheet

2:01 | Problem Solving

Name: _____ **Class:** _____

- **Read the question carefully to decide what you have to find.**
- **Choose a method, which uses the given information.**
- **Set out your work clearly and neatly.**
- **Is your answer reasonable? Answer the question.**

Exercise

1 John is given $3.50 pocket money each week. How much would he be given in 10 weeks?

2 When Jamie went shopping, he bought 2 CDs for $19.95 each, 3 blank tapes for $1.50 each and 5 videotapes for $2.20 each. How much did he spend?

3 How much change should Clive receive if he pays for a $34.50 shirt with a $100 note?

4 How many 15 mL doses of medicine could Peter get from a 500 mL bottle?

5 Calculate the cost of 8 drinks at $1.29 each. How much would I pay in cash for them?

6 Ian played 4 rounds of golf, scoring 81, 86, 92 and 85. Find his total score and his average.

7 Helen, Tina and Janet picked fruit in a local orchard. They counted and found they had 97 apples and 65 pears. They shared the fruit by each taking the same number. How many pieces of fruit did each receive?

8 The distance from the Earth to the Sun is about 150 000 000 km. If the distance for Mercury is $\frac{2}{15}$ as much, find how far it is from the Sun.

9 Kerrie wishes to buy a new saddle priced at $480. How many weeks will it take if she saves $20 a week?

10 Heather pays $70 a month for dancing lessons. What is her cost for the year?

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Foundation Worksheet

3:01A | Grouping Symbols

Name: _____ **Class:** _____

Examples

1 $(6 + 3) \times 8$
$= 9 \times 8$
$= 72$

2 $(10 - 6) \times 5$
$= 4 \times 5$
$= 20$

3 $7 \times (9 - 3)$
$= 7 \times 6$
$= 42$

4 $12 - (8 - 3)$
$= 12 - 5$
$= 7$

Exercise

Find the basic numeral for each expression.

1 **a** $(3 + 2) \times 10$ **b** $(4 + 3) \times 6$ **c** $(2 + 1) \times 3$
d $(5 + 1) \times 5$ **e** $7 \times (3 + 4)$ **f** $4 \times (2 + 5)$

2 **a** $(8 - 2) \times 3$ **b** $(10 - 8) \times 2$ **c** $(6 - 4) \times 1$
d $4 \times (8 - 3)$ **e** $(9 - 4) \times 6$ **f** $8 \times (7 - 5)$

3 **a** $10 - (4 + 3)$ **b** $8 - (4 + 2)$ **c** $9 - (3 + 1)$
d $8 - (3 + 3)$ **e** $9 - (6 - 4)$ **f** $6 - (8 - 2)$

Fun Spot 3:01A | Which month has 28 days?



Work out the answer for each part, and put the letter for that part in the box above the correct answer.

A $(5 + 3) \times 4$ **E** $(7 - 5) \times 4$ **F** $7 - (4 - 2)$ **H** $(7 - 3) \times 6$
L $5 - (3 + 2)$ **M** $(8 - 5) \times 4$ **O** $(7 + 1) \times 2$ **T** $10 - (8 - 4)$

									!
32	0	0	16	5	6	24	8	12	

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Foundation Worksheet

3:03 | Language and Symbols

Name: _____ **Class:** _____

Examples

1 True or false? $16 < 3 \times 7$
This means 16 is less than 3×7.
ie $16 < 21$
$\therefore$ True

2 True or false? $6 \times 3 = 10 + 6$
This means 6×3 is equal to $10 + 6$.
ie $18 = 16$
$\therefore$ False

3 Write using symbols.
The sum of 12 and 8
$12 + 8$

4 Write using symbols.
5 times 6 is greater than 20
$5 \times 6 > 20$

Exercise

1 True or false?

a $3 = 5 \times 2$ **b** $8 > 5$ **c** $10 - 2 < 3 \times 5$

d $7 \times 4 = 24$ **e** $12 < 15$ **f** $10 \div 2 \neq 6 - 3$

2 Use symbols to write these.

a 5 is less than 6 **b** 10 is greater than 4

c 12 minus 5 equals 7 **d** 4 times 3 is greater than 7

e 10 is not equal to 4 times 3 **f** the sum of 6 and 3

Fun Spot 3:03 | Why do cats make the best pets?



Work out each answer. Put the letter for each part in the box above the correct answer.

C 10 less 4 **E** product of 2 and 7 **F** sum of 3 and 7

H decrease 8 by 6 **P** increase 6 by 9 **R** sum of 2, 3, 4

T divide 10 by 2 **U** average of 6 and 10 **Y** 6 times 4

				,											
5	2	14	24		9	14		15	8	9	9	10	14	6	5

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Foundation Worksheet

3:07 | Divisibility Tests

Name: _____ **Class:** _____

- An *even* number is divisible by 2 and ends in 0, 2, 4, 6 or 8.
- An *odd* number ends with an odd digit: 1, 3, 5, 7 or 9.
- A number is *divisible by 3* if the total of its digits is divisible by 3:
  $7320 \rightarrow 7 + 3 + 2 + 0 = 12$ which is divisible by 3.
- A number is *divisible by 5* if it ends in a 0 or 5.

Exercise

1 **a** Write the first 10 even numbers. _____

b Write the first 10 odd numbers. _____

c Write the first 7 numbers divisible by 5. _____
These are called multiples of 5.

2 Write the first 10 multiples of the following.

a 2 _____

b 3 _____

c 4 _____

d 10 _____

e 6 _____

f 7 _____

g 8 _____

h 9 _____

3 Write the numbers below 30 that are divisible by the following.

a 2 _____

b 3 _____

c 10 _____

4 **a** Circle the odd numbers.
15 48 116 5030 54 321

b Circle the numbers divisible by 5.
40 554 125 75 052 100 000

c Circle the numbers divisible by 3.
12 201 755 5333 54 321 12 345

d Circle the numbers divisible by both 2 and 3.
24 8 1216 42 66 888

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Foundation Worksheet

3:08 | Square and Cube Roots

Name: _____ **Class:** _____

Examples

The square root of a number is squared to give that number.

1 $9 = 3 \times 3$
$\therefore \sqrt{9} = 3$

2 $6 \times 6 = 36$
$\therefore \sqrt{36} = 6$

Write $6 \times 6 = 6^2$, for short, and say '6 squared'.

Similarly, a cube root is cubed to give the number.

3 $7 \times 7 \times 7 = 343$
$\therefore \sqrt[3]{343} = 7$

Write $7 \times 7 \times 7 = 7^3$, and say '7 cubed'.

4 If $1000 = 10 \times 10 \times 10$, then $\sqrt[3]{1000} = 10$

Exercise

1 a If $4 \times 4 = 16$, then $\sqrt{16} =$ ☐

b If $2 \times 2 = 4$, then $\sqrt{4} =$ ☐

c If $5 \times 5 = 25$, then $\sqrt{25} =$ ☐

d If $9 \times 9 = 81$, then $\sqrt{81} =$ ☐

e $121 = 11 \times 11$, so $\sqrt{121} =$ ☐

f $400 = 20 \times 20$, so $\sqrt{400} =$ ☐

2 a If $7^2 = 49$, $\sqrt{49} =$ ☐

b If $8^2 = 64$, then $\sqrt{64} =$ ☐

c $100 = 10^2$, so $\sqrt{100} =$ ☐

d $144 = 12^2$, so $\sqrt{144} =$ ☐

e If $1^2 = 1$, then $\sqrt{1} =$ ☐

f If $15^2 = 225$, then $\sqrt{225} =$ ☐

3 a If $5 \times 5 \times 5 = 125$, then $\sqrt[3]{125} =$ ☐

b If $3 \times 3 \times 3 = 27$, then $\sqrt[3]{27} =$ ☐

c If $4 \times 4 \times 4 = 64$, then $\sqrt[3]{64} =$ ☐

d If $729 = 9 \times 9 \times 9$, then $\sqrt[3]{729} =$ ☐

4 a If $6^3 = 216$, then $\sqrt[3]{216} =$ ☐

b If $2^3 = 8$, then $\sqrt[3]{8} =$ ☐

c Since $512 = 8^3$, $\sqrt[3]{512} =$ ☐

d Since $27\,000 = 30^3$, $\sqrt[3]{27\,000} =$ ☐

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Foundation Worksheet

4:01 | Review of Decimals

Name: _____ **Class:** _____

Examples

1 Write as a decimal.

a $\frac{5}{10}$ $= 0{\cdot}5$

b $\frac{8}{100}$ $= 0{\cdot}08$

c $\frac{17}{100}$ $= 0{\cdot}17$

d $\frac{273}{1000}$ $= 0{\cdot}273$

- 1st decimal place is tenths
  2nd decimal place is hundredths
  3rd decimal place is thousandths.
- Fill in empty spaces with zero.
- Simplify fractions.

2 Write as a fraction.

a $0{\cdot}9 = \frac{9}{10}$

b $0{\cdot}01 = \frac{1}{100}$

c $0{\cdot}42 = \frac{42}{100} \begin{pmatrix} \div 2 \\ \div 2 \end{pmatrix} = \frac{21}{50}$

d $0{\cdot}015 = \frac{15}{1000} \begin{pmatrix} \div 5 \\ \div 5 \end{pmatrix} = \frac{3}{200}$

3 Which is larger?

a $0{\cdot}6$ or $0{\cdot}027$
$0{\cdot}600$ or $0{\cdot}027$
$\therefore 0{\cdot}600 = 0{\cdot}6$

b $0{\cdot}37$ or $0{\cdot}5$
$0{\cdot}37$ or $0{\cdot}50$
$\therefore 0{\cdot}50 = 0{\cdot}5$

c $0{\cdot}154$ or $0{\cdot}54$
$0{\cdot}154$ or $0{\cdot}540$
$\therefore 0{\cdot}540 = 0{\cdot}54$

Write each decimal with the same number of places.

Exercise

1 Write as a decimal.

a $\frac{3}{10}$ **b** $\frac{7}{100}$ **c** $\frac{93}{100}$ **d** $\frac{6}{10}$ **e** $\frac{4}{1000}$ **f** $\frac{133}{1000}$

g $\frac{1}{10}$ **h** $\frac{73}{100}$ **i** $\frac{43}{100}$ **j** $\frac{24}{1000}$ **k** $\frac{92}{100}$ **l** $\frac{28}{100}$

m $\frac{36}{100}$ **n** $\frac{555}{1000}$ **o** $\frac{2}{1000}$ **p** $\frac{99}{100}$ **q** $\frac{31}{1000}$ **r** $\frac{607}{1000}$

2 Write as a fraction.

a $0{\cdot}7$ **b** $0{\cdot}03$ **c** $0{\cdot}53$ **d** $0{\cdot}09$ **e** $0{\cdot}123$ **f** $0{\cdot}091$

g $0{\cdot}13$ **h** $0{\cdot}59$ **i** $0{\cdot}563$ **j** $0{\cdot}21$ **k** $0{\cdot}303$ **l** $0{\cdot}299$

m $0{\cdot}06$ **n** $0{\cdot}4$ **o** $0{\cdot}25$ **p** $0{\cdot}64$ **q** $0{\cdot}005$ **r** $0{\cdot}014$

3 Which is larger?

a $0{\cdot}7$ or $0{\cdot}08$ **b** $0{\cdot}67$ or $0{\cdot}71$ **c** $0{\cdot}09$ or $0{\cdot}3$ **d** $0{\cdot}83$ or $0{\cdot}115$

e $0{\cdot}51$ or $0{\cdot}099$ **f** $0{\cdot}12$ or $0{\cdot}104$ **g** $0{\cdot}63$ or $0{\cdot}5$ **h** $0{\cdot}3$ or $0{\cdot}007$

i $0{\cdot}413$ or $0{\cdot}43$ **j** $0{\cdot}6$ or $0{\cdot}92$ **k** $0{\cdot}402$ or $0{\cdot}68$ **l** $0{\cdot}40$ or $0{\cdot}5$

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Foundation Worksheet

4:02 | Addition and Subtraction of Decimals

Name: _____ **Class:** _____

Examples

- Line up the decimal points under each other.
- For whole numbers put the point at the end.
- Fill in spaces with zeros.

1 2·3 + 5·4

$$\begin{array}{r} 2{\cdot}3 \\ 5{\cdot}4 \\ \hline 7{\cdot}7 \end{array}$$

2 9·7 − 3·24

$$\begin{array}{r} 9{\cdot}70 \\ -3{\cdot}24 \\ \hline 6{\cdot}46 \end{array}$$

3 18 + 2·3 + 1·76

$$\begin{array}{r} 18{\cdot}00 \\ 2{\cdot}30 \\ 1{\cdot}76 \\ \hline 22{\cdot}06 \end{array}$$

4 7·674 − 3·26

$$\begin{array}{r} 7{\cdot}674 \\ -3{\cdot}260 \\ \hline 4{\cdot}414 \end{array}$$

Exercise

1 a $\begin{array}{r} 6{\cdot}4 \\ 0{\cdot}2 \end{array}$ **b** $\begin{array}{r} 2{\cdot}6 \\ 6{\cdot}3 \end{array}$ **c** $\begin{array}{r} 3{\cdot}74 \\ 4{\cdot}1 \end{array}$ **d** $\begin{array}{r} 0{\cdot}362 \\ 3{\cdot}504 \end{array}$ **e** $\begin{array}{r} 4{\cdot}04 \\ 3{\cdot}5 \end{array}$

f $\begin{array}{r} 3{\cdot}8 \\ 2{\cdot}73 \end{array}$ **g** $\begin{array}{r} 8{\cdot}26 \\ 9{\cdot}47 \end{array}$ **h** $\begin{array}{r} 12{\cdot}2 \\ 5{\cdot}9 \end{array}$ **i** $\begin{array}{r} 8{\cdot}45 \\ 2{\cdot}718 \end{array}$ **j** $\begin{array}{r} 7{\cdot}623 \\ 6{\cdot}9 \end{array}$

2 a $\begin{array}{r} 4{\cdot}7 \\ -1{\cdot}1 \end{array}$ **b** $\begin{array}{r} 6{\cdot}9 \\ -2{\cdot}7 \end{array}$ **c** $\begin{array}{r} 17{\cdot}23 \\ -5{\cdot}1 \end{array}$ **d** $\begin{array}{r} 2{\cdot}59 \\ -0{\cdot}32 \end{array}$ **e** $\begin{array}{r} 5 \\ -3{\cdot}2 \end{array}$

f $\begin{array}{r} 3{\cdot}64 \\ -2{\cdot}8 \end{array}$ **g** $\begin{array}{r} 19{\cdot}81 \\ -2{\cdot}08 \end{array}$ **h** $\begin{array}{r} 3{\cdot}86 \\ -2{\cdot}77 \end{array}$ **i** $\begin{array}{r} 4{\cdot}2 \\ -3{\cdot}641 \end{array}$ **j** $\begin{array}{r} 9{\cdot}7 \\ -3{\cdot}82 \end{array}$

3 a 3·2 + 0·1 **b** 7·2 + 2·3 **c** 5·9 − 2·3 **d** 18 − 2·32

e 18 + 2·32 + 3·6 **f** 9·5 − 6·375 **g** 8·13 − 6·4 **h** 15·6 − 4·78

i 14·2 + 8·64 + 1·65 **j** 3·1 + 6·26 + 9·5

Fun Spot 4:02 | What is the cure for water on the brain?

Match the letters for these sums to the answers below.

A 2·05 + 2·343 **D** 0·243 + 1·87 **E** 0·63 + 2·442 **H** 0·007 + 2·174

N 0·02 + 1·843 **O** 1·7 + 1·63 **P** 1·03 + 3·96 **T** 2·45 + 1·88

4·393	4·33	4·393	4·99	3·33	1·863	4·33	2·181	3·072	2·181	3·072	4·393	2·113

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Foundation Worksheet

4:03 | Multiplying a Decimal

Name: _____ **Class:** _____

Examples

- When you multiply, there are the same number of figures after the decimal point in the question and answer.
- When multiplying by 10, 100 or 1000, move the decimal point 1, 2 or 3 places to the right.

1 $2{\cdot}7 \times 3$

$$\begin{array}{r} 2{\cdot}7 \\ \times\ 3 \\ \hline 8{\cdot}1 \end{array}$$

2 $4{\cdot}23 \times 5$

$$\begin{array}{r} 4{\cdot}23 \\ \times\ 5 \\ \hline 21{\cdot}15 \end{array}$$

3 $1{\cdot}894 \times 10$

$$\begin{array}{r} 1{\cdot}894 \\ \times\ 10 \\ \hline 18{\cdot}94 \end{array}$$

4 $8 \times 3{\cdot}24$

$$\begin{array}{r} 3{\cdot}24 \\ \times\ 8 \\ \hline 25{\cdot}92 \end{array}$$

5 $18{\cdot}9 \times 100$

$$\begin{array}{r} 18{\cdot}9 \\ \times\ 100 \\ \hline 18\,90{\cdot}0 \end{array}$$

Exercise

1 a $0{\cdot}3 \times 2$ **b** $0{\cdot}6 \times 7$ **c** $0{\cdot}59 \times 3$ **d** $4{\cdot}71 \times 5$ **e** $4{\cdot}8 \times 6$ **f** $17{\cdot}3 \times 4$

g $9{\cdot}2 \times 10$ **h** $7{\cdot}65 \times 8$ **i** $0{\cdot}234 \times 100$ **j** $3{\cdot}21 \times 7$ **k** $0{\cdot}08 \times 4$ **l** $0{\cdot}009 \times 7$

2 a $3 \times 0{\cdot}4$ **b** $5 \times 0{\cdot}63$ **c** $7 \times 0{\cdot}9$ **d** $11 \times 0{\cdot}02$

e $1{\cdot}5 \times 7$ **f** $3{\cdot}61 \times 3$ **g** $4{\cdot}76 \times 2$ **h** $13{\cdot}3 \times 5$

i $2{\cdot}3 \times 9$ **j** $0{\cdot}84 \times 6$ **k** $0{\cdot}027 \times 4$ **l** $5{\cdot}9 \times 2$

3 a $0{\cdot}6 \times 10$ **b** $5{\cdot}2 \times 10$ **c** $0{\cdot}4 \times 100$ **d** $0{\cdot}07 \times 10$

e $10 \times 3{\cdot}54$ **f** $100 \times 3{\cdot}7$ **g** $16{\cdot}6 \times 100$ **h** $0{\cdot}42 \times 100$

i $1000 \times 0{\cdot}414$ **j** $8{\cdot}1 \times 100$ **k** $10 \times 0{\cdot}032$ **l** $1000 \times 0{\cdot}02$

Fun Spot 4:03 | Comment from student to teacher



Complete these products to decode the message.

A $0{\cdot}27 \times 5$ **C** $1{\cdot}41 \times 2$ **D** $10 \times 0{\cdot}12$ **E** $4 \times 0{\cdot}31$ **F** $0{\cdot}012 \times 10$

G $0{\cdot}05 \times 100$ **H** $0{\cdot}5 \times 100$ **I** $0{\cdot}115 \times 10$ **L** $1{\cdot}3 \times 3$ **M** $2{\cdot}12 \times 2$

N $7 \times 0{\cdot}58$ **O** $4 \times 0{\cdot}11$ **P** $8 \times 0{\cdot}51$ **S** $0{\cdot}08 \times 5$ **T** $0{\cdot}063 \times 3$

1·15	1·2 0·44 4·06 0·189	5 1·24 0·189	0·189 50 1·24

4·08 0·44 1·15 4·06 0·189	0·44 0·12	1·2 1·24 2·82 1·15 4·24 1·35 3·9 0·4	

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Foundation Worksheet

4:04 | Dividing a Decimal

Name: _____ **Class:** _____

Examples

When dividing a decimal by a whole number, place the point in the answer above the point in the question.

1 $0{\cdot}9 \div 3$

$$3\overline{)0{\cdot}9} \quad 0{\cdot}3$$

2 $0{\cdot}48 \div 4$

$$4\overline{)0{\cdot}48} \quad 0{\cdot}12$$

3 $5{\cdot}2 \div 4$

$$4\overline{)5{\cdot}2} \quad 1{\cdot}3$$

4 $4{\cdot}84 \div 2$

$$2\overline{)4{\cdot}84} \quad 2{\cdot}42$$

5 $3{\cdot}06 \div 3$

$$3\overline{)3{\cdot}06} \quad 1{\cdot}02$$

Exercise

1 **a** $2\overline{)0{\cdot}8}$ **b** $6\overline{)0{\cdot}6}$ **c** $4\overline{)0{\cdot}8}$ **d** $7\overline{)0{\cdot}07}$ **e** $3\overline{)0{\cdot}03}$

f $5\overline{)0{\cdot}05}$ **g** $2\overline{)0{\cdot}6}$ **h** $3\overline{)0{\cdot}09}$ **i** $4\overline{)0{\cdot}08}$ **j** $2\overline{)0{\cdot}4}$

2 **a** $3\overline{)0{\cdot}69}$ **b** $3\overline{)0{\cdot}36}$ **c** $5\overline{)0{\cdot}45}$ **d** $7\overline{)0{\cdot}77}$ **e** $4\overline{)0{\cdot}52}$

f $2\overline{)0{\cdot}26}$ **g** $8\overline{)0{\cdot}24}$ **h** $6\overline{)0{\cdot}18}$ **i** $4\overline{)0{\cdot}64}$ **j** $3\overline{)0{\cdot}93}$

3 **a** $5\overline{)2{\cdot}5}$ **b** $3\overline{)6{\cdot}3}$ **c** $4\overline{)6{\cdot}8}$ **d** $5\overline{)5{\cdot}5}$ **e** $2\overline{)8{\cdot}6}$

f $2\overline{)3{\cdot}6}$ **g** $6\overline{)7{\cdot}2}$ **h** $3\overline{)9{\cdot}3}$ **i** $2\overline{)4{\cdot}6}$ **j** $4\overline{)8{\cdot}8}$

4 **a** $7\overline{)7{\cdot}14}$ **b** $3\overline{)6{\cdot}36}$ **c** $8\overline{)8{\cdot}08}$ **d** $2\overline{)8{\cdot}26}$ **e** $5\overline{)5{\cdot}75}$

f $6\overline{)2{\cdot}46}$ **g** $3\overline{)1{\cdot}26}$ **h** $4\overline{)4{\cdot}48}$ **i** $9\overline{)9{\cdot}27}$ **j** $7\overline{)7{\cdot}28}$

Fun Spot 4:04 | How is Dorothy the Dinosaur like Conan the Barbarian?



Complete these divisions then match the letters with the answers.

A $2{\cdot}62 \div 2$ **D** $3{\cdot}9 \div 3$ **E** $5{\cdot}6 \div 2$ **H** $0{\cdot}85 \div 5$

I $4{\cdot}08 \div 4$ **L** $0{\cdot}65 \div 5$ **M** $0{\cdot}04 \div 4$ **N** $4{\cdot}8 \div 2$

S $8{\cdot}22 \div 2$ **T** $6{\cdot}6 \div 6$ **V** $7{\cdot}35 \div 5$ **Y** $0{\cdot}84 \div 3$

1·1	0·17	2·8	0·28

0·17	1·31	1·47	2·8

1·1	0·17	2·8

4·11	1·31	0·01	2·8

0·01	1·02	1·3	1·3	0·13	2·8

				!
2·4	1·31	0·01	2·8	

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Foundation Worksheet

4:05 | Using Decimals

Name: _____ **Class:** _____

- Follow the usual rules for decimals.
- Put the unit of measurement in your answers.

Exercise

1 **a** $2 \times 0{\cdot}3$ kg **b** $3 \times \$2.24$ **c** $7{\cdot}3$ cm $\times 6$
d $8{\cdot}22$ L $\times 5$ **e** $\$4.06 \times 4$ **f** $0{\cdot}9$ t $\times 6$
g $3{\cdot}26$ m $\times 7$ **h** $0{\cdot}68$ g $\times 4$ **i** $3 \times 5{\cdot}07$ h
j $8 \times \$3.17$ **k** $0{\cdot}04 \times 6$ **l** $8{\cdot}4$ L $\times 9$

2 **a** $0{\cdot}8$ m + $0{\cdot}3$ m **b** \$1.57 + \$3.24 **c** \$7.20 – \$3.40
d $1{\cdot}7$ cm + $8{\cdot}3$ cm **e** $14{\cdot}4$ t – $3{\cdot}67$ t **f** $0{\cdot}72$ kg + $0{\cdot}255$ kg
g $7{\cdot}501$ kg – $3{\cdot}26$ kg **h** \$7.67 + \$2.94 **i** $0{\cdot}6$ m – $0{\cdot}34$ m
j $7{\cdot}2$ mL + $2{\cdot}66$ mL **k** \$3.74 – \$1.47 **l** $3{\cdot}9$ m + $4{\cdot}73$ m

3 **a** $3\overline{)1{\cdot}8\text{ L}}$ **b** $5\overline{)\$2.35}$ **c** $4\overline{)6{\cdot}28\text{ m}}$
d $2\overline{)0{\cdot}54\text{ kg}}$ **e** $9\overline{)\$9.18}$ **f** $3\overline{)6{\cdot}96\text{ mL}}$
g $6\overline{)0{\cdot}246\text{ m}}$ **h** $7\overline{)0{\cdot}77\text{ t}}$ **i** $3\overline{)6{\cdot}93\text{ cm}}$
j $5\overline{)0{\cdot}405\text{ g}}$ **k** $4\overline{)\$6.88}$ **l** $6\overline{)4{\cdot}8\text{ m}}$

Fun Spot 4:05 | What is grey, has four legs, a tail and a trunk?

Match the calculated answers with the letters to complete the riddle.

A $\$3.42 \times 3$ **D** \$4.26 + \$5.37 **E** \$6.20 + \$2.40 **G** $5\overline{)\$15.05}$
H \$12.35 – \$2.60 **I** $\$5.07 \times 4$ **L** \$10 – \$1.55 **M** \$7 + \$2.25
N \$8 – \$3.80 **O** $4\overline{)\$4.48}$ **S** $6 \times \$2.35$ **U** $8 \times \$1.60$
Y $3\overline{)\$6.27}$

\$10.26	\$9.25	\$1.12	\$12.80	\$14.10	\$8.60	\$3.01	\$1.12	\$20.28	\$4.20	\$3.01				

\$1.12	\$4.20	\$9.75	\$1.12	\$8.45	\$20.28	\$9.63	\$10.26	\$2.09						

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Foundation Worksheet

4:06 | Multiplying Decimals

Name: _____ **Class:** _____

Examples

- **Multiply the numbers ignoring the decimal point.**
- **The number of figures after the decimal point is the same in the answer as the total number in the question.**

1 $0{\cdot}63 \times 4$
$63 \times 4 = 252$
2 figures after the point
$\therefore 2{\cdot}52$

2 $3{\cdot}27 \times 0{\cdot}2$
$327 \times 2 = 654$
3 figures after the point
$\therefore 0{\cdot}654$

3 $4{\cdot}3 \times 0{\cdot}04$
$43 \times 4 = 172$
3 figures after the point
$\therefore 0{\cdot}172$

4 $0{\cdot}6 \times 0{\cdot}9$
$6 \times 9 = 54$
2 figures after the point
$\therefore 0{\cdot}54$

5 $4{\cdot}26 \times 0{\cdot}07$
$426 \times 7 = 2982$
4 figures after the point
$\therefore 0{\cdot}2982$

Exercise

1 **a** $0{\cdot}41 \times 2$ **b** $0{\cdot}27 \times 3$ **c** $1{\cdot}4 \times 6$ **d** $3{\cdot}7 \times 5$ **e** $0{\cdot}234 \times 5$ **f** $4{\cdot}15 \times 7$

g $0{\cdot}83 \times 2$ **h** $0{\cdot}012 \times 3$ **i** $0{\cdot}075 \times 5$ **j** $6{\cdot}1 \times 4$ **k** $3{\cdot}04 \times 7$ **l** $0{\cdot}38 \times 2$

2 **a** $0{\cdot}4 \times 0{\cdot}7$ **b** $0{\cdot}2 \times 0{\cdot}3$ **c** $0{\cdot}5 \times 0{\cdot}07$ **d** $5 \times 0{\cdot}01$

e $0{\cdot}8 \times 0{\cdot}5$ **f** $0{\cdot}4 \times 9$ **g** $0{\cdot}03 \times 0{\cdot}9$ **h** $0{\cdot}06 \times 0{\cdot}08$

i $0{\cdot}3 \times 0{\cdot}3$ **j** $0{\cdot}5 \times 0{\cdot}5$ **k** $0{\cdot}004 \times 0{\cdot}8$ **l** $0{\cdot}3 \times 0{\cdot}02$

3 **a** $7{\cdot}2 \times 0{\cdot}3$ **b** $2{\cdot}5 \times 0{\cdot}5$ **c** $1{\cdot}2 \times 0{\cdot}6$ **d** $0{\cdot}34 \times 0{\cdot}4$ **e** $0{\cdot}61 \times 0{\cdot}7$ **f** $0{\cdot}22 \times 0{\cdot}3$

4 **g** $52 \times 0{\cdot}01$ **h** $0{\cdot}26 \times 0{\cdot}02$ **i** $0{\cdot}014 \times 0{\cdot}06$ **j** $0{\cdot}73 \times 0{\cdot}2$ **k** $7{\cdot}9 \times 0{\cdot}3$ **l** $0{\cdot}48 \times 0{\cdot}5$

Fun Spot 4:06 | Can a match box?



Answer each question then match the letters with the answer to solve the riddle.

A $0{\cdot}5 \times 3$ **B** $0{\cdot}4 \times 0{\cdot}3$ **C** $0{\cdot}2 \times 0{\cdot}06$ **I** $0{\cdot}3 \times 0{\cdot}5$

N $0{\cdot}03 \times 0{\cdot}5$ **O** $0{\cdot}6 \times 2$ **T** $0{\cdot}04 \times 0{\cdot}03$ **U** $0{\cdot}03 \times 0{\cdot}05$

		.												!
0·015	1·2		0·12	0·0015	0·0012		1·5		0·0012	0·15	0·015		0·012	1·5	0·015

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Foundation Worksheet

4:09 | Rounding off

Name: _____ **Class:** _____

Examples

To round off to the nearest whole number, check the first decimal place.
If it is 5 or more, 'round up' (add 1 to the whole numbers).
If it is less than 5, 'round down' (leave the whole number).

1 43·7c
First place is more than 5.
Round up (add 1).
∴ 44c

2 $3.16
First place is less than 5.
Round down (leave).
∴ $3

3 17·2c
First place is less than 5.
Round down.
∴ 17c

4 $8.84
First place is more than 5.
Round up.
∴ $9

Exercise

1 Round off to the nearest cent.

a 84·6c	**b** 7·9c	**c** 60·5c	**d** 19·3c	**e** 39·9c	**f** 53·3c
g 3·1c	**h** 45·7c	**i** 26·5c	**j** 30·2c	**k** 88·4c	**l** 14·4c
m 9·5c	**n** 40·2c	**o** 70·7c	**p** 17·5c	**q** 59·8c	**r** 5·5c

2 Round off to the nearest dollar.

a $7.63	**b** $9.24	**c** $3.90	**d** $4.08	**e** $2.03	**f** $1.92
g $8.49	**h** $8.88	**i** $11.80	**j** $6.30	**k** $5.72	**l** $2.38
m $3.50	**n** $10.05	**o** $5.38	**p** $9.90	**q** $6.87	**r** $4.44

Fun Spot 4:09 | Ever heard of this book?

Calculate the answers. Match each letter with the answer below.

A 1·2 + 2·5	**B** 3·8 ÷ 2	**C** 0·7 × 4	**D** 7 − 2·3
E 0·64 + 0·7	**F** 5·3 × 0·3	**H** 2·48 ÷ 4	**I** 3·7 − 2·6
N 2·73 + 1·64	**R** 2·91 × 0·3	**U** 0·65 ÷ 5	**Y** 0·23 × 0·6

'[ ][ ][ ][ ] [ ][ ][ ][ ]'

0·62 3·7 1·1 0·873 2·8 3·7 0·873 1·34

[ ][ ] [ ][ ][ ] [ ][ ][ ][ ][ ]

1·9 0·138 4·7 3·7 4·37 4·7 0·873 0·13 1·59 1·59

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Foundation Worksheet

4:10 | Application of Decimals

Name: _____ **Class:** _____

Examples

1 51c → 50c **2** $2.32 → $2.30

3 74c → 75c **4** $1.23 → $1.25

5 36c → 35c **6** $5.47 → $5.45

7 49c → 50c **8** $4.88 → $4.90

When rounding to the nearest 5 cents:
- 1c or 2c are rounded down to 0
- 3c or 4c are rounded up to 5
- 6c or 7c are rounded down to 5
- 8c or 9c are rounded up to 10.

Exercise

1 Round off to the nearest dollar.

a $105.56 **b** $89.73 **c** $4.08 **d** $91.48 **e** $24.84

f $66.70 **g** $13.41 **h** $3.29 **i** $7.15 **j** $39.81

2 Round off to the nearest 5 cents.

a 73c **b** 87c **c** 29c **d** 42c **e** 58c

f 12c **g** 64c **h** 23c **i** 96c **j** 7c

3 Round off to the nearest 5 cents.

a $3.98 **b** $4.23 **c** $1.71 **d** $5.58 **e** $3.08

f $7.62 **g** $2.93 **h** $3.33 **i** $8.11 **j** $2.34

Fun Spot 4:10B | Have you read this book?

Complete these calculations. Match the letters with the answers below.

A 0·7 + 0·3 **B** 1·5 − 0·8 **C** 0·25 + 0·65

E 3·7 × 0·2 **F** 6·4 ÷ 2 **H** 2·7 − 1·3

I 0·2 × 0·8 **L** 4 − 2·7 **N** 0·62 + 0·34

O 10 × 3·46 **R** 0·32 × 5 **S** 0·63 × 0·3

U 100 × 4·27 **W** 3·8 + 4·9 **Y** 0·5 × 0·5

‘ | | | | | | | | | ’

1·4 427 1·6 1·6 0·16 0·9 1 0·96 0·74

0·7	0·25

1·6	427	3·2	427	0·189

0·7	1·3	34·6	8·7	0·96	34·6	3·2	3·2

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Foundation Worksheet

5:10 | Division of Directed Numbers

Name: _____ **Class:** _____

Examples

1 $-10 \div 2$
$= -10 \div (+2)$
$= -5$

2 $20 \div (-10)$
$= +20 \div (-10)$
$= -2$

3 $\frac{-18}{-3} = +6$
$= 6$

When dividing:
- **two like signs give a plus**
- **two unlike signs give a minus**

Having no sign is the same as a +.

Exercise

1 **a** $-8 \div 2$ **b** $-12 \div 3$ **c** $-14 \div 7$ **d** $-18 \div 6$ **e** $-9 \div 3$
f $-5 \div 5$ **g** $-16 \div 4$ **h** $-6 \div 3$ **i** $-20 \div 10$ **j** $-15 \div 5$

2 **a** $60 \div (-10)$ **b** $-4 \div (-2)$ **c** $-12 \div (-6)$ **d** $7 \div (-7)$ **e** $30 \div (-3)$
f $-24 \div (-6)$ **g** $21 \div (-3)$ **h** $18 \div (-9)$ **i** $-30 \div (-6)$ **j** $-8 \div (-1)$
k $-40 \div (-5)$ **l** $40 \div (-10)$ **m** $25 \div (-5)$ **n** $-36 \div (-4)$ **o** $-26 \div (-2)$

3 **a** $\frac{-15}{3}$ **b** $\frac{-10}{-2}$ **c** $\frac{-24}{-8}$ **d** $\frac{18}{-3}$ **e** $\frac{13}{-13}$
f $\frac{-22}{-11}$ **g** $\frac{-36}{-6}$ **h** $\frac{36}{-3}$ **i** $\frac{-45}{5}$ **j** $\frac{-7}{1}$
k $\frac{-12}{2}$ **l** $\frac{16}{-8}$ **m** $\frac{-100}{-25}$ **n** $\frac{50}{-5}$ **o** $\frac{-33}{3}$

Fun Spot 5:10 | What did Jack Frost say to the thermometer?

Answer each of these and match the letters with the answers below.

A $10 \div (-2)$ **B** $-3 - 6$ **C** $5 - (-4)$ **E** $8 + (-4)$ **G** $5 \times (-3)$
I $-7 \times (-3)$ **M** $-18 \div 3$ **N** $-3 + 8$ **O** $-3 - 8$ **R** $16 \div (-2)$
S $-20 \div (-2)$ **T** -3×4 **U** $-10 + 10$ **V** $-6 - (-6)$ **Y** $-6 + 4$

−2 −11 0	−9 4 9 −11 −6 4	12 4 −8 −2

5 4 −15 −5 −12 21 12 4	10 −11 −6 4 −12 21 −6 4 10

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Foundation Worksheet

5:11 | Using Directed Numbers

Name: _____ **Class:** _____

Examples

Working from left to right
do ( )
then × and ÷
then + and –

1 $8 - 2 - 10$
$= 6 - 10$
$= -4$

2 $5 - 4 \times 3$
$= 5 - 12$
$= -7$

3 $9 - (5 - 3)$
$= 9 - 2$
$= 7$

4 $5 + (-4)^2$
$= 5 + 16$
$= 21$

Exercise

1 **a** $2 \times 2 - 6$ **b** $5 - 3 - 3$ **c** $8 - (-3) - 2$ **d** $8 \div (-2) + 3$
e $-10 + 8 - 5$ **f** $-3 \times 5 + 8$ **g** $14 + (-5) - (-2)$ **h** $10 \times -4 + 5$
i $-6 - 7 - 8$ **j** $-1 + 4 - (-3)$ **k** $-3 \times -2 - 4$ **l** $3 \times -7 - 2$

2 **a** $(-3)^2 - 1$ **b** $-1 + 2 \times 3$ **c** $-8 - 18 \div 3$ **d** $4 - (-2)^2$
e $5 - 2 \times 3$ **f** $6 + (-2 + 4)$ **g** $20 - 4 \times (-2)$ **h** $-5 + (-7 - 3)$
i $12 - 12 \div 4$ **j** $-6 - (10 - 4)$ **k** $16 - 5^2$ **l** $-10 - (3 + 5)$

3 Complete these patterns.
a 3, 2, 1, 0, –1, . . ., . . ., ... **b** 0, –3, –6, –9, . . ., . . ., ...
c 15, 10, 5, 0, . . ., . . ., ... **d** –7, –4, –1, 2, . . ., . . ., ...
e 10, –20, 30, –40, . . ., . . ., ... **f** –20, –16, –12, –8, . . ., . . ., ...

Fun Spot 5:11 | Did you hear about the silly sailor who was discharged from the submarine service?



Answer these and match the letters below.

D $(-1)^2$ **E** $-6 + 4$ **H** $-4 \times (-7)$ **I** $9 - 12$
L $-6 \div (-1)$ **N** $2 - 9 + 3$ **O** $5 - (-3)$ **P** $-2 + 5 - 4$
S $3 - 2 \times 6$ **T** $2 - 3^2$ **W** $-3 - 5$

28 –2	–9 6 –2 –1 –7	–8 –3 –7 28	–7 28 –2

–8 –3 –4 1 8 –8	8 –1 –2 –4 .

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Foundation Worksheet

6:04 | Addition and Subtraction of Fractions

Name: ______________________________ Class: ______________

Examples

1 $\frac{3}{10} + \frac{4}{10} = \frac{7}{10}$

2 $\frac{9}{13} - \frac{4}{13} = \frac{5}{13}$

3 $\frac{2}{7} + \frac{1}{7} = \frac{3}{7}$

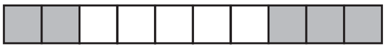
4 $\frac{8}{15} - \frac{3}{15} = \frac{5}{15}$ *Simplify by ÷5*

$= \frac{1}{3}$

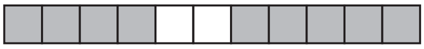
Add or subtract the numerators (tops) of fractions that have the same denominator (bottoms).

Exercise

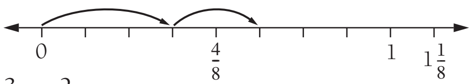
Use the diagrams to calculate the following.

1 **a** 

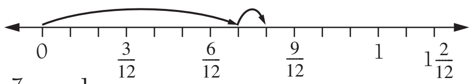
$\frac{2}{10} + \frac{3}{10}$

b 

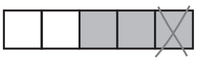
$\frac{4}{11} + \frac{5}{11}$

2 **a** 

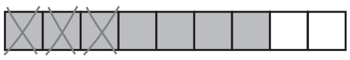
$\frac{3}{8} + \frac{2}{8}$

b 

$\frac{7}{12} + \frac{1}{12}$

3 **a** 

$\frac{3}{5} - \frac{1}{5}$

b 

$\frac{7}{9} - \frac{3}{9}$

4 Write the simplest answer.

a $\frac{2}{8} + \frac{1}{8}$	**b** $\frac{7}{8} - \frac{6}{8}$	**c** $\frac{7}{10} - \frac{4}{10}$	**d** $\frac{13}{20} + \frac{6}{20}$	**e** $\frac{3}{5} - \frac{1}{5}$
f $\frac{7}{15} + \frac{4}{15}$	**g** $\frac{9}{11} - \frac{2}{11}$	**h** $\frac{23}{100} + \frac{4}{100}$	**i** $\frac{1}{5} + \frac{2}{5}$	**j** $\frac{1}{8} + \frac{6}{8}$
k $\frac{3}{13} + \frac{7}{13}$	**l** $\frac{8}{25} + \frac{9}{25}$	**m** $\frac{16}{25} - \frac{2}{25}$	**n** $\frac{9}{20} + \frac{2}{20}$	**o** $\frac{9}{10} - \frac{2}{10}$
p $\frac{8}{15} - \frac{6}{15}$	**q** $\frac{57}{100} - \frac{20}{100}$	**r** $\frac{19}{40} + \frac{10}{40}$	**s** $\frac{4}{12} + \frac{7}{12}$	**t** $\frac{70}{100} + \frac{13}{100}$
u $\frac{3}{10} + \frac{2}{10}$	**v** $\frac{9}{16} - \frac{5}{16}$	**w** $\frac{1}{12} + \frac{3}{12}$	**x** $\frac{7}{20} - \frac{2}{20}$	**y** $\frac{7}{8} - \frac{1}{8}$

Fun Spot 6:04 | Brother: 'Did you just take a shower?'

Simplify each fraction by dividing top and bottom. Match the letters with the answers below.

E $\frac{2}{4}$ **G** $\frac{2}{12}$ **H** $\frac{4}{10}$ **I** $\frac{5}{20}$ **M** $\frac{2}{20}$ **N** $\frac{6}{8}$ **O** $\frac{8}{12}$ **R** $\frac{20}{100}$ **S** $\frac{10}{18}$ **T** $\frac{3}{9}$ **W** $\frac{6}{20}$ **Y** $\frac{15}{25}$

$\frac{5}{9}$ $\frac{1}{4}$ $\frac{5}{9}$ $\frac{1}{3}$ $\frac{1}{2}$ $\frac{1}{5}$: ' $\frac{3}{10}$ $\frac{2}{5}$ $\frac{3}{5}$, $\frac{1}{4}$ $\frac{5}{9}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{1}{2}$ $\frac{1}{10}$ $\frac{1}{4}$ $\frac{5}{9}$ $\frac{5}{9}$ $\frac{1}{4}$ $\frac{3}{4}$ $\frac{1}{6}$?'

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Foundation Worksheet

6:05 | Addition and Subtraction of Mixed Numbers

Name: _____ **Class:** _____

Examples

When adding and subtracting mixed numbers, it is usually simplest to do the whole numbers and fractions separately.

1 $2 + \frac{7}{8}$
$= 2\frac{7}{8}$

2 $3\frac{1}{4} + 2$
$= 3 + 2 + \frac{1}{4}$
$= 5\frac{1}{4}$

3 $5\frac{3}{5} - 2$
$= 5 - 2 + \frac{3}{5}$
$= 3\frac{3}{5}$

4 $1 - \frac{9}{20}$
$= \frac{20}{20} - \frac{9}{20}$
$= \frac{11}{20}$

5 $4 - \frac{7}{15}$
$= 3 + 1 - \frac{7}{15}$
$= 3 + \frac{15}{15} - \frac{7}{15}$
$= 3\frac{8}{15}$

Be careful when you subtract a fraction from a whole number.

Exercise

1 **a** $1 + \frac{3}{4}$ **b** $5 + \frac{1}{10}$ **c** $\frac{5}{9} + 2$ **d** $7 + \frac{13}{20}$ **e** $3 + \frac{7}{8}$
f $\frac{11}{12} + 4$ **g** $10 + \frac{4}{5}$ **h** $6 + \frac{12}{25}$ **i** $\frac{11}{100} + 1$ **j** $\frac{5}{6} + 5$

2 **a** $2 + 1\frac{1}{2}$ **b** $2\frac{1}{4} + 3$ **c** $7\frac{5}{8)} + 2$ **d** $4\frac{3}{10} + 2$ **e** $5 + 3\frac{1}{9}$
f $6\frac{2}{5} + 4$ **g** $8 + 1\frac{17}{20}$ **h** $5\frac{7}{12} + 3$ **i** $1 + 1\frac{8}{15}$ **j** $3 + 2\frac{3}{10}$

3 **a** $1 - \frac{1}{2}$ **b** $2 - \frac{1}{4}$ **c** $3 - \frac{1}{6}$ **d** $4 - \frac{3}{8}$ **e** $2 - \frac{3}{10}$
f $3 - \frac{5}{12}$ **g** $1 - \frac{3}{20}$ **h** $4 - \frac{2}{9}$ **i** $3 - \frac{13}{16}$ **j** $10 - \frac{4}{5}$

4 **a** $3\frac{1}{2} - 1$ **b** $4\frac{3}{4} - 2$ **c** $3\frac{2}{3} - 2$ **d** $6\frac{7}{8} - 4$ **e** $5\frac{1}{5} - 3$
f $2\frac{17}{20} - 1$ **g** $5\frac{8}{15} - 4$ **h** $7\frac{15}{16} - 3$ **i** $1\frac{47}{100} - 1$ **j** $2\frac{8}{9} - 2$

Fun Spot 6:05 | What type of music do mummies prefer?

Simplify each fraction by cancelling down. Match the letters with the answer below.

A $\frac{7}{14}$ **C** $\frac{4}{12}$ **I** $\frac{2}{10}$ **M** $\frac{6}{24}$ **P** $\frac{4}{6}$ **R** $\frac{9}{12}$ **S** $\frac{20}{25}$ **U** $\frac{15}{50}$ **W** $\frac{6}{16}$

									!
$\frac{3}{8}$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{2}{3}$		$\frac{1}{4}$	$\frac{3}{10}$	$\frac{4}{5}$	$\frac{1}{5}$	$\frac{1}{3}$

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Foundation Worksheet

6:06 | Multiplication of Fractions

Name: _____ **Class:** _____

Simple rules to follow when multiplying fractions.

- Write each part as a fraction.
- Simplify by cancelling or dividing top and bottom.
- Multiply numerators and denominators.

Examples

1 $4 \times \frac{1}{7}$

$= \frac{4}{1} \times \frac{1}{7}$

$= \frac{4}{7}$

2 $5 \times \frac{3}{20}$

$= \frac{{}^{1}\cancel{5}}{1} \times \frac{3}{\cancel{20}_{4}}$

$= \frac{3}{4}$

3 $\frac{1}{7} \times \frac{2}{3}$

$= \frac{2}{21}$

4 $\frac{1}{6} \times \frac{18}{20}$

$= \frac{1}{\cancel{6}_{1}} \times \frac{\cancel{18}^{3}}{20}$

$= \frac{3}{20}$

5 $\frac{1}{3} \times \frac{12}{16}$

$= \frac{1}{\cancel{3}_{1}} \times \frac{\cancel{12}^{\cancel{4}^{1}}}{\cancel{16}_{4}}$

$= \frac{1}{4}$

Exercise

1 **a** $2 \times \frac{1}{4}$ **b** $\frac{2}{1} \times \frac{1}{4}$ **c** $5 \times \frac{11}{100}$ **d** $\frac{5}{1} \times \frac{11}{100}$ **e** $\frac{2}{9} \times 3$

f $4 \times \frac{1}{16}$ **g** $5 \times \frac{3}{20}$ **h** $\frac{7}{100} \times 10$ **i** $4 \times \frac{5}{24}$ **j** $10 \times \frac{7}{80}$

k $2 \times \frac{19}{40}$ **l** $\frac{8}{27} \times 3$ **m** $6 \times \frac{1}{30}$ **n** $\frac{13}{60} \times 4$ **o** $7 \times \frac{8}{77}$

2 **a** $\frac{1}{2} \times \frac{4}{8}$ **b** $\frac{1}{2} \times \frac{12}{15}$ **c** $\frac{1}{2} \times \frac{10}{20}$ **d** $\frac{1}{2} \times \frac{6}{10}$ **e** $\frac{1}{4} \times \frac{8}{11}$

f $\frac{1}{3} \times \frac{9}{10}$ **g** $\frac{1}{6} \times \frac{12}{20}$ **h** $\frac{1}{5} \times \frac{15}{16}$ **i** $\frac{1}{10} \times \frac{40}{45}$ **j** $\frac{1}{3} \times \frac{15}{18}$

k $\frac{18}{24} \times \frac{1}{2}$ **l** $\frac{8}{16} \times \frac{1}{4}$ **m** $\frac{1}{8} \times \frac{24}{25}$ **n** $\frac{1}{5} \times \frac{20}{31}$ **o** $\frac{1}{4} \times \frac{28}{100}$

3 **a** $\frac{1}{2} \times \frac{1}{3}$ **b** $\frac{1}{4} \times \frac{1}{3}$ **c** $\frac{1}{10} \times \frac{1}{7}$ **d** $\frac{1}{2} \times \frac{1}{10}$ **e** $\frac{1}{3} \times \frac{1}{5}$

f $\frac{1}{16} \times \frac{1}{2}$ **g** $\frac{1}{10} \times \frac{1}{10}$ **h** $\frac{1}{4} \times \frac{1}{4}$ **i** $\frac{1}{6} \times \frac{1}{5}$ **j** $\frac{1}{4} \times \frac{1}{25}$

Fun Spot 6:06 | Teacher: 'You missed school yesterday, didn't you?'

Match the letters with the answer below.

C $\frac{1}{5} \times \frac{10}{15}$ **D** $\frac{1}{3} \times \frac{3}{10}$ **E** $5 \times \frac{1}{30}$ **H** $\frac{3}{8} \times 2$ **M** $\frac{1}{6} \times \frac{18}{20}$ **N** $\frac{2}{9} \times \frac{1}{2}$

O $\frac{1}{3} \times \frac{1}{6}$ **R** $\frac{1}{2} \times \frac{1}{2}$ **S** $\frac{1}{4} \times 8$ **T** $\frac{1}{8} \times 4$ **U** $\frac{1}{4} \times \frac{8}{16}$ **V** $\frac{1}{10} \times \frac{20}{25}$ **Y** $\frac{8}{11} \times \frac{1}{8}$

							: '												. '
2	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{1}{10}$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{1}{2}$		$\frac{1}{9}$	$\frac{1}{18}$	$\frac{1}{2}$		$\frac{2}{25}$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{11}$		$\frac{3}{20}$	$\frac{1}{8}$	$\frac{2}{15}$	$\frac{3}{4}$

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Foundation Worksheet

6:07 | Division Involving Fractions

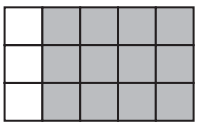
Name: _____ **Class:** _____

Examples

1 Note that $\frac{12}{15}$ is shaded.

How many $\frac{3}{15}$ in $\frac{12}{15}$? 4.

Then $\frac{12}{15} \div \frac{3}{15} = 4$.



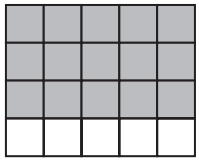
Divisions ask 'how many?'

- **$10 \div 2$ means how many 2s in 10.**
- **$6 \div \frac{1}{4}$ means how many $\frac{1}{4}$s in 6.**
- **$\frac{15}{20} \div \frac{3}{20}$ means how many $\frac{3}{20}$s in $\frac{15}{20}$.**

2 $\frac{15}{20}$ is shaded.

How many $\frac{5}{20}$ in $\frac{15}{20}$? 3.

Then $\frac{15}{20} \div \frac{5}{20} = 3$.



3 $\frac{10}{12}$ is shaded.

How many $\frac{5}{12}$ in $\frac{10}{12}$? 2.

Then $\frac{10}{12} \div \frac{5}{12} = 2$.



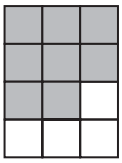
Exercise

1 **a** How many $\frac{1}{12}$ in $\frac{8}{12}$?

$\therefore \frac{8}{12} \div \frac{1}{12} =$

b How many $\frac{2}{12}$ in $\frac{8}{12}$?

$\therefore \frac{8}{12} \div \frac{2}{12} =$

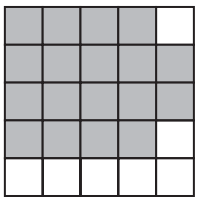


2 **a** How many $\frac{2}{25}$ in $\frac{18}{25}$?

$\therefore \frac{18}{25} \div \frac{2}{25} =$

b How many $\frac{9}{25}$ in $\frac{18}{25}$?

$\therefore \frac{18}{25} \div \frac{9}{25} =$



3 **a** How many $\frac{3}{10}$ in $\frac{6}{10}$?

$\therefore \frac{6}{10} \div \frac{3}{10} =$

b How many $\frac{2}{10}$ in $\frac{6}{10}$?

$\therefore \frac{6}{10} \div \frac{2}{10} =$

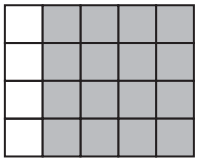


4 **a** How many $\frac{4}{20}$ in $\frac{16}{20}$?

$\therefore \frac{16}{20} \div \frac{4}{20} =$

b How many $\frac{8}{20}$ in $\frac{16}{20}$?

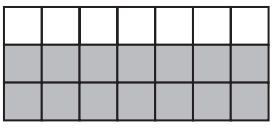
$\therefore \frac{16}{20} \div \frac{8}{20} =$



5 $\frac{14}{21}$ is shaded.

a $\frac{14}{21} \div \frac{7}{21} =$

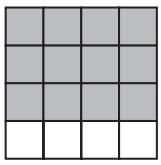
b $\frac{14}{21} \div \frac{2}{21} =$



6 $\frac{12}{16}$ is shaded.

a $\frac{12}{16} \div \frac{4}{16} =$

b $\frac{12}{16} \div \frac{3}{16} =$



When the denominators are the same, just divide the numerators.

7 **a** $\frac{6}{9} \div \frac{2}{9}$ **b** $\frac{15}{17} \div \frac{3}{17}$ **c** $\frac{20}{27} \div \frac{10}{27}$ **d** $\frac{20}{27} \div \frac{4}{27}$ **e** $\frac{5}{8} \div \frac{1}{8}$

f $\frac{40}{100} \div \frac{8}{100}$ **g** $\frac{27}{100} \div \frac{9}{100}$ **h** $\frac{21}{40} \div \frac{7}{40}$ **i** $\frac{7}{19} \div \frac{1}{19}$ **j** $\frac{4}{11} \div \frac{4}{11}$

k $\frac{18}{25} \div \frac{6}{25}$ **l** $\frac{30}{31} \div \frac{10}{31}$ **m** $\frac{30}{47} \div \frac{5}{47}$ **n** $\frac{33}{40} \div \frac{3}{40}$ **o** $\frac{24}{35} \div \frac{12}{35}$

p $\frac{24}{35} \div \frac{8}{35}$ **q** $\frac{18}{20} \div \frac{2}{20}$ **r** $\frac{13}{18} \div \frac{13}{18}$ **s** $\frac{44}{51} \div \frac{11}{51}$ **t** $\frac{50}{57} \div \frac{25}{57}$

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Foundation Worksheet

6:08 | Fractions of Quantities

Name: _____ **Class:** _____

Examples

1 Find $\frac{1}{3}$ of \$60

$\frac{1}{3}$ of \$60
$= \$60 \div 3$
$= \$20$

2 Find $\frac{1}{5}$ of 40 m

$\frac{1}{5}$ of 40 m
$= 40 \text{ m} \div 5$
$= 8 \text{ m}$

3 Find $\frac{3}{5}$ of 35 L

$\frac{1}{5}$ of 35 L
$= 35 \text{ L} \div 5$
$= 7 \text{ L}$
$\therefore \frac{3}{5}$ of 35 L
$= 3 \times 7 \text{ L}$
$= 21 \text{ L}$

4 Find $\frac{5}{8}$ of \$200

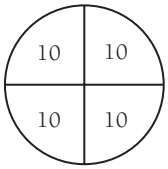
$\frac{1}{8}$ of \$200
$= \$200 \div 8$
$= \$25$
$\therefore \frac{5}{8}$ of \$200
$= 5 \times \$25$
$= \$125$

Fractions are divisions.

Exercise

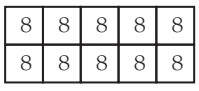
1
a $\frac{1}{2}$ of 20 min **b** $\frac{1}{2}$ of 6 kg **c** $\frac{1}{4}$ of \$20 **d** $\frac{1}{3}$ of 90 m
e $\frac{1}{10}$ of 70 mm **f** $\frac{1}{8}$ of \$24 **g** $\frac{1}{100}$ of \$5000 **h** $\frac{1}{4}$ of 36 km
i $\frac{1}{6}$ of 66 t **j** $\frac{1}{5}$ of 75 kg **k** $\frac{1}{10}$ of 180 L **l** $\frac{1}{3}$ of \$180
m $\frac{1}{2}$ of 44 mm **n** $\frac{1}{8}$ of 960 t **o** $\frac{1}{4}$ of 48 h **p** $\frac{1}{5}$ of 555 min

2



a $\frac{1}{4}$ of 40
b $\frac{2}{4}$ of 40
c $\frac{3}{4}$ of 40

3



a $\frac{1}{10}$ of \$80
b $\frac{3}{10}$ of \$80
c $\frac{7}{10}$ of \$80

4



a $\frac{1}{5}$ of \$30
b $\frac{2}{5}$ of \$30
c $\frac{3}{5}$ of \$30

5
a $\frac{3}{4}$ of \$20 **b** $\frac{3}{10}$ of 70 mm **c** $\frac{5}{8}$ of \$24 **d** $\frac{7}{100}$ of \$5000
e $\frac{7}{11}$ of 44 t **f** $\frac{2}{5}$ of 75 kg **g** $\frac{2}{3}$ of \$180 **h** $\frac{3}{5}$ of 555 min
i $\frac{5}{6}$ of 24 cm **j** $\frac{7}{10}$ of 50 L **k** $\frac{5}{9}$ of 27 kg **l** $\frac{3}{50}$ of 200 h
m $\frac{3}{8}$ of 48 m **n** $\frac{4}{5}$ of \$60 **o** $\frac{19}{25}$ of \$75 **p** $\frac{5}{12}$ of 36 cm

Fun Spot 6:08 | Do you know this book?



Calculate each and place each letter above its answer below.

A $\frac{1}{3}$ of 18 **B** $\frac{1}{4}$ of 28 **E** $\frac{1}{10}$ of 200 **G** $\frac{1}{6}$ of 48 **I** $\frac{1}{5}$ of 60 **L** $\frac{1}{20}$ of 80
M $\frac{4}{5}$ of 20 **N** $\frac{2}{3}$ of 60 **S** $\frac{7}{8}$ of 16 **T** $\frac{7}{20}$ of 60 **U** $\frac{5}{9}$ of 27 **Y** $\frac{3}{4}$ of 32

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4 6 21 20 6 8 6 12 40 7 24 16 12 14 21 24 7 15 14 14

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Foundation Worksheet

6:09 | Review of Percentages

Name: _____ **Class:** _____

Examples

If possible simplify fractions by cancelling.

A percentage is a fraction over 100 or a decimal with 2 places.

1 $9\% = \frac{9}{100}$
$= 0{\cdot}09$

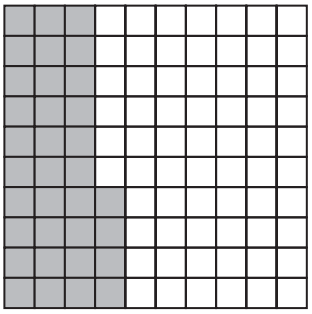
2 $73\% = \frac{73}{100}$
$= 0{\cdot}73$

3 $60\% = \frac{60}{100} = \frac{3}{5}$
$= 0{\cdot}60 = 0{\cdot}6$

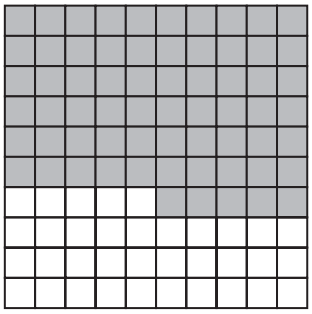
4 $45\% = \frac{45}{100} = \frac{9}{20}$
$= 0{\cdot}45$

Exercise

1 a What percentage is coloured?
b What percentage is not coloured?
c What fraction is coloured?
d What fraction is not coloured?



2 a What percentage is coloured?
b What percentage is not coloured?
c What fraction is coloured?
d What fraction is not coloured?



3 Write as a fraction and as a decimal.

a 3%	**b** 17%	**c** 63%	**d** 99%	**e** 27%	**f** 20%	**g** 5%
h 6%	**i** 15%	**j** 44%	**k** 25%	**l** 50%	**m** 18%	**n** 62%
o 55%	**p** 36%	**q** 83%	**r** 46%	**s** 77%	**t** 1%	

4 Write as a percentage.

a $\frac{7}{100}$	**b** 0·07	**c** $\frac{23}{100}$	**d** $\frac{78}{100}$	**e** 0·52	**f** 0·20
g $\frac{39}{100}$	**h** 0·75	**i** $\frac{90}{100}$	**j** $\frac{16}{100}$	**k** 0·35	**l** 0·45
m 0·02	**n** $\frac{58}{100}$	**o** 0·50	**p** $\frac{47}{100}$	**q** 0·61	**r** $\frac{28}{100}$

Fun Spot 6:09 | Do you know this book?



Write each as a simplified fraction. Match each letter with the answer below.

A 10%	**B** 19%	**D** 40%	**E** 75%	**H** 30%	**I** 16%
L 35%	**O** 42%	**R** 51%	**S** 50%	**T** 4%	**Y** 85%

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$\frac{3}{10}$ $\frac{21}{50}$ $\frac{7}{20}$ $\frac{4}{25}$ $\frac{2}{5}$ $\frac{1}{10}$ $\frac{17}{20}$ $\frac{1}{25}$ $\frac{51}{100}$ $\frac{3}{4}$ $\frac{1}{10}$ $\frac{1}{25}$

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$\frac{19}{100}$ $\frac{17}{20}$ $\frac{51}{100}$ $\frac{3}{10}$ $\frac{21}{50}$ $\frac{2}{5}$ $\frac{1}{10}$ $\frac{3}{10}$ $\frac{21}{50}$ $\frac{51}{100}$ $\frac{1}{2}$ $\frac{3}{4}$

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Foundation Worksheet

6:10 | Changing Fractions and Decimals to Percentages

Name: _____ **Class:** _____

Examples

1 $\frac{27}{100} = \frac{27}{\cancel{100}_{1}} \times \frac{\cancel{100}^{1}}{1}\%$

$= 27\%$

To change to a percentage, multiply by 100.

- For a fraction this means $\times \frac{100}{1}$.
- For a decimal, move the point 2 places to the right.

2 $0{\cdot}42$

$= 0{\cdot}42 \times 100\%$

$= 42\%$

3 $\frac{3}{5} = \frac{3}{\cancel{5}_{1}} \times \frac{\cancel{100}^{20}}{1}\%$

$= 60\%$

4 $0{\cdot}03$

$= 0{\cdot}03 \times 100\%$

$= 3\%$

5 $0{\cdot}7$

$= 0{\cdot}7 \times 100\%$

$= 70\%$

Exercise

1 Write as percentages.

a $\frac{15}{100}$	**b** $\frac{2}{100}$	**c** $\frac{47}{100}$	**d** $\frac{93}{100}$	**e** $\frac{72}{100}$
f $\frac{3}{50}$	**g** $\frac{1}{4}$	**h** $\frac{1}{5}$	**i** $\frac{1}{100}$	**j** $\frac{33}{100}$
k $\frac{1}{20}$	**l** $\frac{84}{100}$	**m** $\frac{9}{10}$	**n** $\frac{9}{20}$	**o** $\frac{6}{25}$
p $\frac{1}{2}$	**q** $\frac{11}{50}$	**r** $\frac{3}{4}$	**s** $\frac{12}{25}$	**t** $\frac{1}{10}$
u $\frac{77}{100}$	**v** $\frac{41}{50}$	**w** $\frac{7}{10}$	**x** $\frac{19}{20}$	**y** $\frac{19}{100}$

2 Write as percentages.

a $0{\cdot}14$	**b** $0{\cdot}07$	**c** $0{\cdot}64$	**d** $0{\cdot}99$	**e** $0{\cdot}02$
f $0{\cdot}31$	**g** $0{\cdot}30$	**h** $0{\cdot}85$	**i** $0{\cdot}80$	**j** $0{\cdot}18$
k $0{\cdot}44$	**l** $0{\cdot}4$	**m** $0{\cdot}53$	**n** $0{\cdot}23$	**o** $0{\cdot}01$
p $0{\cdot}29$	**q** $0{\cdot}71$	**r** $0{\cdot}9$	**s** $0{\cdot}25$	**t** $0{\cdot}58$

Fun Spot 6:10 | Have you heard of this book?

Place these in order from largest (1st) to smallest (11th). Match the letters with the order below to complete the title.

A $0{\cdot}74$ **B** $\frac{41}{100}$ **D** $\frac{78}{100}$ **E** $0{\cdot}7$ **G** $0{\cdot}08$ **H** $0{\cdot}8$ **I** $\frac{3}{4}$ **M** $0{\cdot}88$ **N** $\frac{41}{50}$ **T** 40% **Y** 85%

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10th 4th 8th 9th 6th 11th 9th 7th 3rd 11th

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9th 2nd 5th 6th 3rd 7th 4th 1st 6th 10th 8th

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Foundation Worksheet

6:12 | Finding a Percentage of a Quantity

Name: _____ **Class:** _____

Examples



Change the percentage to a decimal or fraction and multiply. Usually it is easier to change to a decimal, but there are some fractions that are easier than decimals.

1 4% of 200 kg
$= 0{\cdot}04 \times 200$ kg
$= 8$ kg

2 60% of \$700
$= 0{\cdot}6 \times \$700$
$= \$42$

3 10% of 40
$= \frac{1}{10} \times 40$
$= 4$

4 50% of 148 L
$= \frac{1}{2}$ of 148 L
$= 74$ L

5 25% of 64 t
$= \frac{1}{4}$ of 64 t
$= 16$ t

Exercise

1 **a** 50% of \$30 **b** 10% of 80 g **c** 25% of 40 kg **d** 50% of 60 m
e 25% of 16 **f** 25% of 36 L **g** 10% of \$200 **h** 10% of 150 cm
i 50% of 28 t **j** 25% of 48 g **k** 10% of 90 **l** 25% of 88 m
m 50% of 50 h **n** 50% of 422 t **o** 25% of 120 kg **p** 10% of \$450

2 **a** 20% of \$20 **b** 70% of \$50 **c** 3% of 600 t **d** 5% of 700 kg
e 40% of 400 m **f** 6% of 900 g **g** 30% of 200 **h** 20% of 120 s
i 80% of \$50 **j** 2% of 1200 km **k** 60% of 90 d **l** 90% of \$20
m 8% of 3000 m **n** 30% of 180 cm **o** 20% of 800 g **p** 4% of \$5000
q 11% of 300 t **r** 40% of \$70 **s** 60% of 500 L **t** 15% of 200

Fun Spot 6:12 | Teacher: 'What came after the Stone Age and the Bronze Age?'



Change each decimal to a percentage. Match the letters with the answers below.

A 0·05 **D** 0·35 **E** 0·3 **G** 0·53 **H** 0·50
N 0·01 **S** 0·15 **T** 0·1 **U** 0·26

							:
15%	10%	26%	35%	30%	1%	10%	

‘								–			’
10%	50%	30%		15%	5%	26%	15%		5%	53%	30%

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Foundation Worksheet

6:13 | One Quantiy as a Percentage of Another

Name: ______________________________ **Class:** ______________

Examples

Three simple steps:

- Check that units are the same (eg both \$, cm, g).
- Make them into a fraction in the order given.
- Multiply by 100 to make a percentage.

What percentage are the following?

1 20c of 50c
$= \frac{20}{50}$
$= \frac{20}{50} \times \frac{100}{1}\%$
$= 40\%$

2 18 g of 100 g
$= \frac{18}{100}$
$= \frac{18}{100} \times \frac{100}{1}\%$
$= 18\%$

3 14 L of 25 L
$= \frac{14}{25}$
$= \frac{14}{25} \times \frac{100}{1}\%$
$= 56\%$

4 \$10 of \$40
$= \frac{10}{40}$
$= \frac{10}{40} \times \frac{100}{1}\%$
$= 25\%$

Exercise

1 What fraction are the following?

a 10 g of 20 g **b** 20 m of 100 m **c** \$7 of \$100 **d** 15c of 30c
e 20 t of 80 t **f** 10 min of 50 min **g** 33 kg of 100 kg **h** 150 of 300
i \$120 of \$200 **j** 90 L of 100 L **k** 20 mm of 25 mm **l** 50c of 100c
m 66 km of 99 km **n** 80 kg of 240 kg **o** 14 g of 20 g **p** 10 h of 24 h

2 What percentage are the following?

a 10 g of 20 g **b** 20 m of 100 m **c** \$7 of \$100 **d** 15c of 30c
e 20 t of 80 t **f** 10 min of 50 min **g** 33 kg of 100 kg **h** 150 of 300
i \$120 of \$200 **j** 90 L of 100 L **k** 20 mm of 25 mm **l** 50c of 100c
m 7 mL of 10 mL **n** 8 cm of 25 cm **o** 89 L of 100 L **p** \$1 of \$50
q 4 g of 25 g **r** 9 min of 50 min **s** 50 s of 200 s **t** 30 t of 40 t

Fun Spot 6:13 | Teacher: 'If you multiplied 395 by 246 what would you get?'

Match the letter for each question with the simplified fraction below.

A $\frac{15}{25}$ **D** $\frac{10}{40}$ **E** $\frac{20}{24}$ **G** $\frac{20}{100}$ **H** $\frac{16}{20}$ **N** $\frac{24}{50}$

O $\frac{4}{12}$ **R** $\frac{50}{90}$ **S** $\frac{20}{30}$ **T** $\frac{45}{60}$ **U** $\frac{35}{100}$ **W** $\frac{16}{18}$

$\frac{2}{3}$	$\frac{3}{4}$	$\frac{7}{20}$	$\frac{1}{4}$	$\frac{5}{6}$	$\frac{12}{25}$	$\frac{3}{4}$

: '

$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{6}$

$\frac{8}{9}$	$\frac{5}{9}$	$\frac{1}{3}$	$\frac{12}{25}$	$\frac{1}{5}$

$\frac{3}{5}$	$\frac{12}{25}$	$\frac{2}{3}$	$\frac{8}{9}$	$\frac{5}{6}$	$\frac{5}{9}$

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Foundation Worksheet

8:03 | Patterns and Rules

Name: _____ **Class:** _____

Examples

1 Complete this pattern.
6, 12, 18, . . . , . . .
The rule is 'add 6', so the pattern becomes
6, 12, 18, 24, 30

2 Complete the table.

Top	1	2	3	4
Bottom	4	8	12	

The bottom is 4 times the top number, so the missing number is $4 \times 4 = 16$

3 Complete the rule.

Top	0	1	2	3
Bottom	8	9	10	11

Bottom = top + . . .
8 is added each time, so bottom = top + 8

Exercise

1 Complete these patterns.

a 5, 10, 15, . . . , . . .
b 4, 6, 8, . . . , . . .
c 20, 18, 16, . . . , . . .
d 3, 6, 12, . . . , . . .
e 20, 30, 40, . . . , . . .
f 2, 4, 8, . . . , . . .

2 Complete these tables.

a

Top	1	2	3	4
Bottom	2	4	6	

b

Top	1	2	3	4
Bottom	11	12	13	

c

Top	1	2	3	4
Bottom	5	10	15	

d

Top	0	1	2	3
Bottom		4	5	6

e

Top	3	4	5	6
Bottom	9	12	15	

f

Top	1	2	3	4
Bottom		4	5	6

3 Complete the rule used in each table below.

a

Top	1	2	3	4
Bottom	7	8	9	10

Bottom = top + . . .

b

Top	3	4	5	6
Bottom	4	5	6	7

Bottom = top + . . .

c

Top	0	1	2	3
Bottom	0	7	14	21

Bottom = top × . . .

d

Top	1	2	3	4
Bottom	10	20	30	40

Bottom = top × . . .

e

Top	10	9	8	7
Bottom	4	3	2	1

Bottom = top − . . .

f

Top	1	2	3	4
Bottom	2	3	4	5

Bottom = . . . + top

4 Write the rule for each table in question 2.

a Bottom = . . . × top
b Bottom = . . . + top
c Bottom = top × . . .
d Bottom = top + . . .
e Bottom =
f Bottom =

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Foundation Worksheet

9:02 | Making Sense of Algebra

Name: _____ **Class:** _____

Examples

Imagine $\boxed{x}$ is a box containing an unknown number of counters ⬭.

How many counters are here?

1 $\boxed{x}$ ⬭⬭⬭ $x + 4$

2 $\boxed{y}$ ⬭⬭ $y + 2$

3 $\boxed{a}\ \boxed{a}$ ⬭⬭ $2a + 2$

4 $\boxed{a}\ \boxed{a}$ / $\boxed{b}\ \boxed{b}\ \boxed{b}$ $2a + 3b$

Exercise

1 Write the number of counters in each case.

a $\boxed{x}$ ⬭⬭ **b** $\boxed{y}$ ⬭⬭⬭ **c** $\boxed{x}\ \boxed{x}\ \boxed{x}$ **d** ⬭⬭⬭ / ⬭⬭⬭ **e** $\boxed{a}$ ⬭⬭⬭⬭

f $\boxed{y}\ \boxed{y}$ / $\boxed{y}\ \boxed{y}$ ⬭⬭ **g** ⬭⬭ $\boxed{x}$ / ⬭⬭ $\boxed{x}$ **h** ⬭⬭⬭ $\boxed{y}$ **i** ⬭ $\boxed{a}$ / $\boxed{a}$ **j** ⬭ $\boxed{y}\ \boxed{y}\ \boxed{y}$ / $\boxed{y}$ $\boxed{y}$

2 a $\boxed{y}\ \boxed{y}$ ⬭ **b** $\boxed{x}\ \boxed{x}$ / $\boxed{y}$ **c** $\boxed{x}\ \boxed{y}$ **d** $\boxed{x}\ \boxed{x}\ \boxed{x}$ / $\boxed{y}\ \boxed{y}$ **e** $\boxed{a}\ \boxed{b}\ \boxed{b}$ / $\boxed{b}$

f ⬭ $\boxed{x}$ ⬭ / $\boxed{x}$ **g** $\boxed{a}\ \boxed{b}\ \boxed{a}$ / $\boxed{b}\ \boxed{b}$ **h** $\boxed{x}$ ⬭⬭ $\boxed{x}$ / $\boxed{x}$ **i** $\boxed{x}\ \boxed{y}\ \boxed{x}\ \boxed{x}$ / $\boxed{x}\ \boxed{y}\ \boxed{x}$ **j** ⬭ $\boxed{y}\ \boxed{y}\ \boxed{y}$ / ⬭ $\boxed{y}$ ⬭

k $\boxed{x}\ \boxed{y}\ \boxed{x}$ / ⬭⬭ **l** ⬭ $\boxed{a}$ ⬭ / $\boxed{b}\ \boxed{b}\ \boxed{b}$ **m** $\boxed{a}\ \boxed{a}\ \boxed{a}$ / ⬭ $\boxed{b}\ \boxed{b}$ **n** $\boxed{x}$ ⬭⬭ / $\boxed{y}\ \boxed{y}$ **o** $\boxed{m}\ \boxed{n}$ ⬭ / ⬭⬭

Fun Spot 9:02 | What's the difference between a night-watchman and a butcher?

Each expression can be written in a shorter form.
Match the letters to the abbreviations.

A $6 \times a$ **D** $a \times 3$ **E** $1 \times a$ **G** $a \times a$ **H** $a \div 2$ **I** $a \times b$ **K** $y \times x$

N $x \div 4$ **O** $x \times 3$ **R** $y \div 4$ **S** $m \times m$ **T** $m \times 2$ **W** $y \times 1$ **Y** $m \div 10$

$3x$	$\frac{x}{4}$	a	m^2	$2m$	$6a$	$\frac{m}{10}$	m^2	$6a$	y	$6a$	xy	a		
$6a$	$\frac{x}{4}$	$3a$	$2m$	$\frac{a}{2}$	a	$3x$	$2m$	$\frac{a}{2}$	a	$\frac{y}{4}$				
y	a	ab	a^2	$\frac{a}{2}$	m^2	$6a$	m^2	$2m$	a	$6a$	xy	!		

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Foundation Worksheet

9:03 | Substitution

Name: _____ **Class:** _____

Examples

To substitute, put a number in place of a letter, then calculate.

1 Find the value of the pronumeral.

a $5a$, if $a = 6$

$5a = 5 \times a$
$= 5 \times 6$
$= 30$

b $m - 6$ if $m = 4$

$m - 6 = 4 - 6$
$= -2$

2 Given that $x = 8$ and $y = 5$, find the value of these.

a $x + y$

$x + y = 8 + 5$
$= 13$

b $\frac{x}{2}$

$\frac{x}{2} = x \div 2$
$= 8 \div 2$
$= 4$

c $4x + 7$

$4x + 7 = 4 \times x + 7$
$= 4 \times 8 + 7$
$= 39$

d $y^2 - 10$

$y^2 - 10 = 5^2 - 10$
$= 25 - 10$
$= 15$

Exercise

1 Find the value of each expression.

a $3x$, if $x = 12$ **b** $2y$, if $y = 20$ **c** $a + 3$, if $a = 8$ **d** $k - 10$, if $k = 13$

e m^2, if $m = 3$ **f** $t + 11$, if $t = 4$ **g** $d - 2$, if $d = 2$ **h** $7p$, if $p = 9$

i $18 - c$, if $c = 10$ **j** $9x$, if $x = 4$ **k** $10 + w$, if $w = 7$ **l** $8d$, if $d = 3$

m v^2, if $v = 1$ **n** $r + 6$, if $r = 0$ **o** $\frac{x}{3}$, if $x = 12$ **p** $\frac{a}{10}$, if $a = 50$

2 Given that $x = 12$ and $y = 4$, find the value of each expression.

a $\frac{x}{6}$ **b** $x - y$ **c** $5y$ **d** y^2 **e** $y - 7$ **f** $20 - x$ **g** $2x - 5$

h $x + 2y$ **i** $\frac{x}{y}$ **j** $x^2 - 20$ **k** $y^2 + 8$ **l** xy **m** $y - x$ **n** $x - 5y$

o $\frac{y}{2} + 5$ **p** $3x + 4y$ **q** $\frac{x}{3} - 7$ **r** $2x - 3y$ **s** $10y - x$ **t** $3x - 6$ **u** $15 - 3y$

Fun Spot 9:03 | What's the difference between a monster and a biscuit?

Simplify the following. Match the letters to the answers.

A $6 \times a$ **D** $12 \times b$ **E** $3 \times 4 \times a$ **I** $3 \times 8 \times a$ **K** $10 \times 6 \times b$

M $20b + 6b$ **N** $6 \times 8a$ **O** $a \times b$ **R** $b \times 5$ **S** $4 \times 2 \times a$

T $2 \times 3b$ **U** $5 \times 7b$ **V** $8 \times 2 \times b$ **Y** $2 \times a \times b$

$12a$	$16b$	$12a$	$5b$	$6b$	$5b$	$24a$	$12a$	$12b$	$6b$	ab	$12b$	$35b$	$48a$	$60b$	$6a$	$26b$	ab	$48a$	$8a$	$6b$	$12a$	$5b$	$24a$	$48a$	$2ab$	ab	$35b$	$5b$	$6b$	$12a$	$6a$	?

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Foundation Worksheet

9:04 | Simplifying Algebraic Expressions

Name: _____ **Class:** _____

Examples

A lot of algebra is just counting letters.

1 $4c + 3c = 7c$ because $4 + 3 = 7$

2 $10x - x = 9x$ as 'x' means '$1x$' and $10 - 1 = 9$

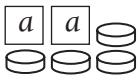
3 $3a + 7a + 4 + 5 = 10a + 9$ because $3a + 7a = 10a$ and $4 + 5 = 9$

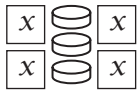
4 $3m + 4n + 5m = 8m + 4n$ because $3m + 5m = 8m$

Exercise

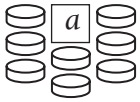
1 Simplify the following.

a 

b 

c 

d 

e 

2 Simplify the following by counting on or counting back.

a $2x + 3x$	**b** $7a - 3a$	**c** $10p + 4p$	**d** $8f - 2f$	**e** $12k - k$
f $4m + 5m$	**g** $y + y$	**h** $6y + y$	**i** $4x - 3x$	**j** $6t + 10t$
k $20w - 5w$	**l** $x + 4x$	**m** $13d - 6d$	**n** $3m - 3m$	**o** $11p + 3p$
p $7y - y$	**q** $f + f$	**r** $26c - 4c$	**s** $15c + 9c$	**t** $8ab + 4ab$

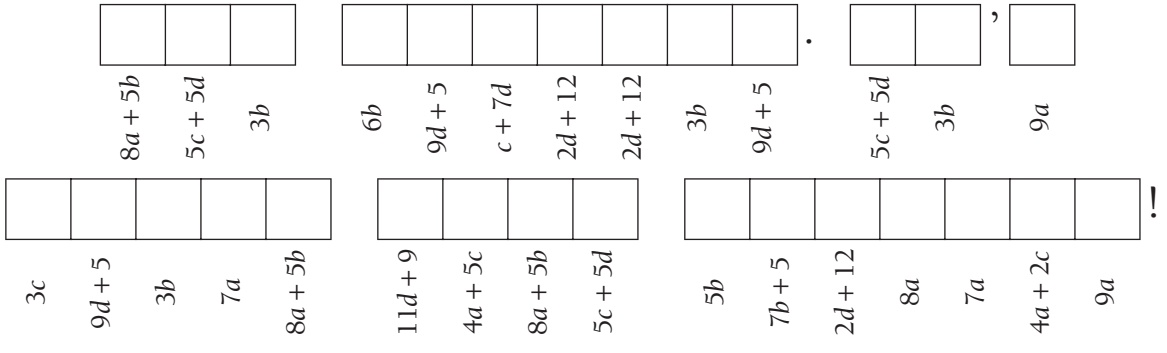
3 Simplify the following by counting like terms.

a $5a + 3 + 4$	**b** $2x + 3y + 4x$	**c** $2a + 5a + 6b$	**d** $7 + 8 + 4h$
e $7c - 3c + 1$	**f** $10t + 8 - 3$	**g** $6m + 7 + 3m$	**h** $10 + 3c + c$
i $6 + 5x - 3x$	**j** $3x - x + 4y$	**k** $10m - 3m + 2n$	**l** $15 - 4 + 7a$
m $5x + 10x + 3y$	**n** $12x + 2y + x$	**o** $4b + 3b + 5c$	**p** $4b + 3c + 5b$
q $10t - 5t + 6u$	**r** $4w + 7w + 9$	**s** $8x - x + 4$	**t** $8 + 3y + 4$

Fun Spot 9:04 | Which band member is really great at algebra?

Simplify each expression, then match its letter with the answers below.

A $3a + 4a$	**B** $7a + a$	**C** $6b - b$	**D** $9b - 3b$
E $2b + b$	**G** $13c - 10c$	**H** $5c + 2d + 3d$	**I** $6a - 2a + 5c$
L $2c + 12a - 8a$	**M** $6d - 4d + 12$	**R** $3 + 9d + 2$	**S** $2a + 3a + 4a$
T $3a + 5b + 5a$	**U** $7d + 3c - 2c$	**W** $3d + 9 + 8d$	**Y** $7 - 2 + 7b$



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Foundation Worksheet

9:05 | Grouping Symbols

Name: _____ **Class:** _____

To expand or remove grouping symbols, multiply each term inside the bracket by the term outside.

Examples

1 $4(a-7)$
$= 4 \times a - 4 \times 7$
$= 4a - 28$

2 $3(x+5)$
$= 3 \times x + 3 \times 5$
$= 3x + 15$

3 $10(2y-3)$
$= 10 \times 2y - 10 \times 3$
$= 20y - 30$

4 $6(4y-1)$
$= 6 \times 4y - 6 \times 1$
$= 24y - 6$

5 $5(4-3y)$
$= 5 \times 4 - 5 \times 3y$
$= 20 - 15y$

Exercise

1 Remove grouping symbols.

a $2(x+3)$	**b** $5(a-4)$	**c** $3(p+2)$	**d** $8(c-1)$
e $10(m+5)$	**f** $4(y+8)$	**g** $7(2-h)$	**h** $2(4+d)$
i $5(x+6)$	**j** $9(4-b)$	**k** $6(3-x)$	**l** $4(y+5)$
m $7(3+t)$	**n** $11(c-3)$	**o** $20(a+5)$	**p** $2(10-x)$
q $1(y+8)$	**r** $10(x-7)$	**s** $3(c-4)$	**t** $6(f+9)$

2 Expand.

a $3(5m-1)$	**b** $2(2x+5)$	**c** $7(3+2x)$	**d** $4(3x+1)$
e $10(2p+9)$	**f** $1(7x-8)$	**g** $5(4x-7)$	**h** $6(2x+1)$
i $8(2-7a)$	**j** $2(3-7x)$	**k** $3(8c-5)$	**l** $11(1-4c)$
m $9(2y+3)$	**n** $4(9+5t)$	**o** $4(3d+1)$	**p** $6(7y+4)$
q $3(7-5x)$	**r** $5(3x+10)$	**s** $2(11p-2)$	**t** $8(2x-6)$

Fun Spot 9:05 | What is the best cure for dandruff?



Expand each of these. Match each letter to the answers below.

A $5(3x+2)$ **B** $2(7x+3)$ **D** $3(5x+4)$ **E** $8(2x+1)$
L $6(3x+2)$ **N** $4(4x+3)$ **S** $3(6x+5)$

$14x+6$	$15x+10$	$18x+12$	$15x+12$	$16x+12$	$16x+8$	$18x+15$	$18x+15$

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Foundation Worksheet

9:08 | Directed Numbers

Name: _____ **Class:** _____

Examples

1 Evaluate, if $a = 5$ and $b = 6$.

a $a - b$
$= 5 - 6$
$= -1$

b ab
$= 5 \times 6$
$= 30$

c $2a + 7$
$= 2 \times 5 + 7$
$= 17$

2 If $x = -8$ and $y = 2$, find

a $x + 3$
$= -8 + 3$
$= -5$

b $\frac{x}{y}$
$= \frac{-8}{2}$
$= -4$

c $x - y$
$= -8 - 2$
$= -10$

Substitute directed numbers (positives or negatives) before you evaluate each expression.

Exercise

1 Evaluate, if $x = 7$ and $y = 3$.

a $2x$	**b** $x + 3$	**c** $x - y$	**d** $x + y$	**e** $5y$
f $x + 3y$	**g** $y - 3$	**h** $3x + 4$	**i** $2y - 5$	**j** $4x + 2y$

2 If $x = -2$ and $y = 3$, find the following.

a $x + 1$	**b** $x + y$	**c** $x - y$	**d** $y - x$	**e** $5x$
f $y - 7$	**g** $x - 5$	**h** $-2y$	**i** $3x - 4$	**j** $x \div 2$
k x^2	**l** $4x + y$	**m** xy	**n** $x + 4y$	**o** $2x + 3y$

3 If $a = 8$ and $b = -2$, find the following.

a $a + b$	**b** $a - b$	**c** $b - a$	**d** $3b$	**e** $b + 5$
f $2a + 3b$	**g** ab	**h** $b - 6$	**i** $3b + 1$	**j** $a + 4b$
k $a \div b$	**l** $10 \div b$	**m** $a - 2b$	**n** $10b$	**o** $3ab$

Fun Spot 9:08 | Why doesn't your sister like peanuts?

Evaluate using $m = -10$ and $n = 4$. Match each letter with the answers below.

A $m + 5$	**E** $m + n$	**H** $2m$	**I** mn	**K** $3n$
L $m - n$	**N** $m - 2$	**O** $n - 4$	**P** $n - m$	**R** $3m + 5$
S $-m$	**T** $-n$	**U** $2n - 3$	**V** $4m + 10$	**Y** $5n - 4$

−20	−5	−30	−6		16	0	5		−6	−30	−6	−25		10	−6	−6	−12

																		?
−5		10	12	−40	−12	−12	16		−6	−14	−6	14	−20	−5	−12	−4		

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Foundation Worksheet

9:09 | Simplifying Expressions with Directed Numbers

Name: _____ **Class:** _____

Examples

1 $4x - 10x$
$= -6x$ because
$4 - 10 = -6$

2 $-2a + 5a$
$= 3a$ because
$-2 + 5 = 3$

The rules for directed numbers are the same in algebra.

3 $-3 \times 2c$
$= -6c$ because
$-3 \times 2 = -6$

4 $2f - (-5f)$
$= 7f$ because
$2 - (-5) = 7$

5 $-7 \times (-3y)$
$= 21y$ because
$-7 \times (-3) = 21$

6 $-8t \div 2$
$= -4t$ because
$-8 \div 2 = -4$

Exercise

1 Simplify.

a $-3 - 2$	**b** $2 - 6$	**c** $4 - (-7)$	**d** $-8 + 3$	**e** $-6 + 11$
f $7 - 13$	**g** $-8 - 6$	**h** $-2 + 7$	**i** $-5 - (-4)$	**j** $-7 - 7$

2 Simplify.

a -4×5	**b** $-2 \times 3x$	**c** $-3 \times (-4)$	**d** $-5 \times -(7x)$	**e** $-12 \div 3$
f $6 \times (-2a)$	**g** $15y \div (-3)$	**h** $-10 \times 2k$	**i** $-18t \div 6$	**j** $-7 \times 3m$
k $-20 \div (-5)$	**l** $-4 \times 9b$	**m** $6 \times (-5n)$	**n** $-24y \div (-8)$	**o** $-4 \times 10x$

3 Simplify.

a $-2x - x$	**b** $2y - 7y$	**c** $5a - 6a$	**d** $-2m + 5m$	**e** $-3h - 7h$
f $7c - (-3c)$	**g** $-5y + y$	**h** $10p - (-p)$	**i** $x - 7x$	**j** $4x - 10x$
k $-4t + 9t$	**l** $d - (-d)$	**m** $-8w + 8w$	**n** $-4x - (-7x)$	**o** $3y - 5y$

Fun Spot 9:09 | What happened to the thief who was caught in the rubber factory?



Simplify each expression. Match each letter with the answers below to answer the riddle.

A $4a - 6a$	**C** $-3 \times 2a$	**E** $-3a + 5a$	**G** $-10a \div 2$	**H** $2a - (-a)$
I $-5a - 2a$	**L** $-7 \times 2a$	**N** $3a - 6a$	**O** $-20a \div (-5)$	**R** $9a \div (-9)$
S $6a - (-2a)$	**T** $a - 9a$	**V** $5 \times (-3a)$	**W** $-4a \times 4$	

$3a$	$2a$	$-16a$	$-2a$	$8a$	$-5a$	$-7a$	$-15a$	$2a$	$-3a$	$-2a$	$-14a$	$4a$	$-3a$	$-5a$	$8a$	$-8a$	$-a$	$2a$	$-8a$	$-6a$	$3a$

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Foundation Worksheet

9:10 | Algebraic Sentences

Name: _____ **Class:** _____

Examples

To find the number that makes the equation true
- **read the sentence to yourself, thinking 'what number . . .'**
- **thinking in reverse can be a help.**

1 $\square + 5 = 9$
(What number add 5 gives 9?)
$\square = 4$

2 $3 \times \triangle = 24$
(3 times what number gives 24?)
$\triangle = 8$
In reverse → [What number goes into 24 three times?]

3 $a - 9 = 7$
(What number take away 9 gives 7?)
$a = 16$
[What number is 9 more than 7?] ← In reverse

4 $\frac{c}{5} = 3$
(What number divided by 5 gives 3?)
$c = 15$

Exercise

Find the number that makes the equation true.

1 **a** $\square + 3 = 10$ **b** $4 \times \square = 20$ **c** $\square + 9 = 11$ **d** $\square \times 3 = 12$ **e** $10 \times \square = 70$
f $5 + x = 11$ **g** $x + 8 = 12$ **h** $8x = 40$ **i** $2a = 18$ **j** $4 + a = 13$
k $6m = 18$ **l** $y + 11 = 18$ **m** $6k = 6$ **n** $7w = 56$ **o** $x + 9 = 30$
p $p + 11 = 33$ **q** $b + 5 = 32$ **r** $7t = 42$ **s** $4y = 36$ **t** $20 + h = 21$

2 **a** $x - 2 = 7$ **b** $a \div 2 = 9$ **c** $\frac{c}{4} = 12$ **d** $y - 9 = 10$ **e** $m - 13 = 18$
f $h - 5 = 6$ **g** $\frac{c}{4} = 2$ **h** $x - 12 = 3$ **i** $a - 7 = 0$ **j** $\frac{x}{10} = 3$
k $a \div 3 = 6$ **l** $k - 6 = 14$ **m** $h - 8 = 15$ **n** $x \div 6 = 8$ **o** $x \div 7 = 9$
p $\frac{n}{5} = 6$ **q** $c - 20 = 10$ **r** $p - 15 = 5$ **s** $m - 8 = 8$ **t** $\frac{a}{7} = 1$

Fun Spot 9:10 | What do you get when you cross a galaxy with a toad?

Solve each equation and match each letter with the answers below.

A $3x = 12$ **R** $x + 7 = 12$ **S** $x - 9 = 5$ **T** $x \div 6 = 2$ **W** $7 + x = 10$

									!
14	12	4	5		3	4	5	12	14

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Foundation Worksheet

9:11 | Solving Problems Using Algebra

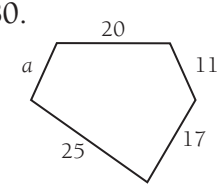
Name: _____ **Class:** _____

Examples

1 The total of 5, 9, 13 and a number is 45. What is the missing number?
Let n = number.
$\therefore n + 5 + 9 + 13 = 45$
$n + 27 = 45$
$n = 18$ *Subtract 27*
$\therefore$ number is 18.

2 The product of a number and 21 is 273. Find the number.
Let x = number.
$21 \times x = 273$
$x = 273 \div 21$
$x = 13$
$\therefore$ number is 13.

3 Find the side a if the perimeter is 80.
$a + 20 + 11 + 17 + 25 = 80$
$a + 73 = 80$ (-73)
$a = 7$
$\therefore$ side is 7.



- **Form an equation.**
- **Solve this equation to answer the problem.**

Exercise

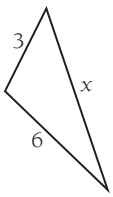
1 Solve each equation.

a $x + 9 = 10$ **b** $5x = 30$ **c** $y - 4 = 11$ **d** $t \div 3 = 6$ **e** $13 + m = 20$

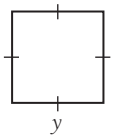
f $a + 5 + 7 = 30$ **g** $15x = 120$ **h** $p + 18 + 13 + 6 = 45$ **i** $\frac{c}{6} = 10$

2 Form an equation, then solve it.

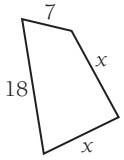
a Perimeter = 18. Find x.



b Perimeter is 24. Find y.



c Perimeter = 50. Find x.



d The sum of two numbers is 40. If one is 18, what is the other?

e If two numbers multiply to give 40 and one number is 8, what is the other?

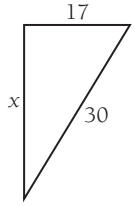
f The difference between a number and 12 is 13, what is the number?

g Five copies of a CD cost me $74.50. Form an equation, then find the cost, c.

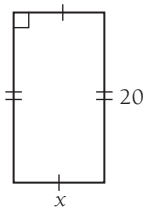
h Three sisters shared $100. If Tina got $30, and Janet got $24, how much did Helen receive?

i An amount of money is divided evenly between 6 people. If they each get $22, how much was the original amount?

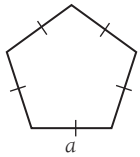
j Perimeter = 70. Find x.



k Area = 180. Find x.



l Perimeter = 85. Find a.



m A box of 144 apples is shared evenly. If each person gets 9 apples, how many people are there?

n Duncan starts shopping with $120. If he ends up with $84, how much did he spend?

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Foundation Worksheet

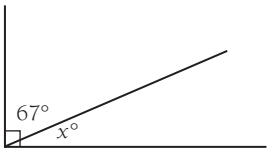
11:04 Finding the Size of an Angle

Name: _____ **Class:** _____

Examples

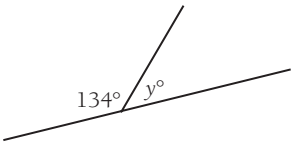
Find the value of each pronumeral.

1 A right angle = 90°.



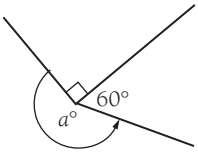
$x = 90 - 67$
$= 23$

2 A straight angle = 180°.



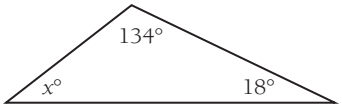
$y = 180 - 134$
$= 46$

3 A revolution = 360°.



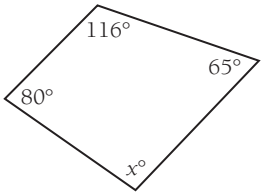
$a = 360 - 90 - 60$
$= 210$

4 The angle sum of a triangle is 180°.



$x = 180 - 134 - 18$
$= 28$

5 The angles in a quadrilateral total 360°.

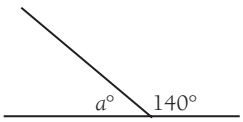


$x = 360 - 80 - 116 - 65$
$= 99$

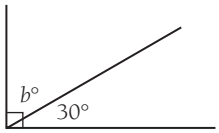
Exercise

Find the value of each pronumeral.

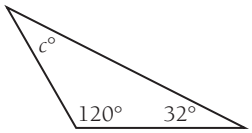
a



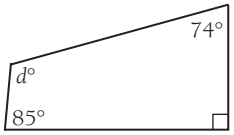
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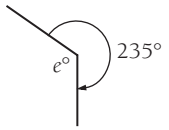
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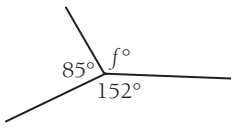
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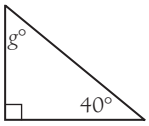
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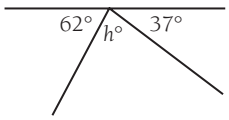
f



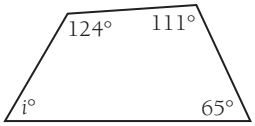
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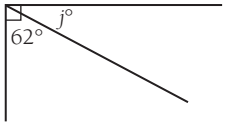
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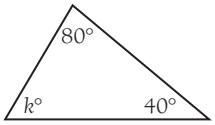
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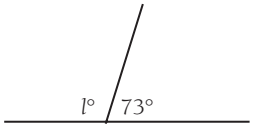
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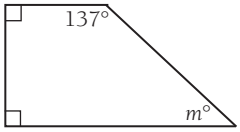
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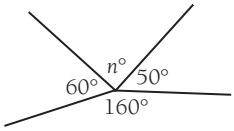
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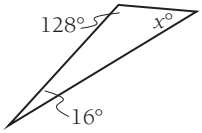
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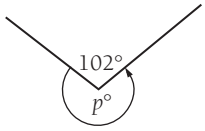
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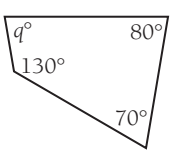
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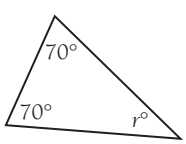
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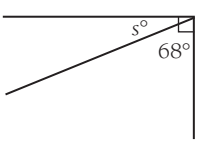
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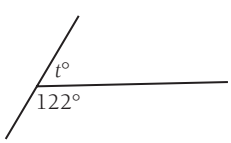
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Foundation Worksheet

12:01 | Measuring Instruments

Name: _____ **Class:** _____

Exercise

1 Measure each length in centimetres.

a _____ **b** _____ **c** _____

d _____ **e** _____

f _____ **g** _____

h _____ **i** _____

2 What time is shown?

a 5:00

b 

c 

d PM 11:02

e 

f AM 10:10

g 

h AM 1:29

i PM 7:01

j 

k AM 3:20

l 

Fun Spot 12:01 | Dad: 'How did you find your maths test?'



Put these times in order, left to right, from earliest to latest in the day, in the boxes below to find the code. Some have been done for you. Use the code to answer the question.

(earliest)	9	11				4			8					10	(latest)
	A	E	F	I	L	N	O	R	S	T	U	W	Y		

1 8 pm **2** 10:15 am **3** 7 o'clock at night **4** noon

5 quarter to seven in the morning **6** 1:30 pm **7** 6:15 am

8 half past three in the afternoon **9** ten minutes past midnight

10 quarter past nine at night **11** 4:36 am

12 two in the afternoon **13** twenty past six in the evening

S		N	:	'		N						N	A		E		Y
8	6	4			3	4	7	6	12	13	3	4	9	13	11	2	10

			A	S	N				S		!'
5	13		1	9	8	4	13		2	6	8	13

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Foundation Worksheet

12:02 | Units of Length

Name: _____ **Class:** _____

Examples

- **When converting or changing to a smaller unit, multiply.**
- **When converting to a larger unit, divide.**

1 5 m = . . . cm
m is larger unit,
multiply by 100
5 × 100 = 500 cm

2 80 mm = . . . cm
mm is smaller unit,
divide by 10
80 ÷ 10 = 8 cm

3 2 min = . . . s
min is larger unit,
2 × 60 = 120 s

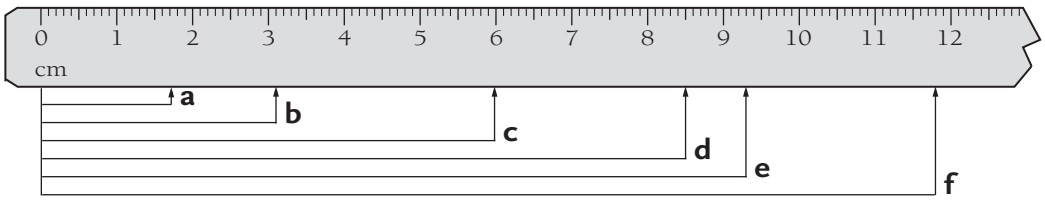
4 360 min = . . . h
min is smaller unit,
360 ÷ 60 = 6 h

10 mm = 1 cm
100 cm = 1 m
1000 mm = 1 m
1000 m = 1 km

1 h = 60 min, 1 min = 60 s

Exercise

1 Write down the length of each interval to the nearest centimetre.



2

a 8 cm = . . . mm	**b** 300 cm = . . . m	**c** 90 mm = . . . cm	**d** 8000 mm = . . . m
e 7 km = . . . m	**f** 9 m = . . . cm	**g** 3 m = . . . mm	**h** 12 cm = . . . mm
i 5000 m = . . . km	**j** 1500 cm = . . . m	**k** 2 km = . . . m	**l** 11 m = . . . mm
m 50 mm = . . . cm	**n** 10 km = . . . m	**o** 8000 m = . . . km	**p** 600 cm = . . . m
q 12 000 mm = . . . m	**r** 20 cm = . . . mm	**s** 62 000 m = . . . km	**t** 6 m = . . . mm

3

a 3 min = . . . s	**b** 2 h = . . . min	**c** 120 s = . . . min	**d** 600 min = . . . h
e 5 min = . . . s	**f** 480 s = . . . min	**g** 24 h = . . . min	**h** 300 min = . . . h
i 20 min = . . . s	**j** 30 min = . . . h	**k** 900 s = . . . min	**l** 7 h = . . . min
m 12 h = . . . min	**n** 10 min = . . . s	**o** 4 h = . . . min	**p** 15 min = . . . h

Fun Spot 12:02 | What do bees do if they want to catch public transport?



Fill in the gaps for each question. Match the letters with the answers below.

A 75 cm = . . . mm	**B** 5 m = . . . mm	**I** 5 m = . . . cm	
O 500 cm = . . . m	**P** 570 mm = . . . cm	**S** 7 m = . . . mm	
T 7 cm = . . . mm	**U** 70 mm = . . . cm	**W** 75 m = . . . cm	**Z** 5 cm = . . . mm

																.
7500	750	500	70	750	70	750	5000	7	50	50	7000	70	5	57		

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Foundation Worksheet

12:03 | Measuring Length

Name: _____ **Class:** _____

Examples

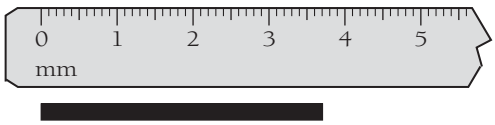
- Measuring is rounding off.
- Write the measurement it is closest to.

1



This measures 3 cm *to the nearest cm*, since it is only a bit more than 3 cm long.

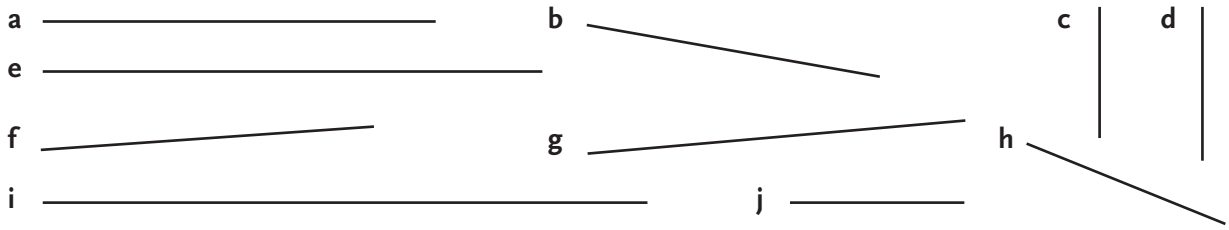
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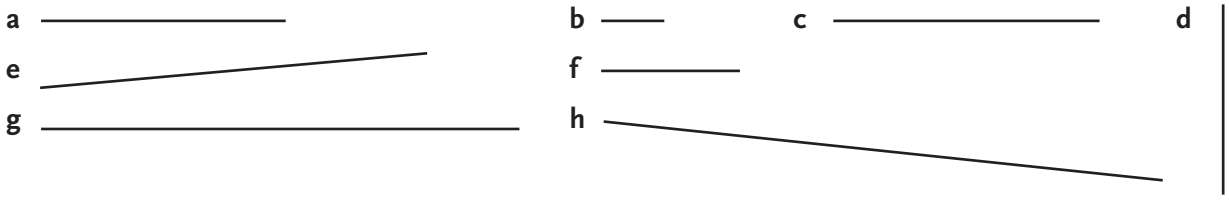
This measures 37 mm to the nearest mm, since the ruler is marked in mm. (It would measure 4 cm to the nearest cm.)

Exercise

1 Measure these lengths correct to the nearest centimetre.



2 Measure these lengths in millimetres.



Fun Spot 12:03 | What's the easiest way to get a day off school?

Put these lengths in order from shortest to longest in the boxes below to find the code. (Change to mm first.) Use the code to solve the riddle.

(shortest) | 11 | | | | | | | | | | | (longest)

A D I L N R S T U W Y

1 19 mm **2** 3 cm **3** 28 mm **4** 6·2 cm **5** 6·7 cm **6** 59 mm

7 1 cm **8** 39 mm **9** 5 cm **10** 42 mm **11** 4 mm

	A		
4	11	1	9

6	2	9	1	3

	A					A		!
10	11	9	6	8	7	11	5	

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Foundation Worksheet

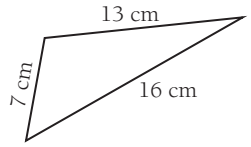
12:05 Perimeter

Name: _____ **Class:** _____

Examples

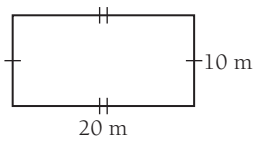
- **Perimeter is the total length around a shape.**
- **The same marking on sides mean they have the same length.**
- **For a square, $P = 4 \times$ side.**
  For a rectangle, $P = 2 \times$ length $+ 2 \times$ breadth.

1



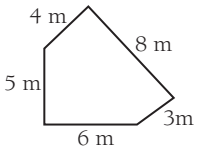
$P = (7 + 13 + 16)$ cm
$= 36$ cm

2



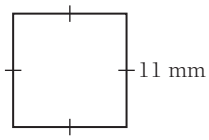
$P = 2 \times 20 + 2 \times 10$ m
$= 60$ m

3



$P = 4 + 8 + 3 + 6 + 5$ m
$= 26$ m

4

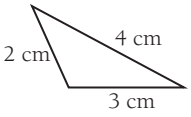


$P = 4 \times 11$ mm
$= 44$ mm

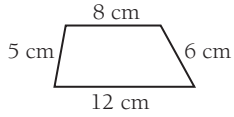
Exercise

1 Calculate each perimeter.

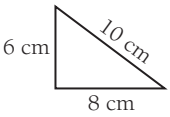
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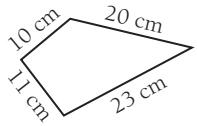
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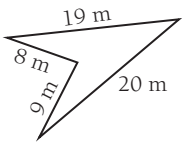
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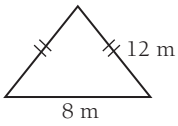
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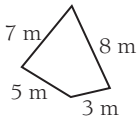
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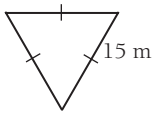
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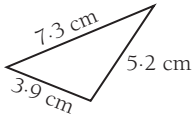
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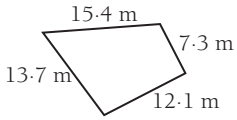
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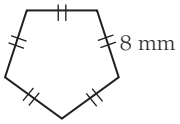
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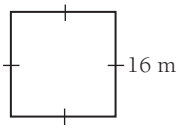
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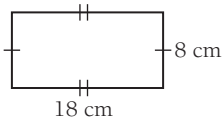
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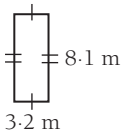
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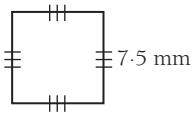
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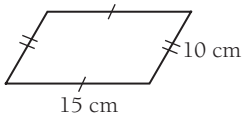
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o



p



2 Find the perimeter of the following shapes.

a a square of side length 10 cm
b a rectangle with a length of 16 m and a breadth of 5 m
c a square with a side length of 50 m
d a rectangle with a length of 1·9 cm and a width of 1·1 cm
e a rectangle with length 37 mm and breadth 30 mm
f a square with sides 3·6 cm long
g a square with each side length 110 m
h a rectangle with length 2·5 cm and breadth 1·7 cm
i a triangle with sides 17 cm, 24 cm and 36 cm
j a quadrilateral with sides of length 9·3 m, 14 m, 16·2 m and 20·7 m

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Foundation Worksheet

12:06 | The Calendar and Dates

Name: _____ **Class:** _____

Examples

365 days = 1 year 2 weeks = 1 fortnight
7 days = 1 week 12 months = 1 year

1 How many days in 3 weeks?
$3 \times 7 = 21$ days

2 How many years in 60 months?
$60 \div 12 = 5$ years

3 How many days in 2 years?
2×365
$= 730$ days

4 How many days from 8 to 20 September?
$20 - 8 = 12$ days

5 How many days from 8 June to 14 July?
8 to 30 June = 22
+ 14 in July = 14
Total = 36 days

6 How many days from 28 April to 17 June?
28 to 30 April = 2
+ May = 31
+ June = 17
Total = 50 days

Exercise

1 **a** How many days in 2 weeks?
b How many years in 36 months?
c How many days in 3 years?
d How many fortnights in 16 weeks?
e How many days in 4 fortnights?
f How many months in 4 years?
g How many days in 10 weeks?
h How many weeks in 84 days?
i How many (full) weeks in a year?
j How many months in 10 years?
k How many weeks in 280 days?
l How many years in 72 months?

2 How many days from:
a 13 to 20 June?
b 1 to 24 April?
c 22 October to 13 November?
d 30 March to 21 April?
e 16 February to 16 March?
f 7 April to 7 July?
g 20 October to Christmas Day?
h 3 August to the end of the month?
i 10 August to 10 October?
j 15 January to 9 February?

Fun Spot 12:06 | What is bigger when it is upside down?



Put each day in order as it comes during the year. The first one has been done for you.

B Christmas Day ☐ **E** April Fool's Day ☐ **H** Fathers' Day ☐
I New Year's Day [1] **M** Australia Day ☐ **N** Anzac Day ☐
R Mothers' Day ☐ **S** Boxing Day ☐ **T** New Year's Eve ☐
U Halloween (31 October) ☐ **X** Valentine's Day (14 February) ☐

Use the letters to complete the riddle.

11	7	4		5	8	2	9	4	6		10	1	3				

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Foundation Worksheet

12:07 | Clocks and Times

Name: ______________________ **Class:** ____________

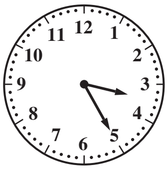
Examples

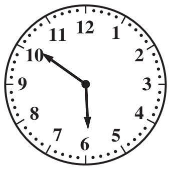
1 How many minutes in the following times?

a 4 hours
4×60
$= 240$ min

b $2\frac{1}{2}$ hours
$2\frac{1}{2} \times 60$
$= 150$ min

2 Read the times in 'minutes to' or 'minutes past'.

a 
25 minutes past 3

b 
10 minutes to 6

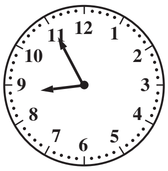
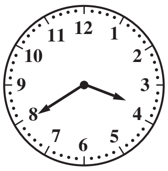
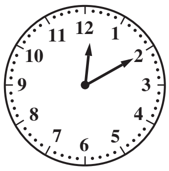
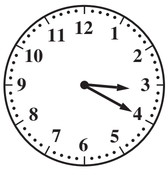
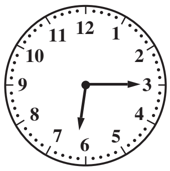
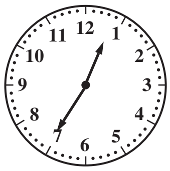
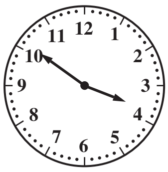
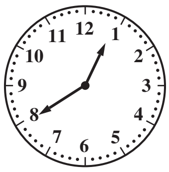
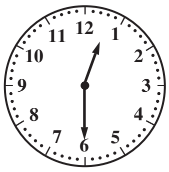
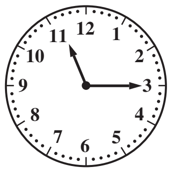
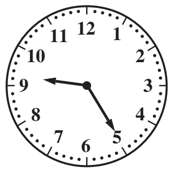
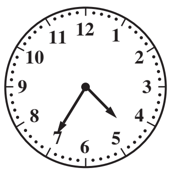
There are 5 minutes between numbers.

Exercise

1 How many minutes in the following lengths of time?

a 1 hour **b** 2 hours **c** 10 hours **d** 24 hours **e** $\frac{1}{2}$ hour **f** $1\frac{1}{2}$ hours
g 5 hours **h** $5\frac{1}{2}$ hours **i** $\frac{1}{4}$ hour **j** $2\frac{1}{4}$ hours **k** 11 hours **l** 7 hours

2 Write each time as 'minutes to' or 'minutes past'.

a 
b 
c 
d 
e 
f 
g 
h 
i 
j 
k 
l 

Fun Spot 12:07 | Why is a belt like a garbage truck?

Fill in the blanks below, then use the letters to answer the riddle.

A 3 h = . . . min **B** 2 days = . . . h **C** 5 min = . . . s **D** 660 min = . . . h
E 30 min = . . . s **G** 300 min = . . . h **H** 10 min = . . . s **I** 7 d = . . . h
N 180 s = . . . min **O** 30 s = . . . m **R** $\frac{1}{2}$ day = . . . h **S** 6 h = . . . min
T 120 s = . . . min **U** 24 h = . . . day **W** $\frac{3}{4}$ h = . . . min

48	1800	300	180	1	360	1800

168	2

5	$\frac{1}{2}$	1800	360

180	12	$\frac{1}{2}$	1	3	11

180	3	11

5	180	2	600	1800	12	360

2	600	1800

45	180	168	360	2

.

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Foundation Worksheet

12:08 | Operating with Time

Name: _____ **Class:** _____

Examples

1 Find 3 h 10 min plus 4 h 20 min.

3 h 10 min
4 h 20 min
7 h 30 min

2 Find the difference between 9 am and 8 pm.

9 am to 12 noon = 3 h
noon to 8 pm = 8 h
∴ difference = 11 h

3 How long is it from

a 8:15 to 8:55?

55 min
−15 min
40 min

b 3:30 to 5:45?

5 h 45 min
−3 h 30 min
2 h 15 min

Exercise

1 **a** 2 h + 5 h 30 min **b** 1 h 20 min + 6 h **c** 4 h 15 min + 2 h 20 min
d 3 h 25 min + 2 h 15 min **e** 2 h 55 min + 8 h **f** 10 h + 4 h 5 min
g 6 h 25 min + 7 h 5 min **h** 1 h 17 min + 3 h 36 min **i** 8 h 7 min + 2 h 19 min
j 4 h 15 min + 27 min **k** 34 min + 2 h 12 min **l** 3 h 5 min + 2 h 9 min

2 What is the difference between the following times (on the same day)?

a 10 am and 1 pm **b** 7 pm and 11 pm **c** 4 am and 10 am
d 8 am and 3 pm **e** 1 am and noon **f** 3 am and 2 pm
g 11 am and 6 pm **h** noon and 5 pm **i** noon and midnight
j 3 pm and 8 pm **k** 6 am and 6 pm **l** 7 am and 4 pm

3 How long is it between the following times?

a 5:35 to 5:50 **b** 1:20 to 1:42 **c** 11:10 to 11:40
d 7:05 to 7:30 **e** 3:10 to 4:30 **f** 10:15 to 11:35
g 9:05 to 11:35 **h** 6:30 to 9:45 **i** 2:16 to 2:50
j 4:30 to 6:50 **k** 3:25 to 4:30 **l** 8:27 to 8:50

Fun Spot 12:08 | My sister went on a crash diet.



Put these times in order from the earliest to the latest in the day (1st to 13th).
Then use the letters to complete the punchline below.

A 2:30 pm **C** noon **E** 7:20 am **H** 6:45 pm **I** 4 am
K 9:16 pm **L** 11:52 am **O** 1 am **R** 3:05 pm **S** 5:55 am
T 9 pm **W** 12:30 am **Y** 11:10 pm

3rd	4th		11th	10th	8th	11th		1st	10th	13th		4th	10th	5th					
6th	2nd	2nd	12th	4th		8th		1st	9th	5th	7th	12th	?						

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Foundation Worksheet

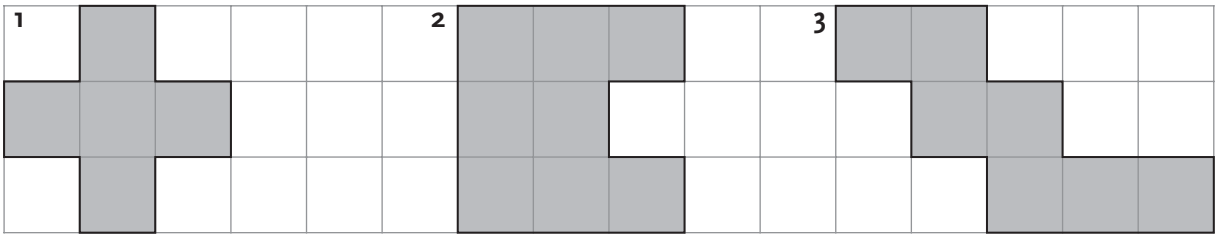
13:01 | The Definition of Area

Name: _____ **Class:** _____

Examples

Find the area of each shape by counting square centimetres.

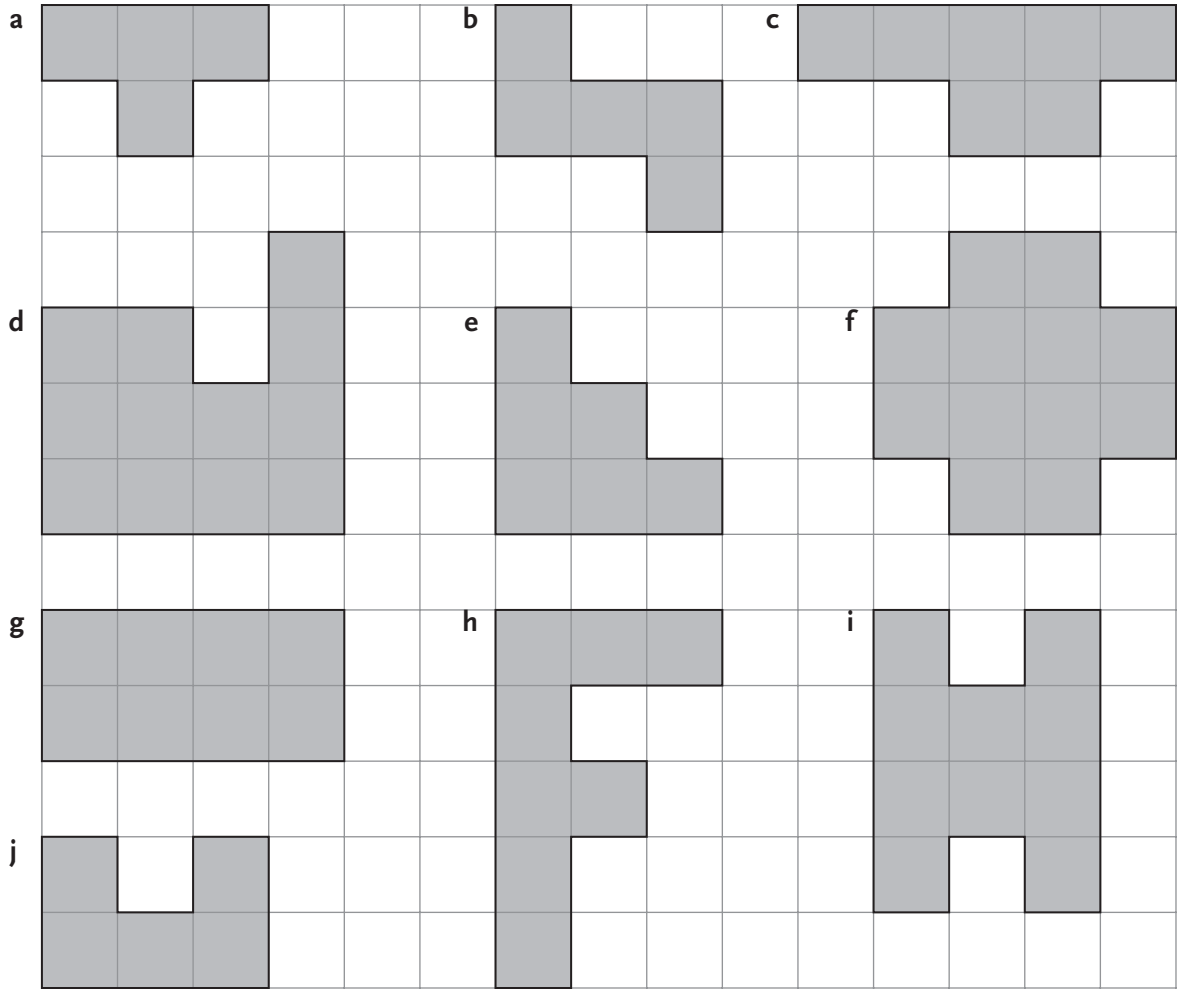
The area of a figure is measured by counting the number of square units it covers.



Area = 5 cm^2 Area = 8 cm^2 Area = 7 cm^2

Exercise

1 These shapes are drawn on centimetre grid paper. Find their areas.



2 **a** On your own grid paper draw 3 different shapes with an area of 6 cm^2.

b Draw shapes with areas of 3 cm^2, 5 cm^2, 7 cm^2, 10 cm^2.

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Foundation Worksheet

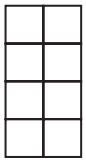
13:02 | Area of a Rectangle

Name: _____ **Class:** _____

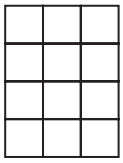
Examples

For rectangles, multiply squares across by squares down: area = length × breadth.

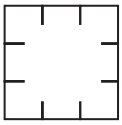
Find the area of each figure.

1 

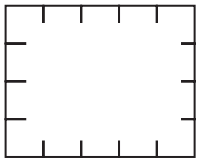
Area = 8 unit^2
(by counting)

2 

Area = 12 unit^2
(3 squares across
by 4 squares down)

3 

Area = 9 unit^2
(3 squares across
by 3 squares down)

4 

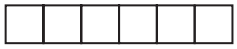
Area = 20 unit^2
(5 across by 4 down)

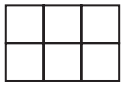
Exercise

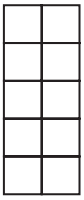
1 Find the area in square units.

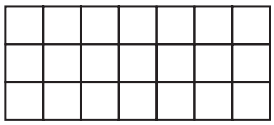
a 

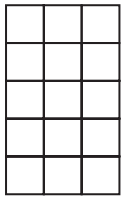
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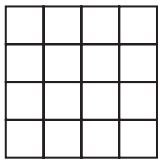
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d 

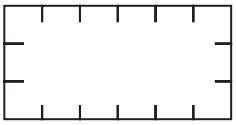
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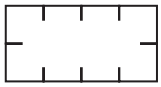
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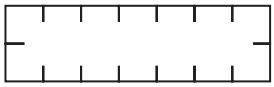
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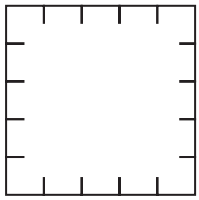
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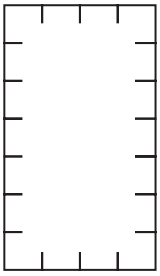
2 Find the area in square units.

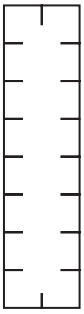
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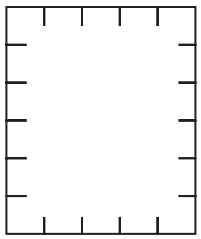
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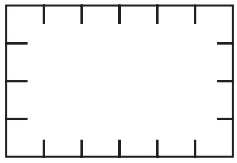
d 

e 

f 

g 

h 

i 

3 Calculate the area of these rectangles.

a length 10 cm, breadth 4 cm
b length 14 cm, breadth 2 cm
c length and breadth both 10 cm
d length 8 cm, breadth 4 cm
e length 5 cm, breadth 9 cm
f length 3 cm, breadth 11 cm
g length and breadth both 6 cm
h length and breadth both 8 cm
i length 20 cm, breadth 5 cm
j length 15 cm, breadth 2 cm

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Foundation Worksheet

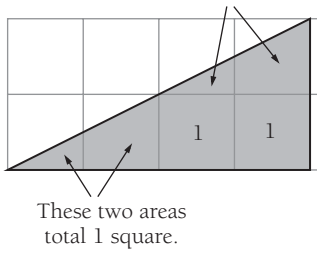
13:03 | Area of a Triangle

Name: _____ **Class:** _____

Examples

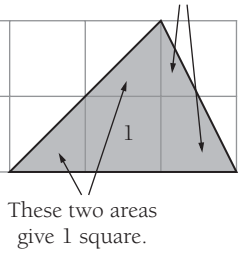
Find the area of these triangles.

1



Area = 4 cm²

2



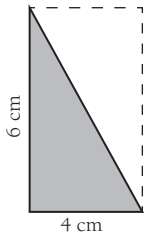
Area = 3 cm²

Easier than counting is to say each triangle is half the rectangle.

Area $= \frac{1}{2} \times (4 \times 2)$ cm²
$= 4$ cm²

Area $= \frac{1}{2} \times (3 \times 2)$ cm²
$= 3$ cm²

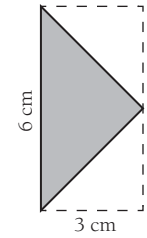
3



The triangle is half the 6 by 4 rectangle.

$\therefore$ Area $= \frac{1}{2} \times (6 \times 4)$ cm²
$= 12$ cm²

4

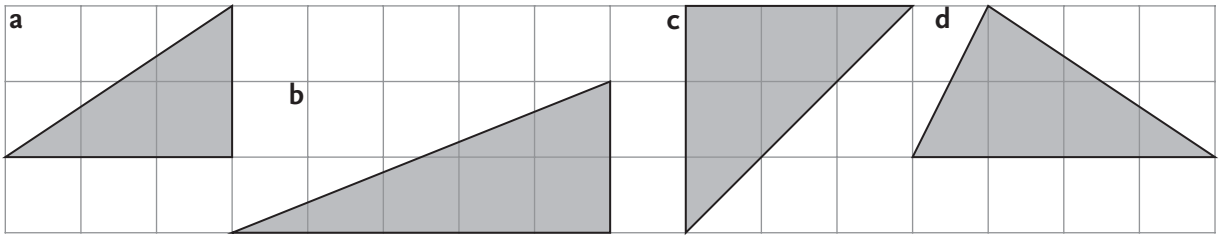


The triangle is half the 3 by 6 rectangle.

$\therefore$ Area $= \frac{1}{2} \times (3 \times 6)$ cm²
$= 9$ cm²

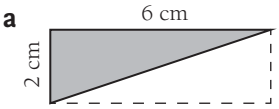
Exercise

1 Use the centimetre grid to find the area of each triangle.

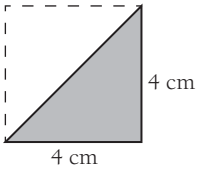


2 Calculate the area of each triangle.

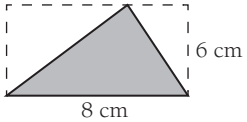
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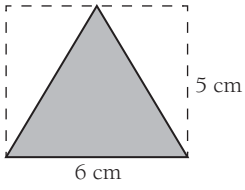
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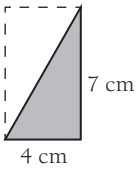
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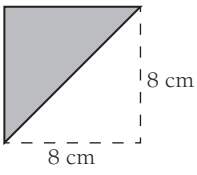
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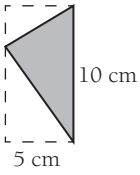
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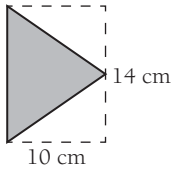
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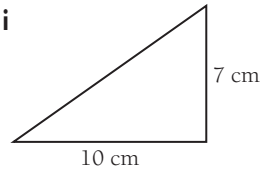
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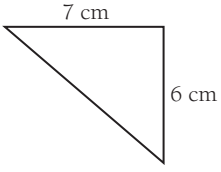
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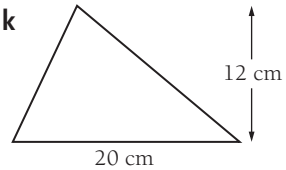
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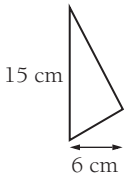
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Foundation Worksheet

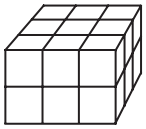
13:06 | Volume of a Rectangular Prism

Name: _____ **Class:** _____

Examples

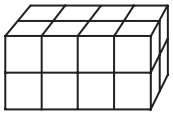
To find the volume of a rectangular prism, count (or imagine) the number of cubes in the box. It's helpful to count layer by layer.

1



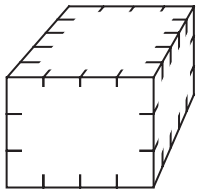
This prism has 2 layers each with 3×3 cubes.
$\therefore$ Volume $= (3 \times 3) \times 2$
$= 18$ cm^3

2



This prism has 2 layers each with 4×2 cubes.
$\therefore$ Volume $= (4 \times 2) \times 2$
$= 16$ cm^3

3



This prism has 3 layers with 4×5 cubes.
$\therefore$ Volume $= 4 \times 5 \times 3$
$= 60$ cm^3

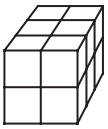
Exercise

1 Find the volume in cm^3.

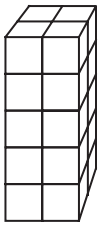
a



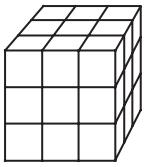
b



c

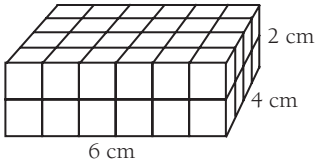


d

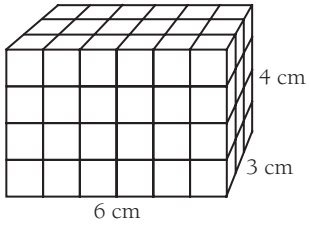


2 Find the volume of each prism in cm^3.

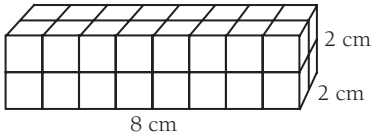
a



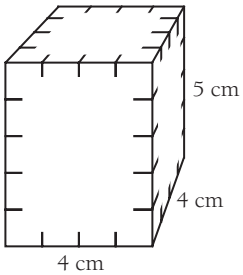
b



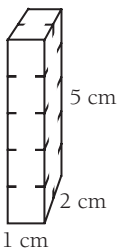
c



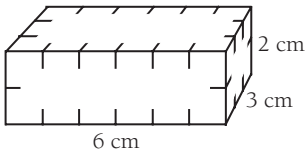
d



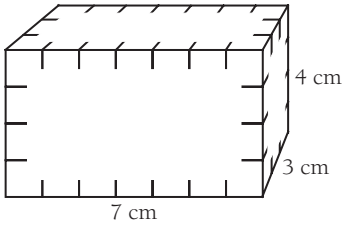
e



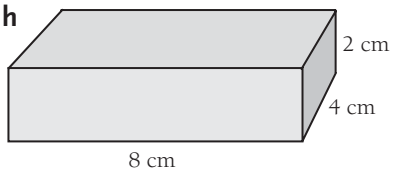
f



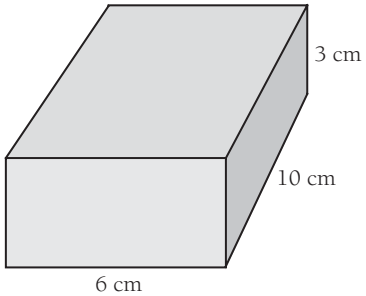
g



h



i



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Foundation Worksheet

13:07 | Capacity

Name: ______________________ **Class:** ____________

Capacity is the volume of liquids. It is measured using the litre (L) as the basic unit.

- **1 litre (L) = 1000 millilitres (mL)**
- **1000 L = 1 kilolitre (kL)**

Volume and capacity are connected:

- **1 cm^3 = 1 mL**
- **1000 cm^3 = 1 L**

Examples

Complete these conversions.

1 5 L = . . . mL
L is larger unit
∴ × 1000
5 L = 5000 mL

2 8000 L = . . . kL
L is smaller unit
∴ ÷ 1000
8000 L = 8 kL

3 25 mL = . . . cm^3
equal units
∴ 25 mL = 25 cm^3

4 2 L = . . . cm^3
L is larger unit
∴ × 1000
2 L = 2000 cm^3

Exercise

Complete.

1 **a** 2 L = . . . mL **b** 7 L = . . . mL **c** 4000 mL = . . . L **d** 11 000 mL = . . . L
e 3000 mL = . . . L **f** 6 L = . . . mL **g** 5000 mL = . . . L **h** 14 000 mL = . . . L
i 10 L = . . . mL **j** 8 L = . . . mL **k** $\frac{1}{2}$ L = . . . mL **l** 20 000 mL = . . . L

2 **a** 3 kL = . . . L **b** 2000 L = . . . kL **c** 9 kL = . . . L **d** 6000 L = . . . kL
e 5000 L = . . . kL **f** 25 kL = . . . L **g** 7 kL = . . . L **h** 10 000 L = . . . kL
i 0·5 kL = . . . L **j** 0·4 kL = . . . L **k** 700 L = . . . kL **l** 4000 L = . . . kL

3 **a** 3 cm^3 = . . . mL **b** 15 cm^3 = . . . mL **c** 100 mL = . . . cm^3 **d** 4000 cm^3 = . . . L
e 400 cm^3 = . . . mL **f** 8 L = . . . cm^3 **g** $\frac{1}{2}$ L = . . . cm^3 **h** 750 mL = . . . cm^3
i 0·1 L = . . . cm^3 **j** 5000 cm^3 = . . . L **k** 3000 cm^3 = . . . mL **l** 1500 cm^3 = . . . L

Fun Spot 13:07 | Why did the girl plant birdseed?

Complete the following, then match the letters with the answers below.

A 35 mL = . . . cm^3 **B** 35 L = . . . mL **C** 3500 mL = . . . L
D 300 L = . . . kL **E** 5 L = . . . mL **H** 350 cm^3 = . . . mL **I** 2·5 L = . . . mL
N 0·5 kL = . . . L **O** 6 mL = . . . cm^3 **R** 4000 L = . . . kL **S** 5·5 L = . . . mL
T 3000 mL = . . . L **U** 50 cm^3 = . . . mL **W** 3 L = . . . cm^3

35 000	5000	3·5	35	50	5500	5000

5500	350	5000

3000	35	500	3	5000	0·3

3	6

4	35	2500	5500	5000

3·5	35	500	35	4	2500	5000	5500

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Challenge Worksheet

Extension 1:09 | Solving Puzzles

Name: _____ **Class:** _____

Examples

Complete this magic square.
You can see from the bottom line **1** that the total is 120. Each row, column and diagonal must have a sum of 120, the magic number.
You find the 26 from column **2**.
Then find the 40 from row **3**.
Then move to column **4** to fill in the empty box, and so on until completed.

			16	42	28
3 120 →	38	14	**40**	**26**	2
			24	0	
	46	22		34	
1 120 →	20	6	32	18	44
			↓ **4**	↓ **2**	
			120	120	

Exercise

1 Find the missing numbers in these giant magic squares.

a

6	32	18	44	30
	16	42	28	
14		26		38
48	24		36	
		34	20	46

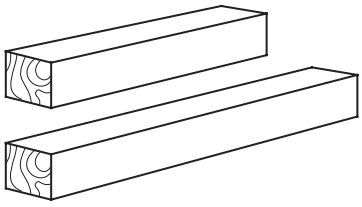
b

	6	19		15
	18	1	14	22
	5	13	21	9
	12		8	16
		7		3

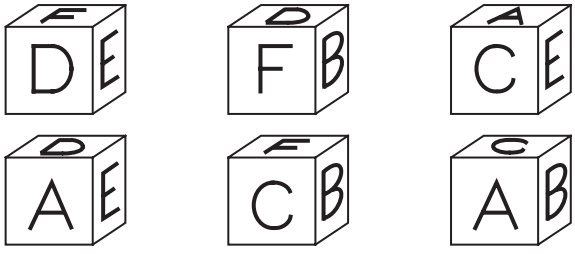
c

3	10	17		26
22				
11		20		
25	7	9	16	
14		23	5	12

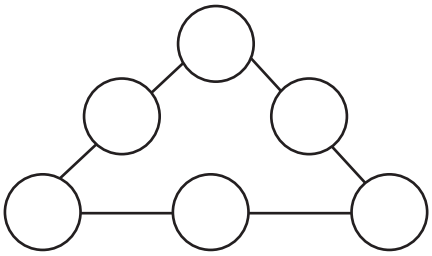
2 A carpenter has two lengths of timber, each with a square cross section of 100 mm by 100 mm. He saws one length into 3 pieces in six minutes. At this rate of sawing how long would it take him to saw the second length into 6 pieces?



3 Here are six different views of the same cube. What letter is on the opposite face To F? To E? To C?
(*Hint:* Make a cube.)



4 How would you place the numbers
1, 2, 3, 4, 5 and 6
in the circles so that the sum of the numbers on each side of the triangle is 10?



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Challenge Worksheet

Extension 3:06 | HCF and LCM by Prime Factors

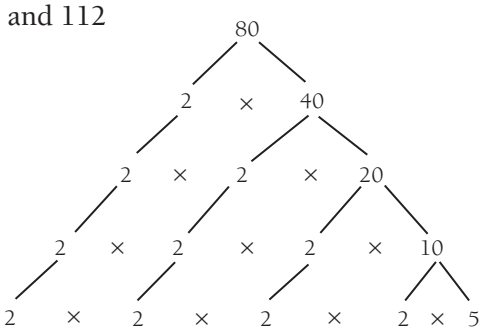
Name: _____ **Class:** _____

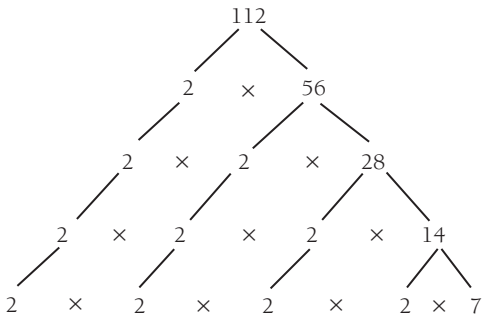
- **HCF is the highest factor common to both numbers and must contain all the common prime factors.**
- **LCM is the lowest number both numbers divide into and may be found by listing the factors of the bigger number and including factors of the smaller number not yet included.**

Examples

Find the HCF and LCM of the following.

1 80 and 112





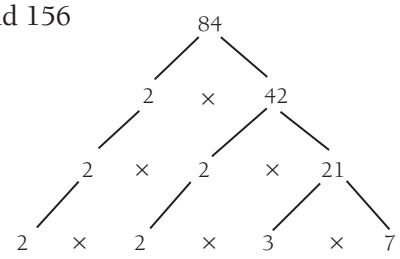
$\therefore 80 = 2 \times 2 \times 2 \times 2 \times 5$

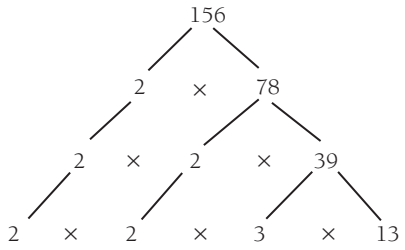
$\therefore 112 = 2 \times 2 \times 2 \times 2 \times 7$

HCF $= 2 \times 2 \times 2 \times 2 = 16$ (all the factors that are common to both)

LCM $= 2 \times 2 \times 2 \times 2 \times 7 \times 5 = 560$ (the factors of 112 and the 5 not included from 80)

2 84 and 156





$\therefore 84 = 2 \times 2 \times 3 \times 7$

$\therefore 156 = 2 \times 2 \times 3 \times 13$

HCF $= 2 \times 2 \times 3 = 12$ (all the factors that are common to both)

LCM $= 2 \times 2 \times 3 \times 13 \times 7 = 1092$ (the factors of 156 and the 7 not included from 84)

Exercise

Find the HCF and LCM of the following pairs of numbers.

1 16 and 24
2 18 and 24
3 36 and 100
4 80 and 140
5 81 and 108
6 48 and 72
7 30 and 75
8 36 and 54
9 90 and 135
10 60 and 84
11 50 and 75
12 70 and 98

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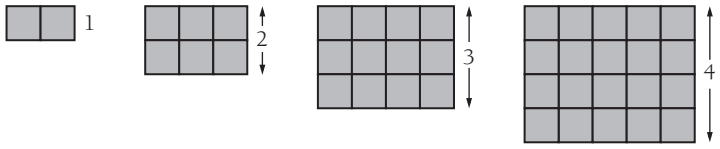
Challenge Worksheet

Extension 8:02 | Describing Number Patterns

Name: _____ **Class:** _____

Exercise

1 This table of values gives the number of small squares in each figure.



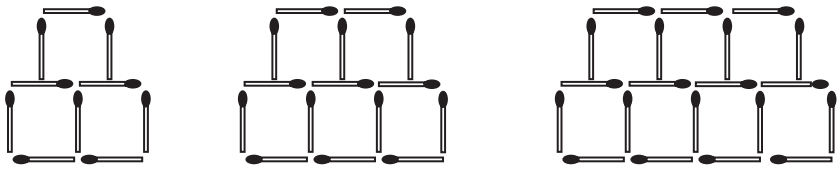
Height of figure	1	2	3	4	5	6	7	8
Number of small squares	2	6	12	20				

Hint: $1 \times 2 \quad 2 \times 3 \quad 3 \times 4 \quad 4 \times 5$

a What rule links the number of small squares to the height of each figure?

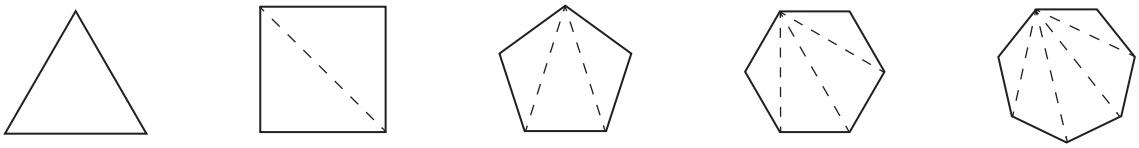
b Complete the table for heights up to 8.

2 **a** Write down the number pattern that these matches represent.



b Write the next two numbers in the pattern.

3



a Complete this table.

Diagonals Drawn from One Vertex

Number of sides (s)	3	4	5	6	7	8	9	10
Number of diagonals (d)								

b The pattern in words is . . . .

c Write the rule using s, d.

4 Find the rule connecting the top row number with the bottom row number. (B = . . .) What would the tenth bottom number be?

a

Top (T)	1	2	3	4
Bottom (B)	1	4	9	16

b

Top (T)	1	2	3	4
Bottom (B)	11	10	9	8

5 Find the pattern, then write the next two numbers.

a 5, 9, 13, 17, . . . , ...

b 8, 4, 2, 1, . . . , ...

c 20, 15, 10, 5, . . . , ...

d 400, 4000, 40 000, 400 000, . . . , ...

6 Write the first four numbers in the pattern with these rules.

a $3n + 2$ **b** $6 - n$ **c** $2n - 7$ **d** $n^2 + 5$

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Foundation Worksheets

Answers

1:03 Know Your Tables

1 7, 8, 11, 10, 14, 9, 15, 13, 12, 16
2 9, 3, 6, 10, 2, 5, 8, 1, 4, 7
3 0, 14, 7, 35, 70, 21, 49, 28, 56, 42, 63, 77
4 12, 24, 8, 28, 32, 4, 16, 36, 20, 40, 0, 44
5 9, 12, 8, 11, 5, 6, 10, 14, 13, 15
6 6, 1, 2, 5, 8, 3, 0, 7, 9, 4
7 24, 12, 18, 27, 21, 15, 6, 3, 0, 33, 9, 30
8 13, 14, 10, 12, 16, 8, 15, 11, 17, 9
9 42, 12, 24, 60, 36, 48, 54, 18, 66, 6, 0, 30
10 9, 8, 12, 5, 7, 10, 3, 13, 6, 11
11 9, 27, 72, 99, 18, 81, 90, 0, 36, 63, 54, 45
12 1, 10, 4, 2, 7, 6, 9, 3, 0, 5

1:06 Powers of Numbers

1	**a** 16	**b** 64	**c** 9	**d** 1	**e** 81	**f** 4	**g** 25	**h** 100
	i 49	**j** 36	**k** 8	**l** 1	**m** 125	**n** 27	**o** 1000	
2	**a** 49	**b** 100	**c** 25	**d** 64	**e** 4	**f** 1	**g** 36	**h** 16
	i 9	**j** 81	**k** 27	**l** 8	**m** 64	**n** 1000	**o** 216	
3	**a** 10^3	**b** 6^2	**c** 2^6	**d** 7^2	**e** 5^4	**f** 9^5	**g** 8^4	**h** 4^3
	i 10^5							

1:07 Rounding Numbers

1	**a** 370	**b** 80	**c** 40	**d** 140	**e** 570	**f** 6080	**g** 590	**h** 8020
	i 40	**j** 880	**k** 60	**l** 210				
2	**a** 8700	**b** 1200	**c** 300	**d** 500	**e** 6600	**f** 900	**g** 2800	**h** 200
	i 500	**j** 9900	**k** 7000	**l** 300				

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2:01 Problem Solving

1 $35
2 $55.40
3 $65.50
4 33
5 $10.32, $10.30
6 344, 86
7 54
8 20 000 000
9 24
10 $840

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3:01A Grouping Symbols

1 **a** 50 **b** 42 **c** 9 **d** 30 **e** 49 **f** 28

2 **a** 18 **b** 4 **c** 2 **d** 20 **e** 30 **f** 16

3 **a** 3 **b** 2 **c** 5 **d** 2 **e** 7 **f** 0

3:03 Language and Symbols

1 **a** F **b** T **c** T **d** F **e** T **f** T

2 **a** $5 < 6$ **b** $10 > 4$ **c** $12 - 5 = 7$ **d** $4 \times 3 > 7$ **e** $10 \neq 4 \times 3$ **f** $6 + 3$

3:07 Divisibility Tests

1 **a** 2, 4, 6, 8, 10, 12, 14, 16, 18, 20 **b** 1, 3, 5, 7, 9, 11, 13, 15, 17, 19
c 5, 10, 15, 20, 25, 30, 35

2 **a** 2, 4, 6, 8, 10, 12, 14, 16, 18, 20 **b** 3, 6, 9, 12, 15, 18, 21, 24, 27, 30
c 4, 8, 12, 16, 20, 24, 28, 32, 36, 40 **d** 10, 20, 30, 40, 50, 60, 70, 80, 90, 100
e 6, 12, 18, 24, 30, 36, 42, 48, 54, 60 **f** 7, 14, 21, 28, 35, 42, 49, 56, 63, 70
g 8, 16, 24, 32, 40, 48, 56, 64, 72, 80 **h** 9, 18, 27, 36, 45, 54, 63, 72, 81, 90

3 **a** 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28 **b** 3, 6, 9, 12, 15, 18, 21, 24, 27 **c** 10, 20

4 **a** 15, 54 321 **b** 40, 125, 100 000
c 12, 201, 54 321, 12 345 **d** 24, 42, 66, 888

3:08 Square Roots

1 **a** 4 **b** 2 **c** 5 **d** 9 **e** 11 **f** 20

2 **a** 7 **b** 8 **c** 10 **d** 12 **e** 1 **f** 15

3 **a** 5 **b** 3 **c** 4 **d** 9

4 **a** 6 **b** 2 **c** 8 **d** 30

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4:01 Review of Decimals

1 **a** 0·3 **b** 0·07 **c** 0·93 **d** 0·6 **e** 0·004 **f** 0·133
g 0·1 **h** 0·73 **i** 0·43 **j** 0·024 **k** 0·92 **l** 0·28
m 0·36 **n** 0·555 **o** 0·002 **p** 0·99 **q** 0·031 **r** 0·607

2 **a** $\frac{7}{10}$ **b** $\frac{3}{100}$ **c** $\frac{53}{100}$ **d** $\frac{9}{100}$ **e** $\frac{123}{1000}$ **f** $\frac{91}{1000}$
g $\frac{13}{100}$ **h** $\frac{59}{100}$ **i** $\frac{563}{1000}$ **j** $\frac{21}{100}$ **k** $\frac{303}{1000}$ **l** $\frac{299}{1000}$
m $\frac{3}{50}$ **n** $\frac{2}{5}$ **o** $\frac{1}{4}$ **p** $\frac{16}{25}$ **q** $\frac{1}{200}$ **r** $\frac{7}{500}$

3 **a** 0·7 **b** 0·71 **c** 0·3 **d** 0·83 **e** 0·51 **f** 0·12
g 0·63 **h** 0·3 **i** 0·43 **j** 0·92 **k** 0·68 **l** 0·5

4:02 Addition and Subtraction of Decimals

1 **a** 6·6 **b** 8·9 **c** 7·84 **d** 3·866 **e** 7·54 **f** 6·53 **g** 17·73 **h** 18·1
i 11·168 **j** 14·523

2 **a** 3·6 **b** 4·2 **c** 12·13 **d** 2·27 **e** 1·8 **f** 0·84 **g** 17·73 **h** 1·09
i 0·559 **j** 5·88

3 **a** 3·3 **b** 9·5 **c** 3·6 **d** 15·68 **e** 23·92 **f** 3·125 **g** 1·73 **h** 10·82
i 24·49 **j** 18·86

4:03 Multiplying a Decimal

1 **a** 0·6 **b** 4·2 **c** 1·77 **d** 23·55 **e** 28·8 **f** 69·2 **g** 92 **h** 61·2
i 23·4 **j** 22·47 **k** 0·32 **l** 0·063

2 **a** 1·2 **b** 3·15 **c** 6·3 **d** 0·22 **e** 10·5 **f** 10·83 **g** 9·52 **h** 66·5
i 20·7 **j** 5·04 **k** 0·108 **l** 11·8

3 **a** 6 **b** 52 **c** 40 **d** 0·7 **e** 35·4 **f** 370 **g** 1660 **h** 42
i 414 **j** 810 **k** 0·32 **l** 20

4:04 Dividing a Decimal

1 **a** 0·4 **b** 0·1 **c** 0·2 **d** 0·01 **e** 0·01 **f** 0·01 **g** 0·3 **h** 0·03
i 0·02 **j** 0·2

2 **a** 0·23 **b** 0·12 **c** 0·09 **d** 0·11 **e** 0·13 **f** 0·13 **g** 0·03 **h** 0·03
i 0·16 **j** 0·31

3 **a** 0·5 **b** 2·1 **c** 1·7 **d** 1·1 **e** 4·3 **f** 1·8 **g** 1·2 **h** 3·1
i 2·3 **j** 2·2

4 **a** 1·02 **b** 2·12 **c** 1·01 **d** 4·13 **e** 1·15 **f** 0·41 **g** 0·42 **h** 1·12
i 1·03 **j** 1·04

4:05 Using Decimals

1 **a** 0·6 kg **b** \$6.72 **c** 43·8 cm **d** 41·1 L
e \$16.24 **f** 5·4 t **g** 22·82 m **h** 2·72 g
i 15·21 h **j** \$25.36 **k** 0·24 **l** 75·6 L

2 **a** 1·1 m **b** \$4.81 **c** \$3.80 **d** 10 cm
e 10·73 t **f** 0·975 kg **g** 4·241 kg **h** \$10.61
i 0·26 m **j** 9·86 mL **k** \$2.27 **l** 8·63 m

3 **a** 0·6 L **b** \$0.47 **c** 1·57 m **d** 0·27 kg
e \$1.02 **f** 2·32 mL **g** 0·041 m **h** 0·11 t
i 2·31 cm **j** 0·081 g **k** \$1.72 **l** 0·8 m

4:06 Multiplying Decimals

1 **a** 0·82 **b** 0·81 **c** 8·4 **d** 18·5 **e** 1·17 **f** 29·05 **g** 1·66 **h** 0·036
i 0·375 **j** 24·4 **k** 21·28 **l** 0·76

2 **a** 0·28 **b** 0·06 **c** 0·035 **d** 0·05 **e** 0·4 **f** 3·6 **g** 0·027 **h** 0·0048
i 0·09 **j** 0·25 **k** 0·0032 **l** 0·006

3 **a** 2·16 **b** 1·25 **c** 0·72 **d** 0·136 **e** 0·427 **f** 0·066 **g** 0·52 **h** 0·0052
i 0·000 84 **j** 0·146 **k** 2·37 **l** 0·24

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4:09 Rounding Off

1 **a** 85c **b** 8c **c** 61c **d** 19c **e** 40c **f** 53c **g** 3c **h** 46c
i 27c **j** 30c **k** 88c **l** 14c **m** 10c **n** 40c **o** 71c **p** 18c
q 60c **r** 6c

2 **a** $8 **b** $9 **c** $4 **d** $4 **e** $2 **f** $2 **g** $8 **h** $9
i $12 **j** $6 **k** $6 **l** $2 **m** $4 **n** $10 **o** $5 **p** $10
q $7 **r** $4

4:10 Application of Decimals

1 **a** $106 **b** $90 **c** $4 **d** $91 **e** $25 **f** $67 **g** $13 **h** $3
i $7 **j** $40

2 **a** 75c **b** 85c **c** 30c **d** 40c **e** 60c **f** 10c **g** 65c **h** 25c
i 95c **j** 5c

3 **a** $4 **b** $4.25 **c** $1.70 **d** $5.60 **e** $3.10 **f** $7.60 **g** $2.95 **h** $3.35
i $8.10 **j** $2.35

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5:10 Division of Directed Numbers

1 **a** −4 **b** −4 **c** −2 **d** −3 **e** −3 **f** −1 **g** −4 **h** −2
i −2 **j** −3

2 **a** −6 **b** 2 **c** 2 **d** −1 **e** −10 **f** 4 **g** −7 **h** −2
i 5 **j** 8 **k** 8 **l** −4 **m** −5 **n** 9 **o** 13

3 **a** −5 **b** 5 **c** 3 **d** −6 **e** −1 **f** 2 **g** 6 **h** −12
i −9 **j** −7 **k** −6 **l** −2 **m** 4 **n** −10 **o** −11

5:11 Using Directed Numbers

1 **a** −2 **b** −1 **c** 9 **d** −1 **e** −7 **f** −7 **g** 11 **h** −35
i −21 **j** 6 **k** 2 **l** −23

2 **a** 8 **b** 5 **c** −14 **d** 0 **e** −1 **f** 8 **g** 28 **h** −15
i 9 **j** −12 **k** −9 **l** −18

3 **a** −2, −3, −4 **b** −12, −15, −18 **c** −5, −10, −15
d 5, 8, 11 **e** 50, −60, 70 **f** −4, 0, 4

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6:04 Addition and Subtraction of Fractions

1 **a** $\frac{5}{10} = \frac{1}{2}$ **b** $\frac{9}{11}$ **2** **a** $\frac{5}{8}$ **b** $\frac{8}{12} = \frac{2}{3}$ **3** **a** $\frac{2}{5}$ **b** $\frac{4}{9}$

4 **a** $\frac{3}{8}$ **b** $\frac{1}{8}$ **c** $\frac{3}{10}$ **d** $\frac{19}{20}$ **e** $\frac{2}{5}$ **f** $\frac{11}{15}$ **g** $\frac{7}{11}$ **h** $\frac{27}{100}$

i $\frac{3}{5}$ **j** $\frac{7}{8}$ **k** $\frac{10}{13}$ **l** $\frac{17}{25}$ **m** $\frac{14}{25}$ **n** $\frac{11}{20}$ **o** $\frac{7}{10}$ **p** $\frac{2}{15}$

q $\frac{37}{100}$ **r** $\frac{29}{40}$ **s** $\frac{11}{12}$ **t** $\frac{83}{100}$ **u** $\frac{5}{10} = \frac{1}{2}$ **v** $\frac{4}{16} = \frac{1}{4}$ **w** $\frac{4}{12} = \frac{1}{3}$ **x** $\frac{5}{20} = \frac{1}{4}$

y $\frac{6}{8} = \frac{3}{4}$

6:05 Addition and Subtraction of Mixed Numbers

1 **a** $1\frac{3}{4}$ **b** $5\frac{1}{10}$ **c** $2\frac{5}{9}$ **d** $7\frac{13}{20}$ **e** $3\frac{7}{8}$ **f** $4\frac{11}{12}$ **g** $10\frac{4}{5}$ **h** $6\frac{12}{25}$

i $1\frac{11}{100}$ **j** $5\frac{5}{6}$

2 **a** $3\frac{1}{2}$ **b** $5\frac{1}{4}$ **c** $9\frac{5}{8}$ **d** $6\frac{3}{10}$ **e** $8\frac{1}{9}$ **f** $10\frac{2}{5}$ **g** $9\frac{17}{20}$ **h** $8\frac{7}{12}$

i $2\frac{8}{15}$ **j** $5\frac{3}{10}$

3 **a** $\frac{1}{2}$ **b** $1\frac{3}{4}$ **c** $2\frac{5}{6}$ **d** $3\frac{5}{8}$ **e** $1\frac{7}{10}$ **f** $2\frac{7}{12}$ **g** $\frac{17}{20}$ **h** $3\frac{7}{9}$

i $2\frac{3}{16}$ **j** $9\frac{1}{5}$

4 **a** $2\frac{1}{2}$ **b** $2\frac{3}{4}$ **c** $1\frac{2}{3}$ **d** $2\frac{7}{8}$ **e** $2\frac{1}{5}$ **f** $1\frac{17}{20}$ **g** $1\frac{8}{15}$ **h** $4\frac{15}{16}$

i $\frac{47}{100}$ **j** $\frac{8}{9}$

6:06 Multiplication of Fractions

1 **a** $\frac{1}{2}$ **b** $\frac{1}{2}$ **c** $\frac{11}{20}$ **d** $\frac{11}{20}$ **e** $\frac{2}{3}$ **f** $\frac{1}{4}$ **g** $\frac{3}{4}$ **h** $\frac{7}{10}$

i $\frac{5}{6}$ **j** $\frac{7}{8}$ **k** $\frac{19}{20}$ **l** $\frac{8}{9}$ **m** $\frac{1}{5}$ **n** $\frac{13}{15}$ **o** $\frac{8}{11}$

2 **a** $\frac{1}{4}$ **b** $\frac{2}{5}$ **c** $\frac{1}{4}$ **d** $\frac{3}{10}$ **e** $\frac{2}{11}$ **f** $\frac{3}{10}$ **g** $\frac{1}{10}$ **h** $\frac{3}{16}$

i $\frac{4}{45}$ **j** $\frac{5}{18}$ **k** $\frac{3}{8}$ **l** $\frac{1}{8}$ **m** $\frac{3}{25}$ **n** $\frac{4}{31}$ **o** $\frac{7}{100}$

3 **a** $\frac{1}{6}$ **b** $\frac{1}{12}$ **c** $\frac{1}{70}$ **d** $\frac{1}{20}$ **e** $\frac{1}{15}$ **f** $\frac{1}{32}$ **g** $\frac{1}{100}$ **h** $\frac{1}{16}$

i $\frac{1}{30}$ **j** $\frac{1}{100}$

6:07 Division Involving Fractions

1 **a** 8 **b** 4 **2** **a** 9 **b** 2 **3** **a** 2 **b** 3

4 **a** 4 **b** 2 **5** **a** 2 **b** 7 **6** **a** 3 **b** 4

7 **a** 3 **b** 5 **c** 2 **d** 5 **e** 5 **f** 5 **g** 3 **h** 3

i 7 **j** 1 **k** 3 **l** 3 **m** 6 **n** 11 **o** 2 **p** 3

q 9 **r** 1 **s** 4 **t** 2

6:08 Fractions of Quantities

1 **a** 10 min **b** 3 kg **c** \$5 **d** 30 m **e** 7 mm **f** \$3

g \$50 **h** 9 km **i** 11 t **j** 15 kg **k** 18 L **l** \$60

m 22 mm **n** 120 t **o** 12 h **p** 111 min

2 **a** 10 **b** 20 **c** 30

3 **a** \$8 **b** \$24 **c** \$56

4 **a** \$6 **b** \$12 **b** \$18

5 **a** \$15 **b** 21 mm **c** \$15 **d** \$350 **e** 28 t **f** 30 kg **g** \$120 **h** 333 min

i 20 cm **j** 35 L **k** 15 kg **l** 12 h **m** 18 m **n** \$48 **o** \$57 **p** 15 cm

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6:09 Review of Percentage

1 **a** 34% **b** 66% **c** $\frac{34}{100} = \frac{17}{50}$ **d** $\frac{66}{100} = \frac{33}{50}$

2 **a** 65% **b** 35% **c** $\frac{65}{100} = \frac{13}{20}$ **d** $\frac{35}{100} = \frac{7}{20}$

3 **a** $\frac{3}{100} = 0{\cdot}03$ **b** $\frac{17}{100} = 0{\cdot}17$ **c** $\frac{63}{100} = 0{\cdot}63$ **d** $\frac{99}{100} = 0{\cdot}99$

e $\frac{27}{100} = 0{\cdot}27$ **f** $\frac{20}{100} = \frac{1}{5} = 0{\cdot}2$ **g** $\frac{5}{100} = \frac{1}{20} = 0{\cdot}05$ **h** $\frac{6}{100} = \frac{3}{50} = 0{\cdot}06$

i $\frac{15}{100} = \frac{3}{20} = 0{\cdot}15$ **j** $\frac{44}{100} = \frac{11}{25} = 0{\cdot}44$ **k** $\frac{25}{100} = \frac{1}{4} = 0{\cdot}25$ **l** $\frac{50}{100} = \frac{1}{2} = 0{\cdot}5$

m $\frac{18}{100} = \frac{9}{50} = 0{\cdot}18$ **n** $\frac{62}{100} = \frac{31}{50} = 0{\cdot}62$ **o** $\frac{55}{100} = \frac{11}{20} = 0{\cdot}55$ **p** $\frac{36}{100} = \frac{9}{25} = 0{\cdot}36$

q $\frac{83}{100} = 0{\cdot}83$ **r** $\frac{46}{100} = \frac{23}{50} = 0{\cdot}46$ **s** $\frac{77}{100} = 0{\cdot}77$ **t** $\frac{1}{100} = 0{\cdot}01$

4 **a** 7% **b** 7% **c** 23% **d** 78% **e** 52% **f** 20%
g 39% **h** 75% **i** 90% **j** 16% **k** 35% **l** 45%
m 2% **n** 58% **o** 50% **p** 47% **q** 61% **r** 28%

6:10 Changing Fractions and Decimals to Percentages

1 **a** 15% **b** 2% **c** 47% **d** 93% **e** 72% **f** 6% **g** 25% **h** 20%
i 1% **j** 33% **k** 5% **l** 84% **m** 90% **n** 45% **o** 24% **p** 50%
q 22% **r** 75% **s** 48% **t** 10% **u** 77% **v** 82% **w** 70% **x** 95%
y 19%

2 **a** 14% **b** 7% **c** 64% **d** 99% **e** 2% **f** 31% **g** 30% **h** 85%
i 80% **j** 18% **k** 44% **l** 40% **m** 53% **n** 23% **o** 1% **p** 29%
q 71% **r** 90% **s** 25% **t** 58%

6:12 Finding a Percentage of a Quantity

1 **a** \$15 **b** 8 g **c** 10 kg **d** 30 m **e** 4 **f** 9 L **g** \$20 **h** 15 cm
i 14 t **j** 12 g **k** 9 **l** 22 m **m** 25 h **n** 211 t **o** 30 kg **p** \$45

2 **a** \$4 **b** \$35 **c** 18 t **d** 35 kg **e** 160 m **f** 54 g **g** 60 **h** 24 s
i \$40 **j** 24 km **k** 54 d **l** \$18 **m** 240 m **n** 54 cm **o** 160 g **p** \$200
q 33 t **r** \$28 **s** 300 L **t** 30

6:13 One Quantity as a Percentage of Another

1 **a** $\frac{1}{2}$ **b** $\frac{1}{5}$ **c** $\frac{7}{100}$ **d** $\frac{1}{2}$ **e** $\frac{1}{4}$ **f** $\frac{1}{5}$ **g** $\frac{33}{100}$ **h** $\frac{1}{2}$
i $\frac{3}{5}$ **j** $\frac{9}{10}$ **k** $\frac{4}{5}$ **l** $\frac{1}{2}$ **m** $\frac{2}{3}$ **n** $\frac{1}{3}$ **o** $\frac{7}{10}$ **p** $\frac{5}{12}$

2 **a** 50% **b** 20% **c** 7% **d** 50% **e** 25% **f** 20% **g** 33% **h** 50%
i 60% **j** 90% **k** 80% **l** 50% **m** 70% **n** 32% **o** 89% **p** 2%
q 16% **r** 18% **s** 25% **t** 75%

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8:03 Patterns and Rules

1	**a**	20, 25	**b**	10, 12	**c**	14, 12	**d**	24, 48	**e**	50, 60	**f**	16, 32
2	**a**	8	**b**	14	**c**	20	**d**	3	**e**	18	**f**	3
3	**a**	6	**b**	1	**c**	7	**d**	10	**e**	6	**f**	1
4	**a**	2	**b**	10	**c**	5	**d**	3	**e**	$b = t \times 3$	**f**	$b = t + 2$

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9:02 Making Sense of Algebra

1 **a** $x + 2$ **b** $y + 3$ **c** $3x$ **d** 6 **e** $a + 8$ **f** $4y + 3$ **g** $2x + 3$ **h** $y + 5$
i $3a + 3$ **j** $5y + 3$

2 **a** $2y + 1$ **b** $2x + y$ **c** $x + y$ **d** $3x + 2y$
e $a + 3b$ **f** $2x + 3$ **g** $2a + 3b$ **h** $3x + 4$
i $5x + 2y$ **j** $4y + 5$ **k** $2x + y + 2$ **l** $a + 3b + 2$
m $3a + 2b + 1$ **n** $x + 2y + 3$ **o** $m + n + 4$

9:03 Substitution

1 **a** 36 **b** 40 **c** 11 **d** 3 **e** 9 **f** 15 **g** 0 **h** 63
i 8 **j** 36 **k** 17 **l** 24 **m** 1 **n** 6 **o** 4 **p** 5

2 **a** 2 **b** 8 **c** 20 **d** 16 **e** −3 **f** 8 **g** 19 **h** 20
i 3 **j** 124 **k** 24 **l** 48 **m** −8 **n** −8 **o** 7 **p** 52
q −3 **r** 12 **s** 28 **t** 30 **u** 3

9:05 Simplifying Algebraic Expressions

1 **a** $2x + 5$ **b** $2a + 4$ **c** $4x + 3$ **d** $x + 3y$ **e** $a + 8$

2 **a** $5x$ **b** $4a$ **c** $14p$ **d** $6f$ **e** $11k$ **f** $9m$
g $2y$ **h** $7y$ **i** x **j** $16t$ **k** $15w$ **l** $5x$
m $7d$ **n** 0 **o** $14p$ **p** $6y$ **q** $2f$ **r** $22c$
s $24c$ **t** $12ab$

3 **a** $5a + 7$ **b** $6x + 3y$ **c** $7a + 6b$ **d** $15 + 4h$ **e** $4c + 1$ **f** $10t + 5$
g $9m + 7$ **h** $10 + 4c$ **i** $6 + 2x$ **j** $2x + 4y$ **k** $7m + 2n$ **l** $11 + 7a$
m $15x + 3y$ **n** $13x + 2y$ **o** $7b + 5c$ **p** $9b + 3c$ **q** $5t + 6u$ **r** $11w + 9$
s $7x + 4$ **t** $12 + 3y$

9:05 Grouping Symbols

1 **a** $2x + 6$ **b** $5a - 20$ **c** $3p + 6$ **d** $8c - 8$ **e** $10m + 50$ **f** $4y + 32$
g $14 - 7h$ **h** $8 + 2d$ **i** $5x + 30$ **j** $36 - 9b$ **k** $18 - 6x$ **l** $4y + 20$
m $21 + 7t$ **n** $11c - 33$ **o** $20a + 100$ **p** $20 - 2x$ **q** $y + 8$ **r** $10x - 70$
s $3c - 12$ **t** $6f + 54$

2 **a** $15m - 3$ **b** $4x + 10$ **c** $21 + 14x$ **d** $12x + 4$ **e** $20p + 90$ **f** $7x - 8$
g $20x - 35$ **h** $12x + 6$ **i** $16 - 56a$ **j** $6 - 14a$ **k** $24c - 15$ **l** $11 - 44c$
m $18y + 27$ **n** $36 + 20t$ **o** $12d + 4$ **p** $42y + 24$ **q** $21 - 15x$ **r** $15x + 50$
s $22p - 4$ **t** $16x - 48$

9:08 Directed Numbers

1 **a** 14 **b** 10 **c** 4 **d** 10 **e** 15 **f** 16
g 0 **h** 25 **i** 1 **j** 34

2 **a** −1 **b** 1 **c** −5 **d** 5 **e** −10 **f** −4
g −7 **h** −6 **i** −10 **j** −1 **k** 4 **l** −5
m −6 **n** 10 **o** 5

3 **a** 6 **b** 10 **c** −10 **d** −6 **e** 3 **f** 10
g −16 **h** −8 **i** −5 **j** 0 **k** −4 **l** −5
m 12 **n** −20 **o** −48

9:09 Simplifying Expressions With Directed Numbers

1 **a** −5 **b** −4 **c** 11 **d** −5 **e** 5 **f** −6
g −14 **h** 5 **i** −1 **j** −14

2 **a** −20 **b** $-6x$ **c** 12 **d** $35x$ **e** −4 **f** $-12a$
g $-5y$ **h** $-20k$ **i** $-3t$ **j** $-21m$ **k** 4 **l** $-36b$
m $-30n$ **n** $3y$ **o** $-40x$

3 **a** $-3x$ **b** $-5y$ **c** $-a$ **d** $3m$ **e** $-10h$ **f** $10c$
g $-4y$ **h** $11p$ **i** $-6x$ **j** $-6x$ **k** $5t$ **l** $2d$
m 0 **n** $3x$ **o** $-2y$

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9:10 Algebraic Sentences

1	**a** 7	**b** 5	**c** 2	**d** 4	**e** 7	**f** 6
	g 4	**h** 5	**i** 9	**j** 9	**k** 3	**l** 7
	m 1	**n** 8	**o** 21	**p** 22	**q** 27	**r** 6
	s 9	**t** 1				
2	**a** 9	**b** 18	**c** 48	**d** 19	**e** 31	**f** 11
	g 8	**h** 15	**i** 7	**j** 30	**k** 18	**l** 20
	m 23	**n** 48	**o** 63	**p** 30	**q** 30	**r** 20
	s 16	**t** 7				

9:11 Solving Problems Using Algebra

1	**a** 1	**b** 6	**c** 15	**d** 18	**e** 7	**f** 18
	g 8	**h** 8	**i** 60			
2	**a** 9	**b** 6	**c** 12	**d** 22	**e** 5	**f** 25
	g $14.90	**h** $46	**i** $132	**j** 23	**k** 9	**l** 17
	m 16	**n** $36				

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11:04 Finding the Size of an Angle

a 40	**b** 60	**c** 28	**d** 111	**e** 125	**f** 123	**g** 50	**h** 81
i 60	**j** 28	**k** 60	**l** 107	**m** 43	**n** 90	**o** 36	**p** 258
q 80	**r** 40	**s** 22	**t** 58				

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12:01 Measuring Instruments

1 **a** 5 cm **b** 3 cm **c** 2 cm **d** 8 cm
e 4 cm **f** 10 cm **g** 1 cm **h** 7 cm
i 6 cm

2 **a** 5 o'clock **b** 9 o'clock **c** half-past three **d** 2 minutes past 11 pm
e 10 to 2 **f** 10 past 10 am **g** 12 o'clock **h** 29 past 1 am
i 1 minute past 7 pm **j** quarter to 6 **k** 20 past 3 am **l** 6 o'clock

12:02 Units of Length

1 **a** 2 cm **b** 3 cm **c** 6 cm **d** 9 cm **e** 9 cm **f** 12 cm

2 **a** 80 **b** 3 **c** 9 **d** 8 **e** 7000 **f** 900 **g** 3000 **h** 120
i 5 **j** 15 **k** 2000 **l** 11 000 **m** 5 **n** 10 000 **o** 8 **p** 6
q 12 **r** 200 **s** 62 **t** 6000

3 **a** 180 **b** 120 **c** 2 **d** 10 **e** 300 **f** 8 **g** 1440 **h** 5
i 1200 **j** $\frac{1}{2}$ **k** 15 **l** 420 **m** 720 **n** 600 **o** 240 **p** $\frac{1}{4}$

12:03 Measuring Length

1 **a** 5 cm **b** 4 cm **c** 2 cm **d** 2 cm **e** 7 cm **f** 4 cm **g** 5 cm **h** 3 cm
i 8 cm **j** 2 cm

2 **a** 32 mm **b** 8 mm **c** 35 mm **d** 25 mm **e** 51 mm **f** 18 mm **g** 63 mm **h** 74 mm

12:05 Perimeter

1 **a** 9 cm **b** 31 cm **c** 24 cm **d** 64 cm **e** 56 m **f** 32 m **g** 23 m **h** 45 m
i 16·4 cm **j** 48·5 m **k** 40 mm **l** 64 m **m** 52 cm **n** 22·6 m **o** 30 mm **p** 50 cm

2 **a** 40 cm **b** 42 m **c** 200 m **d** 6 cm **e** 134 mm **f** 14·4 cm **g** 440 m **h** 8·4 cm
i 77 cm **j** 60·2 m

12:06 The Calendar and Dates

1 **a** 14 **b** 3 **c** 1095 **d** 8 **e** 56 **f** 48 **g** 70 **h** 12
i 52 **j** 120 **k** 40 **l** 6

2 **a** 7 **b** 23 **c** 22 **d** 22 **e** 28 (or 29 in leap year) **f** 91 **g** 66
h 28 **i** 61 **j** 25

12:07 Clocks and Times

1 **a** 60 **b** 120 **c** 600 **d** 1440 **e** 30 **f** 90 **g** 300 **h** 330
i 15 **j** 135 **k** 660 **l** 420

2 **a** 5 minutes to 9 **b** 20 minutes to 4 **c** 10 minutes past 12 **d** 20 minutes past 3
e 15 minutes past 6 **f** 25 minutes to 1 **g** 10 minutes to 4 **h** 20 minutes to 1
i 30 minutes past 12 **j** 15 minutes past 11 **k** 25 minutes past 9 **l** 25 minutes to 5

12:08 Operating With Time

1 **a** 7 h 30 min **b** 7 h 20 min **c** 6 h 35 min **d** 5 h 40 min
e 10 h 55 min **f** 14 h 5 min **g** 13 h 30 min **h** 4 h 53 min
i 10 h 26 min **j** 4 h 42 min **k** 2 h 46 min **l** 5 h 14 min

2 **a** 3 h **b** 4 h **c** 6 h **d** 7 h **e** 11 h **f** 11 h
g 7 h **h** 5 h **i** 12 h **j** 5 h **k** 12 h **l** 9 h

3 **a** 15 min **b** 22 min **c** 30 min **d** 25 min
e 1 h 20 min **f** 1 h 20 min **g** 2 h 30 min **h** 3 h 15 min
i 34 min **j** 2 h 20 min **k** 1 h 5 min **l** 23 min

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13:01 The Definition of Area

1 **a** 4 cm^2 **b** 5 cm^2 **c** 7 cm^2 **d** 12 cm^2 **e** 6 cm^2 **f** 12 cm^2 **g** 8 cm^2 **h** 8 cm^2
i 10 cm^2 **j** 5 cm^2

2 Various possibilities—check with teacher.

13:02 Area of a Rectangle

1 **a** 12 u^2 **b** 4 u^2 **c** 6 u^2 **d** 6 u^2 **e** 10 u^2 **f** 21 u^2 **g** 15 u^2 **h** 16 u^2

2 **a** 18 u^2 **b** 8 u^2 **c** 14 u^2 **d** 25 u^2 **e** 28 u^2 **f** 16 u^2 **g** 30 u^2 **h** 7u^2
i 24 u^2

3 **a** 40 cm^2 **b** 28 cm^2 **c** 100 cm^2 **d** 32 cm^2 **e** 45 cm^2 **f** 33 cm^2 **g** 36 cm^2 **h** 64 cm^2
i 100 cm^2 **j** 30 cm^2

13:03 Area of a Triangle

1 **a** 3 cm^2 **b** 5 cm^2 **c** $4\frac{1}{2}$ cm^2 **d** 4 cm^2

2 **a** 6 cm^2 **b** 8 cm^2 **c** 24 cm^2 **d** 15 cm^2 **e** 14 cm^2 **f** 32 cm^2
g 25 cm^2 **h** 70 cm^2 **i** 35 cm^2 **j** 21 cm^2 **k** 120 cm^2 **l** 45 cm^2

13:06 Volume of a Rectangular Prism

1 **a** 8 cm^3 **b** 12 cm^3 **c** 20 cm^3 **d** 27 cm^3

2 **a** 48 cm^3 **b** 72 cm^3 **c** 32 cm^3 **d** 80 cm^3 **e** 10 cm^3 **f** 36 cm^3
g 84 cm^3 **h** 64 cm^3 **i** 180 cm^3

13:07 Capacity

1 **a** 2000 **b** 7000 **c** 4 **d** 11 **e** 3 **f** 6000
g 5 **h** 14 **i** 10 000 **j** 8000 **k** 500 **l** 20

2 **a** 3000 **b** 2 **c** 9000 **d** 6 **e** 5 **f** 25 000
g 7000 **h** 10 **i** 500 **j** 400 **k** 0·7 **l** 4

3 **a** 3 **b** 15 **c** 100 **d** 4 **e** 400 **f** 8000
g 500 **h** 750 **i** 100 **j** 5 **k** 3000 **l** 1·5

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Challenge

Extension 1:09 Solving Puzzles

1 a

6	32	18	44	30
40	16	42	28	4
14	50	26	2	38
48	24	10	36	12
22	8	34	20	46

b

23	6	19	2	15
10	18	1	14	22
17	5	13	21	9
4	12	25	8	16
11	24	7	20	3

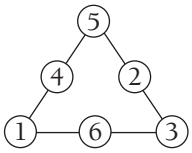
c

3	10	17	19	26
22	24	6	8	15
11	13	20	27	4
25	7	9	16	18
14	21	23	5	12

2 5 cuts → 15 minutes

3 F is opposite A, E is opposite B, C is opposite D

4



Extension 3:06 HCF and LCM by Prime Factors

1 HCF = 8, LCM = 48 **2** 6, 72 **3** 4, 900 **4** 20, 560 **5** 27, 324 **6** 24, 144

7 15, 150 **8** 18, 108 **9** 45, 270 **10** 12, 420 **11** 25, 150 **12** 14, 490

Extension 8:02 Describing Number Patterns

1 a number = $h\,(h + 1)$

b

5	6	7	8
30	42	56	72

2 a 10, 15, 20 **b** 25, 30

3 a

Number of sides (s)	3	4	5	6	7	8	9	10
Number of diagonals (d)	0	1	2	3	4	5	6	7

b The number of diagonals is 3 less than the number of sides.

c $d = s - 3$

4 a $B = T^2$, 100 **b** B = 12 − T, 2

5 a 21, 25 **b** 0·5, 0·25 **c** 0, −5 **d** 4 000 000, 40 000 000

6 a 5, 8, 11, 14 **b** 5, 4, 3, 2 **c** −5, −3, −1, 1 **d** 6, 9, 14, 21

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